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Mixing Under Vibrations in Reduced Gravity

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Abstract The aim of this study is to analyze the physical mechanism by which vibrations affect the mixing characteristic of two initially stratified miscible fluids. The translational periodic vibrations of a rigid cell filled with different mixtures ($Sc = 7125$) are considered. The vibrations with a constant frequency are imposed parallel to the initially planar interface. The ability of the applied vibrations to enhance the flow is examined. At the early stage after imposing the vibrations the Kelvin-Helmholtz instability is observed in absence and at low level of gravity. Later in time the system undergoes a transition to Rayleigh-Taylor instability. With increasing of gravity level the life-time of Kelvin-Helmholtz instability is decreased. We found the critical value of Gr above which this instability do not developed.

Keywords Vibrational convection · Mixing · Instability

Introduction

Transport phenomena, such as heat and mass transfer, are important in energy production technologies. There are many analogies among heat and mass transport processes, both involve molecular and convective transport within the physical system. Here the attention is focused on the mass transfer under vibrations. One of the important industrial applications is mixing in liquids (Nishimura et al. 2004). Two miscible liquids when brought into contact inside a container will mix and eventually become homogeneous via molecular mass diffusion. Depending on the volumes of the liquids, spatial homogenization by random molecular motion occurs over a long time scale since, typically, the binary diffusion coefficient for liquids is on the order of $10^{-10}\ m^2/s$.

Vibrations, acting on density non-uniformity may essentially influence on the fluid dynamics and mass transport (Siddavaram and Homsy 2006; Chang and Alexander 2007). The microgravity environment onboard ISS is characterized by small mean accelerations, which are $10^{-5} - 10^{-6} g_0$, and fluctuations that are two or three orders of magnitude above mean. Interacting with density and concentration gradients, these g-jitters may cause convective flows. In weightlessness, it is an additional way of transporting heat and matter similar to thermo- and solutocapillary (Marangoni) convection.

Formulation of the Problem

Here the results on numerical modeling of vibrational convection under reduced gravity are presented. However, the physical properties and geometry of the system are similar to those, which will be used in ground experiments. The cubic cell of $L = 10\,mm$ length is filled with two miscible liquids: both liquids consist of the same components, water and ethanol, in different proportions. The layer of heavier/denser liquid (51% of water) is at the bottom and lighter (5% of water)

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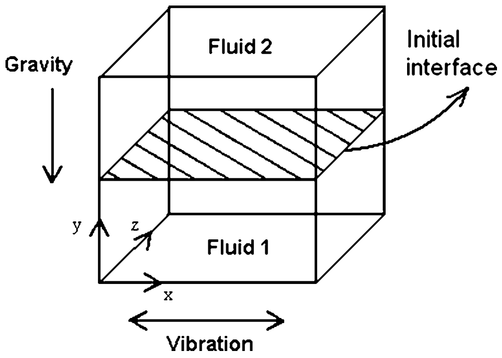


Fig. 1 Geometry of the system

is on the top (gravitationally stable configuration). The system is isothermal. The vibrations are imposed parallel to the horizontal interface, see Fig. 1. Since it is assumed that the two fluids are miscible, there is no discontinuity in the concentration at the interface ($y = 0.5\,L$). Because we assumed that interface is sharp, the width of the region over which the initial concentration changes from 1 to 0 is assumed to be $0.03\,L$.

This system is subjected to periodical oscillations of the vessel along the x-axis according to the law $Af(\tilde{\omega}t)$. Here A is vibration amplitude and f is a periodical function:

$$\langle f \rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\tau)d\tau = 0.$$

