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SMOOTH MOTION CONTROL OF THE ADAPTIVE CRUISE CONTROL SYSTEM BY A VIRTUAL LEAD VEHICLE

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ABSTRACT−The adaptive cruise control system maintains the appropriate distance to the lead vehicle when the lead vehicle exists and maintains the desired speed when no lead vehicle is detected. A virtual lead vehicle scheme is introduced to make the switching between the speed control algorithm and the distance control algorithm unnecessary and simplify the structure of the control system. The speed and the position of the virtual vehicle can be decided by the control system according to the current situation. Smoother responses are achieved by the virtual lead vehicle scheme compared to the conventional mode switching scheme. This method is also shown to provide a good reaction for when a lead vehicle cuts in or out. A linear quadratic controller with variable weights is suggested to control the virtual lead vehicle. This scheme shows improved performance in terms of passenger comfort and fuel efficiency of the host vehicle.

KEY WORDS : Automotive control, Cruise control, Vehicle dynamics, Switching algorithms, Control algorithm

1. INTRODUCTION

The adaptive cruise control (ACC) system is a driver assistance feature in modern vehicles which can relieve the driver`s fatigue and enhance safety. The conventional cruise control system assists the driver by maintaining a desired speed set by the driver. However, the driver is still responsible for reacting to other vehicles on the road and maintaining an appropriate relative distance to the lead vehicle. The ACC system relieves the driver of this responsibility by detecting the lead vehicle and maintaining an appropriate relative distance to the lead vehicle. With the stop-and-go function, the ACC system is capable of stopping completely and is utilizable in city traffic (Vahidi and Eskandarian, 2003).

The ACC system is also known as the intelligent cruise control system, autonomous intelligent cruise control system, or active cruise control system (Ioannou and Chien, 1993). Considerable research about the ACC system have been done since 1990`s and miscellaneous controllers have been designed to make the performance of the ACC system stable and efficient (Zhang *et al*., 1999; Ioannou and Xu, 1994; Hedrick *et al*., 1991).

The ACC system not only reduces the driver`s fatigue by automatically maintaining the relative distance to the lead vehicle, but can also improve the fuel efficiency and reduce vehicle emissions. With properly designed controllers, a platoon of vehicles equipped with the ACC system shows better fuel efficiency and traffic performances (Ioannou and Stefanovic, 2005; Bose and Ioannou, 2000).

The ACC system should be able to perform two main modes. If a lead vehicle is detected in front of the host vehicle with which the ACC system is equipped, the ACC system should follow the lead vehicle at an appropriate distance. When no lead vehicles are detected, the ACC system should maintain the desired velocity set by the driver like the conventional cruise control system. It is important that both modes should not only operate properly and stably, but also the transient performance should be smooth and safe when the mode is switched from one to the other.

A mode switching scheme is one method which deals with the presence of a lead vehicle. The control algorithm switches between the speed control and the distance control modes based on the relative distance to the lead vehicle (Yi *et al*., 2001; Venhovens *et al*., 2000). The speed control algorithm is used when no lead vehicle is detected or the lead vehicle is farther than a specified relative distance. When the lead vehicle is close, the distance control algorithm is used. One downside of this method is that two different feedback loops are required: velocity and distance control loops. If conventional PID feedback controllers are used, integrator windup must also be properly taken into consideration.

Another drawback of the mode switching scheme is that the transient maneuver is not considered. When the speed control algorithm is activated, if a new lead vehicle cuts in

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from a side lane or a slower lead vehicle approaches from the front, the mode is abruptly changed to the distance control which can cause unpleasant motion of the host vehicle. A model predictive control method was studied to control the transitional maneuver of the ACC system (Bageshwar *et al*., 2004). In this study, the optimal motion of the host vehicle is calculated by solving an optimization problem over a time horizon and the initial optimal control input was utilized as the control command. The model predictive controller is appropriate only for the distance control mode or the transition from the speed control to the distance control mode. Still, a separate speed controller is necessary to control the host vehicle during the speed control mode. Also, the model predictive control requires significant computational efforts and a fast processor is necessary for real time application.

A virtual lead vehicle scheme (Kim *et al*., 2009) is a method which can replace the complicated switching mechanism. When no lead vehicle is detected, the controller generates a virtual vehicle whose speed is the same with the desired speed. When there is a lead vehicle, the speed and the position of the virtual lead vehicle are set to be those of the actual vehicle. The host vehicle always follows the virtual lead vehicle with an appropriate distance control algorithm (i.e. a PID controller) and no switching scheme between the speed and the distance control is necessary.

Additional advantage of the virtual lead vehicle scheme is that the motion of the host vehicle can be smoothly controlled when a new lead vehicle cuts in or the current lead vehicle cuts out. If the host vehicle follows the actual lead vehicle, the cutting in/out of the lead vehicle is a step change of the reference signal. However, if the host vehicle follows the virtual lead vehicle and the speed and the position of the virtual lead vehicle merge smoothly from those of the old lead vehicle to those of the new lead vehicle, the host vehicle moves smoothly.

