# DEVELOPMENT AND ANALYSIS OF AN AIR SPRING MODEL

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ABSTRACT+The mainly can be effectively used for a spring can be effectively used for a behavior and spring can be effectively used for the design of air spring can be effective which are not at the design of air spring th provide better ride and handling characteristics along with various functions for passenger convenience. However, establishing a general model of an air spring poses particular difficulties due to the severe nonlinearities in the stiffness and the hysteresis effects, which are hardly observed in conventional coil springs. The purpose of this study is to develop a general analytic model of an air spring - one which represents the main characteristics of stiffness and hysteresis and which can be connected − one which represents the main characteristics of stiffness and hysteresis and which can be connected ic systems desigined to control air spring height. To this end, the mathematical model was established dynamics with t to a model of pneumatic systems desigined to control air spring height. To this end, the mathematical model was established on the basis of thermodynamics with the assumptions that the thermodynamic parameters do not vary with the position inside the air spring, that the air has the ideal gas property, and that the kinetic and potential energies of the air are negligible. The analysis of the model has revealed that the stiffness is affected by the volume variation, the heat transfer, and the variation of the air mass and the effective area. However, the hysteresis is mainly affected by the heat transfer and the variation of the effective area. In particular, it was revealed that the increase of the volume due to the cross-sectional area increases the stiffness, while the increase of the volume due to the other reason decreases it. In addition, the model was used to develop the sufficient stability condition, and the stability of the model was analyzed. The paper also presents the comparison between the simulation and experimental results to validate the established model and demonstrates the potential of the model to be usefully employed for the development of the air spring and its algorithm for use in a pneumatic system.

- : air mass flow rate flowing into air spring
- : air mass flow rate flowing out of air spring
- : air mass inside air spring  $\frac{m_{in}}{m_{ov}}$  $m_{\infty}$ m<sub>cv</sub><br>17
- : control volume of air spring  $m$ <br> $m$ <br> $V_c$
- <sub>heat</sub> : heat transfer rate
- $A_{heat}$  : area of heat transfer<br> $h_{c}$  : heat transfer coeffici  $A_{\text{box}}$ -<br>-<br>-<br>-
- $h_c$  : heat transfer coefficient<br> $W$  : work performed on air s
- : work performed on air spring
- $h_{in}$  : enthalpy flowing into air spring<br>  $h_{out}$  : enthalpy flowing out of air sprin
- $h_{out}$  : enthalpy flowing out of air spring<br> $U_{cv}$  : internal energy inside air spring
- $U_{\text{cv}}$  : internal energy inside air spring<br>  $P_{\text{cv}}$  : pressure inside air spring
- $P_{cv}$  : pressure inside air spring<br> $P_{dm}$  : pressure of environment
- $P_{\text{atm}}$  : pressure of environment<br> $T_{\text{cv}}$  : temperature inside air sp
- 
- $T_{cv}$  : temperature inside air spring<br> $T_{in}$  : temperature of air flowing in  $T_{in}$  : temperature of air flowing into the air spring<br> $T_{env}$  : temperature of environment around air spring
- $T_{\text{env}}$  : temperature of environment around air spring  $c_{\text{v}}$  : specific heat at constant volume
- $c_v$  : specific heat at constant volume<br>  $c_p$  : specific heat at constant pressure
- $c_p$  : specific heat at constant pressure<br>  $k$  : specific heat ratio
- : specific heat ratio
- $R$  : ideal gas constant
- $F_{as}$  : force applied to vehicle body by air spring<br>  $A_{ef}$  : effective area of air spring
- : effective area of air spring
- z : vertical displacement
- $z_0$  : magnitude of displacement sinusoid<br>  $f$  : frequency of displacement sinusoid
- : frequency of displacement sinusoid
- $t$  : time
- $V_{\text{cv0}}$  : fixed volume of air spring<br> $A_{cs}$  : cross-sectional area of the
- $A_{cs}$  : cross-sectional area of the air spring<br> $z_{\text{max}}$  : maximum displacement of bottom of
- $z_{\text{max}}$  : maximum displacement of bottom of air spring<br>  $z_{\text{curr}}$  : current displacement of bottom of air spring
- : current displacement of bottom of air spring

**KEY WORDS:** Air spring, Analytic model, Stiffness, Hysteresis, Thermodynamic model, Stability<br>  $\text{AENCLATURE}$ <br>  $\therefore$  air mass how rate flowing into air spring<br>  $\therefore$  air mass how rate flowing out of air spring<br>  $\therefore$  air ma Air springs have been primarily applied to commercial vehicles and luxury passenger cars because they are costly. They have many advantages, however, compared with conventional coil springs. Air springs provide better comfort and improvement in the handling performance because they can have relatively low stiffness and enable a vehicle to maintain optimum wheel alignment. In addition, air springs can protect the body of a vehicle on rough roads and make the task of loading baggage into the trunk of a vehicle more convenient (Figure 1) because the heights of the air springs can be adjusted through supplying and exhausting the air via the pneumatic circuit connected to the air spring (Jang et al., 2007; Hyundai Motor Company, 2009; Kia Motor Company, 2009). Figure 2 shows an air spring and its relevant pneumatic system.

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The analytic model can be usefully employed for the design of an air spring, the related pneumatic system, and the algorithm for the operation of the pneumatic system (Jang et al., 2007; Kim et al., 2001). However, it is very difficult to develop an accurate air spring model due to its severe nonlinearities, which are not found in conventional coil springs. More specifically, the stiffness of an air spring, which has a significant effect on the ride and handling characteristics of a vehicle, varies nonlinearly with the frequency of the road excitation. The hysteresis characteristics of an air spring, which provides a vehicle with the additional damping force, cannot be neglected compared with the force of the damper, and it also varies with the frequency of the road excitation (Nieto et al., 2008; Chang and Lu, 2008).

Some research works (Kim et al., 2001; Nieto et al., 2008; Chang and Lu, 2008; Kim and Kim, 2005; Quaglia and Sorli, 2001; Seong et al., 2008; Cha et al., 2006) have been carried out to develop an analytic model for an air spring involving these nonlinear characteristics. Kim et al. (2001) have developed a model of an air spring and a vehicle with a flexible body using ADAMS, which has been used to estimate the performance of a vehicle with a control algorithm for the pneumatic system. The stiffness of the air spring model is expressed as a function of pressure, volume, area, and the polytropic index, but the process that determines the pressure of the air spring is not described. Nieto et al. (2008) derived a nonlinear model of an air spring on the basis of thermodynamics, assuming adiabatic or isothermal conditions, and analyzed the stiffness, the damping factor, and the transmissibility using the derived model. Chang and Lu (2008) also developed a





Figure 2. Air spring and its pneumatic supply system (Folchert, 2006; Jang et al., 2007). Figure 3. Control volume of the air spring.

model of an air spring on the basis of thermodynamics, which consisted of two steps. First, the air spring pressure is obtained using the adiabatic condition, and then it is corrected by considering the temperature obtained by the heat transfer equation. Because the model does not consider the air supply or the air exhaust to/from the air spring, it cannot be employed in the design of the pneumatic system or its control algorithm. In addition, it is difficult to employ the model for the stability analysis because the model is expressed by algebraic equations.

The objective of this study is to develop the general air spring model on the basis of the thermodynamic equation without the assumption of adiabatic or isothermal conditions and with the variation of air mass. The analysis of the developed model will reveal the important factors that have a significant effect on the stiffness and hysteresis of an air spring. The author of this paper performed the study on the air spring model and its analysis in previous research (Cha et al., 2006). The current study enhances the previous model of an air spring. The further analysis is performed on the basis of the enhanced model. Moreover, the stability of the air spring model is analyzed in this paper.

The rest of this paper is organized in the following order. In Section 2, the generalized model of the air spring is derived on the basis of the thermodynamic equation. In Section 3, the derived model is validated by experimental results, and the stability and important characteristics of the air spring such as the stiffness and hysteresis are analyzed. Finally, Section 4 presents a summary of the results and draws the conclusions.

