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VEHICLE MODELING WITH NONLINEAR TIRES FOR VEHICLE STABILITY ANALYSIS

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ABSTRACT−The dynamic stability of a vehicle depends on various maneuvering features, such as traction, braking, and cornering. This study presents nonlinear vehicle models for estimating the stability region and simulating the dynamic behavior of a vehicle. Two types of vehicle models were found by considering the degrees of freedom and linearity. A simple model with nonlinear tire dynamics is useful for determining the stability region, while a complex model (a multi-body dynamic model in MSC.ADAMS) is appropriate for carrying out accurate simulations. Actual data for a mid-sized passenger car were used, and the models were validated by comparison with test results.

KEY WORDS : Vehicle stability, Nonlinear tire dynamics, Vehicle modeling, Model validation, MSC.ADAMS

1. INTRODUCTION

The dynamic stability of a vehicle depends on various maneuvering features, such as traction, braking, and cornering (Allen *et al*., 1990, 1991; Allen and Rosenthal, 1993). Two types of vehicle motion are important factors for achieving dynamic stability: roll and yaw. Excessive roll during sharp cornering often causes a vehicle to overturn; the major problem of roll stability is the prevention of rollover. Yaw may occur in any driving situation. In traction and braking, the main yaw stability issue is the prevention of any type of yaw motion, whereas in cornering, the primary concern is controlling the magnitude of the yaw rate. The most popular method for analyzing yaw during cornering is in terms of steer tendencies, categorized as neutral-steer, under-steer, and over-steer. The steer tendency is determined by the direction of the yaw moment, which is induced by lateral forces generated at each tire. The idea originated in the linear analysis technique applied to cornering maneuvers with less than 0.3 g of lateral acceleration. This technique was borrowed from aerospace dynamics and uses a linearized vehicle model to obtain the sensitivity equations of the system states. The steer tendencies can be derived from these equations. The linear analysis technique is capable of providing useful information in the neighborhood of a specific operating point and hence is appropriate for studying normal driving situations but not critical ones. During critical cornering, the determination of whether a vehicle is in the stable region is the key factor for analyzing yaw stability. In this situation, the stability potential of a vehicle is also significant. The size and pattern of its stability region can be used as the criteria to analyze the stability of a vehicle during critical cornering (Ko *et al*., 2002). Dynamic stability is a very important factor in vehicle design and control because it has a strong influence on overall vehicle safety. The more dynamically stable a vehicle is, the smaller the likelihood that the driver will lose control of it. As a nonlinear dynamical system, every vehicle has a definite stability region.

Most of the existing studies (Gillespie, 1992; Milliken and Milliken, 1995) on vehicle yaw stability focus on the concept of steer tendency (neutral-steer, under-steer, and over-steer). This concept was derived from the force analysis of a linearized vehicle model with two degrees of freedom and is used as a key parameter in fine-tuning the handling characteristics of a vehicle. The linear analysis technique was originally introduced in connection with the stability derivative in aerospace dynamics. Riekert and Schunck (1940) proposed a bicycle model with three degrees of freedom, Radt *et al*. (1996) used the stability derivative to obtain a linearized vehicle model with three degrees of freedom, and Mashasi *et al*. (Mashadi and Crolla, 1996) developed a model with four degrees of freedom. Zeng *et al.*(2006) proposed a dynamic model for lateral and yaw motions with 10 degrees of freedom. They showed that linearized models are useful for studying vehicle parameters, such as the under-steer gradient, steering sensitivity, and roll and sideslip gradients. The results are still applicable in the linear range when the lateral acceleration is less than 0.3 g.

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This study presents nonlinear vehicle models for vehicle stability analysis, focusing on degrees of freedom and linearity. A simple model with nonlinear tire dynamics is useful for determining the stability region, while a complex model (a multi-body full-vehicle model with nonlinear tires in MSC.ADAMS (ADAMS, 2005; Song, 2003; Song and Cho, 2007) is appropriate for carrying out accurate dynamic simulations. These models are applicable in the nonlinear and linear ranges.

2. ISSUES IN VEHICLE MODELING

In this section, we discuss some issues pertaining to the modeling of a vehicle, specifically the degrees of freedom and the linearity of the model. The dynamic components of a vehicle can be treated as rigid bodies for the purpose of analyzing vehicle dynamics. An unconstrained rigid body has six degrees of freedom: three translational and three rotational. For a complete expression of the general vehicle dynamics, the model must include rigid bodies, kinematic constraints, and force elements. The model should incorporate sprung-mass, wheels, and suspension links as rigid bodies. Proper modeling of kinematic joints, rubber bushings, springs, and shock absorbers is also required.

