OBJECTIFICATION OF ON-CENTER HANDLING CHARACTERISTICS BASED ON SPRING-MASS-DAMPER SYSTEM

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(Received 23 February 2010; Revised 10 June 2011)

ABSTRACT-As one of the major handling performance factors of vehicles and tires, on-center handling is very important at high speeds where safety and refinement are major concerns for the driver. In this paper, the steering wheel torque and vehicle response characteristics that play an important role in on-center handling performance were quantified using a spring-mass-damper (SMD) system. Using this system, the characteristics of steering wheel torque and vehicle response could be objectified with SMD parameters such as those for the spring and damping. Experimental objective tests were performed by considering the process by which the on-center handling is evaluated subjectively, and the SMD parameters were extracted from the measurement data. From a statistical analysis of the subjective and SMD parameters for several vehicles and professional drivers, it was found that the subjective assessment of on-center handling could be successfully explained using the suggested parameters.

KEY WORDS : On-center handling, Spring-mass-damper system, Parameter identification, Correlation coefficient

1. INTRODUCTION

One of the most common driving situations at high speeds is driving on a straight section of road, such as motorways. In this situation, the ease and confidence with which a vehicle can be driven is of great importance to vehicle users and vehicle manufactures alike. According to the terminology currently used in vehicle and tire tests, this kind of vehicle performance is referred to as the steering feel and is characterized by two distinct behaviors: oncenter and off-center steering handling.

A vehicle is considered to have good on-center handling if it requires minimal correction and if it instructs the driver on how much correction to apply and then proceeds to execute the driver's command accurately. Excessive steering wheel (handwheel) activity, uninformative steering, and imprecise vehicle response are all contributing factors to poor on-center handling. These factors contribute to additional driving loads and reduce the driver's ability to focus on driving demands.

Many researchers, including international groups such as Norman (1984), Farrer (1993), Kim (2007), Higuch and Sakai (2001), ISO (2002; 2003), and Tokunaga *et al.* (2004), have attempted to objectify on-center handling by means of several objective test methods and numerous objective parameters. Farrer (1993) suggested that oncenter handling was identified as a function of three characteristics: steering activity, steering feel, and vehicle response. Here, for an objective measure of the steering activity, steering wheel angle, rate, and torque activities were used. For the steering feel, objective measures of steering friction, torque dead-band, and steering stiffness were used. For an objective measure of the vehicle response, response dead-band, response gain, and response time lag were used. However, these researchers attempted to quantify on-center handling using many objective parameters obtained from objective measurements with special procedures and complicated data-processing. Consequently, it was not only inconvenient to obtain the objective parameters but there were also problems in applying the vehicle and tire designs practically.

This paper tries to quantify on-center handling with objective parameters that can be obtained without difficulty. For this purpose, the subjective test and assessment methods of on-center handling were first studied. Then, by investigating the ergonomic perception nature of the driver, a SMD (spring-mass-damper) system can be constructed. The on-center handling characteristics were approximated by the SMD parameters. For the validation of the proposed methods, both subjective assessment and objective measurement tests were executed simultaneously. In the subjective assessment, a special survey that featured eight questionnaires of evaluation and observation items for describing on-center handling was used, and drivers were asked to answer these items for various test vehicles. In the objective test, a weave test was employed to obtain

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measurement data from which the SMD parameters were determined. Finally, by the correlation analysis between the subjective grade and SMD parameters, it was shown that the on-center handling of the tested vehicles and tires could be successfully represented using the suggested SMD parameters.

2. ON-CENTER HANDLING

On-center handling is normally estimated at high speeds from 80 kph to the speed limit. The type of steering wheel input is sinusoidal, with a small amplitude around 10 deg. The rate of the steering wheel input is quasi-static state, namely, 0.2 Hz. In this study, the vehicle moved along a nearly straight line. The effort of the driver and the response characteristics of the vehicle over a small range of steering wheel input has a great effect on the on-center handling (Farrer, 1993; Kim, 2007).