# 2. MATHEMATICAL MODEL OF AIR SPRING

Figure 3 shows the control volume of the air spring, the main variables of which are pressure, absolute temperature, air mass and volume. The mathematical model of the air spring can be derived using the energy conservation law.

The flow of the air mass into or out of the control volume, shown in Figure 3, is controlled by the operation of the control valve in the pneumatic circuit, as shown in Figure 2. The flow of the air accompanies the enthalpy. In addition, work is performed on the control volume by the Figure 1. Adjustment of the height of a vehicle.<br>vehicle body and the wheel, and the difference of temper-



atures between the inner and the outer sides of the control volume generates some heat transfer between them. These power flows can be modeled by the following energy conservation equation (Fernandez and Woods, 1999; Cha et al., 2006).

$$
\dot{Q}_{\text{heat}} + \dot{W} + (h_{\text{in}} m_{\text{in}} - h_{\text{out}} m_{\text{out}}) = \dot{U}_{\text{cv}} \tag{1}
$$

where the time derivative of work that is performed on the control volume,  $W$ , is defined using the pressure inside the ·

$$
W=-P_{c}V_{c} \tag{2}
$$

The enthalpies flowing into and out of the control volume, by

$$
h_{in}=c_p T_{in}
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h_{out}=c_p T_{cv}
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\n(3)

*et al.*, 2006).<br>  $\dot{Q}_{heat} + \dot{W} + (h_{in})$ <br>
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volume,  $V_{cv}$ , l<br>  $\dot{W} = -P_{cv}\dot{V}_{cv}$ <br>
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pressure,  $c_p$ ,  $z_p$ <br>
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by<br>  $h_{in} = c_p T_{in}$ <br> control volume,  $P_{c,9}$  and the time derivative of the control<br>volume,  $V_{c,9}$  by<br> $\dot{W}=-P_{c,7}\dot{V}_{eq}$  (2)<br>The enthalpies flowing into and out of the control volume,<br> $h_a$  and  $h_{qo}$ , are expressed using the specific he volume,  $V_{ev}$ , by<br>  $\dot{W} = -P_{ev} \dot{V}_{ev}$ <br>
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power flows. T<br>  $h_n$  and  $h_{mn}$ , are expressed using the specific heat at constant<br>pressure,  $c_p$ , and the temperatures of the air mass flowing<br>into and inside the control volume,  $T_n$  and  $T_{-n}$ , respectively,<br>by<br>by<br> $h_m = c_p T_m$ <br> $h_{mn} = c_p T$ pressure,  $c_p$ , and the temperatures of the air mass flowing<br>into and inside the control volume,  $T_n$  and  $T_{cs}$ , respectively,<br>by<br>by<br> $h_n = c_p T_w$  (3)<br> $h_{\infty} = c_p T_w$  (3)<br> $\frac{1}{2}$ <br> $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{6}$  enthalpies mult into and inside the control volume,  $T_n$  and  $T_{cs}$ , respectively,<br>by<br>by<br>by<br> $h_n = c_p T_w$ <br> $h_{ow} = c_p T_c$ <br>(3)<br> $\hat{n}_{bw} = c_p T_c$ <br>(3)<br>The enthalpies multiplied by the air smass flow rate flowing<br>into and out of the air spring,  $\dot{m}_m$ The enthalpies multiplied by the air mass flow rate flowing into and out of the air spring,  $m_m$  and  $m_{out}$ , represent power flows. The internal energy of the control volume, volume by  $\omega$  wild converse  $\vec{W}$ . The primary  $h_n$  in The production of  $U_{\alpha}$  or  $\vec{Q}$ . The conversion of  $Q$ the the theorem is the three than the end to the end of  $n = c_p T_b$ <br>  $u = c_p T_c$ <br>  $u = c_p$ time ume ume ume ume ume  $\frac{1}{n}$ , by pies are  $\frac{1}{n}$ , and side pies ansf trol effice  $\frac{1}{n}$  and  $\frac{1}{n}$  he p Fivative is a set of the set of th  $(n_{in}m_{in}-n_{out}m_{out})=O$ <br>time derivative of  $\cdot$ <br>lume,  $W$ , is defined<br>plume,  $P_{ev}$ , and the  $\cdot$ <br>lpies flowing into a<br>*c*<sub>e</sub>, by<br>lpies flowing into a<br>*c*<sub>e</sub>, and the tempera<br>nside the control vo<br>lpies multiplied by  $\cdot$ <br>out  $P_{\epsilon}$ <br>  $\infty$  to  $\infty$ <br>  $\infty$  the  $\infty$ <br>  $\infty$ The  $h_{m}$  and  $h_{n}$  and  $h_{n}$  and  $h_{n}$  and  $h_{n}$  and  $h_{n}$  and  $h_{n}$  and  $\mathcal{Q}_{n}$  are  $\mathcal{Q}_{n}$  and  $\mathcal{$ e entha<br>and  $h_{ou}$ <br>and  $h_{ou}$ <br>sssure,<br> $\bar{=}c_pT_m$ <br> $\bar{=}c_pT_{cv}$ <br> $\bar{=}c_pT_{cv}$ <br>ie entha<br>io and wer flo<br>on and wer flo<br>is detice entha<br> $\bar{=}c_pT_{cv}$ <br>ie heat the consfer c<br>dening by  $\bar{=}c_vm_{cv}$ <br> $\bar{=}k_cA$ <br>derive dening by  $\bar{=}k$  $h_{out}=c_pT_c$ <br>  $h_{out}=c_pT_c$ <br>
The enth into and power fl<br>
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power flict entha<br>  $U_{cv}$ , is de:<br>  $U_{cv} = c_v m_{cv}$ <br>
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To derive<br>
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following<br>  $T_{cv} = \frac{1}{R} \frac$ 

$$
U_{c\mathbf{v}} = c_{\mathbf{v}} m_{c\mathbf{v}} T_{c\mathbf{v}} \tag{4}
$$

 $U_{\nu}$ , is defined using the specific heat at constant volume,<br>  $U_{\nu}$  he air mass,  $m_{\omega}$ , and the temperature inside the control<br>
volume by<br>
volume by<br>  $U_{\nu} = c_{\nu} m_{\omega} T_{\nu}$ . (4)<br>
The heat transfer rate between the c<sub>v</sub>, the air mass, m<sub>cv</sub>, and the temperature inside the control<br>volume by<br> $U_{co} = c_{c} m_{ce} T_{c}$  (4)<br>The heat transfer rate between the inner and the outer sides<br>of the control volume,  $Q_{base}$ , is expressed using the heat<br>t The heat transfer rate between the inner and the outer sides of the control volume,  $Q_{\textit{heat}}$ , is expressed using the heat and temperatures of the outer and the inner sides of the m<br>
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n m on side that the magnetic result of The heat transfer coef<br>transfer coef<br>and tempera<br>control voluing  $Q_{head} = h_c A_{head}$ <br>To derive the<br>To derive the privation of the private  $T_{cv} = \frac{1}{R} \frac{V_{cv} P_{cv}}{m_{cv}}$ <br>where R is temperature<br>to time, as f<br>temperature can be de transfer coefficient,  $h_c$ , the area of the heat transfer,  $A_{heat}$ ,

