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RESEARCH ARTICLE

Can the link between functional and personal income distribution enhance the analysis of inequality?

Marisa Civardi[1] · **Renata Targetti Lenti**[2]

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Abstract The aim of this paper is to present a framework which links functional and personal income distribution. In the first part of the paper, Piketty's book "Capital in the XXI Century" is briefly reviewed. Piketty's framework is discussed arguing that it can only partially explain levels and changes within personal income distribution. Piketty links the returns from capital r to the rate of growth of national income g in a very innovative way comparing them within a macroeconomic framework. He claims that when returns on capital rise more quickly than the overall economy and taxes on capital remain low, a vicious circle of ever-growing dynastic wealth and growing concentration of wealth takes place. However, the rise in the inequality of personal income distribution cannot only be explained by the rise of capital incomes. An analysis of the generation of personal incomes, and consequently of inequality, requires a suitable framework that links incomes at the macroeconomic level (national accounts) and incomes at the level of the individual/household. It is possible to set up this framework starting from individual endowments and their link to the productive structure: that is to what can be called the "generating function of personal income." This function transforms personal endowments into personal earnings, given the productive structure, the technologies, and the market rules that determine the functional distribution. Personal income distribution and its inequality are linked to the functional one through the shares of capital and labor owned by each individual. The framework introduced here seems to be a suitable tool to account for the fact that personal income distribution is inextricably tied up to different sources of inequality in the distribution

✉ Marisa Civardi
marisa.civardi@unimib.it

Renata Targetti Lenti
targetti@unipv.it

[1] Università degli studi di Milano-Bicocca, Milan, Italy

[2] Università degli studi di Pavia, Pavia, Italy

of national income. Sources come from institutional and productive structures (matrix Y), but also from the distribution of endowments and of individual/household entitlements (matrix S). This approach, we argue, allows for the assessment and evaluation of the effects of "ambitious new policies," aimed at reducing poverty and inequality ex-ante, as suggested by Atkinson in his last book.

Keywords Capitalism · Inequality · Income distribution

JEL Classification P16 · P48 · D31

1 Introduction

Inequality in income distribution has long been one of the major themes of economic and sociopolitical debate, but interest in this topic within the economic field has waned over time. A very substantial body of research on inequality has been accumulated, building on the potential of improved data and focused on clarifying concepts and measures, capturing trends, and understanding the causal processes operating in the economy. Inequality has now become the focus of remarkably wide-ranging attention, from International Institutions, Government Reports and finally, also academic journals across a variety of disciplines. As Tony Atkinson observed in a "pioneering" article (Atkinson 1997), the personal income distribution was brought in "from the cold" only at the end of the 1990s. The question is "why should economists 'care' about inequality"? Equity is a fundamental value in democracies. It is connected to the functioning of the economic system (Atkinson 1997). Economic inequalities can be conceived of as the outcome of the underlying economic process which produces other economic effects. Individual differences provide behavioral incentives to work, save, or take an entrepreneurial risk. Inequalities tend to be self-enforcing and self-sustaining. Inequality in one space or for one group may connect with equality in another space or for other groups. Linkages and effects among and between inequalities are not necessarily obvious, and their revealing requires analytical effort.

More recently, considerable attention has been paid to the ways in which higher inequality could act as an obstacle to growth. Social protection and more broadly the Welfare State (e.g., via education and health care) can potentially provide an environment that stimulates rather than undermines economic growth. Even more important it is the understanding that policies might improve both equity and efficiency simultaneously and avoid the trade-off between them. Special attention has been paid to the role of institutions. Differences in institutions and policies are likely to play an important role in explaining the different levels and trends of wage and capital income inequality across countries. Inequality in income distribution in industrialized countries is very high and increases with the rise of per-capita income. A forty-year trend of increasing inequality, common to many advanced economies, deserves research to discover the forces which are deeply rooted "within

modern industrial capitalism" (Solow 2014, p. 2). Growing inequality leads to the discussion of issues such as how different sources of inequality can be analyzed and what set of concrete proposals aimed at reducing it can be put forward. We can find some answers and proposals in two recent books: Piketty's "Capital in the Twenty-First Century" (2014a) and Atkinson's "Inequality. What can be done?" (2015).

Piketty's work is an impressive empirical research on inequality and its persistence over time, with particular reference to the "sustainability" of capitalist systems in which inequality is increasing. An empirical analysis of inequality in wealth and income distribution in the long run had never been carried out before Piketty's book (Piketty 2014a, b). Therefore, it has already sparked and will continue to promote a very rich and innovative debate in the future. "Post-Piketty, the public-intellectual debate over inequality, economic policy and equitable growth ought to focus differently" argue the editors of a very recent book collecting essays from economists and sociologists (Boushey et al. 2017, p. 1). "Capital in the Twenty-First Century" and in the many reviews, comments, and critics that followed its publication are packed with so many topics, insights, comments, and observations. In this essay, we will focus mainly on the factors that can explain the trends concerning the concentration of wealth and personal income distribution. The functional distribution of income between capital and labor, the increase in capital share in national income, and the concentration of wealth are of course very important factors. The relation between them, which pushes toward greater inequality in income and wealth, is well identified in Piketty's macro-setting. "Capital in the XXI Century" discusses trends in inequality of income and wealth in many countries, and in different periods, taking into account the relation between capital and growth. However, the personal income-generating process, and its links with inequality, is not really analyzed.