The on-center effort perceived by the driver can be related to the steering wheel torque reaction with respect to the steering wheel angle input. The torque pattern can be approximated as possessing a "U" shape, which is normally symmetrical about the y-axis as shown in Figure 1(a). Here, the predominant features are dead-band, steering stiffness and torque transition. The dead-band is the width of the angle of the steering wheel that produces a flat feel and no variation in the magnitude of the torque of the steering wheel. This property can be approximated by a flat bottom in the curve of the torque of the steering wheel versus the



(c) Response time delay

angle of the steering wheel, as shown in Figure 1(a). The steering stiffness is the gradient of the side in the "U"shaped curve, where drivers can perceive the torque feel as being stiff or soft. This is related to the stiffness or compliance of the steering system. The torque transition is the maximum radius of curvature linking the flat bottom and the side of the "U" curve.

Similarly, the vehicle response perceived by a driver can be related to the vehicle response with respect to the angle of the steering wheel in the form of "U" shapes, as shown in Figure 1(b). The predominant features are response dead-band, response transition and response gain. The response dead-band is the width of the angle of the steering wheel that produces no response in the vehicles. This property can be represented by the flat bottom in the "U"shaped curve.

As the steering wheel angle required to generate the vehicle motion increases, the dead-band becomes larger. The response transition is the characteristic that links the flat bottom to the side of the "U" curve. The response gain is the magnitude of the vehicle response that corresponds to the angle of the steering wheel around the on-center region. The response time delay is the short period of time lag between the steering wheel angle and yaw response, as shown in Figure 1(c).

Early work has reported that the steering stiffness, the friction in steering system, response gain and response time delay are the most significant influences on the on-center quality.

3. SPRING-MASS-DAMPER SYSTEM

The SMD system is of considerable significance in engineering because many simple systems in the real world are very closely approximated by a SMD system. Milliken and Milliken (1995) explained that the dynamics of a vehicle are closely approximated by those of a SMD system, including the lateral/directional response of a simple automobile to steering wheel inputs. Similarly, oncenter handling characteristics can be approximated by a SMD system.

3.1. Steering Wheel Torque - Steering Wheel Angle In Figure 2, by replacing the applied force F(t) with steering wheel torque T(t) and the output x with the steering



Figure 1. On-center parameters perceived by driver (Farrer, 1993).

Figure 2. Spring-mass-damper system.

wheel angle δ , a SMD system representing the relationship between steering wheel torque and steering wheel angle can be constructed as follows:

$$\ddot{\delta} + c_{\delta} \dot{\delta} + k_{\delta} \delta = T(t) \tag{1}$$

where *m* can be set to unit mass because there is no change in mass in the system.

In Equation (1), the damping parameter c_{δ} is related to the expression for the damping feel of steering wheel in the subjective evaluation of on-center handling. Moreover, the spring parameter k_{δ} is the gain of the steering wheel angle against the input of the steering wheel torque. This is related to the expression for steering wheel torque stiffness in the subjective evaluation of on-center handling. Namely, the greater k_{δ} is, the stiffer the steering wheel torque is.

3.2. Steering Wheel Angle - Yaw Rate

Similarly, by replacing the applied force F(t) with the steering wheel angle $\delta(t)$ and the output *x* with the yaw rate *y* in Figure 2, a SMD system representing the relationship between yaw rate and steering wheel angle can be constructed as follows:

$$\ddot{y} + c_v \dot{y} + k_v y = \delta(t) \tag{2}$$

where m is unit mass. Equation (2) is very similar to the simple dynamics equation of the 2-degrees-of-freedom bicycle model between yaw rate and steering wheel angle justified by Milliken and Milliken (1995).

In Equation (2), the SMD parameters can be explained by the subjective feeling of on-center handling, where the damping parameter c_y is related to the yaw response lag level of the input of the steering wheel angle.

Moreover, the spring parameter k_y is related to the gain of the yaw rate against the input of steering wheel angle. This is related to the expression of vehicle response gain in the subjective evaluation of on-center handling.

