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PREDICTION OF VIBRATING FORCES ON MESHING GEARS FOR A GEAR RATTLE USING A NEW MULTI-BODY DYNAMIC MODEL

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ABSTRACT–An efficient multibody dynamic model was developed to predict the vibrating transmitted gear forces of loaded and unloaded pairs of helical gears simultaneously at all speeds. The model can also calculate the bearing forces of a manual transmission that, in turn, may be converted to rattling noises. The bending of meshing gear teeth and torsional flexibility of transmission shafts were considered and embodied effectively in the multibody dynamic model by calculating the tooth bending stiffness and adding a torsion spring on a shaft section between two gears, respectively. The reactive forces on teeth and bearings were calculated and compared using three different models that were developed for this study - an equivalent model, a rigid-body model, and a frequency-based model. The equivalent model took only 58% computation time, compared to the frequency-based method, even though the two showed very similar results.

KEY WORDS : Rattle noise, Fluctuating gear force, Manual transmission, Multi-body dynamics, Backlash

1. INTRODUCTION

Gear rattle is a typical gear noise phenomenon that is generated under the existence of torsional fluctuations, which occur due to the irregular combustion power of engines, as shown in Figure 1, and in turn, lead to the gear tooth impact of unloaded gears that fluctuate within tooth backlash (Seaman *et al.*, 1984; Padmanabhan *et al.*, 1995; Kamo *et al.*, 1996). The fluctuating impact is transmitted to the bearings (Fujimoto and Kizuka, 2001), and the bearing reactions can be calculated and converted to the gear rattle noise intensity (Sakai *et al.*, 1981; Wang *et al.*, 2001, 2002). It is critical to consider the gear rattle noise of an automobile transmission from the preliminary design phase in order to obtain first-time-capable designs, since diesel engines are widely used in passenger cars. To achieve this, efficient numerical models that can predict vibrating gear forces in meshed and unmeshed states at each speed shift and under fluctuating reactive forces on transmission shaft bearings need to be developed.

This problem has attracted significant attention. Kubur *et al.* (2004) proposed a dynamic model of a multi-shaft helical gear reduction unit formed by several flexible shafts. The model consists of a finite element model of shafts combined with a 3-D discrete model of gear pairs. It is used to analyze the free and forced vibration of the system. Park *et al.* (1994) developed a finite element model of a geared rotor system with flexible bearings to investigate the effects of bearing coefficients on the dynamic behavior of the system. The gear mesh was modeled by a pair of rigid disks with a spring.

The stiffness variation of meshed spur gear teeth due to elastic deformations as a function of a contact point along a line of action was predicted by finite element analysis (Kim *et al.*, 2000). The gear stiffness was considered to be a time-varying coefficient (Blankenship and Singh, 1995; Theodossiades and Natsiavas, 2000). The nonlinear dynamic behaviors of a gear driving system due to transmission errors and backlash were analyzed, and a nonlinear motion equation was developed based on the calculated transmission error and backlash (Choi *et al.*, 1997; Singh *et al.*, 1989).

An analytical relationship between gear error and shaft deformation has been developed (Park and Cho, 2001). In the study, the shaft and support bearing deformations were

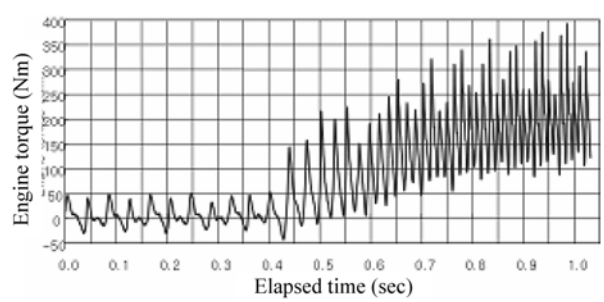

Figure 1. Engine torque fluctuation with time.