Since the virtual vehicle has no internal dynamics or delay, the position and the velocity of it can be set without any physical limitations. However, a sudden change of the position or velocity of the virtual lead vehicle can cause an undesirable motion of the host vehicle which can make the passengers uncomfortable. To prevent this, the virtual lead vehicle is modeled as a double integrator whose input is the acceleration and outputs are the velocity and the position. A smooth switching can be achieved by controlling the virtual lead vehicle with limited acceleration and jerk.

This paper is organized as follows. In Section 2, the modes of the ACC systems and the concept of the virtual lead vehicle scheme are introduced. In Section 3, the linear quadratic controller is designed to control the virtual lead vehicle. In Section 4, variable weights are utilized for the linear quadratic controller to better control the virtual lead vehicle during transient motion. The stability of the suggested controller is analyzed in Section 5. The simulation results are shown to support the suggested ideas.

2. CONCEPT OF THE VIRTUAL LEAD VEHICLE SCHEME

The ACC system should be able to perform two modes: speed control and distance control. The modes depend on the desired and actual distance between the lead and host vehicles. One way to implement these two modes is by designing two different controllers and switching between them. However this can cause various problems. Another way is the virtual lead vehicle scheme which makes the switching between the speed control algorithm and the distance control algorithm unnecessary (Kim *et al*., 2009). With the virtual lead vehicle scheme, the control system structure is simplified.

2.1. Mode Decision of the ACC System

One popular way to decide the appropriate relative distance is to use the constant time headway which can prevent the string instability common to platooning vehicles (Rajamani 2006). When the host vehicle follows a lead vehicle, the desired distance is given by:

$$d_{desired} = t_{hw} v_{host} + d_{offset} \tag{1}$$

where t_{hw} is the time headway, v_{host} is the velocity of the host vehicle, and d_{offset} is the safety distance to be maintained in case of a complete stop. The time headway is the time to take for the host vehicle to collide with the lead vehicle when the lead vehicle is suddenly stopped and the host vehicle maintains its speed. The desired time headway is decided by the driver when the ACC system is activated.

Using Equation (1), a simple scheme to determine the proper mode is given by:

$$\text{mode} = \begin{cases} DC & d_{rel} \le d_{desired} \\ SC & d_{rel} > d_{desired} \end{cases} \tag{2}$$

where DC is the distance control, SC is the speed control, and d_{rel} is the relative distance to the lead vehicle. Using this scheme, however, may cause chattering in the algorithm if the distance between the lead and host vehicle is too close to the desired distance. To prevent chattering, marginal distances are introduced as:

$$\text{mode} = \begin{cases} SC \rightarrow DC & d_{rel} - d_{m1} \le d_{desired} \\ DC \rightarrow SC & d_{rel} + d_{m2} > d_{desired} \end{cases} \tag{3}$$

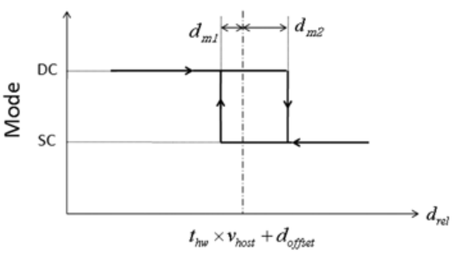

Figure 1. Mode switching logic.

where d_{m1} and d_{m2} are marginal distances. This introduces hysteresis in the mode switching as shown in Figure 1. If there is no lead vehicle, d_{rel} is infinite and the speed control mode is utilized.

2.2. Mode Switching Scheme

Figure 2 shows the block diagram of the mode switching scheme. The input signal to the distance control algorithm is the distance error which is given by:

$$e_1 = \begin{cases} d_{rel} - d_{desired} & \text{mode} = DC \\ 0 & \text{mode} = SC \end{cases} \quad (4)$$

The input signal to the speed control algorithm is the speed error which is given by:

$$e_2 = \begin{cases} v_{desired} - v_{host} & \text{mode} = SC \\ 0 & \text{mode} = DC \end{cases} \quad (5)$$

where is the desired speed set by the driver.

There is an additional switch to cancel the output of the inactivated mode:

$$a_d = \begin{cases} a_1 & \text{mode} = DC \\ a_2 & \text{mode} = SC \end{cases} \quad (6)$$

If PID controllers are used for both algorithms, the integral controller of the inactivated mode should be initialized when the mode is switched.