In his last book, Atkinson (2015) brings his theoretical and practical experience to bear on the multifaceted features of income inequality. He presents a comprehensive set of policies that could bring about a genuine shift in the distribution of income in developed countries. Atkinson claims that fresh ideas are needed in order to reduce inequality. It is necessary to go beyond imposing new taxes on the wealthy. His quite ambitious set of policies requires an analytical innovative framework aimed at identifying the micro- and the macro-variables which are sources of personal income distribution and therefore of inequality. It is necessary to introduce a framework in order to "make a link between incomes at the macroeconomic level (national accounts) and incomes at the level of the household" (Atkinson 2009, p. 1). As Atkinson claims, the relationships between "factor shares … and inequality in the personal distribution of income … is the principal problem of political economy" (Atkinson 2009, p. 1). This framework, as we will show, can be set up starting from individual endowments and their links to the productive structure, which is what we can call the "generating function of personal income."

The outline of the paper is the following. After Introduction (Sect. 1), Sect. 2 introduces Piketty's analytical framework. Section 3 discusses the "fundamental contradiction of capitalism," that is how the inequality $r > g$ in the long run leads to a higher concentration of wealth. In this section, the authors argue that Piketty's approach is only partially suitable to explain growing inequality in personal income distribution. Section 4 introduces a general framework that can link functional and

personal income distribution. The framework introduced here seems to be a suitable tool to take into account the fact that personal income distribution is inextricably tied up with different sources of inequality in the distribution of income from factors such as those related to the institutional and productive structures and those that come from the distribution of endowments and from individual/household entitlements. Remarks on the relevance of the proposed framework for the assessment of reducing inequality' policies conclude the paper (Sect. 5).

2 Piketty's analytical framework

"Capital in the Twenty-First Century" provides a general theory of the functioning of a capitalist economy. Issues on inequality are only one aspect of that general theory. Piketty is not interested in explaining the role of capital accumulation *on* economic growth, but rather the inverse relation, that is the role of economic growth *on* the returns to capital, on the concentration of wealth and the inequality of income in capitalist economies. The forces that shape the concentration of income are economic but also political and institutional such as wars, taxation, and inflation (Milanovic 2014, p. 529). In a society where "patrimonial capitalism" prevails, wealth concentration mainly contributes to the inequality in the distribution of personal income. When the percentage of people who do not need to work in order to earn their living (the rentiers) goes up, the distribution of personal income will become even more unequal.

Piketty uses very simple economic models to explain what is happening. He reverses the relationship between income distribution and growth as it had been interpreted in the traditional Keynesian models. In such models the rate of economic growth is obtained from the ratio between the saving rate and the capital–output ratio. Piketty, on the other hand, investigates how the ratio between the saving rate and the rate of growth of the economy determines the capital–output ratio, and consequently the share of the income from capital in the national product. The definition of capital follows the SNA guidelines (United Nations 1993). It includes all forms of assets (housing, land, machinery, financial assets in the form of cash, bonds and shares, intellectual property) that generate a return/rent as the result of the functioning of a "pure and perfect" market for capital. Durable goods are not included.

The analytical framework consists of two models: (1) the first is a standard Solow macro-model aimed at determining the capital–income ratio β and the capital share on national income α; (2) the second model is aimed at linking the concentration of wealth to the growth rate of the average rate of return to capital r compared to the rate of growth of the economy g. The first model leads to the "first and the second laws of capitalism." According to the first law, α is linked to β and to r ($\alpha = r \times \beta$). The capital–output ratio β measures the overall importance of wealth in a given society, as well as the capital intensity of production (Piketty and Saez 2014, p. 840). Assuming, as Piketty does, that in perfectly competitive markets r is equal to the marginal productivity of capital, it decreases when β increases. In a more complex economy, where there are many diverse uses of capital, the rate of return

on capital r may be higher or lower than the marginal productivity of capital. It can be determined by other forces: firstly by technology and secondly by the abundance of capital stock (Piketty 2014a, p. 154). Thirdly, the owner of capital, who is in a monopolistic position, can impose a rate of return greater than the marginal productivity of the capital itself.

The central question becomes how much the rate of return on capital r decreases when the capital–income ratio β increases (Piketty 2014a, p. 155). This depends on the elasticity of substitution (σ) between capital K and labor L in a CES production function. The standard assumption is that the production function is a Cobb–Douglas where σ is equal to 1: as the stock of capital rises, the rate of return on capital r decreases exactly in the same proportion, so that α does not change (Piketty 2014a, p. 154). If the rate of return on capital r falls more than proportionately when the capital–income ratio β increases, then the share of income from capital in national income decreases. On the other hand, if the rate of return r falls less than proportionately when β increases, then the capital share of income increases with β. Piketty introduces the hypothesis that the elasticity of substitution (σ) between capital K and labor L is greater than one ($\sigma > 1$) so a rise of capital–income ratio β also leads to a rise of the share of capital in the national income. Intuitively, it makes sense to assume that σ tends to rise over the development process, as there are more diverse uses and "forms" for capital and more possibilities to substitute capital for labor (Piketty and Saez 2014, p. 841).