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modeled by FEM. Kim and Singh (2001) suggested an analytical model that could explain the dynamic interaction between loaded and unloaded gear pairs in the drive rattle mode and compared their results with experiments. Park *et al.* (2007) recently developed a numerical dynamic model based on a single pair of loaded gears and a rigid shaft to study the gear rattle condition of a manual transmission.

Yakoub *et al.* (2004) developed a numerical model to predict the gear rattle noise that radiates from manual gearboxes using DADS computational flexible multibody dynamics and vibro-acoustics. Four methods for modeling gear mesh excitations in simple and compound planetary gear sets were developed and proved to be applicable for the assessment of operating rattle noise and vibration levels (Morgan *et al.*, 2007). Recently, a tribo-dynamic model of a front wheel drive manual transmission has been developed to study idle rattle and consider the hydrodynamic contact film reaction and flank friction (Tangasawi *et al.*, 2007). FEM and BEM were applied to reduce gear whine noise in an axle system (Kim *et al.*, 2007).

However, because most previous studies only focused on a single gear pair or were based on simple finite element analyses, the combined effects of loaded and unloaded gears on vibrating forces could not be considered efficiently during multi-speed gear shifts. The objective of this study is to develop an efficient multibody dynamic model to predict the fluctuating loaded and unloaded gear forces at each gear speed and the reactive bearing forces, which may be converted to rattling noises. The bending of meshing gear teeth and the torsional flexibility of transmission shafts were effectively considered and embodied in the multibody dynamic model by calculating the tooth bending stiffness and adding a torsion spring in a shaft section between two gears, respectively. The reactive forces on teeth and bearings were calculated and compared using three different methods developed for this study - an equivalent model, a rigid-body model, and a frequency-based model.

2. MULTIBODY EQUIVALENT MODEL

A realistic multibody dynamic model for a transmission system, which may reflect real working conditions, should be constructed for an accurate loading analysis. An FF manual transmission consists of a clutch, input and main shafts, mating helical gears, final-drive gears in the differential section, and housing. The 3-D six-speed manual transaxle model, which combines the manual transmission, final drive gearing, and differential into a single unit, is shown in Figure 2. A multibody dynamic analysis model for Figure 2 was constructed using MSC/ADAMS and is represented in Figure 3. This model is based on the following three assumptions: (1) shafts and gear teeth are flexible, and bearings are considered to be bushings with 6 degrees of freedom; (2) gear meshing stiffness due to bending varies along a moving contact point between two helical teeth; and (3) fluctuating torque or acceleration is transmitted through the clutch input to the transmission input shaft.

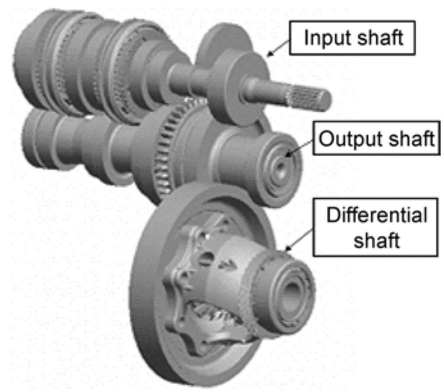


Figure 2. 3-D gear train model of a manual transmission.

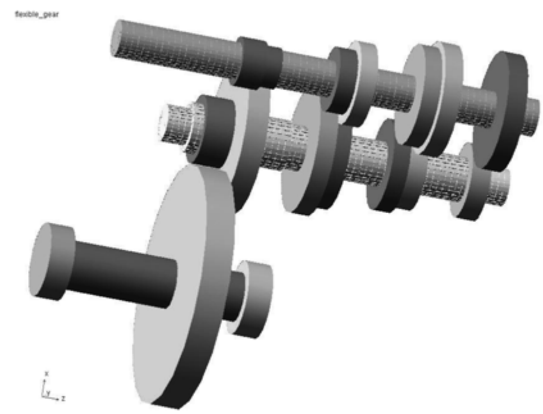


Figure 3. Multi-body analysis model of a manual transmission.