3.3. Parameter Identification

The SMD system can be represented by a LIT (linear timeinvariant) continuous system. The LIT system, with model and system errors w and v, respectively, can be described by a state-space system such as (Ljung, 2004)

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{w}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{v}$$
 (3)

where \mathbf{x} is the vector of state variables, \mathbf{y} is the sensor measurement, \mathbf{u} is the input vector, and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1/m \\ 0 \end{bmatrix}$$
(4)

In Equation (3), **C** is the output measurement matrix and **D** is the influence of **u** on **y**. $\mathbf{D} = 0$ means that there is no direct influence of **u** on **y**. Thus, the effect of the input on



Figure 3. Estimated results for (a) overdamped and (b) underdamped systems.

Table 1. Comparison between the theoretical value and estimated value of the SMD system parameters.

| Parameters | Theoretical | Estimated | Error (%) | |
|-------------|-------------|-----------|--------------|--|
| | value | value | | |
| Overdamped | | | | |
| Damper, c | 3 | 2.9902 | 0.33 | |
| Spring, k | 4 | 4.0248 | 0.62 | |
| Underdamped | | | | |
| Damper, c | 7 | 6.9822 | 0.25 | |
| Spring, k | 4 | 4.0459 | 1.15 | |

the output passes via \mathbf{x} and will be delayed by at least one sample.

The SMD parameters in Equation (4) can be estimated by the system identification method (Ljung, 2004). The estimation of the SMD parameters is demonstrated via a simulation study. Figure 3 shows the responses of underdamped and overdamped systems with m = 1 kg, $F(t) = F_0 \cos(2\pi f t)$, $F_0 = 10 N$, f = 0.2 Hz. To estimate the SMD parameters, the response signals were sampled at 100 Hz and featured by adding white Gaussian noise. Then, using Equation (3), the SMD parameters were estimated. The estimation results of the SMD parameters are listed in Table 1. From these results, it was found that *c* and *k* could be estimated accurately with small error.

4. TEST

An experimental objective test was conducted by considering the process by which the on-center handling is



Figure 4. Measurement devices for (a) steering wheel angle and torque and (b) vehicle motions.

evaluated subjectively.

4.1. Instrumentation

To measure the various signals related to on-center handling, many sensors were attached to the vehicles. A specially designed steering wheel sensor was used to measure the steering wheel angle, steering wheel speed, and steering wheel torque, as shown in Figure 4. This sensor consists of four parts: sensor jig, sensors, angle limiter, and stop-bar. Using the jig, the steering wheel sensor can be mounted on the conventional steering wheel. Using the encoder and strain gage inserted into the inner part of the steering wheel sensor, the angle and torque can also be measured. In addition, using the angle limiter and strop-bar, the desired amplitude of steering wheel rotation can be precisely controlled.

The vehicle speed was measured by a GPS. Additionally, to measure the vehicle's motion, inertial sensors with 3-axes acceleration (vertical, longitudinal, lateral) and 3-axes angular rate (roll, pitch, yaw) detection were attached inside the vehicle as shown in Figure 4.

4.2. Objective Test Technique

To fully describe the subjective test procedure in the

evaluation of the on-center handling, a weave steering test was used. The test was carried out under nearly ideal weather conditions, typically when the road surface was dry and the wind calm. Breezes up to 10 kph were tolerated, but under no circumstances were tests permitted when the road was wet or there was a moderate wind. Variation in the position of the accelerator pedal was kept to a minimum while maintaining the vehicle's longitudinal velocity at 120 kph. The longitudinal velocity that did not vary by more than $\pm 3\%$ from the suggested velocity during the test sequences used for data analysis. The tires were warmed up prior to testing to achieve a steady temperature and pressure defining normal driving conditions. For example, driving at the test speed for a distance of 10 km was performed to warm up the tires. This test commenced with the vehicle traveling initially along a straight line. The driver then proceeded to steer the vehicle first in the clockwise direction, then in the anti-clockwise direction, returning finally to the straight position. The steering wheel angle amplitude was set to generate 0.1 g of lateral acceleration. Using a specially designed steering wheel sensor with an angle limiter and stop-bar, the amplitude of the steering wheel angle could be successfully controlled at every test.