The "second fundamental law of capitalism" determines the value of β in the long run. The hypothesis is that the capital–income ratio converges toward $\beta = s/g$, where s is the long-run annual saving rate (savings being invested in domestic or foreign assets) and g is the long-run annual total growth rate of the economy (as the sum of the population and the productivity growth rate (Piketty and Saez 2014, p. 840). The higher the savings rate and the lower the growth rate, the higher the capital–income ratio β will be (Piketty 2014a, p. 44). In a stagnant economy, where the growth of the economy is relatively slow because of both demographic and technical factors, the rate of saving exceeds the rate of growth, ($s > g$) so that β will be high and increasing. If both α and β are increasing, the concentration of wealth will also be high and increasing. This process tends to give lasting, disproportionate importance to wealth created in the past, and therefore to inheritance (Piketty 2014a, p. 267).

Several daily and weekly newspapers have hosted reviews of "Capital in the Twenty-First Century," which have almost always been extremely positive (Stiglitz 2014a; Solow 2014). However, some reviews have been critical. The reliability of the data sources and the estimates presented has been challenged (Giles 2014). Magness and Murphy (2015, p. 1) claim that "they have found evidence of pervasive errors of historical fact, opaque methodological choices, and the cherry-picking of sources to construct favorable patterns from ambiguous data." These authors also argue that Piketty used a highly distortive data assumption including the capital global ratio global index data from the Soviet Union and other communist regimes. This choice "exerts a strong downward pull upon the "U-shaped" curve" (Magness and Murphy 2015, p. 36), so that one of his main historical claims about global "capitalism" in the twentieth century is accentuated.

Criticism on the sources and the data, however, does not appear to have significantly weakened the volume's contribution to inequality studies. Sutch (2017a, p. 2), examining Piketty's US data for the period 1810–2010 for the top 10% and the top 1% of wealth distribution, concludes that Piketty's data for "the wealth shares of the top 10% for the period 1870–1970 are unreliable." Sutch Geloso et al. (2017a) also question Piketty's data.

Piketty has revolutionized the field of research on income distribution with the use of fiscal sources and with his focus on top income shares. In collaboration with Alvaredo et al. (2013), he uses very detailed fiscal data to trace the evolution of income and wealth distribution in different countries. This choice is in line with the previous research carried out by Simon Kuznets (1953) who first used tax data. The use of fiscal data, however, can be questioned as the sole (or even the best) approach for the analysis of income distribution (Mechling et al. 2017; Geloso and Magness 2017b). Historically, income tax returns have been filed by a small percentage of the population even in today's rich countries, so the long-term series can be of dubious quality. The same is true in developing countries now. We might know something about those at the top of an income distribution (the richest tax filers), but we do not have any information about the bulk of the population. Whether the highest tax filers are really the richest people is also questionable. Not only because of the obvious incentive to underreport income, but because in the past some particularly rich classes were exempt from taxation. Taxes are paid by fiscal units, not by individuals: the richest fiscal units may change depending on the tax rules (e.g., whether it is more advantageous to file jointly or separately). The income that is reported to tax authorities is market and not disposable income, that is, income not including transfers and taxes. The concentration of market income among fiscal units may, or may not, tell us much about the inequality of disposable income among individuals, which is ultimately the concept we are interested in. Piketty mentions some of these caveats but essentially, he ignores them. Many authors have raised a variety of questions concerning the sources and construction of the data. Our paper does not focus on this issue. It can be useful, however, to remember the claims of Auerbach and Hassett (2015) that "in translating data from other sources on US wealth concentration, Piketty changes the timing of observations in a manner that overstates the strength of the recent upward trend and obscures a downtrend present in his source data at the end of the sample period. Other data sources—largely from administrative databases—suggest dramatically different trends in inequality" (Bricker et al. 2016).

More recently, also the theoretical framework and the methodological approach adopted by Piketty have been challenged. French economists, in particular, have been critical of Piketty's approach. Although reviewing these topics is out of our scope here, it may be useful to summarize some of the most important critical remarks. (1) the definition of "capital as wealth" has been criticized as being too ambiguous and not compatible with an approach based on a CES production function (Raval 2017); (2) the definition of "capital" and "rent" and the choice to include houses in total capital has been questioned (Bonnet et al. 2014); (3) a radical critic of the Piketty's "second fundamental law of capitalism" has been advanced by Krusell and AA (2015). They argue that this law, "which embeds a theory of

saving, is rather implausible"; (4) some doubts have been advanced on the hypothesis that the elasticity of substitution between capital and labor is likely to remain higher than one (Rognlie 2016). (5) Acemoglu and Robinson (2015) argue that Piketty did not take into account the role of institutions in shaping the inequality.

Piketty replied to the numerous and different remarks by writing some new papers and arguing that the issues of the book had been "simplified in the telling and retelling" so that the original message had been misunderstood. Piketty argues that there are many factors that generate inequality in income and wealth: institutional, socioeconomics, and demographic (Piketty 2015b). The factors which explain the dynamics of wealth (accumulation of capital) are different from those which explain the dynamics of labor income (demand and supply of skills and education, technology). It is very difficult, therefore, to reach a consensus on a general and "shared theory" of the generation of wealth and income concentration. This issue deserves deeper discussion starting from the "fundamental contradiction of capitalism": the inequality $r > g$. This inequality can be considered the most important relation between r and g which explains the rising concentration of wealth and therefore of income.

