International Journal of Automotive Technology, Vol. 10, No. 2, pp. 167−174 (2009)
DOI 10.1007/s12239−009−0020−6

1229−9138/2009/045−05

DESIGN OF ANFIS NETWORKS USING HYBRID GENETIC AND SVD METHODS FOR MODELING AND PREDICTION OF RUBBER ENGINE MOUNT STIFFNESS

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(Received 9 August 2007; Revised 19 July 2008)

ABSTRACT−Genetic Algorithm (GA) and Singular Value Decomposition (SVD) are deployed for optimal design of both the Gaussian membership functions of antecedents and the vector of linear coefficients of consequents, respectively, of ANFIS networks. These networks are used for stiffness modelling and prediction of rubber engine mounts. The aim of such modelling is to show how the stiffness of an engine mount changes with variations in geometric parameters. It is demonstrated that SVD can be optimally used to find the vector of linear coefficients of conclusion parts using ANFIS (Adaptive Neuro-Fuzzy Inference Systems) models. In addition, the Gaussian membership functions in premise parts can be determined using a GA. In this study, the stiffness training data of 36 different bush type engine mounts were obtained using the finite element analysis (FEA).

KEY WORDS : ANFIS (Adaptive Neuro-Fuzzy Inference Systems), Engine mount; Genetic algorithms (GAs), SVD (Singular Value Decomposition), FEA (Finite Element Analysis)

1. INTRODUCTION

Process modelling and system identification using input-output data have always been the focus of many research efforts. Astrom and Eykhoff (1971) described system identification techniques that are applied in many fields to model and predict the behaviours of unknown and/or very complex systems based on given input-output data. Theoretically, to model a system, a precise understanding of the explicit mathematical input-output relationship is required. Alternatively, Sanchez *et al*. (1997) presented soft-computing methods that involve computation in imprecise environments and have gained significant attention. The main components of soft computing, namely, fuzzy-logic, neural networks, and genetic algorithms, have a great ability to solve complex non-linear system identification and control problems. Among these methodologies, Porter and Nariman-Zadeh (1994) proposed evolutionary methods that have mostly been used as effective tools for both system identification and optimal design of fuzzy and neural network systems. Fuzzy rule-based systems have been an active field of research because of their unique ability to build models based on experimental data. Lee (1990) presented the concept of fuzzy sets, which deal with uncertain or vague information, and paved the way for applying them to real and complex tasks. Indeed, fuzzy-logic, together with rule-based systems, has the ability to model the approximate and imprecise reasoning processes, which are common in human thinking or human problem solving. This results in a policy that can be mathematically evaluated using fuzzy set theory. Therefore, Wang (1992) stated that fuzzy systems could be effectively employed as universal approximators to perform input-output mapping. Porter and Nariman-Zadeh (1995) showed that such fuzzy systems could be iteratively designed using different evolutionary search methods, where such genetic-fuzzy systems continue to grow in visibility, as noted by Cordon *et al*. (2001). In fact, these fuzzy systems are trained by examples (X_i, y_i) (i=1, 2, …, m) in terms of input-output pairs. Recently, Wang *et al*. (1999) proposed the use of a combination of orthogonal transformation and back propagation methods to train a candidate fuzzy model and to remove its unnecessary fuzzy rules. In other recent works, Darvizeh *et al.* (2001) showed that Singular Value Decomposition (SVD) can be used to enhance the performance of both fuzzy and GMDH-type (Group Method of Data Handling) neural network models obtained using simple heuristic approaches. In such networks, every two input neurons are connected to produce a hidden or output neuron using a linear or, more commonly, a non-linear quadratic form of function.

Moreover, Nariman-Zadeh *et al*. (2002) recently applied SVD in combination with a genetic algorithm to optimally design a fuzzy system for modelling purposes that demonstrated its superior performance in comparison with previous works. However, a fuzzy model consisting of a large number of IF-THEN rules to map inputs to outputs is not