Multi-body dynamic packages, such as MSC.ADAMS, are widely used to model the dynamic elements of a vehicle accurately. A multi-body dynamic model of a vehicle usually has more than a hundred degrees of freedom and allows the motions of the vehicle to be realistically animated. However, these multi-body models are incapable of using equations of motion to analyze the system directly because the equations are generated automatically by the package and are very complex. A simple vehicle model can be very useful for clearly understanding the relationship between different states and for easily tracking their changes. Moreover, it is readily available for use by other applications. The number of degrees of freedom in a simple vehicle model usually does not exceed 15 to 17, which is sufficient to describe the global motions of a vehicle. Most of the degrees of freedom pertain to the motions of the sprung-mass and the wheels, while those related to suspension kinematics are omitted. When the focus is on handling (the lateral dynamics), the simple model shown in Figure 1 is generally used. This model describes the global motion of a vehicle in a plane and provides major dynamic characteristics, such as lateral acceleration, yaw rate, and sideslip angle. To account for the vertical motion of the sprung-mass, the roll motion has to be added. This research considers only the global motion of a vehicle in a plane.

All multi-body dynamic models are nonlinear and complicated. The nonlinearity of a dynamic model originates primarily in the tires and suspension linkages. Tires induce nonlinearity related primarily to the plane motion of the vehicle, and suspension linkages induce nonlinearity related to the vertical and rotational motions of the sprung-mass. In this study, the multi-body model reflects the nonlinearities of the tires as well as those of the suspension linkages, whereas the simple model expresses only tire nonlinearity (because this research focuses only on the plane motion of a vehicle).

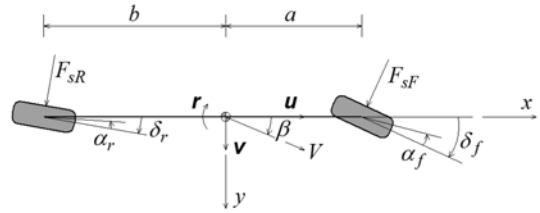

Figure 1. Simple vehicle model (3 degrees of freedom).

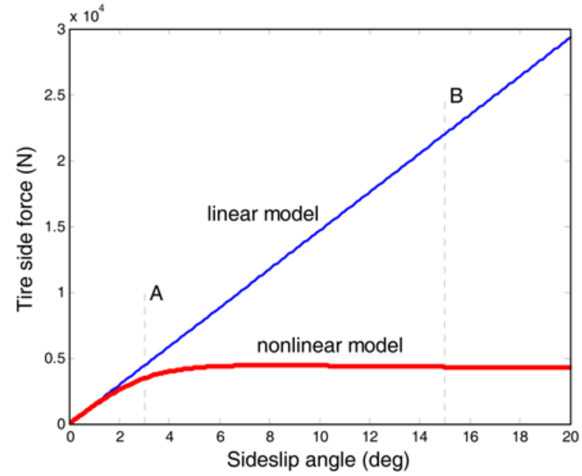

Figure 2. Linear versus nonlinear model.

The importance of tire nonlinearity in the simple model (shown in Figure 1) can be observed in Figure 2. Two dot-lines, denoted by A and B, are considered. While the tire side force of the linear model at A, with 3 degrees of side slip angle, has the nearly same level as that of the nonlinear model at A, that of the linear model at B, with 15 degrees of side slip angle, has a different level than that of the nonlinear model at B. If the tire side force at B is treated as linear, the results will be very inaccurate. It is well known that the simple model is linear only for cornering maneuvers with less than 0.3 g of lateral acceleration.

3. VEHICLE MODELING

Actual data for a mid-sized passenger car, given in Table 1, were used in this study.

3.1. Simple Model with Nonlinear Tires

The simple vehicle model with three degrees of freedom (longitudinal, lateral and yaw) depicted in Figure 1 was considered, and nonlinear tire dynamics were added to the model. The model had zero width and two wheels per axle. Therefore, the weight transfer among the wheels was never considered. In other words, this model did not include the

Table 1. Vehicle data.