3 The "fundamental contradiction of capitalism": the inequality $r > g$ in the long run

Piketty links the returns from capital to the rate of growth of national income in an innovative way. Trends of capital rent r and income growth rate g are compared within a macroeconomic framework. From this relation Piketty derives the value of the concentration of wealth, that of capital incomes and, finally, from the inequality in the distribution of capital income, the inequality in personal income distribution. Piketty shows how the gap $r - g$ is directly linked to the inverted Pareto coefficient of wealth distribution. The analytical model (rather mathematical in nature) that explains the relationship between the rate of return of capital, the rate of growth of the economy, and the concentration of wealth, is developed in two papers following on from "Capital in the Twenty-First Century" (Piketty 2015a; Piketty and Zucman 2015). Piketty claims that when $r > g$ capital grows faster than the economy the concentration of capital will attain extremely high levels (Piketty 2014a, p. 25). Piketty also argues that the effect "of $r - g$ on inequality follows its dynamic cumulative effects in wealth accumulation models with random shocks, and the quantitative magnitude of this impact seems to be sufficiently large to account for very important variations in wealth inequality" (Piketty 2015b, p. 75–76). Over a wide range of models, the long-run magnitude and concentration of wealth and inheritance are a decreasing function of g and an increasing function of r. Under fairly general conditions, if shocks take a multiplicative form, the top tail of the distribution of wealth converges toward a Pareto distribution. The inverted Pareto coefficient (measuring the thickness of the upper tail and hence the inequality of the distribution) increases with the gap between r and g. A decrease in g is inevitable once countries have reached a very high level of income. Capital

accumulation generates changes in the functional distribution of income in favor of capital. Since incomes from capital are more concentrated than incomes from labor, the personal income distribution will also become more unequal. A vicious circle of ever-growing dynastic wealth starts, which cannot be reduced by an additional level of competition. This inequality is the fundamental contradiction of patrimonial capitalism (Piketty 2014a, p. 298).

The relationship between the inequality $r > g$ and a rising concentration of wealth has been criticized for different reasons. Firstly, it has been considered weak from a theoretical point of view. Debraj Ray (2015, p. 9) argues that the inequality $r > g$ is mainly a condition of economic efficiency. An economy where $r < g$ is inefficient in the sense that too much has been saved. The $r > g$ relationship does not tell us anything more. It does not tell us anything about increasing inequalities in wealth distribution. Only a detailed study of inheritance can enlighten us as to whether inheritance is a key factor in explaining rising inequalities. Mankiw (2015, p. 43) argues that $r > s$ is an equilibrium condition in neoclassical models. Their existence means that "we live in a world in which any dynamic Pareto improvements has been unexploited… There is, moreover, good reason to doubt that $r > g$ leads to the 'endless inegalitarian spiral' that Piketty describes… Piketty reasons that resources of the wealthy would grow relative to the labor income if $r > g$. We can now see, however, that this condition is not sufficient once consumption, procreation, and taxation are accounted for. Instead, to obtain the worrisome 'endless inegalitarian spiral,' we would need the return on capital r to exceed the growth of the economy g by at least 7% points per year" (Mankiw 2015, p. 44). Furthermore, recently some authors (Sucth 2017b; Korom et al. 2017) have claimed that the massive fortunes created in the past are now largely depleted so that "Dynastic wealth accumulation is simply a myth" (Arnott et al. 2015, p. 2).

In Piketty's model, capital income grows as share of national income because the rate of return does not fall sufficiently fast with the increase in capital–output ratio. This argument is based on the empirical evidence collected by Piketty, but it is running against one of the fundamentals of economic theory: decreasing returns to an abundant factor of production (Milanovic 2014). When the capital–output ratio increases, the marginal return to capital r should go down. Stiglitz (2014b) argues that institutions and policies (banking and finance) can explain "Piketty puzzle" of a rising wealth–income ratio together with a rise in r and stable wages. Piketty's argument can also be considered weak from an empirical point of view. Rognlie (2016) argues that when converted from gross to net terms, standard empirical estimates of the elasticity of substitution between capital and labor are well below those assumed by Piketty. Rising capital–income ratio, which depends partly on rising market prices, is more likely to be associated with a substantial decrease in capital's net share of income and not with an increase. In this case $r - g$ is likely to fall and not to increase.

Piketty's time series, based on after-tax rates of return, also faces two shortcomings. "First, the tax rates used to calculate the after-tax returns on capital in Piketty and Zucman (2014) are average tax rates. But given the book's focus on the share of wealth held by the top 1% and even the top 0.1%, the relevant tax policy parameter seems to be the top marginal tax rate rather than the average tax rate.

Secondly, Piketty calculates the return on capital based on national accounts data and does not adjust for risk" (Auerback and Hasset 2015, p. 39). Relying on an alternative time series on the after-tax rates of return in the USA that takes into account marginal tax rates and is adjusted for risk, Auerback and Hasset (2015, p. 39) find that the post-tax rate of return remains consistently lower than GNP growth. From this perspective, the apocalyptic $r > g$ "exploding wealth inequality" scenario does not look particularly likely.