2.1. Bending Stiffness of a Gear Tooth

Figure 4 shows a schematic of three components of forces acting against a helical gear tooth. The tangential component is also a transmitted load, which results in a transmitted torque. The important tangential force has the following relationships:

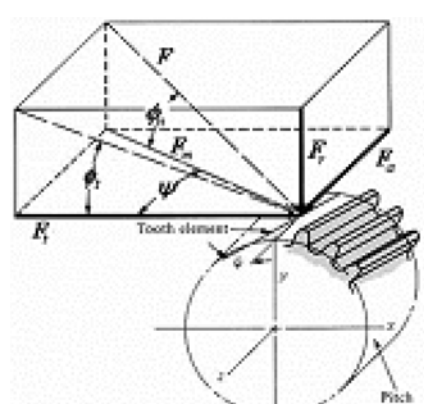


Figure 4. Force components acting on a helical gear tooth.

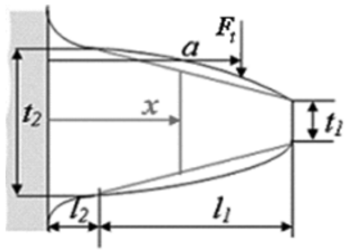


Figure 5. Nomenclature of a gear tooth.

$$F_t = k(r_g\theta_g + r_p\theta_p) \tag{1}$$

$$hp = \frac{F_t V}{745.7} = \frac{Tn}{7121} \tag{2}$$

where r is the gear radius; θ is the tooth rotational angle (bending slope) due to bending at a moving contact point during one rotation; the subscripts g and p are a gear and a pinion, respectively; F_t is the transmitted tangential load (N); n is speed (rpm); T is torque (Nm); and V is pitch line velocity (m/s).

The equivalent tooth bending stiffness k in equation (1) can be obtained analytically by Castigliano's theorem, which is based on a single gear tooth (Figure 5).

$$k = \left\{\frac{4l_2}{Ebt_2^3}(a^2 - al_2 + l_2^2) + \frac{12}{Eb}\frac{1}{c^3}\ln\frac{d - cl_2}{d - ca} - \frac{1}{2c}\frac{(a - l_2)^2}{(d - cl_2)} - \frac{1}{c^2}\frac{(a - l_2)}{(d - cl_2)}\right\}^{-1} \tag{3}$$

where $c = (t_2 - t_1)/l_1$, $d = t_2(l_1 + l_2)/l_1$, l_2 is the clearance under a base circle, l_1 is the working depth, E is Young's modulus and b is the gear face width.

Consequently, the transmitted tangential force (F_t) may be calculated by ADAMS (with the option of a function input) in conjunction with k in equation (3), and it varies as gears rotate. Considering k, the influence of tooth bending deflections can be included in the equivalent model.

2.2. Torsional Stiffness of Shafts

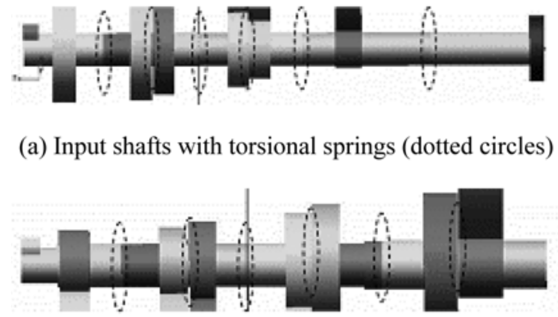
In the equivalent model, the torsional stiffness of transmission shafts is represented in ADAMS as several torsional springs with torsional rates at each shaft section:

$$k_t = \frac{GJ}{L} \tag{4}$$

where L is the solid shaft length between two gears, G is the shear modulus, and J is the polar area moment of inertia. Figure 6 represents the locations of the torsional springs on the equivalent model shafts.