The exact type of function f is not specified; it can be $cos(\tilde{\omega}\,t)$ or $sin(\tilde{\omega}\,t)$ or any other periodical function with zero mean value. The imposed excitations induce the oscillatory acceleration that is directed along the x-axis:

$$\hat{F} = a_{os} f''(\tilde{\omega}\,t), \quad a_{os} = A\tilde{\omega}^2.$$

It is assumed that density difference between liquidsis small, i.e. $\Delta\rho/\rho_0 << 1$ and that the Boussinesq approximation is valid, $\rho = \rho_0(1 + \beta_c(C - C_0))$. Here $\beta_c = 1/\rho_0(\partial\rho/\partial C)$ is the solutal expansion coefficient. It is known that for aqueous solutions of alcohols, the viscosity and the diffusion coefficient are strongly depending on composition. Therefore the viscosity and the diffusion coefficients are considered as function of concentration (Pratt and Wakeham 1974):

$$\begin{aligned}\nu(c) = \big(&90.092c^6 - 264.46c^5 + 312.43c^4 - 192.57c^3 \\ &+ 54.93c^2 - 2.486c + 3.0901\big)\cdot 10^{-6}\big[m^2/s\big] \\ D(c) = \big(&-11.561c^5 - 13.101c^4 + 69.684c^3 \\ &- 28.216c^2 - 4.963c + 10.568\big)\cdot 10^{-10}\big[m^2/s\big].\end{aligned}$$

All reference values, noted by subscript "0" are taken at the equilibrium conditions, i.e. the mean values for two mixtures at the initial state. These values of physical properties, used in calculations, are listed in Table 1.

In the stated above assumptions the governing equations in moving coordinates system will be written as:

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} &= -\frac{1}{\rho_0}\nabla P + \nabla(\nu\nabla\mathbf{V}) \\ &\quad + g_0\beta_c C\mathbf{n} + a_{os}\beta_c C f''(\tilde{\omega}t)\mathbf{s}, \qquad (1) \\ \frac{\partial C}{\partial t} + \mathbf{V}\cdot\nabla C &= \nabla(D\nabla C), \\ \mathrm{div}\,\mathbf{V} &= 0.\end{aligned}$$

Here $\mathbf{V}$ is the vector of velocity; $\mathbf{s} = (1, 0)$, $\mathbf{n} = (0, -1)$ are unit vectors; $P = P' - g_0\rho_0 y$ is the pressure; D is the molecular diffusion and ν is the kinematic viscosity. The boundary conditions are no-penetration for the concentration and no-slip condition for the velocity on solid walls. The initial conditions are: $\mathbf{V} = 0$, $C = C_0$, for $0 \leq y < 0.5L$ and $C = C_0 - \Delta C$ for $0.5L \leq y < L$.

Hereafter we will consider high frequency vibrations, i.e. period of oscillations is smaller than viscous ($\tau_{vis} = L^2/\nu$) and diffusion ($\tau_D = L^2/D$) times. The viscous time is at least two order of magnitude smaller than the diffusion time. Thus, the limitations for the frequency is based on the viscous time. Besides, the oscillation amplitude is assumed to be finite i.e.

$$\Omega = \frac{\tilde{\omega}L^2}{\nu} \gg 1, \quad \frac{A\beta_c\Delta C}{L} \ll 1, \qquad (2)$$

here Ω is the dimensionless frequency, Δc is the initial concentration difference. Then all physical quantities may be presented as the superposition of a slow (the characteristic time τ_{vis} is large with respect to the vibration period) and fast (the characteristic time $1/\omega$ is of the order of the vibration period) parts (Zenkovskaya and Simonenko 1965; Gershuni and Lyubimov 1998). The variables ($\mathbf{V}$, P, C) take form:

$$\mathbf{V} = \overline{\mathbf{V}} + \mathbf{V}_f, \quad P = \overline{P} + P_f, \quad C = \overline{C} + C_f \qquad (3)$$

where ($\overline{\mathbf{V}}$, $\overline{P}$, $\overline{C}$) are the slow components and ($\mathbf{V}_f$, P_f, C_f) are the fast components.