Piketty replied to these critics claiming that he was well aware that $r > g$ could not be the only or even the primary tool for explaining changes in wealth and income inequality. Institutional changes and political shocks, which can be considered endogenous to the inequality and to the development process itself, are also very important (Piketty 2015a, p. 48). In the real world, many shocks to the wealth trajectories of families can contribute to making wealth distribution highly unequal. These shocks are related to financial or estates rates of return, demographic factors, differences in saving behavior, differences in people's propensity to invest, differences in taste parameters, differences in labor market features, differences in the institutional and political setting. Differences in earnings to be saved and cumulated are also important shocks. Wealthier people can obtain higher average returns than less wealthy people. Unequal returns on capital, therefore, are a divergence force that significantly amplifies and aggravates the effects of the inequality depending on $r > g$. (Piketty 2015a, p. 50). He also claims: "I certainly do not believe that $r > g$ is a useful tool for the discussion of rising inequality of labor income: other mechanisms and policies are much more relevant here, e.g., supply and demand of skills and education" (Piketty 2015a, p. 48). He goes on to say "the rise in labor income inequality in recent decades explains why total income inequality is now substantially higher in the United States than in Europe" (Piketty 2015a, p. 49). Piketty argues that the two dimensions of inequality (from labor income and capital ownership) do interact in important ways: for example, rising inequality in labor earnings over a certain period of time might tend to fuel rising wealth concentration in future decades or generations (Piketty 2015b).

One important issue, however, is still not explained in Piketty' approach: how to derive the inequality in personal income distribution from capital and labor shares in national income and subsequently how to highlight the different factors behind the level of inequality. As Pier Luigi Porta (2014) argues, Piketty's analysis focuses too much on the symptoms rather than going directly to the heart of the inequality generation process. The move from a macro-approach (the $r > g$ inequality) to a micro-one (inequality in personal income distribution) is very complex and in Piketty's book this is not really explained. However, even in the literature functional and personal income distributions have always been analyzed separately and, furthermore, since the 1960s, the relevance of these issues has been increasingly downplayed. As Lindert (2014, p. 5) argues "the shares of labor versus capital in current income… have never proved to be a good predictor of inequality." Personal income distribution has almost always been analyzed emphasizing "that there is no direct link with factor shares" (Atkinson 2009, p. 4). However, there is a very strong link and it must be highlighted with suitable frameworks.

4 From functional to personal income distribution: what kind of linkage?

Atkinson (2009) argues that a new approach to functional and personal income distribution is needed for three reasons: (1) to make a link between incomes at the macroeconomic level (national accounts) and household incomes; (2) to help understand inequality in the personal distribution of income; and (3) to address the concern of social justice with the fairness of different sources of income. "In making the link between national income and the income of the household sector, the breakdown by sources is, indeed, necessary since the different sources raise different issues… The link between factor shares and personal distribution is more complex than in the days of classical economists for two important reasons: people have multiple sources of income … and there is considerable inequality within categories of income" (Atkinson 2009, p. 4).

An analysis of the process which links functional to personal income distribution requires a suitable framework. The effects of a rise of the capital income share (π) on personal income distribution depend not only on its amount, as Piketty argues, but also on the inequality within capital and labor income distributions and on the correlation between the two distributions. In today's world, where people have earnings from both labor and capital, all effects can be highlighted, following Atkinson (2009, p. 10), with a simple decomposition of the squared coefficient of variation V^2. The overall inequality of total income, measured by V^2, can be decomposed as a function of the share of capital income π, of the squared coefficient of variation of labor income V_w^2, of the squared coefficient of variation of capital income V_k^2, and of the correlation ρ between wage income and capital income distributions:

$$V^2 = (1-\pi)^2 V_w^2 + \pi^2 V_k^2 + 2\pi(1-\pi)\rho V_k V_w \tag{1}$$

Equation (1) shows that "an upward trend in the share of capital income… does not necessarily lead to a rise in overall income inequality… for the right side of (1) to increase with the share of capital income"; it is necessary that:

$$\pi > \left[1 - \lambda\rho\right]/\left[1 + \lambda^2 - 2\lambda\rho\right] \tag{2}$$

where λ is defined "as the ratio of the coefficient of variation of capital income to that of wages (i.e., $\lambda = V_k/V_w$)."

Condition (2) implies that the effect on inequality of a rise in the capital share depends both on the degree of correlation ρ and on the ratio between the inequality of the capital income V_k and the inequality of the labor income V_w. In particular, if the two distributions are uncorrelated, "a rise in the capital share increases inequality where it exceeds $1/(1+\lambda^2)$… (It is assumed throughout that λ is greater than 1) where the correlation is positive, the critical value is reduced still further, and if ρ is greater than $1/\lambda$, an increase in the capital share always increases inequality" (Atkinson 2009, p. 11).

The decomposition proposed by Atkinson becomes more relevant when it is considered in a more general framework which could explain how personal incomes are generated by both capital and labor in the productive sector. This means linking personal income distribution and its inequality to the functional one, through the shares of capital and labor owned by each individual. As a matter of fact, the factors that can be considered as sources of individual incomes, in the short run, are not only the amount of the endowments (physical and human capital) but also the "ability" to transform these endowments into personal income. Of course, in the long run, the accumulation of capital/wealth (earned or inherited) plays a very important role in generating earnings, income, and inequality. However, it must be considered as only one of the factors. The ownership of the factors by individuals grouped into family units, of different composition and size, determines the distribution of market incomes.