2.3. Virtual Lead Vehicle Scheme

Figure 3 shows the conceptual sketch of the virtual lead vehicle scheme. In the virtual lead vehicle scheme, the host vehicle follows the virtual lead vehicle with a distance control algorithm in the absence of the actual lead vehicle. When a lead vehicle is detected inside the desired relative distance and the speed of the lead vehicle is slower than the desired speed, the position and the velocity of the virtual lead vehicle is set to be those of the actual lead vehicle. When no lead vehicle is detected or if the speed of the lead vehicle is too fast, the virtual lead vehicle scheme prevents switching to the speed control algorithm by limiting the velocity of the virtual lead vehicle to the desired speed. In that case, the position of the virtual lead vehicle is updated by integrating the velocity.

As a result, the speed control algorithm is not utilized with the virtual lead vehicle scheme and the overall controller structure has only one loop which is the distance control algorithm as described in Figure 4.

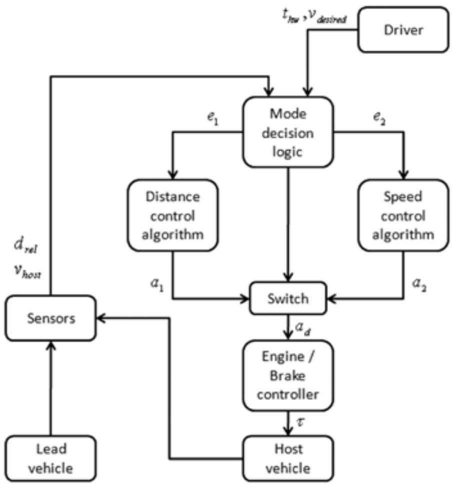

Figure 2. Block diagram of the mode switching scheme.

Figure 3. Concept of the virtual lead vehicle scheme.

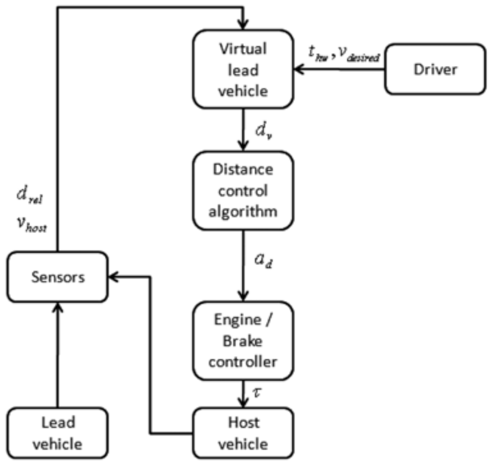

Figure 4. Block diagram of the virtual lead vehicle scheme.

3. CONTROL OF THE VIRTUAL LEAD VEHICLE IN TRANSIENT

In this section, a double integrator model will be suggested as the model of the virtual lead vehicle and a linear quadratic controller will be designed to control the speed and the position of the virtual lead vehicle in transient motions. The control scheme deciding the speed and position of the virtual lead vehicle during the speed control and the distance control algorithms are described in Section 2.3. However, when the mode is changed from one to the other due to cutting in or out of the lead vehicle, the speed and the position of the virtual lead vehicle should be smoothly change from those of the old lead vehicle to those of the new lead vehicle. A double integrator model and a linear quadratic control scheme are strong candidates to achieve this desired behavior.

3.1. Double Integrator Model for the Virtual Lead Vehicle

When the current lead vehicle cuts out from the current lane or a new lead vehicle cuts in from a side lane, there are step changes in the position and the speed of the lead vehicle. If the host vehicle directly follows the actual lead vehicle, undesirable transient oscillation or overshoots can be generated. With the virtual lead vehicle scheme, however, the virtual vehicle`s position and speed are controlled to merge smoothly with those of the new lead vehicle. Hence, the host vehicle will also move smoothly.

In this paper, the goal is to design a controller which makes the virtual lead vehicle moves smoothly from an initial state to the desired state. The position and the speed of the virtual lead vehicle can be set arbitrarily by the controller. However, the host vehicle should react slowly and smoothly given any initial conditions to avoid producing unnecessary uncomfortable motions. The virtual lead vehicle is considered as a double integrator whose input is the acceleration. The continuous time model of the virtual lead vehicle is:

$$\begin{aligned}\begin{bmatrix}\dot{x}_{vl}(t)\\ \dot{v}_{vl}(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_{vl}(t)\\ v_{vl}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}a_{vl}(t)\\ &= A\begin{bmatrix}x_{vl}(t)\\ v_{vl}(t)\end{bmatrix}+Ba_{vl}(t)\end{aligned} \tag{7}$$

where x_{vl}, v_{vl} and a_{vl} are the position, speed, and the acceleration of the virtual lead vehicle, respectively.

The task of the controller is to make the virtual vehicle follow the actual lead vehicle smoothly. In this paper, to 'follow' means that both the position and the velocity of the virtual vehicle converge to those of the actual vehicle. The errors are defined as:

$$\begin{bmatrix}e_x(t)\\ e_v(t)\end{bmatrix}=\begin{bmatrix}x_{vl}(t)-x_{al}(t)\\ v_{vl}(t)-x_{al}(t)\end{bmatrix} \tag{8}$$

where x_{al} and v_{al} are the position and the speed of the actual lead vehicle, respectively.