3. FREQUENCY-BASED MODEL

Another approach, the *frequency-based model*, has been developed. The natural frequencies and modes of manual transmission input and output shafts (shown in Figures 7 and 8) were calculated based on finite element analyses and were read by an ADAMS model in order to reflect the stiffness of shafts. It takes three to four hours for this model, which is usually three-dimensional, to read and handle all elemental stiffnesses from the finite element shaft meshes. Four hours of computation time is too long to check a design concept quickly; thus, a more efficient model that runs faster is required. The natural frequencies of two shafts are shown in Table 1. Equation (3) was used again to calculate the tooth bending stiffness in this model.



(a) Input shafts with torsional springs (dotted circles)

(b) Output shafts with torsional springs (dotted circles)

Figure 6. Dynamic model of input and output shafts with torsional springs.

Figure 7. FE model of an output shaft of a manual transmission.

Figure 8. FE model of an input shaft of a manual transmission.

Table 1. Natural frequencies of shafts.

Freqs	Input shaft (Hz)		Output shaft (Hz)	
	Bending	Torsion	Bending	Torsion
1st	1705	5035	1549	5362
2nd	4575	11482	1969	11231
3rd	8196		5300	

4. ANALYSES AND RESULTS

The three different analysis models were developed for comparison - the equivalent model, a rigid body model, and a frequency-based model. The rigid body model consisted entirely of rigid shafts and gears and was only implemented in ADAMS.

4.1. Rigid Body and Frequency-based Models

The rigid body and frequency-based dynamic models were compared based on the numerical results of the angular velocities of input, output, and differential shafts, the fluctuating forces between meshed and unmeshed gears, and the fluctuating reaction forces on bearings.

The calculated angular velocity profiles by the two models of three different shafts are quite similar to each other, as shown in Figure 9. This plot was drawn based on a dynamic simulation in which the gear was shifted consecutively from idling through six different speeds every two seconds. It is noticed from the plot that many small impulses exist along the input and output shafts velocity lines, where gears are directly in the mesh. The angular velocity of the input shaft looks like a sawtooth function. The first rise represents an rpm increase while idling for one second just before torque transfer; therefore, the angular velocities of output and differential gear shafts are zero. It can be seen in Figure 9 that the angular speed of the output shaft is lowest and the torque is highest at the first speed. The negative speeds at the output mean that the input shaft is rotating in the opposite direction. As the speed shifts towards high gears, the rotation of the differential gear shaft gradually increases.

Figure 10 shows the fluctuating gear forces on a pair of the 1st speed gears as the speed shifts from the 1st to the 6th gears for 15 seconds consecutively. The large fluctuating forces at early times are due to the transferred torque, and later, they rattle when the 1st speed gear pair is unmeshed. It can be seen from Figure 10 that the amplitudes of forces by the flexible frequency-based model are a

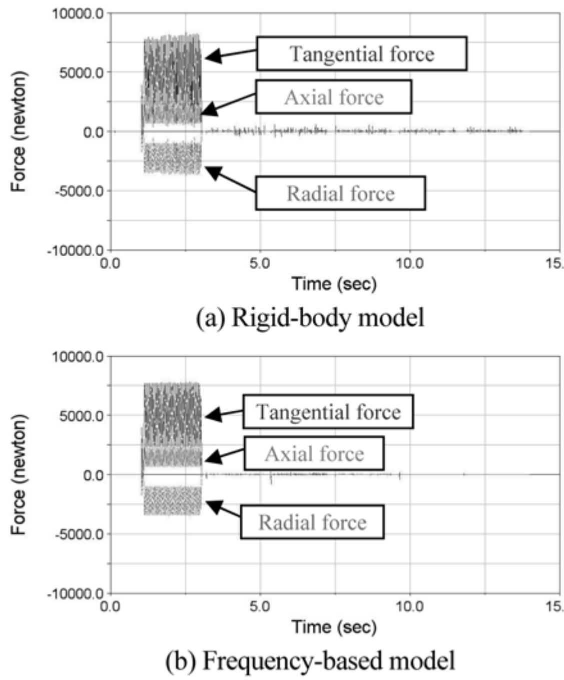


Figure 10. Axial, radial, and tangential forces at the 1st speed of a gear mesh with a time history by two models.