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desired due to the phenomenon of over fitting, which reduces the generalising property of the fuzzy model to predict the unforeseen data. Similarly, the Takagi-Sugeno-Kang (TSK) type fuzzy models are widely used for control and modelling because of their high accuracy and relatively small model size. In the TSK models, which are also known as neuro-fuzzy systems, the consequents of the fuzzy rules are explicit functions, usually with linear relationships, of the input variables rather than fuzzy sets. In other words, as presented by Hoffmann and Nelles (2001) the crisp linear relation portion of the consequents of a TSK fuzzy rule describes the underlying model in the local multi-dimensional region specified in the premise part of that fuzzy rule. Therefore, two types of tuning procedures are required for proper partitioning of the input space and number of fuzzy rules, known as structural tuning, and for the parameters of the consequent parts of the fuzzy rules, known as parametric tuning. In recent years, different approaches have been adopted for optimal tuning of such models based on either heuristic search or fuzzy clustering for the premise part and least squares for linear parameters in the conclusion of the fuzzy rules. Genetic algorithms have received a great deal of attention for the optimal selection of the premise part of TSK type fuzzy rules in Wang and Yen's 1999 literature. In order to identify the parameters of the consequents, there have been attempts in the literature to use SVD as a linear optimisation technique. Jang (1993) proposed an equivalent approach to the TSK models as an Adaptive Neuro-Fuzzy Inference System, ANFIS. In this model a hybrid learning method is used for tuning parameters in both antecedents and consequents of embodied TSK-type fuzzy rules.

The engine is the largest concentrated mass in a vehicle and will cause vibration in the vehicle's body if it is not properly isolated and constrained. An ideal engine mount system isolates engine vibration caused by unbalanced forces and prevents engine bounce from shock excitation. Yu *et al*. (2001) successfully used modern engine mounting systems to isolate the driver and passenger from both noise and vibrations generated from the engine. The proper design of rubber mounts may be the most effective engineering approach to improving the ride of a vehicle. The analysis of engine mounting rubber components should be accompanied by an analysis of the vibration of the engine mount system. Bernuchon (1984) showed that it is necessary not only to know the properties of the rubber and where to place the mounts but also to determine the optimum design of a rubber part to achieve the desired properties along with the required load-bearing capacity resulting from the system vibration analysis.

This paper models the stiffness variation with the geometric parameters of a typical engine mount using the ANFIS network. Using this method, 36 different geometry bush type engine mounts are analysed by FEA to obtain their corresponding stiffness in 3orthogonal directions (k_x, k_y, k_z). The parameters involved in the geometry of an engine mount are regarded as inputs, whilst the stiffness values computed by FEA are regarded as outputs. The ANFIS network identifies the input-output relationship that is a set of TSK-type fuzzy rules for the modelling of the engine mount stiffness. Such an ANFIS identification process needs, in turn, optimisation methods to find both the Gaussian membership functions of the antecedents and the vector of linear coefficients of the consequents. For this reason, a hybrid genetic algorithm and SVD are used for the optimal selection of Gaussian membership functions of premise parts and linear parameters of the ANFIS's conclusion part, respectively.

2. MODELLING USING ANFIS

An ANFIS consisting of a set of TSK-type fuzzy IF-THEN rules can be used in modelling to map inputs to outputs. The formal definition of such identification problem is to find a function $\hat{f}$ so that it can be approximately used instead of the actual one, f. This is done in order to predict output $\hat{y}$ for a given input vector $X=(x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output y. Therefore, given m observations of multi-input-single-output data pairs so that

$$y_i=f(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i=1, 2, \ldots, m), \tag{1}$$

a look-up table can now be built in order to train a fuzzy system to predict the output values $\hat{y}_i$ for any given input vector $X=(x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$; that is

$$\hat{y}_i=\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i=1, 2, \ldots, m). \tag{2}$$

The problem is now to determine an ANFIS that minimises the difference between the actual and predicted output, that is

$$\sum_{i=1}^{m} [\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in})-y_i]^2 \rightarrow \min. \tag{3}$$

In this way, a set of linguistic TSK-type fuzzy IF-THEN rules is designed to approximate f by $\hat{f}$ using m observations of n-input-single-output data pairs (X_i, y_i) $(i=1, 2, \ldots, m)$. The fuzzy rules embodied in such ANFIS models can be conveniently expressed using the following generic form

$$Rule_l: IF\ x_1 \text{ is } A_l^{(j_1)} \text{ AND } x_2 \text{ is } A_l^{(j_2)} \text{ AND},$$

$$\ldots, x_n \text{ is } A_l^{(j_n)} \text{ THEN } y=\sum_{i=1}^{n} w_i^l x_i+w_0^l \tag{4}$$

in which $j_i \in \{1,2,..., r\}$ and $W^l=\{w_1^l, w_2^l,..., w_n^l, w_0^l\}$ are the parameter set of the consequent of each rule. The entire fuzzy set in x_i space is given as