$$
Q_{heal} = h_c A_{heal} (T_{env} - T_{cv})
$$
\n<sup>(5)</sup>

transfer coefficient, h,, the area of the heat transfer,  $A_{head}$ <br>and temperatures of the outer and the inner sides of the<br>control volume,  $T_{env}$  and  $T_{cv}$ , in the following form.<br> $\dot{Q}_{head} = A_{head} (T_{env} - T_{cv})$  (5)<br>To derive the control volume,  $T_{ew}$  and  $T_{ev}$ , in the following form.<br>  $\dot{Q}_{hew} = h_c A_{hewl}(T_{ew} - T_{cv})$ <br>
To derive the pressure dynamic equation from the eq<br>
(1), the temperature inside the control volume is re<br>
with the pressure inside To derive the pressure dynamic equation from the equation (1), the temperature inside the control volume is replaced with the pressure inside the control volume using the following ideal gas equation. Q  $\epsilon$  T(1 w)  $T_c$  w) the case to  $\frac{T_c}{T_c}$  w) the m heat  $n_cA_{heat}(T_{env} - T_{cv})$ <br>
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here *R* is the ideal<br>
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n be derived by diff<br>
time, as follows

$$
T_{cv} = \frac{1}{R} \frac{V_{cv} P_{cv}}{m_{cv}}
$$
\n
$$
\tag{6}
$$

where *R* is the ideal gas constant. The derivative of the<br>temperature inside the control volume with respect to time<br>can be derived by differentiating equation (6) with respect<br>to time, as follows:<br> $\frac{\dot{T}_{cv}}{T_{cv}} = \frac{\dot{P}_{$ temperature inside the control volume with respect to time can be derived by differentiating equation (6) with respect to time, as follows:  $T_{cv} = \frac{1}{R}$ <br>where<br>tempe can be<br> $\frac{\dot{T}_{cv}}{T_{cv}} = \frac{\dot{P}}{P}$ <br>where<br>the ain<br>the ba<br> $m_{cv} =$ re bet m  $\overrightarrow{P}$  re un particular definition of  $\overrightarrow{P}$  required to  $\overrightarrow{P}$  $\frac{1}{R} \frac{C}{m_{ev}}$ <br>re R is<br>be derived in the special special special specified  $\frac{\dot{P}_{ev}}{P_{ev}} \frac{m_{ev}}{m_{ev}}$ <br>re the a space of the space of  $\int m_{ev} dt$ R is turn as  $-\frac{m}{m}$  he mass or  $n_{c}$   $\infty$  G

$$
\frac{\dot{T}_{cv}}{T_{cv}} = \frac{\dot{P}_{cv}}{P_{cv}} - \frac{\dot{m}_{cv}}{m_{cv}} + \frac{\dot{V}_{cv}}{V_{cv}}
$$
\n
$$
\tag{7}
$$

where the air mass inside the control volume varies with the air mass flowing into and out of the control volume on the basis of the mass conservation law, as follows:  $\frac{1}{T}$  wh the m  $x$  where  $x$  and  $x$  and  $m$  $\frac{T_{c}}{T_{c}} = \frac{P}{P}$ <br>where air<br>the air<br> $m_{c} =$ re 1<br>ir 1<br>bas:  $\int h$  $\frac{P_{cy}}{P_{cy}}\frac{m}{m}$ <br>re the<br>ir mas<br>aasis o<br>i  $\int m_{cy}$  $\frac{1}{x}$  ass of  $\frac{1}{x}$  $m_{ev} + \overline{V}$ <br>e air r<br>ass flo<br>of the<br> $v/dt = \int$  $\lim_{x \to 0}$ <br>low<br>he

$$
m_{cv} = \int m_{cv} dt = \int (m_{in} - m_{out}) dt
$$
 (8)

Finally, the first order differential equation for the pressure of the air in the control volume can be obtained from

$$
\dot{P}_{cv} = -k P_{cv} \frac{V_{cv}}{V_{cv}} \n+ \frac{k-1}{V_{cv}} h_{c} A_{heal} \left( T_{env} - \frac{V_{cv}}{R m_{cv}} P_{cv} \right) \n+ \frac{k R}{V_{cv}} \left( T_{in} \dot{m}_{in} - \frac{P_{cv} V_{cv}}{m_{cv}} \dot{m}_{out} \right)
$$
\n(9)

equations (1), (6), (7), the specific heat ratio  $k=c_p/c_r$ , and<br>  $R/c_c=k-1$  as follows:<br>  $\dot{P}_{c_n}=-kP_c\frac{\dot{V}_{c_r}}{V_{c_n}}$ <br>  $+\frac{k}{V_{c_n}}h_cA_{koul}\left(T_{em}-\frac{V_{cr}}{Rm_c}P_{cr}\right)$  (9)<br>  $+\frac{kR}{V_{c_n}}\left(T_{um}/n_m-\frac{P_{cv}V_{cv}}{m_cR}m_{am}\right)$ <br>
Equation  $R/c_v=k-1$  as follows:<br>  $\dot{P}_{cv}=-kP_{cv}\frac{\dot{V}_{cv}}{V_{cv}}$ <br>  $+\frac{k-1}{V_{cv}}h_cA_{heal}(T_{env}-$ <br>  $+\frac{kR}{V_{cv}}(T_{in}m_{in}-\frac{P_{cv}V}{m_{cv}F})$ <br>
Equation (9), which the specific he<br>
the air spring, cc<br>
which the specific he<br>
the area of heat transf<br>
an Equation (9), which represents the mathematical model for the air spring, consists of two kinds of variables, in which the specific heat ratio, heat transfer coefficient, and the area of heat transfer are the parameters, and the volume and the rate of change of the volume in the air spring, the air mass flow rates, and the temperature of the environment are the variables determined by the components connected to the air spring. Each parameter was obtained through the following methods. The ideal gas constant was selected from the property of the air, and the air mass inside the air spring was calculated from the ideal gas equation. The specific heat ratio was estimated from the comparison between the experimental and simulation results. The area of heat transfer and the volume of the air spring were calculated from the measured geometric data and adjusted through the comparison between the experimental and simulation results. The heat transfer coefficient was selected from the well-known heat transfer coefficients and adjusted through the comparison between the experimental and simulation results.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 &$ cv  $kT_c$  +  $\frac{k-1}{V_{cv}}$  +  $\frac{kR}{V_{cv}}$  (1)<br>+  $\frac{kR}{V_{cv}}$  (1)<br>Fermion the air of  $\frac{kR}{V_{cv}}$  Equation<br>in the air should be read of the read of the air s<br>existing the value of  $\frac{kR}{V_{cv}}$  and  $\frac{k}{V_{cv}}$  and  $\frac{k}{V_{cv}}$  =  $a(t)$  $\overline{V}$   $h_c$   $r_b$   $n$  is  $\overline{Y}$  the object of  $\overline{Y}$  is  $\overline{Y}$  and  $\overline{Y}$  is  $\overline{Y}$  is  $\overline{Y}$  and  $\overline{Y}$  is  $\$  $h_{e}A$   $\Gamma_{in}n$  n (special species overlapping set  $\Gamma$  is a set  $\Gamma$  if the contribution of  $\Gamma$  is the set of special spec +  $\frac{V_{ex}}{V_{ex}}$ <br>+  $\frac{kR}{V_{ex}}$  (quatic ain the air and the air and the range of the range of the range of the range was fitted in the transfer of the range of the range of  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2$  $\frac{kR}{V_{c}}$  (at i e a c e r s s i air ingered when  $\frac{kR}{l}$  and  $\frac{k}{l}$  and  $\frac{k}{l}$  and  $\frac{k}{l}$  and  $\frac{k}{l}$  and  $\frac{k}{l}$  and  $\frac{k}{l}$  $V_{cv}$ <br>  $V_{cv}$   $V_{c}$   $\frac{kR}{V_{cv}}$   $(T_{in} \dot{m}_{in} - \frac{P_{cv}}{m_{cv}})$ <br>
unation (9), while early  $T_{m}$ <br>
unation (9), while early  $\frac{1}{T_{cv}}$  and  $\frac{1}{T_{cv}}$  the specific h<br>
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ne rate of changes are variables detai  $\overline{m}$ ,  $\overline{m}$ ,  $\overline{n}$  represents in a arc of the number as  $\overline{m}$  as  $\overline{m}$  and  $\overline{m}$  as  $\overline{m}$  and  $\overline{m}$  and  $\overline{m}$  arc  $\overline{m}$  and  $\overline{m}$  arc  $\overline{m}$  arc  $\overline{m}$  arc  $\overline{m}$  arc  $\overline{m}$  arc  $\$  $\sum_{i=1}^{n} m_{out}$ <br>  $\sum_{i=1}^{n} m_{out}$  $(T_{env} - \frac{P_{cv}V_{cv}}{Rm_{cv}})$ <br>  $-\frac{P_{cv}V_{cv}}{m_{cv}R}m_{out}$ <br>
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Equation (9) can be transformed into the following state space form.