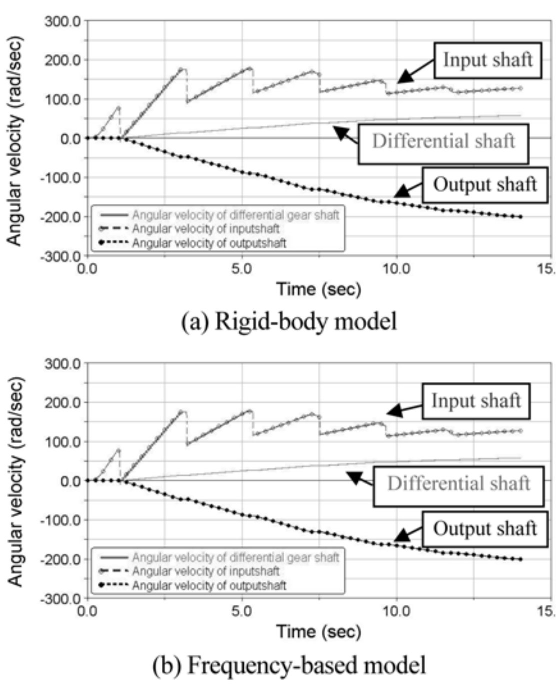


Figure 9. Variations in angular velocities of input, output, and differential shafts by rigid and frequency-based models.

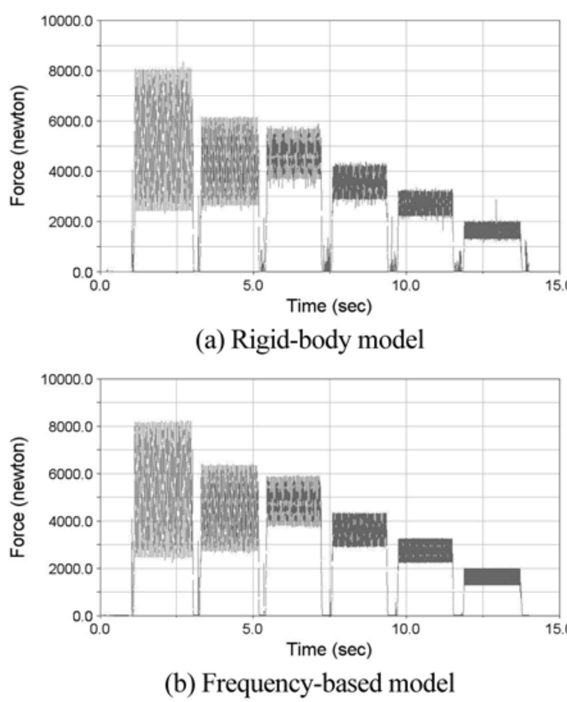
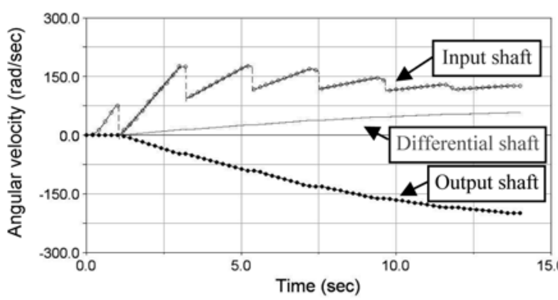