The mechanisms that regulate the distribution of market income among the various units must be brought back to the variables that contribute to the "generation" of the value added of the economic system into different sectors of production, business units, and workers. The ways in which the individual endowments translate into earnings depend on their prices as a result of structural and cyclical market equilibria. This process, following Dagum (1999), can be represented by a "generating function of income," that is, by a function f which ensures the correspondence between the market income y_{mi} of each unit i and the endowments owned, given their prices. In a first very simplified specification, it can be assumed that endowments are human capital cu and physical capital c_k. This function, for each unit i, can be expressed as:

$$y_{\mathrm{mi}} = f\left(c_{\mathrm{ui}}, c_{\mathrm{ki}}\right) \tag{3}$$

Two groups of variables, micro and macro, determine the generating function of the market income. On the one hand, the value of the endowments can be considered the result of the factors that determine the earning capacity of a subject such as personal abilities (innate or acquired through education), age, ownership of capital assets accumulated or inherited. All these factors are considered important variables to explain the earning capacity of the subjects, and therefore the inequality in personal income distribution by traditional theories which stress the influence of microeconomic variables. Occupational choice, and the consequent social position, in turn depends not only on the level of education, but also on the family background and paternal prestige. In a system where incomes are earned on the market, capabilities are enhanced by the social and economic organization. Not only does the unequal distribution of abilities tend to produce an unequal distribution of incomes, but other factors such as "fate" and luck are also important variables. Luck, however, has a very important role in the accumulation process of wealth and heritage, as Piketty has shown.

On the other hand, the ways in which resources are exchanged on the market and translated into income also depend on macroeconomic variables. In particular, the prices of the endowments synthesize the structural economic characteristics of the different markets. The capability of earning a given level of market income y_m

depends on the position of each unity in a specific socioeconomic setting, that is on the relations between the characteristics of each subject and the productive sectors in which he operates: not only on market conditions, but also on the structure of property, the localization as well as on the legal rules and the social, cultural, and religious features of a country. In a market economy these conditions can be expressed by a vector of prices and the final result will be a flow of earned incomes. If the focus of the analysis is on the disposable income y_{di}, that is, the income post-taxes and transfers, it is necessary to consider as a supplementary endowment the capability to obtain transfers from the government or from other subjects t_{ri}.

$$y_{di} = f(c_{ui},\ c_{ki},\ t_{ri}) \tag{4}$$

The distribution of the disposable income y_{di} is obtained from the distribution of market income taking into account the impact of redistributive policies. This impact reflects the structure of the redistributive mechanisms that operate through the tax system (more or less progressive) and through social security. Each economic unit (individual or household) according to its position within the economic system and its interactions with the other units (enterprises, government) will contribute to the inequality in the distribution of both market and disposable income. The inequality of the market income will be higher when the ownership of capital/wealth is more concentrated, when the dispersion of wages and salaries is wider. Furthermore, the inequality will be higher when the exclusion from the market and the marginalization that systematically affects some components of the labor force, specific industrial sectors or regions is higher.

Traditionally, the analysis of personal income distribution considers households and not individuals as the reference units. This choice is justified from both a methodological and an empirical point of view: (1) some of the endowments are owned at the household level, and (2) available data generally come from sample surveys conducted at the household level. For any household h, the level of income earned in the production activities (y_{mh}), in every period, can be expressed as:

$$y_{mh} = f(c_{uh},\ c_{kh}) \tag{5}$$

The function f is the "household income generation function" which transforms personal endowments, owned at the household level c_{uh}, c_{kh} in household earnings y_{mh}, given the productive structure, the technologies, and the market rules that determine the functional distribution of the value added. Atkinson (2015, p. 103) argues that "household income, and its distribution, derives not only on macroeconomic factors" but also on other factors that have been called "entitlement rules." These rules have defined as the "mechanisms regulating the appropriation of the output of the economy, or as the 'filter' between the production and its distribution among people" (Brandolini 1992, p. 5).

Households can be classified into different socioeconomic groups in order to be meaningful and homogenous from the point of view of the process of generation and distribution of income. The choice of the level of disaggregation and the type of socioeconomic groups depends above all on the goals of the researcher. Household classification has to be chosen in accordance with the overall analytical or policy

focus and to a degree that can be supported by the data (United Nations 1993, SNA chapter XX). Many different criteria can be selected including geographical location (e.g., urban–rural), assets (e.g., wealth, size of land holding) and the socioeconomic characteristics of a representative individual (e.g., household head or principal earner).

A suitable classification of the productive factors which make up the households' endowments should be chosen in order to identify the different factor markets. This classification should allow assessing the effects of policies aimed to increase the level of competitiveness of the system, to promote the compliance for the rules of the labour market and to favour technological changes that "increases the employability of workers" (Atkinson 2015, pp. 118–119). The labor factor can be cross-classified by location (e.g., urban–rural, or geographical region), skills or qualifications obtained, employment status (e.g., employee, own account worker, employer), and gender. This factor generates different types of labor income depending on employment status: dependent or independent. Mixed income, a category suggested in the 1993 SNA (United Nations 1993) is also frequently chosen as a category to represent the income of household enterprises (where it is difficult to distinguish the returns from labor from the returns of other factors). Generally there is no distinction made between different types of capital and natural resources (Table 1).