The frequency of the steering wheel input was set to match the main frequency component of the data collected during subjective on-center test, namely, 0.2 Hz. This setting is performed to improve the probability of correlating the subjective and objective data, considering that performance at the main frequency may have the greatest influence on the subjective assessment. Audible timing pulses were provided by a laptop computer at every



Figure 5. Objective test results for weave steering.

half cycle to prompt the driver to turn the steering wheel at the specified frequency. The ISO (2002, 2003) suggest that the results are more sensitive to variations in the frequency of the steering wheel input than those in the amplitude. Tolerances of ± 0.02 Hz and ± 0.01 g, respectively, were therefore specified. Using the steering wheel angle limiter, stop-bar and audible timing pulse, the on-center weave steering test with 0.1 g of lateral acceleration and 0.2 Hz steering wheel input could be successfully performed, as shown in Figure 5.

4.3. Subjective Questionnaire Design

To evaluate the on-center handling quantitatively, a specially designed questionnaire featuring eight inquiries was used. Subjective problems were grouped into two parts; the first part, denoted "observation", asked for the driver's judgement of the vehicle's real characteristics, as shown in Figure 6. During this part of the assessment, the test driver was asked to ignore the driver's personal preferences. The second part, denoted "evaluation", asked how the driver would rate the vehicle in performance.

The "observation" grades were tested later for their correlation with the objective parameters. The on-center handling was based on the evaluation of two major

| | | | | | Scale | | | | | |
|------------------------------------|----------|------|-------|------------|----------|----------|------------|-------|------|----------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | Extremly | Very | Quite | Moderately | Slightly | Slightly | Moderately | Quite | Very | Extremly |
| On-center steering effor | t | | | | | | | | | |
| Hollow-band | Small <- | | | | | | | | | -> Large |
| Friction | Small <- | | | | | | | | | -> Large |
| Stiffness | Light <- | | | | | | | | | -> Heavy |
| ON-CENTER EFFORT | Poor <- | | | | | | | | | -> Good |
| On-center steering resp | onse | | | | | | | | | |
| Dead-band | Small <- | | | | | | | | | -> Large |
| Response gain | Small <- | | | | | | | | | -> Large |
| ON-CENTER RESPONSE (Time delay) | Poor <- | | | | | | | | | -> Good |
| ON-CENTER FEEL | Poor <- | | | | | | | | | -> Good |

Figure 6. Problem sheet for the subjective evaluation of oncenter handling.

categories: on-center effort and on-center response. The oncenter effort was the first category, and its "observation" item comprised four inquiries: hollow-band, friction, and stiffness "observations" along with a conclusive "evaluation". Each question prompted the driver to an answer in terms of two opposing adjectives. Five modifiers were then combined with each adjective to form ten progressive options, producing grades from one to ten, respectively, as shown in Figure 6.

4.4. Test Vehicles

In the subjective test, a specially designed questionnaire with eight inquiries was used for the evaluation of the oncenter handling using nine vehicles and eight drivers. Various kinds of passenger vehicles—luxury, sport-utility, medium, and small—were used as listed in Table 2. It is preferable in a subjective study to treat the problem as a statistical one and employ a large number of drivers of various ages, abilities and backgrounds. However, because of the requirement of highly technical skill in the subjective grade of the steering behavior, this approach was considered inappropriate. Therefore, a small group of eight professional drivers working as test engineers in vehicle and tire companies were employed.

5. ANALYSIS

In this section, the estimated results for the SMD parameters and the correlation between the subjective grade and SMD parameter are discussed by a statistical analysis method.