Wheel base	2.700 m
Wheel tread (front/rear)	1.540 m/1.520 m
Distance from the front axle to the center of gravity	1.003 m
Distance from the rear axle to the center of gravity	1.697 m
Steering gear ratio	16:1
Total vehicle mass	1500 kg
Sprung mass	1325 kg
Yaw moment of inertia of vehicle	2975 kg m^2
Roll moment of inertia of sprung-mass	348 kg m^2
Spring constant of front suspension	18540 N/m
Spring constant of rear suspension	18050 N/m
Damping coefficient of front suspension	2894 N s/m
Damping coefficient of rear suspension	2286 N s/m

effects of roll or pitch. Neither forces nor tire slips were assumed in the longitudinal direction. The plane motion of the vehicle could then be expressed by the following equations:

$$m(\dot{u}-vr)=-2F_{sF}\sin\delta_f-2F_{sR}\sin\delta_r \tag{1}$$

$$m(\dot{v}+ur)=2F_{sF}\cos\delta_f-2F_{sR}\cos\delta_r \tag{2}$$

$$I_{zz}\dot{r}=2aF_{sF}\cos\delta_f-2bF_{sR}\cos\delta_r \tag{3}$$

where a, b, F_{sF}, F_{sR}, δ_f, and δ_r are the distance from the front axle to the center of gravity, the distance from the rear axle to the center of gravity, the side force of the front tire, the side force of the rear tire, the steer angle of the front wheel, and the steer angle of the rear wheel, respectively; m denotes the total vehicle mass, I_{zz} is the yaw moment of inertia, and u, v, and r are the longitudinal velocity, lateral velocity, and yaw rate of the vehicle, respectively.

The sideslip angle and yaw rate were selected for the state plane analysis. Because these two states can effectively be used to explain the plane dynamics of a vehicle, they are the ones most frequently used in state plane analyses. The sideslip angle β and the vehicle speed V were defined by

$$\beta=\tan^{-1}\left(\frac{v}{u}\right) \tag{4}$$

$$V=\sqrt{u^2+v^2} \tag{5}$$

Combined with Equations (4) and (5), Equations (1)~(3) can be written as

$$\dot{\beta}=-r+\frac{2}{mV}F_{sF}\cos(\delta_f-\beta)+\frac{2}{mV}F_{sR}\cos(\delta_r-\beta) \tag{6}$$

$$\dot{r}=\frac{2a}{I_{zz}}F_{sF}\cos\delta_f-\frac{2b}{I_{zz}}F_{sR}\cos\delta_r \tag{7}$$

To deal with the nonlinear characteristics of tires, the front and rear tire side forces were expressed in the 'Magic

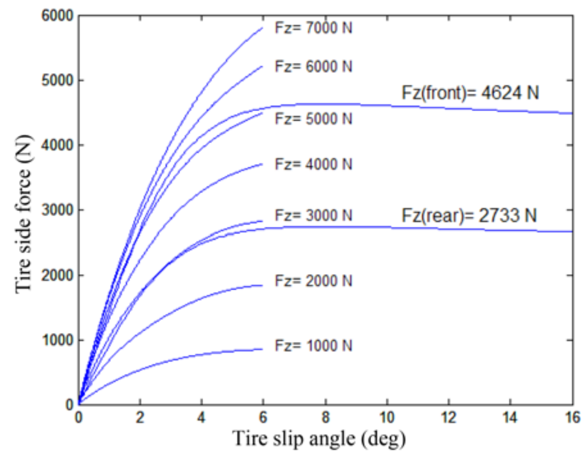

Figure 3. Tire modeling. Experimental results (up to 6°) and the Magic Formula model (μ = 1.0, α_{max}=8°, R_{sp}=0.9, S_{tF}=83074 N/rad, and S_{tR}=53680 N/rad).