$$A^{(i)}=\{A^{(1)}, A^{(2)}, A^{(3)},..., A^{(r)}\} \tag{5}$$

These fuzzy sets are assumed to have a Gaussian shape defined on the domains $[-\alpha_i, +\beta_i]$ $(i=1, 2,..., n)$. In this way, the domains are appropriately selected so that all the fuzzy sets are complete; that is, for any $x_i \in [-\alpha_i, +\beta_i]$, there exists $A^{(j)}$ in equation (5) such that the degree of membership function is non-zero, $\mu_{A^{(j)}}(x_i) \neq 0$. Each fuzzy set $A^{(j)}$ in which $j \in \{1,2,..., r\}$ is represented by Gaussian membership functions in the form

$$\mu_{A^{(j)}}(x_i) = Gaussian \quad (x_i;\, c_j,\, \sigma_j)$$

$$= \exp\left((-1/2)\left(\frac{x_i - c_j}{\sigma_j}\right)^2\right) \tag{6}$$

where c_j, σ_j are adjustable centres and variances in antecedents, respectively. It is evident that that the number of such parameters involved in the antecedents of ANFIS models can be readily calculated as nr, where n is the dimension of input vector and r is the number of fuzzy sets in each antecedent. The fuzzy rule expressed in equation (4) is a fuzzy relation in $R \times \Re$ in which $A^{(i)}$ are fuzzy sets in U_i so that $U = U_1 \times U_2 \times U_3 \times \ldots \times U_n$ and $Rule = A^{(j_1)} \times A^{(j_2)} \times A^{(j_3)} \times \ldots \times A^{(j_n)} \rightarrow y$.

It is evident that the input vector $X = (x_1, x_2, x_3, ..., x_n)^T \in U$ and $y \times \Re$. Using Mamdani algebraic product implication, the degree of such local fuzzy IF-THEN rule can be evaluated in the form

$$\mu_{Rule_k} = \mu_U(x_1, x_2, ..., x_n) \tag{7}$$

where

$$U = A_l^{(j_1)} \times A_l^{(j_2)} \times \ldots \times A_l^{(j_n)} \quad \text{and}$$

$$\mu_U(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} \mu_{A_l^{(i)}}(x_i)\,. \tag{8}$$

In these equations, $\mu_{A_l^{(j_i)}}(x_i)$ represents the degree of membership of input x_i regarding their l_{th} fuzzy rule's linguistic value, $A_l^{(j_i)}$. Using a singleton fuzzifier and a product inference engine and, finally, aggregating the individual contributions of rules leads to the fuzzy system in the form

$$f(X) = \frac{\sum_{l=1}^{N} y_l \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i)\right)}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i)\right)}\,, \tag{9}$$

when a certain set containing N fuzzy rules in the form of equation (4) is available. Equation (9) can be alternatively represented in the following linear regression form

$$f(X) = \sum_{l=1}^{N} p_l(X) y_l + D\,, \tag{10}$$

where D is the difference between $f(X)$ and corresponding actual output, y, and

$$p_l(X) = \frac{\prod_{i=1}^{n} (\mu_{A_l^{(j_i)}}(x_i))}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i)\right)} \tag{11}$$

It is therefore evident that equation (10) can be readily expressed in a matrix form for given M input-output data pairs (X_i, y_i) $(i=1, 2, \ldots, m)$ in the form

$$Y = PW + D \tag{12}$$

where $W = [w_1, w_2, ..., w_s]^T \in \Re^S$, $S = N(n+1)$ and $P = [p_1, ..., p_s]^T \in \Re^{m \times S}$.

It should be noted that each $(n+1)$ component of vector w_i corresponds to the conclusion part of a TSK-type fuzzy rule such that the firing strength matrix P is obtained when input spaces are partitioned into a certain number of fuzzy sets. It is evident that the number of available training data pairs is usually larger than all the coefficients in the conclusion part of all TSK rules when the number of such rules is sufficiently small, that is, $m \geq S$. This situation turns the equation (12) into a least squares estimation process in terms of unknowns, $W = [w_1, w_2, ..., w_s]^T$, so that difference D is minimised. The governing normal equations can be expressed in the form

$$W = (P^T P)^{-1} P^T Y \tag{13}$$

Such modification of coefficients in the conclusion part of TSK rules leads to better approximation of the data pairs given, in terms of minimisation of the difference vector D. However, such direct solution of normal equations is susceptible to round-off error and, more importantly, to the singularity of these equations.

Therefore, in this paper, singular value decomposition is used as a powerful numerical technique to optimally determine the linear coefficients embodied in the conclusion part of the ANFIS model and deal with probable singularities in equation (12). However, in this work, a hybridisation of genetic algorithms and SVD is proposed to model the rubber engine mount stiffness for the optimal design of ANFIS. Such combination of genetic algorithms and SVD were described in previous sections.