5.1. Estimation

The estimated results for the SMD parameters for various vehicles are shown in Table 3. An analysis of variance (ANOVA) study was performed on these data. In the estimated spring and damping parameters for the test group, *F-value* >> 1, which casts doubt on the null hypothesis that all samples are drawn from the same

Table 2. Test vehicles and tires.

| Vehicle — | | Weight (kgf) | | Wheelberge (m) | Dim | Inflation pressure of tire (kgf/cm ²) | |
|------------|-------|--------------|-------|-------------------|-------------|---|------|
| | Front | Rear | Total | - wheelbase (III) | KIII | Front | Rear |
| Vehicle #1 | 993 | 944 | 1937 | 2.94 | 225/55R17V | 7.5J | 2.25 |
| Vehicle #2 | 1326 | 912 | 2238 | 2.84 | 235/55R17H | 7.0J | 2.10 |
| Vehicle #3 | 1116 | 740 | 1856 | 2.62 | P235/60R16T | 6.5J | 2.10 |
| Vehicle #4 | 1095 | 776 | 1871 | 2.64 | P235/60R16T | 6.5J | 2.10 |
| Vehicle #5 | 1056 | 710 | 1766 | 2.75 | P205/65R16H | 6.5J | 2.00 |
| Vehicle #6 | 771 | 801 | 1572 | 2.70 | 205/55R16V | 6.5J | 2.46 |
| Vehicle #7 | 881 | 754 | 1636 | 2.70 | 205/55R16V | 6.5J | 2.10 |
| Vehicle #8 | 799 | 511 | 1310 | 2.50 | P185/65R14T | 5.5J | 2.10 |
| Vehicle #9 | 616 | 401 | 1017 | 2.35 | 155/65R13T | 4.5J | 2.10 |

population. This result indicates that at least the means of the SMD parameters for one test set are significantly different from those of other test sets. Moreover, the estimation ranges at a 95% confidence level about the means were calculated. The estimation ranges of the spring parameters were smaller than those of damping parameters.

The SMD parameters for steering wheel angle-steering wheel torque and yaw rate-steering wheel angle are mapped in Figures 7 and 8, respectively. These maps are well suited for showing on-center targets across different classes of vehicle. From these maps, vehicles in the same class but from different manufacturers show widely different results. For instance, some manufacturers prefer high steering damping (Vehicle #1) while others prefer low steering damping (Vehicle #2).

5.2. Correlation

From Bendat and Piersol (2000), the correlation coefficient between the subjective item and the SMD parameter is given by

$$R_{xy} \equiv \frac{C_{xy}}{\sqrt{C_{xx}C_{yy}}} \tag{4}$$

where C_{xy} is the covariance between x and y variable, and C_{xx} and C_{yy} are the variances of x and y variable, respectively. The correlation coefficient has the range

$$-1 \le R_{xy} \le 1 \tag{5}$$

As an obvious way to reveal the influence of the SMD parameters on the subjective observations in the on-center handling in a statistical sense, p-values used to test the hypothesis with no correlation between the SMD parameters and observation were investigated. The p-values indicate the probability of being wrong in stating that there is no correlation. If a p-value is small, for example less than 0.05, then the correlation between the

Table 3. SMD parameter estimation results



Figure 7. SMD parameters between steering wheel angle and steering wheel torque.



Figure 8. SMD parameters between yaw rate and steering wheel angle.