$$
\dot{P}_{cv} = a(t)P_{cv} + u(t) \tag{10}
$$

$$
a(t) = -k \frac{\dot{V}_{cy}}{V_{cy}} - \frac{(k-1)h_{c}A_{heat}}{R} \frac{1}{m_{cy}} - k \frac{\dot{m}_{out}}{m_{cy}}
$$
(11)

$$
u(t)=(k-1)h_{c}A_{heal}\frac{T_{env}}{T_{cv}}+kR\frac{T_{in}\dot{m}_{in}}{V_{cv}}
$$
\n(12)

Table 1. Parameters for the air spring model.

$\mathbf{1}_{CV}$ $\mathbf{u}(t)$ $\mathbf{1}_{CV}$ $\mathbf{u}(t)$	$\cdots$
where t stands for time, and $a(t)$ and $u(t)$ represent functions of time defined by	
$a(t) = -k\frac{V_{cv}}{V_{cv}} - \frac{(k-1)h_cA_{heat}}{R} \frac{1}{m_{cv}} - k\frac{m_{out}}{m_{cv}}$	(11)
$u(t)=(k-1)h_{c}A_{heal}\frac{T_{env}}{T_{}}+kR\frac{T_{in}m_{in}}{V}$	(12)
This representation is used for the stability analysis of the dynamic equation. Although this equation is a first order	
Table 1. Parameters for the air spring model.	
R 287 J/kgK	
293.15 K $T_{\scriptscriptstyle env}$	
$0.012$ kg $m_{\sim}$	
$0.050$ m <sup>2</sup> (at maximum) $A_{heat}$	
k $1.17~1.3$ (for $0.05~5$ Hz)	

l,

s.<br>S.<br>Sign of the time constant,  $a(t)$ , because the dynamic equa-<br>sign of the time constant,  $a(t)$ , because the dynamic equa-474<br>
S.<br>
linear system, its stability cannot be determined by only the<br>
sign of the time constant,  $a(t)$ , because the dynamic equa-<br>
tion (10) is not a time invariant system. The stability for the<br>
dynamic system will be 474<br>linear system, its stability cannot be determined by c<br>sign of the time constant,  $a(t)$ , because the dynami<br>tion (10) is not a time invariant system. The stability<br>dynamic system will be analyzed in the next secti<br>The 474<br>
linear system, its s<br>
sign of the time co<br>
tion (10) is not a tii<br>
dynamic system v<br>
The pressure in<br>
by equation (10), is<br>
vehicle body<br>  $F_{as} = A_{eff}(P_{cv} - P_{atm})$ <br>
where  $F_{as}$  is the f linear system, its stability cannot be determined by only the sign of the time constant,  $a(t)$ , because the dynamic equation (10) is not a time invariant system. The stability for the dynamic system will be analyzed in the next section.

The pressure inside the air spring, which is determined by equation (10), is transformed into the force acting on the vehicle body

$$
F_{as}=A_{\text{eff}}(P_{\text{cv}}-P_{\text{atm}}) \tag{13}
$$

where  $F_{as}$  is the force,  $A_{\text{eff}}$  is the effective area of the air spring, through which pressure is transformed into the force, and  $P_{\text{atm}}$  is the pressure of the environment.

### 3. ANALYSIS AND VALIDATION OF AIR **SPRING MODEL**

The air spring model of equation (9) was developed to describe the important characteristics such as the hysteresis and nonlinear spring stiffness. The experimental results for the air spring validate the mathematical model, and the factors that affect the stiffness and the hysteresis of the air spring are analyzed in this section.

### 3.1. Experiments of the Air Spring

Figure 4 briefly shows the experimental setup where a sinusoidal displacement is vertically applied to the air spring by a linear actuator, which is positioned in the lower part of the air spring instead of the road excitation. The force, which is applied to the vehicle body by the air spring, is measured by the sensor which is positioned in the upper part of the air spring. The pressure of the air spring is measured by the sensor, which is positioned in the air passage between the air spring and the control valve. Experiments in which the air spring is excited at various frequencies are performed. Through the experiments, the signals such as vertical displacement which represents the vertical movement of the wheel with respect to the vehicle



Figure 4. Schematic representation of the experimental setup.



body, the pressure inside the air spring and the force are measured to validate the established model.

0.05 Hz,  $0.5$  Hz,  $5$  Hz (frequency)

#### 3.2. Analysis and Validation for Hysteresis

input (sinusoid)

The experimental data on the forces generated by the air spring are plotted versus the vertical displacement in Figure 5, which clearly shows the hysteresis. Because the force is



Figure 5. Experimental results of force versus vertical displacement for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 N lower and higher than the actual values for ease in distinction between the different data plots, respectively).



Figure 6. Simulation results for pressure response due to only variation of the volume for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 mbar lower and higher than the actual values, respectively).

the effective area times the pressure, the cause of the hysteresis can be found in the pressure dynamic equation  $(9)$ . The inspection of equation  $(9)$  reveals that the hysteresis can be caused by three terms.

The first term,  $-kP_{cv}V_{cv}/V_{cv}$ , occurs due to the volume<br>change in an air spring according to the vertical displace-–kPcvV ccord the process of the process of the process of the process of the set of or only the betwood by the three process of  $A_{head}$   $\frac{A_{head}}{v}$  or  $\frac{A_{head}}{v}$  in a of  $\varepsilon$  in the currence of the process of the currence of ment. The hysteresis on the pressure response can occur when the volume is a function of other variables as well as the vertical displacement. However, for the fixed air mass and the given temperature, the volume of the air spring is assumed to be a function of only the vertical displacement on the basis of test results on some air springs. Hence, the pressure responses due to only the variation of the volume, which is a function of only the vertical displacement, do not show the hysteresis as in Figure 6.

The second term, 
$$
\frac{k-1}{V_{cv}}h_cA_{hea}\left(T_{env}-\frac{V_{cv}}{Rm_{cv}}P_{cv}\right)
$$
, which occurs

due to the heat transfer between the air spring and the environment, is a function of the pressure as well as the vertical displacement. The hysteresis due to this second term can be analyzed through the pressure dynamic equation, which consists of the first term and the second term as follows:

 $\sqrt{2}$ 

$$
P_{cv} = a(t)P_{cv} + u(t)
$$
  
\n
$$
a(t) = -k \frac{\dot{V}_{cv}}{V_{cv}} \frac{(k-1)h_c A_{heat}}{Rm_{cv}}
$$
  
\n
$$
u(t) = (k-1)h_c A_{heat} \frac{T_{env}}{V_{cv}}
$$
\n(14)

hysteresis can be found in the pressure dynamic equation<br>(a). The inspective of equation of equation (b) reveals that the hysteresis<br>can be caused by three terms.<br>The first term,  $-AP_1, P_2, P_3, P_4, P_5, P_6, P_7, P_7, P_8,$ , o (6). The inspection of equation (9) reveals that the hysteresis<br>can be caused by three terms.  $\epsilon_{\text{H}}m$ ,  $\epsilon_{\text{H}}F_{\text{eff}}F_{\text{eff}}F_{\text{eff}}$ , occurs due to the volume<br>change in an in expire according to the vertical displac can be caused by three lemns.<br>
The first term,  $\pi P_x / F_x / F_x$ ,  $\pi$ , occurs due to the volume<br>
Then first term,  $-\pi P_x / F_x / F_x$ , occurs due to the volume<br>
ment. The hysteresis on the presenct response can occur<br>
when the volum The first term,  $-kP_{cr}V_{c\ell}/V_{cr}$ ,<br>change in an air spring accordinent. The hysteresis on the polytometric ment.<br>then the volume is a function the vertical displacement. How<br>and the given temperature, the assumed to be a ange in an air spring according to the vertical displanearit. The hysters is on the pressure response can occur with the hysters is the pressure response can occur with the hyster is a function of other variables as well when the volume is a function of other variables as well as<br>shown the verical displacement. However, for the fixed urims<br>and the given remperature, the volume of the air messions and the basis of test results on some of t the vertical displacement. However, for the fixed air mass<br>assumed the given temperature, the volume of the air spring is a<br>susumed to be a function of only the vertical displacement<br>on the basis of test results on some a and the given inepreature, the volume of the air spring is<br>and the given temperature, the volume of the air spring is<br>ansumed to be a function of only the vertical displacement, do<br>norm be basis of test results on some ai assumed to be a finction of only the vertical displacement<br>on the basis of test results on one air springs. Hence, the<br>pressure responses due to only the vertical displacement<br>on the basis of test results on some air spri on the basis of test results on some air springs. Hence, the<br>pressure responses the to only the variation of the volume,<br>which is a function of only the vertical displacement, do<br>not show the hysteresis as in Figure 6.<br>Th pressure responses due to only the variation of the volume,<br>which is a function of only the vertical displacement, do<br>not show the hysteresis as in Figure 6.<br>The second term,  $\frac{k-1}{k-2}h$ ,  $A_{\text{ave}}\left(T_{\text{ave}}-\frac{V_{\text{ave}}}{N_{\$ which is a function of only the vertical displacement, do<br>not show the hysteresis as in Figure 6.<br>The second term,  $\frac{k-1}{k}$ ,  $\frac{1}{2}$ ,  $\frac{1}{$ not show the hysteresis as in Figure 6.<br>
The second term,  $\frac{k-1}{k}h_c A_{sea}(\overline{I_{sun}} - \frac{I_c}{Nm_{on}}, \sigma)$ , which occurs<br>
due to the heat transfer between the air spring and the<br>
environment, is a function of the pressure as well The second term,  $\frac{k-1}{V_{cr}}h_cA_{\text{av}}\left(T_{\text{em}}-\frac{V_{\text{ex}}}{Rm_e}$ <br>due to the heat transfer between the<br>environment, is a function of the pressure inversion the<br>environment, is a function of the pressure<br>experient displacemen  $k-1$ <br>  $\overline{V}_{\infty}$ <br>  $\overline{V}_{\infty}$ <br>  $k$ <br>  $\overline{R}$ <br>  $(1_{em} - \frac{P_{cv}}{Rm_{cv}})$ <br>veen the air s<br>the pressure due to the pressure dyn<br>md the second<br>md the second<br>in the second<br>and the second<br>in the second<br>in generates the signal. The<br>quent signal. The<br>quent signal in generates th  $\overline{km}$ ,  $\frac{V_c}{V_c}$   $h_c A_{head}$  ( $I_{em}$ <br>
ansfer between<br>
inster between<br>
inster between<br>
inster between<br>
insterm and the<br>
first term and the<br>
first term and the<br>  $\frac{T_{em}}{V_{cv}}$ <br>  $\frac{F_{em}}{H}$ <br>  $\frac{F_{em}}{V_{cv}}$ <br>  $\frac{F_{em}}{H}$ <br>  $\frac{F_{em}}{V_{$  $V_{B\pi\pi}^{(m)}$ <br>
to the heat transfer between the air spring and the<br>
vironment, is a function of the pressure as well as the<br>
divisor of the pressure as well as the condition<br>
of the pressure dynamic equation,<br>  $-a(t)P_{\pi\$  $\text{L}$  is first contract  $\frac{-1}{R}$ ,  $\text{L}$  is  $\text{L}$  the a second order and the second order and the second order and the second of the second of the second the second the second the second second the second seco environment, is a function of the pressure as well as the<br>vertical displacement. The hysteresis due to this second term<br>team be analyzed through the pressure dynamic equation,<br>which consists of the first term and the seco vertical displacement. The hysteresis due to this second term<br>which consists of the first term and the second term<br>which consists of the first term and the second term<br>which consists of the first term and the second term can be analyzed through the pressure dynamic equation,<br>which consists of the first term and the second term as follows:<br> $\dot{P}_{\alpha} = a(t)P_{\alpha} + u(t)$ <br> $a(t) = -k\frac{V_{\alpha\alpha}}{V_{\alpha}} - Rm_{\alpha\alpha}$  (14)<br> $u(t) = (k-1)h_{\alpha}A_{\alpha\alpha\alpha} - Tm_{\alpha\alpha}$  (14 which consists of the first term and the second term as follows:<br>  $\dot{P}_{\alpha} = a(t)P_{\alpha} + u(t)$ <br>  $a(t) = -k\frac{\dot{V}_{\alpha}}{V_{\alpha}}$   $Rm_{\alpha}$  (14)<br>  $u(t) = (k-1)h_{\alpha}A_{\alpha\alpha}\frac{T_{\alpha\alpha}}{V_{\alpha}}$  (14)<br>  $u(t) = (k-1)h_{\alpha}A_{\alpha\alpha}\frac{T_{\alpha\alpha}}{V_{\alpha}}$ <br>
This eq  $\vec{P}_{\alpha} = a(t)P_{\alpha} + u(t)$ <br>  $a(t) = -k \frac{\vec{V}_{c\alpha}}{V_{c\tau}} \frac{(k-1)\hbar_{\alpha}A_{box}}{(Rm_{\alpha})}$  (14)<br>  $u(t) = (k-1)\hbar_{\tau}A_{box}\frac{T_{c\alpha}}{V_{\alpha}}$ <br>
This equation has the form of a first-order low-pass filter in<br>
which  $a(t)$  is the cut-off frequency,  $ac(t) = -k \frac{\dot{V}_{cy}}{V_{cy}}$ <br>  $a(t) = -k \frac{\dot{V}_{cy}}{V_{cy}}$ <br>  $u(t) = (k-1)$ <br>
his equation has<br>
hich  $a(t)$  is the fill<br>
the d P<sub>cv</sub> is the fill<br>
and P<sub>cv</sub> is the fill<br>
the d P<sub>cv</sub> is the fill<br>
coutput signal,<br>
(1) is time-vary<br>
is time-va To the set of  $\frac{d}{dt}$  $A_{heat}$ <br>  $R_{me}$ <br>  $A_{heat}$ <br>  $V_{ev}$ <br>
the form of frequency<br>
the form of frequency<br>
the form of the state of frequency<br>
ignificantly<br>
the state output and the state output with<br>
the frequency<br>
the frequency<br>
which decreasure, a  $\overline{V}$   $\overline{V}$   $+$   $u(t) = (k \text{ equation})$ <br>  $u(t) = (k \text{ equation})$ <br>  $\frac{1}{a(t)}$  is the physical is the physical intervalsed in the physical interval ter in gnal, filter<br>text to grad in the same and the same that<br>the same than the same of the same of the same of the same of the same<br>dative of the same of the same of the same of the same of 15) -1) has example and the set of  $\frac{dV}{dt}$  and  $\frac{dV}{dt}$  $\frac{T_{en}}{V_{en}}$  form<br>off fr d out of 1<br>off fr d out of 1<br>in respuasification<br>in the dividend frequence,<br>in the best on side of the shonsidion<br> $(2\pi$  $\overline{V_{cy}}$  orm of t in responsive the control of the second put of the short  $\overline{V_{cy}}$  or  $\overline{V_{cy}}$  or  $\overline{V_{cg}}$  or  $\overline{V_{cg}}$  or  $(2\pi)$  $u(t)$  ( $x = 1\pi_c$  Aheat<br>equation has the form of  $\Gamma_{c}$  is the cut-of  $P_{c}$  is the filtered<br>is the phase shift input signal, which<br>utput signal, which<br>utput signal, which<br>utput signal with a<br>s time-varying, equivalent in Figure 1 in the state of the control of the control of the control of the short of  $\frac{1}{\sqrt{V_{\text{eff}}}}$ This equation has the form of a first-order low-pass filter in which  $a(t)$  is the cut-off frequency,  $u(t)$  is the input signal,<br>and  $P_{\varphi}$  is the filtered output signal. The low-pass filter<br>causes the phase shift of the output signal, which respect to<br>the input signal, which in tur which  $a(t)$  is the cut-off frequency,  $u(t)$  is the input signal, which a  $\alpha(t)$  is the cut-off frequency,  $\alpha(t)$  is the filtered output signal, The low-pass filter<br>causes the phase shift of the output signal with respect to<br>the input signal, which in turn generates the hysteresis of<br>th and  $P_{\infty}$  is the filtered output signal. The low-pass filter and  $T_a$  is the intertor output signal. The low-pass finct<br>causes the phase shift of the output signal with respect to<br>the input signal, which in turn generates the hysteresis of<br>the output signal with respect to the inpu causes the phase shift of the output signal with respect to the input signal, which in turn generates the hysteresis of<br>the output signal with respect to the input signal. Because<br> $a(t)$  is time-varying, equation (14) represents the charac-<br>teristics that are significantly differen the input signal, which in turn generates the hysteresis of the output signal with respect to the input signal. Because<br>  $a(t)$  is time-varying, equation (14) represents the charac-<br>
teristics that are significantly different from the low-pass<br>
filter with a time-invariant cut-off the output signal with respect to the input signal. Because  $t = \frac{dP_{\text{ex}}}{dt}$  is time-varying, equation (14) represents the characteristics that are significantly different from the low-pass filter with a time-invariant cut-off frequency. However, the first order dynamics of equa  $a(t)$  is time-varying, equation (14) represents the characand the transmistrian of the plane of the charac-<br>terristics that are significantly different from the low-pass<br>filter with a time-invariant cut-off frequency. However, the<br>first order dynamics of equation (14) generate t teristics that are significantly different from the low-pass filter with a time-invariant cut-off frequency. However, the<br>first order dynamics of equation (14) generate the phase<br>shift between the input and the output, which causes the<br>hysteresis in the pressure output with respect filter with a time-invariant cut-off frequency. However, the first order dynamics of equation (14) generate the phase<br>shift between the input and the output, which causes the<br>hysteresis in the pressure output. More specifically, the phase<br>shift of the pressure output with respect t first order dynamics of equation (14) generate the phase shift between the input and the output, which causes the hysteresis in the pressure output. More specifically, the phase shift of the pressure output with respect to the displacement input decreases as the frequency of th shift between the input and the output, which causes the hysteresis in the pressure output. More specifically, the phase<br>shift of the pressure output. More specifically, the phase<br>shift of the pressure output with respect to the displacement<br>input decreases as the frequency of hysteresis in the pressure output. More specifically, the phase shift of the pressure output with respect to the displacement<br>input decreases as the frequency of the input increases from<br>0.05 Hz to 5 Hz, which decreases the magnitude of the<br>hysteresis in the pressure, as shown in Figu shift of the pressure output with respect to the displacement input decreases as the frequency of the input increases from<br>0.05 Hz to 5 Hz, which decreases the magnitude of the<br>hysteresis in the pressure, as shown in Figure 7. (Pressures<br>of all the figures in this paper represent th input decreases as the frequency of the input increases from 0.05 Hz to 5 Hz, which decreases the magnitude of the hysteresis in the pressure, as shown in Figure 7. (Pressures of all the figures in this paper represent the relative pressure.) In addition, the variation of the magni 0.05 Hz to 5 Hz, which decreases the magnitude of the hysteresis in the pressure, as shown in Figure 7. (Pressures<br>of all the figures in this paper represent the relative<br>pressure.) In addition, the variation of the magnitude of the<br>hysteresis can also be shown in the follow hysteresis in the pressure, as shown in Figure 7. (Pressures of all the figures in this paper represent the relative<br>pressure.) In addition, the variation of the magnitude of the<br>hysteresis can also be shown in the following equation,<br>which is derived by considering only the first of all the figures in this paper represent the relative pressure.) In addition, the variation of the magnitude of the<br>hysteresis can also be shown in the following equation,<br>which is derived by considering only the first term and the<br>second term of equation (9) when the sinuso pressure.) In addition, the variation of the magnitude of the hysteresis can also be shown in the following equation, which is derived by considering only the first term and the second term of equation (9) when the sinusoidal vertical displacement,  $z=z_0 \sin(2\pi f t)$ , is applied to the air spring.