3. APPLICATION OF GENETIC ALGORITHM (GA) TO ANFIS DESIGN

The incorporation of a genetic algorithm into the design of such ANFIS models starts by representing the $N(n+1)$ real-value parameters of $\{c_j, \sigma_j\}$ as a string of concentrated sub-strings of binary digits. Thus, each such sub-string represents the fuzzy partitioning of antecedents of fuzzy rules embodied in such ANFIS models in a binary coded form. The fitness (Φ) of each string of binary digits that represents an ANFIS system, which models the engine mount stiffness, is readily evaluated in the form of where E is the

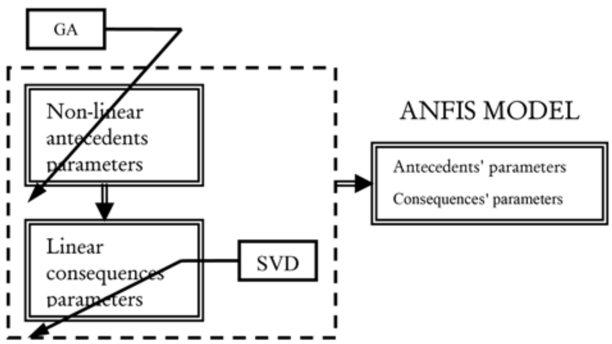

Figure 1. Schematic diagram of hybrid GA/SVD design method.

$$\Phi = 1/E \tag{14}$$

objective function given by equation (3) and is minimised through an evolutionary process by maximisation of fitness, Φ. The evolutionary process starts by randomly generating an initial population of binary strings, each representing a candidate solution of the fuzzy partitioning of the premise part of the rules. Goldberg (1989) stated that using the standard genetic operations of roulette wheel selection, crossover, and mutation causes entire populations of binary string to gradually improve. Simultaneously, linear coefficients of the conclusion parts of TSK rules corresponding to each chromosome representing the fuzzy partitioning of the premise parts are optimally determined using SVD. Therefore, ANFIS models of engine mount stiffness that have progressively increasing fitness (Φ) are produced, while their premise and conclusion parts are simultaneously determined by genetic algorithms and SVD, respectively. In other words, each chromosome that represents the fuzzy partitioning of antecedents is related to the corresponding linear coefficients of consequents obtained by the SVD method. Figure 1 shows a schematic code for this design process and a general ANFIS model algorithm. The following section provides a detailed summary of the SVD application used to optimally determine the linear coefficients in the linear equations.

4. APPLICATION OF SINGULAR VALUE DECOMPOSITION TO ANFIS DESIGN

In addition to the genetic information gained from the antecedents of fuzzy sets involved in ANFIS networks, singular value decomposition is also deployed for the optimal design of consequents of such fuzzy systems. Singular value decomposition is the most common method used for solving linear least squares problems, although some singularities may exist in the normal equations. The SVD of a matrix, $P \times \Re^{M \times S}$, is a factorisation of the matrix into the product of three matrices, a column-orthogonal matrix $U \times \Re^{M \times S}$, a diagonal matrix $Q \times \Re^{S \times S}$ with non-negative elements (singular values), and an orthogonal matrix $V \times \Re^{S \times S}$ such that

$$P = U\ Q\ V^T \tag{15}$$

Golub and Reinsch (1970) originally proposed the most popular technique for computing the SVD. The optimal selection problem of W in equation (12) is first reduced to finding the modified inversion of diagonal matrix Q, in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal W are obtained using the following relation

$$W = V[diag(1/q_j)]U^T Y \tag{16}$$

5. MODELLING AND ANALYSIS OF ENGINE MOUNT

This study considers a bush type engine mount. Figure 2 shows the shape of the engine mount of a passenger car and its geometry. An engine mount should be specifically designed to provide vibration isolation for machinery while also providing high resistance to movement caused by dynamic loads. An engine mount system used in a passenger car was chosen for the application model. The resulting shape of the parameter optimisation is determined as a final model with some modifications.

For convenience, local coordinate axis shown in Figure 2 will be used throughout this paper. There are six geometric parameters used to define the shape of the bush type engine mounts, as shown in Figure 2. However, r_i, r_o and θ are known from the layout design and are fixed for a certain engine mount. Therefore, three parameters, t_r, t_s and t_Z, are considered to be design variables.