| Test set – | Steering wheel angle vs. steering wheel torque | | | | Yaw rate vs. steering wheel angle | | | |
|------------|--|---------------------|-------------------|---------------------|-----------------------------------|---------------------|-------------------|---------------------|
| | Spring parameter | % Conf. interval | Damping parameter | % Conf. interval | Spring parameter | % Conf. interval | Damping parameter | % Conf. interval |
| Vehicle #1 | 0.347 | 1.1 | 0.117 | 4.9 | 5.502 | 1.7 | 0.687 | 9.9 |
| Vehicle #2 | 0.185 | 1.3 | 0.052 | 5.2 | 6.126 | 1.6 | 0.626 | 2.0 |
| Vehicle #3 | 0.249 | 1.1 | 0.057 | 9.4 | 6.73 | 2.2 | 0.551 | 10.2 |
| Vehicle #4 | 0.264 | 1.1 | 0.051 | 4.9 | 5.648 | 1.2 | 0.636 | 5.5 |
| Vehicle #5 | 0.247 | 0.7 | 0.062 | 3.3 | 4.394 | 0.9 | 0.506 | 10.2 |
| Vehicle #6 | 0.344 | 2.5 | 0.057 | 5.2 | 4.405 | 2.0 | 0.372 | 11.4 |
| Vehicle #7 | 0.299 | 1.9 | 0.046 | 7.1 | 4.842 | 5.5 | 0.625 | 15.5 |
| Vehicle #8 | 0.27 | 1.4 | 0.035 | 4.3 | 4.547 | 2.0 | 0.355 | 13.1 |
| Vehicle #9 | 0.221 | 1.0 | 0.034 | 5.3 | 5.156 | 0.9 | 0.506 | 7.2 |

| Subjective observation | Objective parameters | Corr. coeff. | p-value |
|------------------------|---|--------------|---------|
| Friction | Damping parameter in steering wheel angle-torque, $c_{\!\delta}$ | 0.76 | 0.02 |
| Stiffness | Spring parameter of steering wheel angle-torque, k_{δ} | 0.89 | 0.00 |
| On-center gain | Spring parameter in yaw rate-steering wheel angle, $\boldsymbol{k}_{\boldsymbol{y}}$ | -0.77 | 0.02 |
| Time delay | Damping parameter in yaw rate-steering wheel angle, $\boldsymbol{c}_{\boldsymbol{y}}$ | 0.36 | 0.34 |

Table 4. Correlation results between the SMD parameters and subjective observation item.



Figure 9. Correlation results between the SMD parameter and subjective observation item.

subjective grade and SMD parameter is considered significant with a 95% confidence level. Table 4 and Figure 9 show the correlation between the subjective grades of observation and the SMD parameters. Here, p-values are listed together with the correlation coefficient. Most of the p-values with high correlation coefficients were less than 0.05, except for the "Time delay" item.

In the case of the "Friction", the subjective grades show high positive correlation with the $c_{\delta}(R_{xy} = 0.76, p = 0.02)$. Similarly, with the k_{δ} high positive correlation with the subjective observation of the "Stiffness" can be found $(R_{xy} = 0.89, p = 0.00)$.

In the subjective observation of the "On-center gain", the k_y shows high negative correlation ($R_{xy} = 0.77$, p = 0.02). On the other hand, the c_y has low correlation with the subjective observation of the "Time delay" ($R_{xy} = 0.36$, p = 0.34). This result shows that in evaluating the "Time delay", most of the drivers were influenced by other phenomena during the subjective test. To determine the reason for this result, the correlation between "Time delay" and k_y was obtained and a high correlation was observed ($R_{xy} = 0.79$, p = 0.01). These results show that the subjective observation of the time delay was influenced more by the magnitude of the vehicle response than by the response lag time. These results agree with the results reported by Kim (2007).

6. CONCLUSION

Regarding the characterization of the on-center handling of a passenger car in the vehicle and tire design process, a simple method based on a SMD system was proposed. Using the SMD parameters, the on-center handling characteristics of various vehicles could be obtained, and a correlation with the subjective assessment was obtained. The results are summarized as follows.

- Using the SMD parameters, on-center handling characteristics for various types of passenger cars could be successfully identified.
- (2) It was found that the subjective feeling of the on-center handling could successfully be explained by the SMD parameters.
- (3) In case of the subjective feeling of the time delay, much higher correlation with the magnitude of vehicle response was found than with the response lag time. This shows that human perception of the time delay of the vehicle response is mostly influenced by the magnitude of the vehicle response.

ACKNOWLEDGEMENT-The author wish to thank the test engineers of the R&D center of HANKOOK Tire Co., Ltd., for their participation in the subjective test and valuable suggestions.

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