$$
\begin{aligned}\n\text{hysteresis can also be shown in the following equation,} \\
\text{which is derived by considering only the first term and the second term of equation (9) when the sinusoidal vertical displacement, } z = z_0 \sin(2\pi f t), \text{ is applied to the air spring.} \\
\frac{dP_{cv}}{dz} = -\frac{kP_{cv}}{V_{cv}} \frac{dV_{cv}}{dz} \\
&+ \frac{(k-1)h_c A_{heat}}{2\pi f z_0 \cos(2\pi f t) V_{cv}} \left( T_{env} - \frac{V_{cv}}{R m_{cv}} P_{cv} \right) \tag{15}\n\end{aligned}
$$



Figure 7. Simulation results of pressure versus vertical displacement for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 mbar lower and higher than the actual values, respectively).

where  $z_0$  and f represent the magnitude and the frequency of the vertical displacement sinusoid, respectively. This equation indicates that the increase of the frequency reduces the effect of the hysteresis due to the second term.

The simulated pressure responses shown in Figure 7 are compared with the experimental results shown in Figure 8, which are obtained without the air mass flowing into or out of the air spring. The similarity between the simulation and experimental results validates the air spring model developed and its analysis.

 $\overline{V_{cv}}\backslash^{I}$  in $^{III}$  in $^{-}$ 



displacement for sinusoidal motion excitation at 0.05 Hz,<br>0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented<br>200 mbar lower and higher than the actual values, respec-<br>tively).  $0.5$  Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 mbar lower and higher than the actual values, respectively). 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz and 1.05 Hz Data are represented 200 mbar lower and higher than the actual values, respectively). tively).

This term is also a function of the pressure as well as the vertical displacement and is expressed including the first term by the first order filter form as follows:

$$
\dot{P}_{cv} = \left(-k\frac{\dot{V}_{cv}}{V_{cv}} - k\frac{\dot{m}_{out}}{m_{cv}}\right)P_{cv} + kR\frac{T_{in}\dot{m}_{in}}{V_{cv}}
$$
\nUnlike the second term, the third term does not show the

Unlike the second term, the third term does not show the typical form of a hysteresis but shows the pressure responses presented in Figure 9. This figure represents the pressure responses due to the first term and the third term when the upper part of the air spring is lowered by 10 mm, which means that the vehicle height is lowered when the lower part of the air spring is excited at 0.5 Hz. In Figure 9, the thin lines represent the pressure responses before and after the variation of the air mass inside the air spring, and the thick line stands for the pressure response while the air mass varies. This figure indicates that the variation of the Uty<br>spiww<br>Piww<br>Fith<br>fith



Figure 9. Simulation results for pressure responses due to the variations of the volume and the air mass inside the air spring.



Figure 10. Force responses for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 N lower and higher than the actual values, respectively). Simulation results do not include the hysteresis of the effective area while experimental results include it.



Figure 11. Force response for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz (the 0.05 Hz and 5 Hz data are represented 200 N lower and higher than the actual values, respectively). Simulation results include all effects except the air mass variation.

air mass inside the air spring has an effect on the pressure response, but no hysteresis occurs due to the variation of the air mass.