The stiffness values, derived from the system vibration analysis, are known as “dynamic” stiffness. However, the “static” stiffness can be simply computed by

$$k_d = \eta k_s \tag{17}$$

where k_d and k_s are the dynamic stiffness and static stiffness, respectively, and η is a correction factor as stated by Kim *et al.* (1992). The correction factor is generally in the range of 1.2 to 1.6. In this paper, the static stiffness is modelled, and, consequently, the dynamic stiffness can be computed by a proper value of correction factor that can be selected for a particular rubber material.

The static stiffness value is obtained from the quasistatic

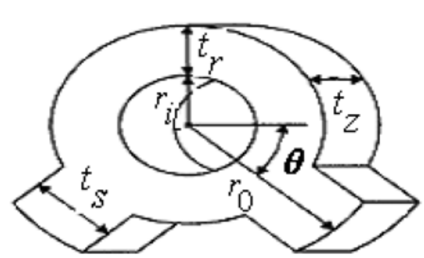

Figure 2. Schematic diagram of an engine mount.

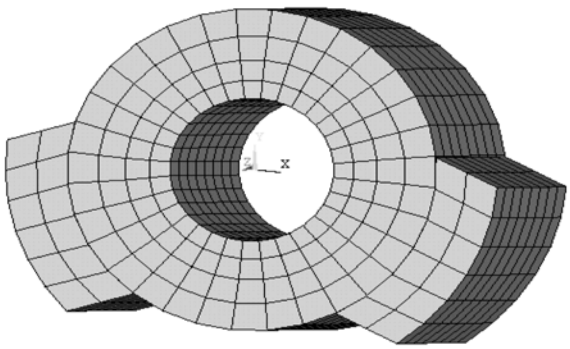

Figure 3. Finite element model of an engine mount.

nonlinear finite element analysis by appropriate boundary conditions included in the model. Since the strain is relatively small in this analysis, the classical Mooney-Rivlin (1992) form of the strain energy is sufficient to describe the fully incompressible hyperelastic material behaviour. The Mooney-Rivlin form of strain energy (U) is expressed by

$$U = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \tag{18}$$

where I_1 and I_2 are the first and second strain invariants, respectively. The coefficients C_{10} and C_{01} are determined from the results of the unaxial tension test. For the special engine mount used in this paper, these values are: C_{10}= 0.03622 and C_{01}=−0.00335. These values are then used by the FEA to obtain the stiffness in different directions. The ranges of design variables that were used in this study are $10 \le t_r \le 15$, $25 \le t_s \le 40$ and $25 \le t_z \le 40$.

The finite element model (FEM) of an engine mount with different dimensions, in ranges of design variables, is built using ANSYS software. A typical engine mount FEM is shown in Figure 3. The structure is represented quantitatively as finite collections of elements whose deformations and stiffness can then be computed using linear algebraic equations. In these analyses, the fixed geometry values are r_i=10 mm, r_i=30 mm, θ=12 degrees. The Mooney-Rivlin coefficients expected from stress-strain relationship have been given as C_{10}=0.03622 and C_{01}=−0.00335.

6. GENETIC/SVD BASED ANFIS MODELLING AND PREDICTION OF RUBBER ENGINE MOUNT STIFFNESS

In this paper, 36 different engine mount geometries were considered in the FEA in order to obtain their stiffness in 3-orthogonal directions. The stiffness values, together with their corresponding geometry dimensions, were given by Nariman-Zadeh *et al*. (2004).

In order to model a 3-input-single-output dataset, an ANFIS with 2 linguistic terms in each antecedent, equivalent to 2 Gaussian membership functions for each input variable, was considered; that is, n=3 and r=2. It should be noted that the number of parameters in each coefficient vector in the conclusion part of each TSK-type fuzzy rule is

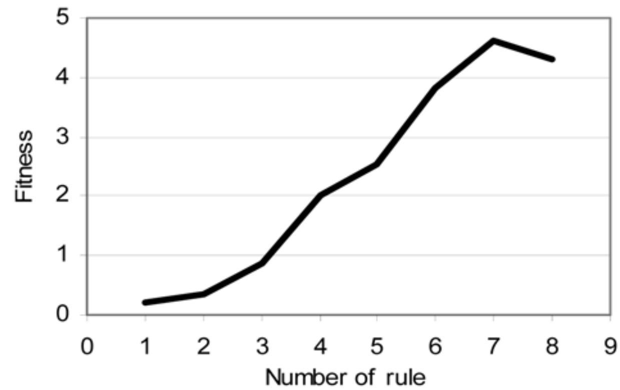

Figure 4. Training errors of GA/SVD-designed ANFIS with different number of rules (k_x modelling).