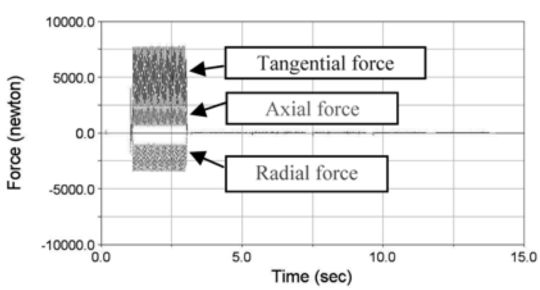
Figure 11. Dynamic loads at a front bearing on an input shaft by two models.

little bit larger than those of the rigid body model because it considers elastic deformations. Figure 11 shows the fluctuating reaction forces at a front bearing on the input shaft by the two models as the speed shifts from the 1st to the 6th. Slightly higher amplitudes are also observed in the frequency-based model. Many peaks (impulses) among speed shifts and at the speeds from the 3rd to 6th are shown in the rigid body model, but on the other hand, no peaks appear in the other model. This is because the frequency-based model considers the bending and twist of gears and shafts, and as a result, the flexibility mitigates excessive peak forces.

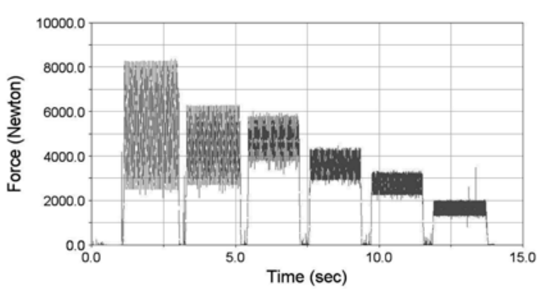
The calculated results from the frequency-based model seem closer to the realistic situations; however, the computation usually requires 3 to 4 hours. The long computational time is not efficient and is not practical for application to the design of a transmission; thus, a new model that is as accurate as the frequency-based model and that does not require a long computational time needs to be developed.



(a) Angular velocities of input, output, and differential shafts



(b) Forces at the 1st speed of a gear mesh



(c) Dynamic loads at a front bearing on an input shaft

Figure 12. Results from the equivalent model.

Table 2. Computational time taken by 3 models.

Time	Dynamic models		
	Rigid	Freq-based	Equivalent
(sec)	1,573	3,276	1,896

4.2. Equivalent Model

The equivalent model, which simultaneously takes the bending stiffness of a gear tooth and the torsional stiffness of shafts consisting of a manual transmission into account was developed as described in Section 2. The results of this method appear to be more accurate than the other two models.

Figure 12(a) shows the angular velocities of the input, output, and differential shafts in a power train by the equivalent model, and they are quite similar to the other two results. Figure 12(b) shows the fluctuating tangential, radial, and axial forces on a pair of the 1st speed gears as the speed shifts from the 1st to the 6th continuously for 15 seconds. The trend is very close to the other two cases. Figure 12(c) shows the fluctuating reaction forces calculated at a front bearing on the input shaft as the speed shifts from the 1st to the 6th. Unlike the rigid body model, there are almost no force peaks between each speed. Even at each speed, only a small number of impulses are shown. Such abnormal force peaks usually lead to an erroneous prediction of a reaction force.

Table 2 represents computational times required by the 3 models for a PC (Pentium IV, 3 GHz, 1 GHz RAM). The frequency-based model takes the longest time -- about 1.7 times the equivalent model.

5. CONCLUSIONS

An efficient dynamic model was developed to predict the vibrating meshed and unmeshed gear forces and reactive bearing forces. These can be directly converted to rattling noises, based on ADAMS advanced functions that are linked with the equations for gear tooth bending and shaft torsion. This method was compared to a rigid body model and a frequency-based model to demonstrate its efficiency. The developed equivalent model shows almost no unnecessary peaks between each speed, even though the rigid model that is widely used results in many peaks. Considering computational time, this method takes 58% of the time required by the frequency-based method. However, two results show little difference. The developed model considers the fluctuating forces of loaded and unloaded pairs of gears at all speeds simultaneously. This method can be readily applied to analyze rattle noises that occur in an automotive transmission.

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