The error dynamics is:

$$\begin{bmatrix}\dot{e}_x(t)\\ \dot{e}_v(t)\end{bmatrix}=A\begin{bmatrix}e_x(t)\\ e_v(t)\end{bmatrix}+B(a_{vl}(t)-a_{al}(t)) \tag{9}$$

where a_{al} is the acceleration of the actual lead vehicle.

It is assumed that the relative distance and velocity of the actual lead vehicle are measurable. Since the position and velocity of the virtual lead vehicle is either known and fixed, or dependant on the actual lead vehicle, the relative distance and the relative velocity between the virtual and the actual lead vehicle can be computed. However, the behaviour of the lead vehicle is unknown and hence the actual lead vehicle acceleration a_{al} is considered as a disturbance.

3.2. Linear Quadratic Control

The goal is to make the states in Equation (9) converge zero. One promising way to control this system is doing state feedback using the steady state Linear Quadratic (LQ) optimal controller. The cost function is a quadratic sum of the position error, the velocity error, and the acceleration given as:

$$J=\frac{1}{2}\int_{t_0}^{\infty}\lambda_x e_x(t)^2+\lambda_v e_v(t)^2+\lambda_a a_{vl}(t)^2 dt \tag{10}$$

Without the constraints, the problem can be formulated as a stationary LQ problem.

$$J=\frac{1}{2}\int_{t_0}^{\infty}X(t)^T QX(t)+a_{vl}(t)^T Ra_{vl}(t)dt \tag{11}$$

where $X(t)=[e_x(t) \quad e_v(t)]^T$, $Q=\begin{bmatrix}\lambda_x & 0\\ 0 & \lambda_v\end{bmatrix}$, and $R=\lambda_a$.

Since the system is time invariant, controllable, and observable, the optimal solution of the stationary LQ problem is given as:

$$a_{vl}(t)=-R^{-1}B^T P_+ X(t) \tag{12}$$

where P_+ is the positive (semi-)definite solution of the algebraic Riccati equation

$$A^T P+PA-PBR^{-1}B^T P+Q=0 \tag{13}$$

In this case, there are only two states and the Riccati equation can be algebraically solved. The feedback control law is given as:

$$a_{LQ}(t)=-\begin{bmatrix}\sqrt{\dfrac{\lambda_x}{\lambda_a}} & \sqrt{2\sqrt{\dfrac{\lambda_x}{\lambda_a}}+\dfrac{\lambda_v}{\lambda_a}}\end{bmatrix}X(t) \tag{14}$$

The solution given in Equation (14) is for the case without constraints. For actual implementation, however, acceleration and jerk have to be constrained to limit passenger discomfort. These constraints are given by:

$$a_{\min}\le a_{vl}(t)\le a_{\max} \tag{15}$$

$$j_{\min}\le j_{vl}(t)\le j_{\max} \tag{16}$$

The acceleration of the virtual lead vehicle is given as:

$$\begin{aligned}a_{vl}(t)=\max\{&\min(a_{LQ}, a_{\max}, a(t-\Delta t)+j_{\max}\Delta t),\\ &a_{\min}, a(t-\Delta t)+j_{\min}\Delta t\}\end{aligned} \tag{17}$$

where a_{LQ} is the solution of the unconstrained problem given in Equation (14), and Δt is the controller sampling time.

3.3. Comparison with the Model Predictive Control

In Equations (15) ~ (17), the acceleration and the jerk constraints are applied to only the current time and not the whole time horizon. Ideally, the constraints should be applied to the whole time horizon. To do so the Model Predictive Control (MPC) method can be applied. The cost function for MPC is the same as Equation (10) and the inequality constraints described in Equations (15) ~ (16) are applied to the whole time horizon. Figure 5 compares the simulation results with MPC and LQ control. As shown in the figure, the two results are very similar to each other. However, the MPC method requires a lot more calculation efforts than the LQ. As a result, using the MPC method will require a faster processor for real time application. Since the results are almost identical, the LQ method can efficiently

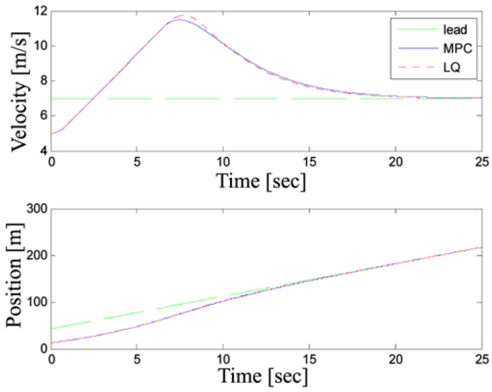

Figure 5. Simulation results with MPC and LQ Control - Position and velocity of the virtual lead vehicle.