4, according to the assumed linear relationship of input variables in the consequents. Consequently, 2^3=8 TSK-type fuzzy rules were identified using the ANFIS given in MATLAB fuzzy-logic toolbox. The corresponding fitness is calculated as 1.022 and 1.125 for k_x and k_y, respectively.

In order to demonstrate the effectiveness of the hybrid design method, the genetic algorithm and singular value decomposition that was developed in this work was applied for the modelling of 3-input-single-output set of data of engine mount for k_x and k_y. The number of Gaussian membership functions for each input variable in the premise part of the rules was considered to be 2. During the evolutionary process, the population size, mutation probability, crossover probability, and generation number were selected to be: 30, 0.07, 0.7, and 150, respectively. It should be noted that 4 bits were chosen to be the binary representation of each variable, making the length of a chromosome 48 bits with respect to 3×2×2=12 parameters. Figures 4 and 5 show the performance of different genetically obtained hybrid ANFIS networks, each having a different number of rules in the cross-validation process for k_x and k_y, respectively. It is evident from these figures that an ANFIS with only 7 rules (considering the exact value of fitness) is sufficient to

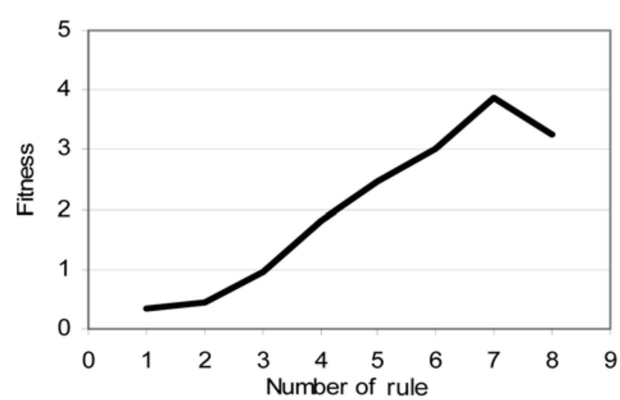

Figure 5. Training errors of GA/SVD-designed ANFIS with different number of rules (k_y modelling).

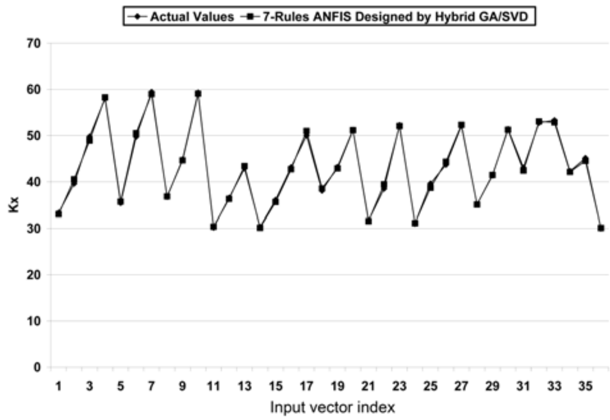

Figure 6. Variation of k_x with input data samples (GA/SVD-ANFIS with 7-rules).

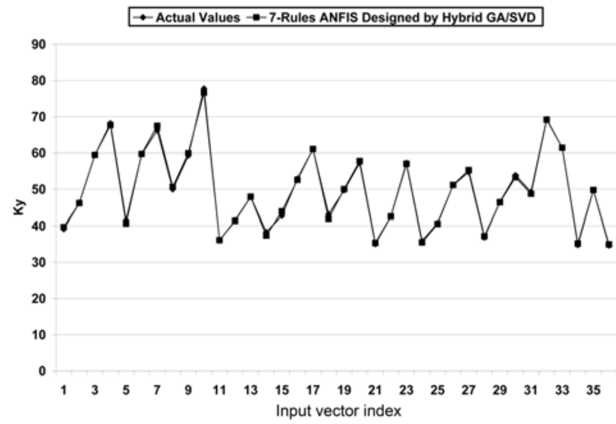

Figure 7. Variation of k_y with input data samples (GA/SVD-ANFIS with 7 rules).

effectively model the 36 3-input-single-output dataset of engine mount stiffness. The corresponding fitness is 4.618 and 3.875 for k_x and k_y, respectively, which is considerably smaller than previously obtained with a larger number of rules. The good behaviour of the ANFIS network designed by hybrid GA/SVD with 7 TSK-type fuzzy rules to model the data of engine mount stiffness in the x and y directions is shown in Figures 6 and 7, respectively.