Because the force is defined by the effective area times the pressure such as expressed in equation (13), the effective area in addition to the pressure has an effect on the hysteresis of the force response. The effective area varies with the vertical displacement because it varies with the vertical shape of the contour of the piston in the lower part of the air spring, which changes the ride comfort of the vehicle. In addition, the effective area varies with the pressure at the same displacement, which yields the hysteresis of the effective area. In Figures 10 and 11, the effect of this hysteresis is represented. Simulation results in Figure 10 do not include the hysteresis of the effective area, while those in Figure 11 include it. These figures show that the hysteresis of the effective area enlarges the hysteresis of the force response. Finally, the comparison between the simulated and the experimental results in Figure 11 confirms the validity of the air spring model.

#### 3.3. Analysis and Validation of Stiffness

The stiffness of the air spring can be obtained by differentiating equation (13) with respect to the vertical displacement, as follows:

$$
k_{as} = \frac{dF_{as}}{dz} = A_{\text{eff}} \frac{dP_{\text{cv}}}{dz} + (P_{\text{cv}} - P_{\text{atm}}) \frac{dA_{\text{eff}}}{dz}
$$
(17)

where  $k_{as}$  represents the stiffness of the air spring, and z the vertical displacement. This equation indicates that the stiffness of the air spring varies with the derivatives of the pressure and the effective area with respect to the vertical displacement. whe<br>wertiness<br>pres<br>disp<br>In<br>of th  $\frac{dE}{dz}$ <br>re  $k_a$ <br>ical of<br>sure<br>sure<br>lace:<br>lace: e *k*<br>cal<br>of uro<br>ace<br>eq<br>e v dz<br>re  $k_{as}$  repr<br>ical displa<br>of the  $\epsilon$ <br>sure and<br>lacement.<br>equation<br>ne volume  $\frac{d^2y}{dz^2}$  esen cem<br>ir sphe e<br>(9),<br>var ser<br>r s<br>ne<br>9)<br>va the stiff<br>the stiff t. This example varies<br>ective are first terminon, is a  $\frac{dy}{dz}$  as of uatio with  $x$ , where  $\frac{dy}{dz}$ s dati<br>wi wi<br>wi w<br>, w<br>e c

In equation  $(9)$ , the first term, which represents the effect of the volume variation, is one of the factors that change

the stiffness of the air spring expressed in equation (17). The variation of the pressure due to the first term is rewritten in the following equation. DEVELOPMENT AND ANAL<br>the stiffness of the air spring expressed in equation (17).<br>The variation of the pressure due to the first term is<br>rewritten in the following equation.<br> $\frac{dP_{ex}}{dz} = -kP_{cy}\frac{1}{V_{cy}}\frac{dV_{cy}}{dz}$  (18)<br> $= k$ 

$$
\frac{dP_{cv}}{dz} = -kP_{cv}\frac{1}{V_{cv}}\frac{dV_{cv}}{dz}
$$
\n(18)

$$
=kP_{cv}\frac{A_{cs}}{V_{cv0}+\int_{curr}^{r_{\text{max}}}A_{cs}dz}
$$
 (19)

DEVELOPMENT AND ANAL<br>the stiffness of the air spring expressed in equation (17).<br>The variation of the pressure due to the first term is<br>rewritten in the following equation.<br> $\frac{dP_{cy}}{dz} = -kP_{cy}\frac{1}{V_{cy}}\frac{dV_{cy}}{dz}$  (18)<br> $= k$ **Space 1.1** DEVELOPMENT AND ANAL<br>the stiffness of the air spring expressed in equation (17).<br>The variation of the pressure due to the first term is<br>rewritten in the following equation.<br> $\frac{dP_{cy}}{dz} = -kP_{cy}\frac{1}{V_{cy}}\frac{dV_{cy}}{$ DEVELOPMENT AND ANAL<br>
the stiffness of the air spring expressed in equation (17).<br>
The variation of the pressure due to the first term is<br>
rewritten in the following equation.<br>  $\frac{dP_{cc}}{dz} = -kP_{\text{ev}}\frac{1}{V_{\text{ev}}} \frac{dV_{\text{ev$  $\frac{d}{dz}$ <br>where  $\frac{d}{dz}$ <br>where  $\frac{d}{dz}$ <br>spring the  $\frac{d}{dz}$ <br>of thought  $\frac{d}{dz}$ <br>the  $\frac{d}{dz}$ <br>wolu  $\frac{d}{dz}$ <br>wolu  $\frac{d}{dz}$ <br>if  $\frac{d}{dz}$ <br>  $\overline{V_{cv}}$   $\overline{dz}$ <br> $\frac{A}{V_{cv0} + \int_{cu}^{u} \frac{A}{dz}}$ <br> $\frac{A}{V_{cv0} + \int_{cu}^{u} \frac{A}{L_{cu}}$ <br>tands f bed to repres<br>tands f bed to repress and<br>the personal displace is value<br>of the vertice example of the contribution in the section<br>of  $dz$ <br>  $= k P_{cy} - \frac{1}{l}$ <br>
where  $V_{cvo}$  is<br>
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However, a<br>
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respect to the stiffine<br>
erivative of the s wholm<br>on principal principal for the server of the ser  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  is a subset of the set of We due to the control of th **Account** the state of the  $V_{\text{cv0}}$  and the attention at the analysis of the set stands for  $z_{\text{max}}$ <br>stands for ched to so  $z_{\text{max}}$  and  $z_{\text{c}}$ <br> $z_{\text{max}}$  and  $z_{\text{c}}$ . It is well<br>spring red close inspective in the vertical<br>server of the increases express<br>which increase of the veven thou  $\int_{\text{curr$ the f me a<br>the f me a<br>ts the metric of the metric of the displication is a set of the displication<br>is a set of the metric of the filt<br>of the filt of the filt<br>of the filt of the filt of the filt<br>which the filt of the set w where  $V_{\text{cv0}}$  stands for the fixed volume like the additional volume attached to some air spring in order to improve the comfort,  $A_{cs}$  represents the cross-sectional area of the air spring, and  $z_{\text{max}}$  and  $z_{\text{curr}}$  are the maximum displacement and the current displacement of the bottom of the air spring, respectively. It is well known that an increase of the volume of the air spring reduces the stiffness of the air spring. However, a close inspection of equation (18) reveals that the absolute value of the derivative of the pressure with respect to the vertical displacement, which represents a part of the stiffness expressed in equation (17), decreases as the entire volume of the air spring increases but increases as the derivative of the volume with respect to the vertical displacement increases. Hence, the derivative of the pressure increases, which increases the stiffness, if the increment of the derivative of the volume is larger than the increment of the volume even though the entire volume increases. The term,  $V_{\text{cv0}}$ +  $\vert$ <sup>max</sup>  $A_{\text{cs}}$  dz, in the denominator of equation (19)  $\int_{curr}^{\text{max}}$ <br>che represents the entire volume, and the cross-sectional area,  $A_{cs}$ , in the numerator stands for the derivative of the entire volume with respect to the vertical displacement. When the cross-sectional area increases, the increment of the derivative of the volume is larger than the increment of the entire volume. For example, when the cross-sectional area increases by 50 percent, the entire volume cannot increase by up to 50 percent because the fixed volume does not increase. Hence, the increase of the volume due to the crosssectional area increases the stiffness of the air spring, while the increase of the volume without the variation of the cross-sectional area decreases the stiffness.