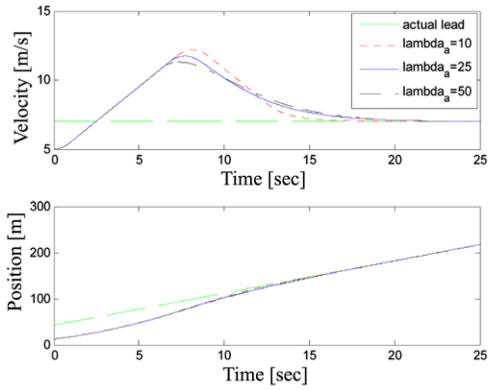

Figure 7. Simulation results with constant weights – position and velocity of the virtual lead vehicle.

replace the MPC scheme and will be used in this paper.

4. LINEAR QUADRATIC CONTROL WITH VARIABLE WEIGHTS

In the cost function of the LQ problem given in Equation (10), the weights λ_x, λ_v and λ_a are parameters to be tuned by the controller designer. Without the constraint given in Equation (15) and (16), the state feedback gains are functions of the weights. To control the motion of the virtual lead vehicle smoothly, state dependent weights are suggested and simulation result shows better performance in the viewpoint of passenger comfort and fuel efficiency (Kim *et al*., 2010).

4.1. Tuning of the LQ Weights

The weights of the cost functions can be tuned using the return difference equation. The closed loop poles of the system are the left half plane solution of the return difference equation (Kwakernaak and Sivan, 1972):

$$s^4 - \frac{\lambda_v}{\lambda_a}s^2 + \frac{\lambda_x}{\lambda_a} = 0 \tag{18}$$

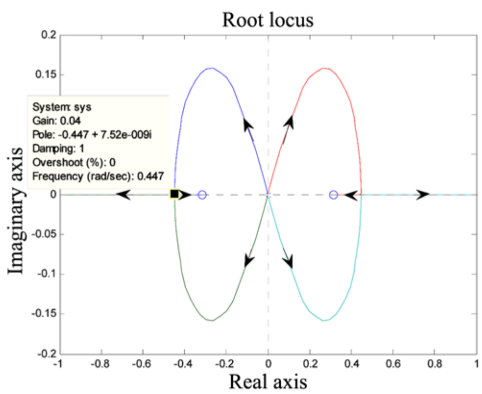

Figure 6. Symmetric root locus - Arrows indicate decreasing λ_a.

The symmetric root locus is shown in Figure 6 λ_a varying from 0 to ∞. Considering the passenger`s comfort and the fuel efficiency, the damping should be large to prevent a large overshoot. Also it is desirable to have the real part of the poles to be as small as possible for faster responses. The point marked in Figure 6 is such a point and the weights are:

$$\lambda_x = 1,\ \lambda_v = 10,\ \lambda_a = 25 \tag{19}$$

4.2. Simulation Results with Constant Weights

Figure 7 shows the simulation results with the constant weights given in Equation (19). The simulation describes the case when the current lead vehicle cuts out and a new lead vehicle is detected. As shown in the figure, the virtual lead vehicle initially accelerates to chase the new lead vehicle and then decelerates to the speed of the new lead vehicle. The initial acceleration is constrained by the acceleration and the jerk constraints. However, the large velocity overshoot is undesirable. To prevent the overshoot, the acceleration is penalized more and the simulation results show that the velocity overshoot is reduced as λ_a is increased.

However, if λ_a is increased, the response of the closed loop system gets slower. This can be also found in Figure 6 where two poles are moving close to the origin if λ_a is increased more than the critical value. Slow response can make the driver uncomfortable especially when a new lead vehicle cuts in and the relative distance to the new lead vehicle is closer than the desired distance. Another solution is to increase λ_v. In that case, when the velocity error is positive, the feedback term of the velocity error is negative and the acceleration can be reduced. However, when a new lead vehicle cuts in and the position error is positive, the large λ_v can cause undesirable system behavior.

4.3. LQ with Variable Weights

To prevent the large velocity overshoot while maintaining a fast response in the cutting in situation, variable weights

Table 1. Desired performance and weights.

Sign of errors	Desired performance	Desired λ_x	Desired λ_v	Desired λ_a
e_x<0, e_v<0	Do not accelerate	small	large	large
e_x>0, e_v<0	Decelerate smoothly	large	-	large
e_x>0, e_v>0	Decelerate fast	large	large	small

are introduced. The desired performance of the virtual lead vehicle is described in Table 1. When the position error is negative and the velocity error is positive, it is desirable to maintain the speed to prevent a large velocity overshoot. To do that the position error should be less penalized while the velocity error and the acceleration should be penalized more. When the position error is positive and the velocity error is negative, which describes the case when a lead vehicle faster than the virtual lead vehicle cuts in, the virtual lead vehicle should smoothly decelerate and merge to the new lead vehicle. The position error should be more penalized than the speed error while the acceleration is penalized more to make the motion smooth. When both the position and the velocity errors are positive, it is a dangerous situation and it is desirable to decelerate the host vehicle quickly. In that case, both errors should be penalized while the acceleration should be less penalized to ensure a fast response.