Table 1 Physical properties of the water/ethanol mixture

ν_0 (m^2/s)	β_c	D_0 (m^2/s)	ρ_0 (kg/m^3)	Sc
$3.75\cdot 10^{-6}$	0.189	$3.35\cdot 10^{-10}$	$0.902\cdot 10^3$	$7.125\cdot 10^3$

The choice of the average approach is due to the nature of the problem. Because several very different time scales are involved in the process, the complexity of simulations can be underestimated. To properly resolve the transport phenomena one should perform calculations with a time step smaller then any of the characteristic physical times, i.e. less than viscous time or period of imposed oscillations. However, mass transport is significantly slower than the viscous process and its characteristic time is determined by diffusion time, $\tau_D >> \tau_{vis}$. The calculations should cover a long period of physical time, at least by the order of magnitude the final time should be comparable with diffusion time. If someone would like to perform parametric study of physical phenomena in a reasonable CPU time, it might be a problem. Thus instead direct numerical simulations the averaging approach is used, which allow the use of relatively large time steps.

Let us look in the principal of averaging approximation (Zenkovskaya and Simonenko 1965; Gershuni and Lyubimov 1998). Taking into account the relations (2), the viscous terms in the equations for the fast velocity can be neglected and the fast flow is considered as potential. Then for the uniform vibrations the governing equations in the framework of the average approximation can be written as

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nabla \left(\frac{\nu(c)}{\nu_0} \nabla \mathbf{v} \right)$$

$$+ Gr\, c\, \mathbf{n} + \frac{Ra_{vib}}{Sc} ((\mathbf{w}, \nabla) \nabla \Phi) \qquad (4)$$

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c = \nabla \left(\frac{D(c)}{\nu_0} \nabla c \right),$$

$$\text{div } \mathbf{v} = 0, \quad \mathbf{w} = -\nabla \Phi - c\mathbf{s}, \quad \text{div } \mathbf{w} = 0.$$

Here $\mathbf{v}$, c are velocity and concentration of the *mean* flow (the corresponding capital letters denoted dimension quantities). Hereafter the overbar is dropped. $\mathbf{w}$ is the amplitude of the fast (pulsating) velocity and Φ is the amplitude of fast (pulsating) pressure. The last term in momentum equation describes the vibration force. The scales of length, time, velocity, pressure, and concentration are L, L^2/ν_0, ν_0/L, $\rho_0 \nu_0^2/L^2$ and ΔC.

Boundary conditions are: (a) no-penetration for the concentration: $\partial c/\partial \mathbf{n} = 0$; (b) no slip condition for the mean velocities at the rigid walls $\mathbf{v}$, and for the fast velocity $\mathbf{w_n} = 0$. The initial conditions are: $\mathbf{v} = 0$, $c = 1$, for $0 \leq y < 0.5$ and $c = 0$ for $0.5 \leq y < 1.0$

The problem is governed by three parameters: vibrational Raleigh number, $\mathrm{Ra_{vib}}$, the Schmidt number, Sc, the Grashof number, Gr or its analog, the Raleigh number, $Ra = Gr \cdot Sc$

$$\mathrm{Ra_{vib}} = \frac{(A \tilde{\omega} \beta_c \, \Delta C \, L)^2}{2 \nu_0 D_0}, \quad \mathrm{Sc} = \frac{\nu_0}{D_0}, \quad \mathrm{Gr} = \frac{g_0 \beta_c \Delta C L^3}{\nu_0^2}. \qquad (5)$$

The target of this study is to examine the mixing of realistic fluid, when the Schmidt number is very large, $Sc = 7.125 \cdot 10^3$ (see Table 1). For this purpose, we will restrict our study by 2D calculations and converse the problem into stream-function vorticity formulation. For the mean fields a stream function, ψ, such that $v_x = \partial\psi/\partial y$, $v_y = -\partial\psi/\partial x$ and a vorticity, $\omega = \partial v_y/\partial x - \partial v_x/\partial y$ are introduced. The equations Eq .4 become

$$\frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega = \nabla \left(\frac{\nu(c)}{\nu_0} \nabla \omega \right) - \mathrm{Gr} \frac{\partial c}{\partial x}$$

$$+ \frac{\mathrm{Ra_{vib}}}{\mathrm{Sc}} \left(\frac{\partial c}{\partial y} \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial c}{\partial x} \frac{\partial^2 \Phi}{\partial x \partial y} \right), \qquad (6)$$