In a first, very simplified framework, taking into account only the relations between households and the productive sectors, the set of all household market incomes can be represented as a block matrix **D**, where, following a functional criteria, the incomes are only split into three categories. Of course the number of

Table 1 Block matrix **D**

Factors		Income from		
Households groups	Households	Dependent labor	Independent labor	Capital
1	1	y^d_{11}	y^i_{11}	k_{11}
	2	y^d_{12}	y^i_{12}	k_{12}
	….	.....	.....	.....
	n_1	y^d_{1n1}	y^i_{1n1}	k_{1n1}
2	1	y^d_{21}	y^i_{21}	k_{21}
	2	y^d_{22}	y^i_{22}	k_{22}
	…	…	…	…
	n_2	y^d_{2n2}	y^i_{2n2}	k_{2n2}
…	…	…	…	…
H	1	y^d_{H1}	y^i_{H1}	k_{H1}
	2	y^d_{H2}	y^i_{H2}	k_{H2}
	…	…	…	…
	n_H	y^d_{HnH}	y^i_{HnH}	k_{HnH}

productive factors considered could be more than the three factors used here. The first column shows the blocks of income from dependent employment of each group of households, the second column shows the blocks of income from independent employment, and the third column shows the blocks of income from capital. In matrix **D** some elements can be equal to zero when the household of the row does not have all types of income considered in the functional distribution. The households have been grouped according to their level of income in decile/percentile groups.

In order to determine the income received by each of the H socioeconomic group from the factors of production, we must multiply each block of the matrix **D** by a unit row vector $\mathbf{e'_{nh}}$, (Bottiroli Civardi and Targetti Lenti 1980, pp. 712–714) obtaining a new matrix **T** with H rows (corresponding to the household groups) and F columns (three in this case) (Table 2).

T is the matrix of the market households' income distribution and can be considered as the product of two matrices: the matrix **Y** of the composition of the factorial income and the matrix **S** of the structure of the ownership of the factors by the different groups of households:

$$\mathbf{T} = \mathbf{S}.\mathbf{Y} \tag{6}$$

If we consider the factorial income classified in F components, we can define the matrices **Y** and **S** as shown in Table 3.

The matrix **Y** is a block diagonal matrix whose elements are the total amount of value added earned by different factors of production (dependent workers, independent workers, physical capital, and so on). The values of **Y** are determined by the macroeconomic variables which generate the factorial income distribution. However, each element $s_h^f = y_h^f / Y^f$ of matrix **S** represents the share of hth group of households to the fth type of income (wages, salaries, income from autonomous work, etc.) according to the different ownership of human and physical capital. These values can be considered as the result of all those variables that influence the earning capability of people such as native or acquired personal abilities, level of education, age, and ownership of physical capital. In other words, they depend on all

Table 2 Matrix **T**

Household groups	Production Factors		
	Income from dependent labor d	Income from independent labor i	Income from capital k
1	T_1^d	T_1^i	T_1^k
2	T_2^d	T_2^i	T_2^k
…			
H	T_H^d	T_H^i	T_H^k

Table 3 Matrix **Y**

$$Y = \begin{vmatrix} Y^1 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & Y^2 & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & \ldots & Y^f & \ldots & 0 \\ 0 & 0 & \ldots & \ldots & \ldots & Y^F \end{vmatrix}$$

Table 4 Matrix **S**

$$s = \begin{vmatrix} s_1^1 & s_1^2 & \ldots & s_1^f & \ldots & s_1^F \\ s_2^1 & s_2^2 & \ldots & s_2^f & \ldots & s_2^F \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ s_h^1 & s_h^2 & & s_h^f & \ldots & s_h^F \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ s_H^1 & s_H^2 & & s_H^f & \ldots & s_H^F \end{vmatrix}$$

the variables that in the "traditional" theories of personal income distribution are considered as causal factors (Lydall 1979) (Table 4).

To assume that matrix **T** is the result of the product of the two matrixes **S** and **Y**, allows us to separate two components in the process of the market distribution of income to the households. This breakdown allows us to identify the different effects of changes in the market, employment structure, and public policies on personal income distribution. First of all, income distribution is influenced by a change in the factorial distribution: this happens, for instance, when technological processes change in the production sectors. The income distribution can change, however, changing the distribution of human capital and wealth in terms of the properties of the factors. These are macroeconomic variables which cannot be controlled by single individuals and/or household.

This framework can be considered the theoretical approach for building a Social Accounting Matrix (SAM). A SAM is basically a matrix combining the flows in value of an economic system as well as showing who pays what to whom in transactions. The elementary flows, which interrelate the economic units aggregated at different level, are the starting point. The SAM captures and shows the entire circular flow of income from its production to its distribution, its redistribution, and its expenditure. Most of the SAM, in particular, has been oriented toward an analysis of linkages between the structural features of an economy and the distribution of incomes and expenditures among household groups. Matrix T, which is introduced here, is the "core" of the Institution Household account.