Formula' form (Bakker *et al*., 1989):

$$F_s=D\sin[C\tan^{-1}\{B(1-E)\alpha+E\tan^{-1}(B\alpha)\}] \tag{8}$$

where B, C, D, and E are constant coefficients that vary with the vertical load. These coefficients were calculated from experimental data (Schuring *et al*., 1993, 1997):

$$D=-F_{s\_max}$$

$$C=2[1-(1/\pi)\sin^{-1}R_{sp}]$$

$$B=S_t/(C\cdot D)$$

$$E=\frac{(\pm)\tan(\pi/2C)-B\alpha_{max}}{\tan^{-1}(B\alpha_{max})-B\alpha_{max}}$$

Figure 3 shows a plot of tire side force versus tire slip angle for Fz, the tire vertical load, obtained using the "Magic Formula". Experimental data are presented, as well as the results of calculations performed with Equation (8). The nonlinear kinematics related to the tire slip angles were expressed by

$$\alpha_f=\tan^{-1}\left(\frac{v+ar}{u}\right)-\delta_f \tag{9}$$

$$\alpha_r=\tan^{-1}\left(\frac{v-br}{u}\right)-\delta_r \tag{10}$$

3.2. Multi-body Full-vehicle Model for Dynamic Simulations

A multi-body dynamic model was constructed in MSC.ADAMS (ADAMS, 2005; Song, 2003; Song and Cho, 2007), permitting exact simulation of a vehicle performing various maneuvers. The model vehicle had a double-wishbone suspension for the front axle, a multi-link suspension for the rear axle, a power-assisted steering subsystem, and a powertrain subsystem, which were used to represent detailed characteristics of vehicle maneuvers. The multi-body model consisted of 51 parts and 58 kinematic constraints and had 119 degrees of freedom. The tire model expressed tire behavior using experimental data. This model was compared statically and dynamically with the results of the kinematic and compliance (K&C) and J-turn tests, respectively. In all cases, there was a good correlation with the results of tests using a suspension parameter measurement device (SPMD).

4. ESTIMATION OF THE STABILITY REGION

A nonlinear dynamic system of the type described by Equations (6) and (7) can usually be represented by an autonomous set of nonlinear differential equations having the form

$$\dot{x}=f(x) \tag{11}$$

where the state x is $\{\beta, r\}^T$. A state $\hat{x}$ is an equilibrium point (or equilibrium state) of the system if $f(\hat{x})=0$.

The stabilities of the equilibrium points of a nonlinear system are defined as follows. (Figure 5 depicts these notions graphically.)

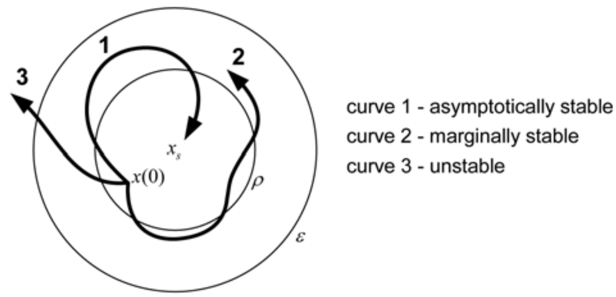

Figure 4. Concepts of stability.

To prove that an equilibrium point is stable, it is necessary that for any given ε, a ρ must be found such that, if the perturbation is initially inside the ρ neighborhood of the point, it will never leave the ε neighborhood of the point.

A typical state space obtained from the simple model with nonlinear tire dynamics. Here, sideslip angle and yaw rate are the states, and the stability region is estimated using the method of Ko and Lee (2002).

In Figure 5, the stability regions according to changes in vehicle speed and road conditions are outlined by two