Figures 8 and 9 show the Gaussian membership functions of input variables. The obtained set of TSK-type fuzzy rules for the modelling of engine mount stiffness in the x and y directions is as follows:

Rule1: If t_r is A1 and t_s is A3 and t_z is A5 then k_x=1 $-1.6315* t_r +0.5421* t_s +0.0003* t_z -0.2453$

Rule2: If t_r is A1 and t_s is A4 and t_z is A5, then k_x=$0.9321* t_r -0.1726* t_s +0.0078* t_z -0.2431$

Rule3: If t_r is A1 and t_s is A4 and t_z is A6, then k_x= $-0.1592* t_r -0.0011* t_s -0.0078* t_z -2.1769$

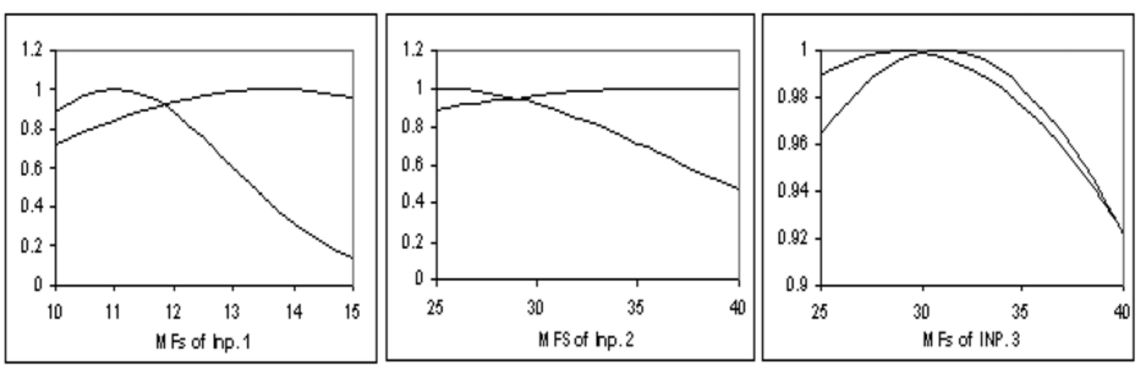

Figure 8. Genetically evolved Gaussian membership functions of input variables for k_x modelling (GA/SVD-designed ANFIS with 7 rules).

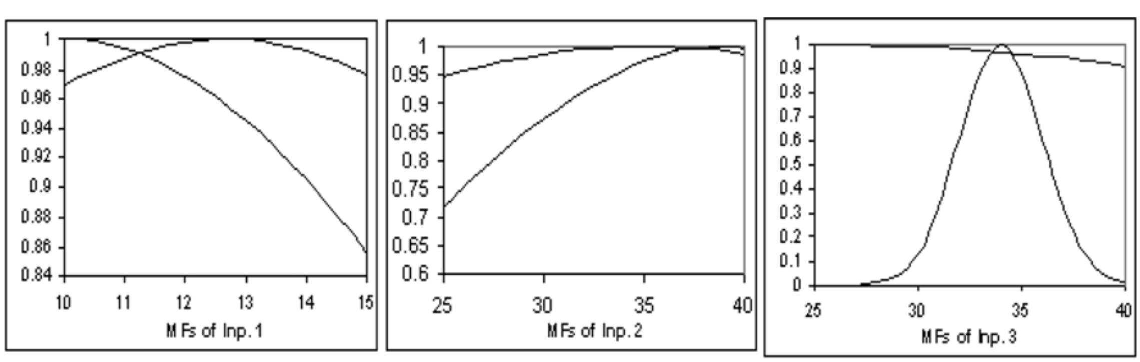

Figure 9. Genetically evolved Gaussian membership functions of input variables for k_y modelling (GA/SVD-designed ANFIS with 6 rules).

Table 1. Training fitness comparisons.