The heat transfer, which is included in the second term of equation (9), also has an effect on the variation of the stiffness. Equation (15) clearly shows the variation of the stiffness due to the heat transfer. In equation (15), the derivative of the pressure with respect to the displacement due to the heat transfer is added to that due to the variation of the volume, which changes the stiffness due to the variation of the volume. In addition, because the term due to the heat transfer is divided by the frequency of the displacement in equation (15), the stiffness due to the heat transfer is reduced as the frequency increases. The stiffness variation with frequency is shown in Figure 12, which represents the pressure responses due to only the heat transfer. The entire pressure response in equation (15) is determined by the sum of Figure 6 and Figure 12. Hence, the heat transfer at the low frequency has a significant effect on the entire stiffness, unlike that at the high frequency. More Fevor  $J_{curr}$ <br>ents the e<br>ents the e<br>the nume<br>e with res<br>sectional  $\varepsilon$ <br>f the volum<br>e. For examperent, t<br>cent because of the incomendation of the intervalsed in the lead t<br>bead the heat t<br>volume, v ss. Equative of the hea ntire<br>tractor pect t<br>me is imple, en unse rease<br>crease de area (<br>terminal per press the )<br>is present also on (1<br>e press vince area (<br>vince )<br>is press to read as<br>reque ssure of as reque<br>sure ressum of he loves, un<br>he loves,

specifically, the heat transfer at the low frequency significantly reduces the stiffness due to the variation of the volume, while at the high frequency it slightly reduces the stiffness. The negative pressure in Figure 12 occurs for the following reason. When the air spring is compressed, the pressure increases due to the compressed volume. The resulting increment of the pressure increases the temperature of the air spring in equation (6) and decreases the rate of change of pressure due to the second term in equation (15). Consequently, when the temperature of the air spring becomes larger than that of the environment, the pressure due to heat transfer decreases, which can yield a negative pressure. However, the entire pressure increases because the increase of the pressure due to the volume variation is larger than the decrease of the pressure due to the heat transfer even when the temperature of the air spring is larger than the environment.

The pressure response due to the variation of the air mass, which is included in the third term of equation (9), is also added to that due to the other two terms, which in turn changes the stiffness which is determined by the other terms. The equation (9) shows that the mass flow rate flowing into the air spring,  $m_{in}$ , increases the stiffness, while the mass flow rate flowing out of the air spring,  $m_{out}$ , decreases it. Figure 9 represents the pressure response due to variations of the volume and the air mass when the air mass is flowing out of the air spring.

In addition to the pressure variation, the variation of the effective area has an effect on the stiffness. As mentioned earlier, the contour of the piston of the air spring is manufactured in order to obtain the optimum ride comfort, which yields the variation of the effective area. Hence, the large variation of the effective area can have a significant effect on the variation of the stiffness. This study on the air spring which was employed in this experiment shows that most of values of the stiffness due to the second term in equation (17), which represents the effect of the effective area variation, vary within 40% of the entire stiffness.  $m_{out}$ <br>e du ai f th one anu for hannu for he ai hannu for he ai hannu for he ai hannu is that is seen to see the  $\sim$  0.0



Figure 12. Simulation results for pressure response due to only heat transfer for sinusoidal motion excitation at 0.05 Hz, 0.5 Hz and 5 Hz.

The slopes of the force curves in Figure 11 represent the entire stiffness, which includes the rate of change of the effective area as well as the pressure. Figure 11 indicates that the stiffness varies with the displacement and frequency. In addition, it is observed that the stiffness of the simulation curves is similar to that of experimental curves in the full range of the displacement.

#### 3.4. Stability Analysis of the Air Spring System

The stability of the air spring is very important for the vehicle stability. However, the stability of the pressure dynamics of the air spring is not simply determined like a time-invariant system because  $a(t)$  in the air spring model equation (10) is time-varying.

For the stability analysis of the dynamic equation (10), linear time-invariant dynamic systems are introduced as follows:

$$
\dot{P}_{\text{max}} = a_{\text{max}} P_{\text{max}} + u(t)
$$
\n
$$
\dot{P}_{\text{min}} = a_{\text{min}} P_{\text{min}} + u(t)
$$
\n
$$
(20)
$$

where  $P_{\text{max}}$  and  $P_{\text{min}}$  are the pressure variables, and  $a_{\text{max}}$  and  $a_{\min}$  are the constant maximum and minimum values of  $a(t)$ . Because  $a(t)$  is bounded by  $a_{\text{max}}$  and  $a_{\text{min}}$ ,  $P_{cv}$  is also bounded by  $P_{\text{max}}$  and  $P_{\text{min}}$  as follows:

$$
P_{\min} \leq P_{cv} \leq P_{\max} \tag{21}
$$

Because equation (20) is a linear time-invariant system,  $P_{\text{max}}$  and  $P_{\text{min}}$  are bounded if  $a_{\text{max}}$  and  $a_{\text{min}}$  are negative values and the input,  $u(t)$ , is bounded (Khalil, 1996).  $u(t)$  in the equation (12) is bounded when the air mass flow rate flowing into the air spring,  $m_{im}$ , is bounded. Consequently, the pressure of the air spring,  $P_{\text{ex}}$  is bounded, when  $m_{in}$  is<br>bounded and the following condition is satisfied bounded and the following condition is satisfied.

$$
a(t) = -\left(k\frac{V_{cv}}{V_{cv}} + \frac{(k-1)h_cA_{heal}}{Rm_{cv}} + k\frac{m_{out}}{m_{cv}}\right) < 0\tag{22}
$$

This stability criterion is a sufficient condition because it



Figure 13. Values of the variable,  $a(t)$ , of equation (22) for sinusoidal motion excitation at 0.5 Hz when there is no air mass flowing into or out of the air spring.

has not been proven that the air spring model is unstable if the derived stability condition is not satisfied.

Equation (22) indicates that the increase of the fixed volume of the air spring, the heat transfer coefficient and area, and the air mass flow rate flowing out of the air spring are helpful in satisfying the stability condition (22), while the increase of the negative rate of change of the volume and the air mass prevent the stability condition (22) from being satisfied. Figure 13 shows that  $a(t)$  has the negative values for sinusoidal motion excitation at 0.5 Hz. For other sinusoidal motion excitation of 0.05 Hz and 5 Hz,  $a(t)$  also has the negative value.

### 4. CONCLUSION

This research developed a general model of an air spring based on thermodynamics. This model was derived from the energy conservation law. The thermodynamic parameters inside the air spring are assumed to be uniform, which means that the parameters do not vary with the position inside the air spring. The air inside an air spring is also assumed to have an ideal gas property, and the kinetic and potential energies of the air are neglected. However, the resulting model can represent all processes ranging from isothermal to adiabatic conditions because the assumption that the process is adiabatic or isothermal has not been employed. In addition, the model can be used to simulate the system with the pneumatic circuit able to adjust the vehicle height because it includes the air mass flowing into and out of the air spring.

The analysis of the established model revealed that the volume variation, the heat transfer, the variation of the air mass and the effective area have an effect on the stiffness and hysteresis. The heat transfer yields the larger hysteresis under the input with the low frequency than that with the high frequency, and the effective area enlarges the hysteresis. In addition, the heat transfer significantly reduces the stiffness at the low frequency, and the air mass flow rate flowing into the air spring increases the stiffness, while the air mass flow rate flowing out of the air spring decreases it. In particular, the increase of the volume due to the crosssectional area increases the stiffness, while the increase of the volume due to the other reason decreases it. Additionally, most of the stiffness due to the effective area varied within about 40% of the entire stiffness for the air spring used in this study.

The stability condition for the air spring was also derived from the study regarding the established time-varying model. The inspection of the stability condition revealed that the increases of the fixed volume, the heat transfer coefficient and area as well as the air mass flow rate flowing out of the air spring have a positive effect on the stability, while the increases of the negative rate of change of the volume and the air mass have a negative effect on it. However, the analysis for the parameters has some limitation because the derived stability criterion is a sufficient condition.

Simulation results of the model were presented, and these were in substantial agreement with experimental measurements of force and pressure with respect to displacement excitations of 0.05 Hz, 0.5 Hz, and 5 Hz, which validates the modeling approach presented here for the air spring. The resulting validated model will be especially useful for the study of air spring systems including the pneumatic circuit and its control algorithm.

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