The SAM can be considered an extension of the traditional input–output framework. This format adds some matrices, not included in the Leontief schema, which take into account the relationships between factorial distribution of income, income distribution to institutions, and the final demand. The introduction of accounts referred to institutions (households, private companies, government, and rest of the world) captures the link between factors of production and the institutions, which own the different factors of production. The secondary distribution of income is also introduced as the result of transfers between different Institutions, mainly between private institutions and the Government. The disposable income of institutions is the starting point for sustaining the final

demand. In particular the household, grouped into different socioeconomic groups, sustains the demand for consumption. The SAM can be considered not only as a database and an accounting tool, but in an even wider sense as a macroeconomic model that can be used both for structural analysis and for fiscal and expenditure policies simulations. It can also be used as the starting point for building computable general equilibrium (CGE) models. The literature on the SAM and CGE has been growing over the last few years at both theoretical and empirical levels. Some empirical applications have been carried out by the authors (Bottiroli Civardi and Targetti Lenti 1980, 1988, 1990, 2008; Civardi et al. 2010).

An innovative research agenda has been launched as a step toward the creation of Distributional National Accounts (DINA), whose aim is to decompose key National Accounts Aggregates (in particular household incomes) by percentiles of the population. This step will allow us to link deciles of personal income distribution to different "items" of value added (Bourguignon 2015). Piketty et al. (2016, 2017) proposed some guidelines in an attempt to construct a prototype distributional national account. Such a prototype could be "improved upon as more data becomes available, new knowledge emerges on who pays taxes and who benefits from government spending, and refined estimation techniques are developed just as today's national accounts are regularly improved" Piketty et al. (2017, p. 3). The authors argue that methodologically, their contribution is "to construct micro files of pre-tax and post-tax income consistent with macro aggregates." See also the results of a recent workshop organized by the Bertelsmann Stiftung, OECD and the High Level Expert Group on the Measurement of Economic Performance and Social Progress (Bourguignon 2015).

5 Concluding remarks

Traditional redistributive policies which transform market income in disposable income act, first of all, through the tax system by adopting progressive taxation not only on income but also on different types of wealth. The set of fiscal policies, suggested by Piketty's, for curbing income inequality, is the result of his model of "patrimonial capitalism." The increase in the tax burden must not be considered, as generally happens, as a measure of equalization, but rather as a tool to finance a significant expansion of social security and the redistributive transfer of income and subsides. It is necessary to reform the tax system by adopting progressive taxation not only on income but also on the different types of wealth: it would be especially necessary to introduce a progressive tax on estates and inheritances and to standardize the taxation of capital worldwide. A progressive wealth tax on a global scale, based on the automatic exchange of bank information, is suggested by Piketty not only as "useful utopia," but as a proposal to think about and discuss.

According to Atkinson (2015), the sources of inequality in the industrialized countries are today so many and multifaced that traditional fiscal and redistributive policies will not go to the root of the problem. Taxation must be considered only one tool, among many others, to increase social mobility. The latter asks for the introduction of more radical policies (ex-ante) than the redistributive (ex-post)

traditional ones (Atkinson 2015). Piketty's approach has a limited efficacy because it is effective only on the ex-post inequality level. Innovative and structural policies must be associated with traditional ones (Atkinson 2015). Social security and taxation are policies aimed at redistributing income and wealth between people. They are very important to reduce inequalities in the distribution of disposable incomes *ex-post*, but certainly they cannot reduce inequality in the distribution of market income, namely on the *ex-ante* income-generating process. This kind of inequality can decrease with policies aimed at changing the composition of the ownership of endowments by different group of households (matrix **S**). The "sharing of capital" could be one of these policies. Social mobility and better opportunities for poor people can be obtained by improving the quality of schools and easing access to all. These policies have a positive impact on the endowment of human capital of different household groups.

Atkinson recommends ambitious new policies in at least five areas: technology, employment, social security, the sharing of capital, and taxation (Atkinson 2015, p. 15). In our opinion, the effects of some of these policies on personal income distribution could be better assessed and evaluated within an analytical framework which links the individual/household market income to functional distribution. The framework introduced here seems to be a suitable tool to take into account the fact that personal income distribution is inextricably tied up with different sources of inequality in the distribution of national income such as what comes from the distribution of endowments and of individual/household entitlements (matrix **S**) but also what comes from the institutional and productive structures (matrix **Y**). Some empirical studies show that "labour market institutions exhibit significant correlations with the distribution of income across countries and over time" (Checchi and García-Peñalosa 2008, p. 25). Stronger institutions (strong unions, high employment protection legislations, support to unemployed) are, however, often associated with higher unemployment so their final impact is an open question. Another study uses two different datasets on income shares of the top 10%, applying the same empirical model, to estimate the effect of market institutions on income inequality. In this case the market institutions are specified as the degree of "market freedom." The results are for the two datasets so that the authors claim that with reference to: "the paper's research question about whether market institutions lead to greater income inequality, the conclusion is ambiguous" (Holcombe and Boudreaux 2016, p. 276). In any case such institutional approach can be very useful in the design of policies aimed at assuring more equality not only in outcomes but also in opportunities, not only in industrialized countries such as UE or USA, but also in developing ones. Our approach can be a starting point for collecting data in a way more suitable to capture and to enlighten the links between household endowments and value added.

Compliance with ethical standards

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