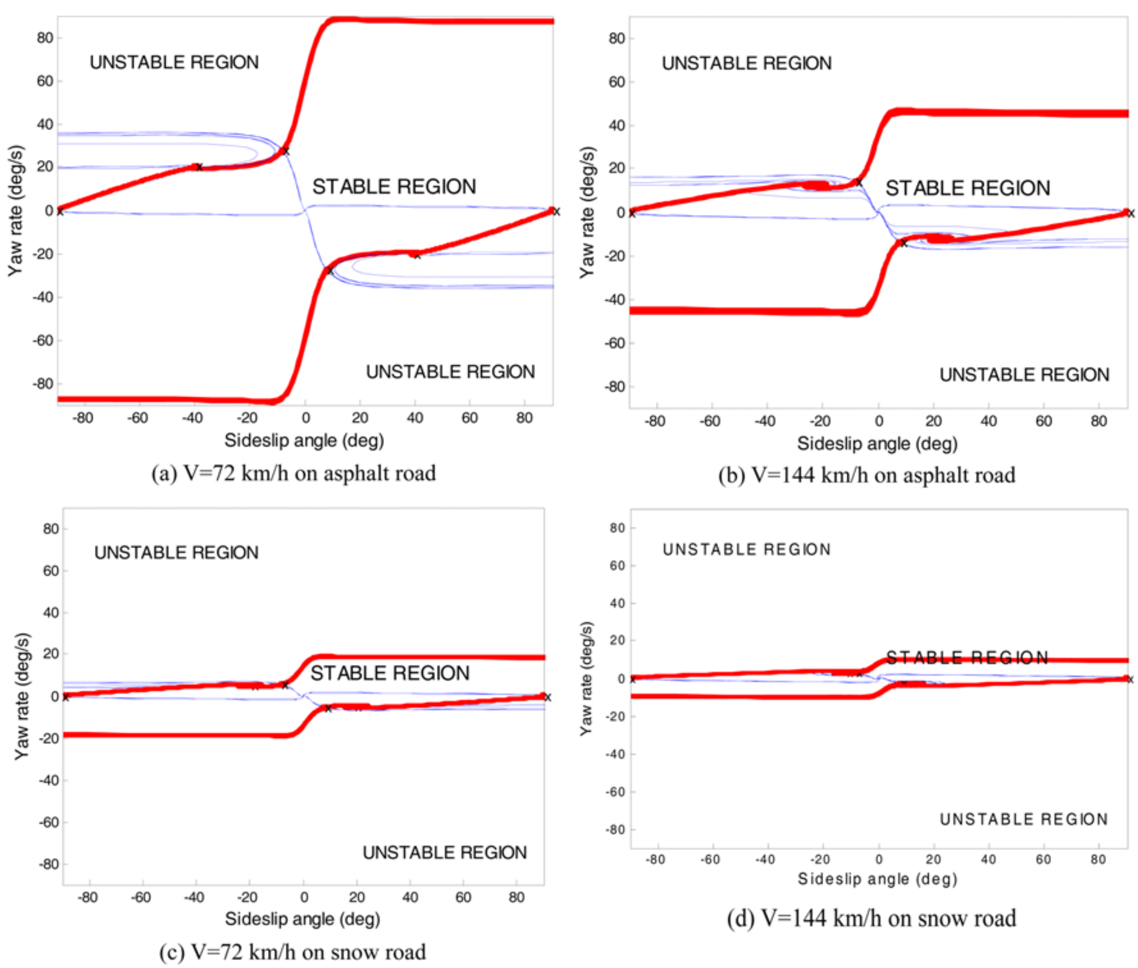

Figure 5. State space of a simple model for plane motion.

symmetric boundaries. As the vehicle speed increases, the stability region becomes narrow, and the stability region on a snowy road is narrower than that on an asphalt road.

Thus, a simple model with nonlinear tire dynamics can readily be used to estimate the stability region.

5. DYNAMICS VERIFICATION OF THE VEHICLE MODELS

Field tests were performed to check the validity of the models. Because we are interested in lateral dynamics, the verification of the models focused on the appropriate parameters, namely the lateral acceleration, yaw rate, and roll angle. The J-turn is a typical test for lateral dynamics and facilitates the tracking of transient changes in these parameters.

Figure 6 illustrates the results of the experiment for the J-turn maneuver as well as computer simulations using the two models. The same vehicle speed (72 km/h) and steering wheel input (measured during the experiment and plotted in Figure 6(a)) were applied in both the test and the simulations. To execute the J-turn, the driver proceeds in a straight line and then turns the wheel quickly to reach the final steering angle.

The simulation results using the multi-body model were in close agreement with those of the experiment in both the transient range and the steady-state range. The absence of a 'speed controller' in the multi-body model led to an additional slight decrease in steady-state values. The multi-body model was constructed for the simulation of a vehicle with a control system. On the other hand, the simple model yielded somewhat excessive responses, as Figures 6(b) and 6(c) indicate, which is only a natural result of neglecting weight transfer effects. During a cornering maneuver, there are definite weight transfers among the wheels. In particular, lateral weight transfer causes the total lateral force of one axle to decrease. Both the development of lateral forces at the rear wheels and the decrement of the total lateral force at the front axle (due to lateral weight transfer) follow the initial development of lateral forces at the front wheels. Balancing the forces between the front and rear axle lessens the exaggerated estimates of Figure 6. However, in the simple model, the tire forces keep their initial values, and there is no decrease of the lateral force at the