Since the desired values of the weights are changing according to the sign of the errors, the variable weights are suggested as:

$$\lambda_x = \lambda_{x0}\left(1+\frac{2}{\pi}\tan^{-1}(\phi_x e_x(t))\right) \quad (20)$$

$$\lambda_v = \lambda_{v0}\left(1+\frac{2}{\pi}\tan^{-1}(\phi_v e_v(t))\right) \quad (21)$$

$$\lambda_a = \lambda_{a0}\left(1-\frac{4}{\pi^2}\tan^{-1}(\phi_x e_x(t))\tan^{-1}(\phi_v e_v(t))\right) \quad (22)$$

The arctangent function is introduced instead of the sign function to make it continuous and ϕ's are constants describing the slope of the arctangent function near the origin.

4.4. Simulation Results with Variable Weights

Figure 8 ~ 10 shows the simulation results with the variable weights given in Equation (20) ~ (22). As shown in the figure, the velocity peak is reduced with the variable weights. Figure 9 shows the weights. As shown in Figure 9, when the velocity error becomes positive, the weights on the velocity and the acceleration are increased to prevent the velocity overshoot. Figure 10 shows the velocity and the fuel consumption of the host vehicle. The engine and the brake dynamics are assumed to be a first order system with time constants 0.5 and 0.04 respectively. The fuel consumption is calculated with a VT model (Ahn *et al.*, 1999). As shown in Figure 10, with the variable weights, the large velocity overshoot is avoided and fuel efficiency is improved. The square sums of the acceleration and the jerk, which can represent a measure of passenger comfort, are 0.2014 [m^2/s^4] and 0.0814 [m^2/s^6] respectively with variable weights, and 0.4003 [m^2/s^4] and 0.7638 [m^2/s^6] respectively with the constant weights.

Figure 11 ~ 13 shows the simulation with different situation where a slower lead vehicle cuts in. As shown in the figures, with variable weights, the motions of the virtual lead vehicle and the host vehicle are smoother. Also the fuel efficiency is better than the constant weights. The square sums of the acceleration and the jerk are 0.1050 [m^2/s^4] and 0.0586 [m^2/s^6] respectively with the variable weights, and 0.1527 [m^2/s^4] and 0.1865 [m^2/s^6] respectively with the constant weights.

5. STABILITY ANALYSIS OF THE LINEAR QUADRATIC CONTROLLER WITH VARIABLE WEIGHT

Since the weights in the cost function of the linear quadratic controller are state dependent, the stability and the robust features of the general linear quadratic controller are not guaranteed anymore. Also the stability of the

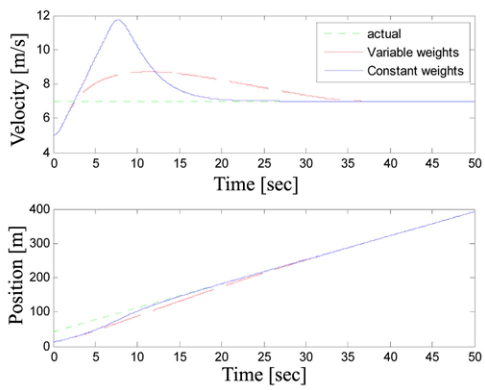

Figure 8. Simulation results: position and velocity of the virtual lead vehicle.

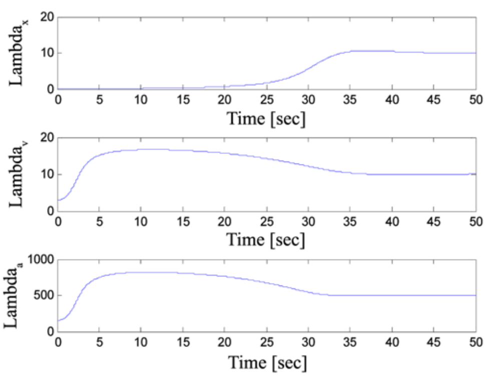

Figure 9. Simulation results: variable weights.

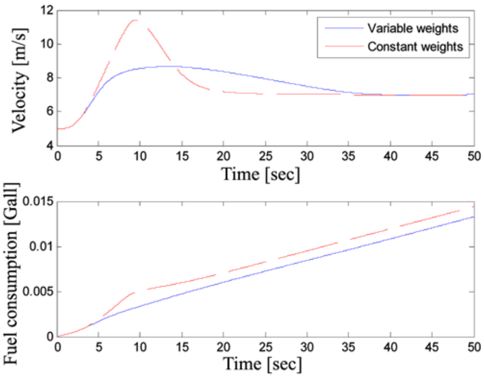

Figure 10. Simulation results: Position and velocity of the host vehicle.

suggest scheme should be proved.