	Training fitness		Number of rules
	K_x Modelling	K_y Modelling	
ANFIS, MATLAB	1.022	1.125	8,8
ANFIS, This work	4.618	3.875	7,7
GMDH-type neural networks,	0.294344	0.231172	–

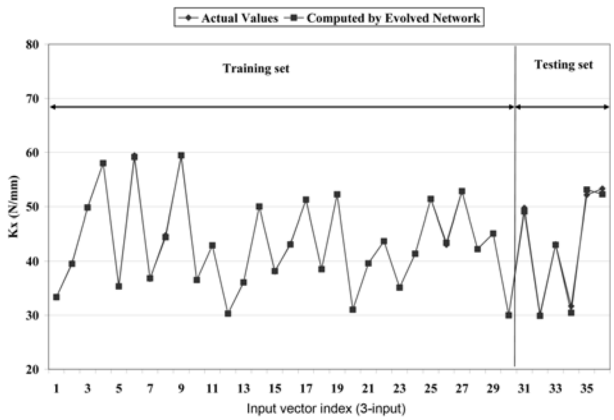

Figure 10. Variation of k_x with input data samples (GA/SVD-ANFIS with 8 rules in modelling & Prediction).

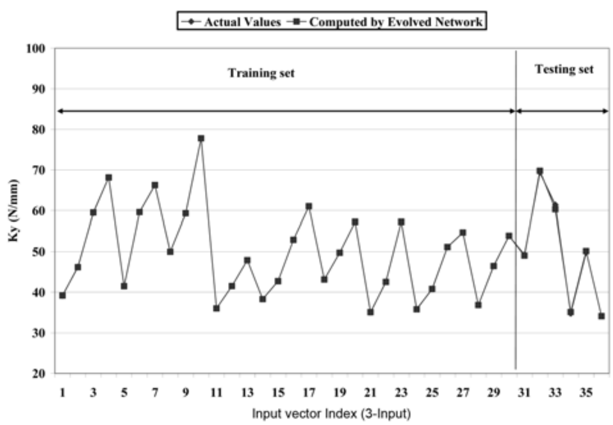

Figure 11. Variation of k_y with input data samples (GA/SVD-ANFIS with 8 rules in modelling & Prediction).

Rule4: If t_r is A2 and t_s is A3 and t_z is A5, then k_x=0.5022* t_r −0.2369* t_s −0.0181* t_z −0.2732
Rule5: If t_r is A2 and t_s is A3 and t_z is A6, then k_x= −0.0362* t_r −0.2072* t_s +0.0179* t_z −9.1570
Rule6: If tr is A2 and ts is A4 and t_z is A5, then k_x= −0.2749* t_r +0.0529* t_s +0.0102* t_z −0.3721
Rule7: If tr is A2 and ts is A4 and t_z is A6, then k_x= −0.0374* t_r +0.0891* t_s −0.0091* t_z −4.5923

And for k_y :

Rule1: If t_r is A1 and t_s is A3 and t_z is A5, then k_y=0.6533* t_r +1.3451* t_s −1.1725* t_z +1.3232
Rule2: If t_r is A1 and t_s is A3 and t_z is A6, then k_y= −0.2702* t_r −0.0295* t_s +0.1642* t_z −1.9258
Rule3: If t_r is A1 and t_s is A4 and t_z is A5, then k_y= −0.2087* t_r −1.3363* t_s +1.0240* t_z −1.2141
Rule4: If t_r is A2 and t_s is A3 and tz is A5, then k_y= −0.6422* t_r −1.4143* t_s +1.1748* t_z −1.2013
Rule5: If tr is A2 and ts is A3 and t_z is A6, then k_y=0.1515* t_r +0.1617* t_s −0.1630* t_z −1.8712
Rule6: If tr is A2 and ts is A4 and t_z is A5, then k_y=0.1910* t_r +1.4047* t_s −1.0262* t_z −1.1685
Rule7: If t_r is A2 and ts is A4 and t_z is A6, then k_y=0.1227* t_r −0.1305* t_s −0.0006* t_z +3.6719

The comparisons of ANFIS network fitness with different numbers of rules are summarised in Table 1. The superiority of the hybrid GA/SVD design approach presented in this paper is clearly evident from these results.

In order to demonstrate the prediction ability of the GA/SVD-designed ANFIS model, the training and testing procedure was performed in a cross-validation process on the datasets consisting of 20 and 16 data samples, respectively. In these cases, the measure of performance in the cross-validation process is accomplished on the 20-data training set and 16-data test set, respectively. The good behaviour of the ANFIS network designed by the hybrid GA/SVD with 8 TSK-type fuzzy rules to model and predict the data of engine mount stiffness is depicted in Figures 10 and 11.

7. CONCLUSION

Hybrid GA/SVD-designed ANFIS networks have been successfully used to model the very complex process of rubber engine mount stiffness. ANFIS was shown to be an effective means to model the stiffness in x and y direction according to different inputs. It has been demonstrated that the methodology of hybrid GA/SVD in the design of ANFIS presented in this work is remarkably effective in terms of both training errors and rule number.

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