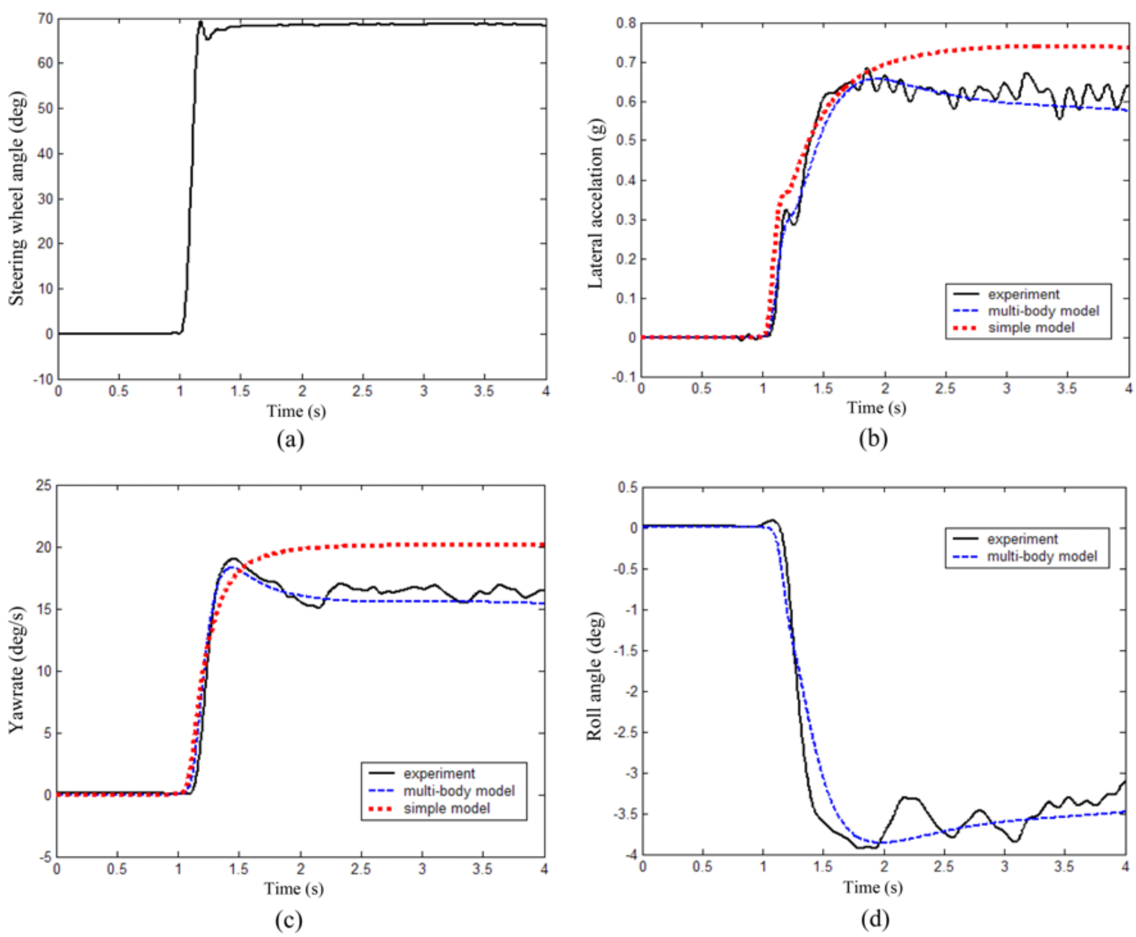

Figure 6. Test and simulation results for the J-turn maneuver: (a) steering input, (b) lateral acceleration, (c) yaw rate, and (d) roll angle.

front axle. Hence, the simple model has a tendency to overestimate the dynamic responses of the cornering maneuver. Figure 6(d) shows the experimental results, as well as those of the multi-body model simulation, for the roll angle. The simple model is not included in this figure because it cannot predict the effects of roll or pitch.

6. CONCLUSION

In this study, two types of vehicle models were obtained by considering degrees of freedom and linearity. Using actual data for a mid-sized passenger car, the two models were validated with field test results for a J-turn maneuver. The results of the multi-body model simulation were in close agreement with those of the experiment in both the transient and steady-state ranges. The simple model with nonlinear tire dynamics could easily be used to determine the stability region, while the complex model (a multi-body dynamic model in MSC.ADAMS) was appropriate for accurate simulations of the vehicle stability behavior.

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