The closed loop system dynamics is given by:

$$\dot{e}_x(t) = e_v(t) \tag{23}$$

$$\dot{e}_v(t) = -\sqrt{\frac{\lambda_x(e_x)}{\lambda_a(e_x, e_v)}}\, e_x(t) - \sqrt{2\sqrt{\frac{\lambda_x(e_x)}{\lambda_a(e_x, e_v)}} + \frac{\lambda_v(e_v)}{\lambda_a(e_x, e_v)}}\, e_v(t) \tag{24}$$

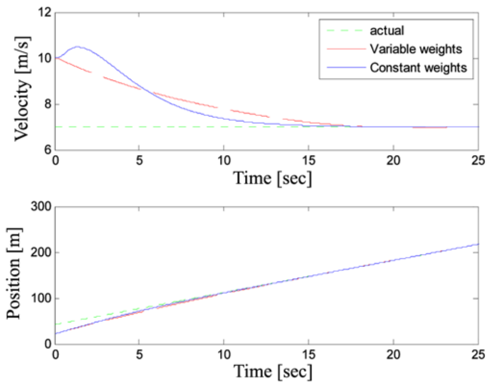

Figure 11. Simulation results: Position and velocity of the virtual lead vehicle.

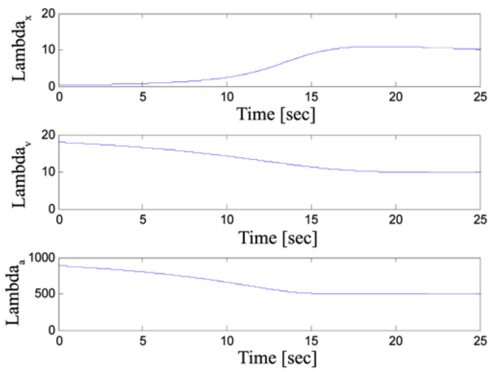

Figure 12. Simulation results: variable weights.

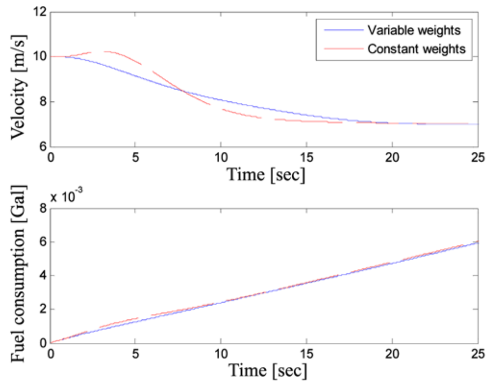

Figure 13. Simulation results: Position and velocity of the host vehicle.

Since $\sqrt{\frac{\lambda_x}{\lambda_a}}$ and $\sqrt{2\sqrt{\frac{\lambda_x}{\lambda_a}} + \frac{\lambda_v}{\lambda_a}}$ are always positive, the equilibrium point of the system is only the origin and the origin is a stable equilibrium point. However that does not guarantee that the solution converges to zero with an arbitrary initial condition. To show that, a phase plane analysis is done.

Figure 14 shows the phase plane plot of the closed loop system. As expected from Equation (23) the solution trajectory moves to the positive x direction in the first and the second quadrants, and to the negative x direction in the third and the forth quadrants. Also as expected from Equation (24) the solution trajectory goes to the negative v direction in the first quadrant and to the positive v direction in the third quadrant. As a result the solution will meet the x axis at some point. The solution from a point on the positive x axis will meet the positive x axis again. If the second point is closer to the origin than the first point, it can conclude that all initial points converge to the origin and the system is stable.

Since the system is nonlinear, it is hard to find the solution explicitly. However it is possible to find a linear switching system which always behaves worse than the

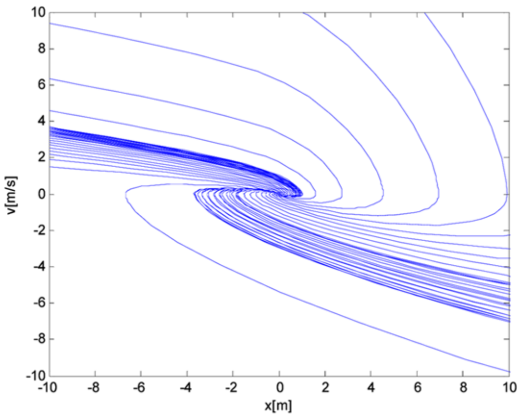

Figure 14. Phase plane plot of the closed loop system.

original system. From Equation (23) and (24), the slope on the phase plane is:

$$\frac{de_v}{de_x} = -\sqrt{\frac{\lambda_x(e_x)}{\lambda_a(e_x,e_v)}\frac{e_x(t)}{e_v(t)}} - \sqrt{2\sqrt{\frac{\lambda_x(e_x)}{\lambda_a(e_x,e_v)}} + \frac{\lambda_v(e_v)}{\lambda_a(e_x,e_v)}} \tag{25}$$

The convergence is slower if the slope is large (steep when positive and flat when negative). With the weights given in Equation (20) ~ (22), the behavior of the system is always better than the worst case system given by:

$$\dot{e}_x(t) = e_v(t) \tag{26}$$

$$\dot{e}_v(t) = -a_1 e_x(t) - a_2 e_v(t) \tag{27}$$

$$a_1 = \begin{cases} \sqrt{\lambda_{x0}/\lambda_{a0}} & x>0, v>0 \\ \sqrt{2\lambda_{x0}/\lambda_{a0}} & x>0, v<0 \\ \sqrt{\lambda_x(x_{min})/\lambda_{a0}} & x<0, v<0 \\ \sqrt{\lambda_{x0}/\lambda_{a0}} & x<0, v>0 \end{cases} \tag{28}$$

$$a_2 = \begin{cases} \sqrt{2\sqrt{\lambda_{x0}/\lambda_{a0}} + \frac{\lambda_{v0}}{\lambda_{a0}}} & x>0, v>0 \\ \sqrt{2\sqrt{\lambda_{x0}/2\lambda_{a0}} + \frac{\lambda_v(v_{\min})}{2\lambda_{a0}}} & x>0, v<0 \\ \sqrt{2\sqrt{\lambda_x(x_{\min})/\lambda_{a0}} + \frac{\lambda_v(v_{\min})}{\lambda_{a0}}} & x<0, v<0 \\ \sqrt{2\sqrt{\lambda_x(x_{\min})/2\lambda_{a0}} + \frac{\lambda_{v0}}{2\lambda_{a0}}} & x<0, v>0 \end{cases} \tag{29}$$

where $x_{\min}$ and $v_{\min}$ are the minimum position error and the minimum velocity error, respectively. In our case $v_{\min}$ is -15m/s and $x_{\min}$ is -32m.

Figure 15 shows the simulation results comparing the worst case gains and the original gains. As shown in the figures, with the worst case gains the performance is worse at all four quadrants in the sense of converging to the origin.

Figure 16 shows the simulation result with the worst case system described in Equation (21) ~ (24). The initial point is (10, 0) and the simulation results shows that the next intersection with the positive x-axis is closer to the origin. Since the system is linear at each quadrant, the sequence of intersections with the positive x-axis converges to the origin and any initial states will converge to the origin. The original system behaves better than the worst case system at each quadrant, i.e. the solution of the original system is confined by the solution of the worst case system, and the states will converge to the origin faster than the worst case system.

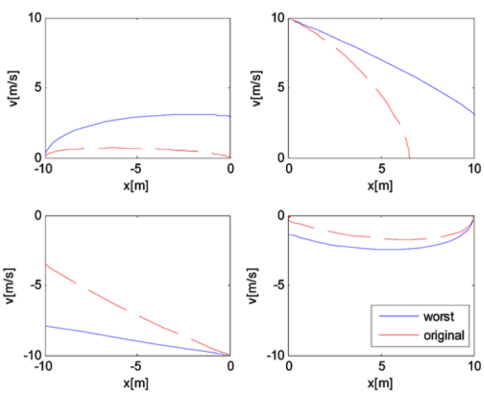

Figure 15. Phase plane plot of the closed loop system at each quadrant comparing the worst case gains and the original gains.

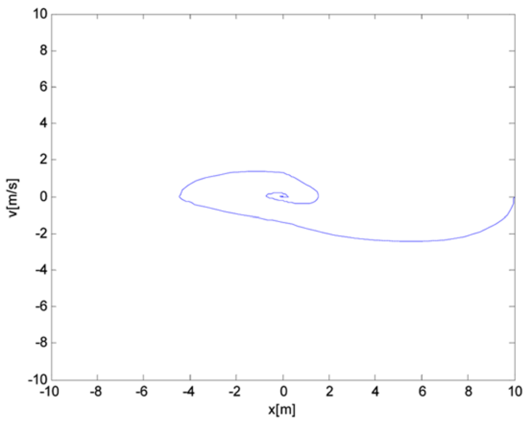

Figure 16. Phase plane plot of the closed loop system with the worst case gains.

6. CONCLUSION

The virtual lead vehicle scheme makes the motion of the host vehicle smooth when a new lead vehicle cuts in or the current lead vehicle cuts out. A LQ controller with variable weights controls the virtual lead vehicle in transient motions. The host vehicle follows the virtual lead vehicle and the suggested controller shows better performance in terms of passenger comfort and fuel efficiency of the host vehicle.

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