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c = \nabla \left(\frac{D(c)}{\nu_0} \nabla c \right),$$

$$\nabla^2 \psi = -\omega, \quad \nabla^2 \Phi = -\nabla c \, \mathbf{s}.$$

The boundary conditions could be written in the form:

$$x = 0, 1 : \psi = \partial_x \psi = 0, \;\; w_x = 0, \;\; \partial_x c = 0, \;\; \partial_x \Phi = -c;$$

$$y = 0, 1 : \psi = \partial_y \psi = 0, \;\; w_y = 0, \;\; \partial_y c = 0, \;\; \partial_y \Phi = 0$$

Initially fluids are at rest, then $\psi =$, $w = 0$, $c = 1$, for $0 \leq y < 0.5$ and $c = 0$ for $0.5 \leq y < 1.0$

A finite-difference method in both directions is utilized. The time derivatives are forward differenced and for the convective and diffusive terms are central approximated. The Poisson equation for the stream function ψ and for the amplitude Φ of fast pressure were solved by introducing an artificial iterative term, analogous to the time-derivative one. ADI method is used to solve the time-dependent problem for vorticity, the concentration, the pulsatory pressure amplitude and the stream function. More detail about numerical procedure one may find in (Gaponenko et al. 2006).

Results

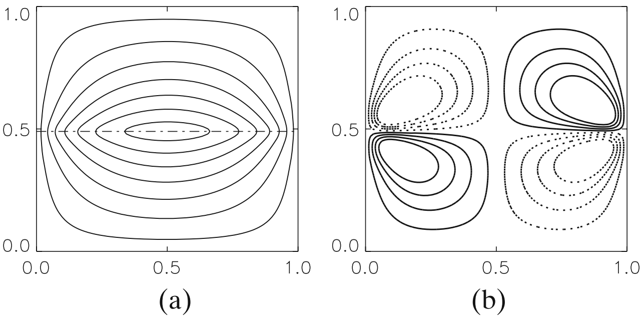
The problem is governed by three parameters, and two of them are fixed, $Sc = 7.125 \cdot 10^3$, $Ra_{vib} = 5.597 \cdot 10^7$. This value of Ra_{vib} corresponds to the following parameters of the experiment: $f = 5\,Hz$, $A = 1\,cm$, $\Delta C = 0.46$. The third parameter, Gr, is changing. The gravity level, expressed via the Grashof number, is varied from

$g = 0$ until $g = 0.165\, g_0$. Different flow regimes were observed depending on the ratio of Gr and Ra_{vib}.

Vibrations act on the non-uniformity of density and generate an oscillatory convection, which starts at the cross-section of the interface with solid walls. The net flow, see Eq. 3 consists of one vortex, which occupies the whole system and half of the period rotates to the one side and another half of the period to the opposite side, see Fig. 2a. For relatively strong external excitations (see Eq. 2) due to inertia the fluid cannot immediately return to its initial position and convective mean flow is created. The vibrations cause mean flow in such a way, that heavy/denser liquid moves up along the both solid walls, $x = 0$ and $x = 1$ and less dense moves down. Two weak vortexes with the opposite direction of the circulation are formed in each fluid, see Fig. 2b. Further development of the mass transfer is strongly depends on the gravity level.

Absence of Gravity: $Gr = 0$

Here we will discuss the fluid behavior in the upper liquid until it is not stated otherwise. From the very beginning the concentration front moves along the solid walls, creating a head, see Fig. 3 ($t = 1 \div 20$). The leading part of the front (we consider isolines $c = 0.3$) expands and rolls up, and a denser liquid intrudes into a less dense region, see Fig. 3 ($t = 50$). The flow resembles a Kelvin-Helmholtz instability which is observed in free shear layers and gravity currents. The instability appears almost immediately after imposing vibrations and persists during certain time interval. For example, for Gr=0 it exists during 30–40 viscous times. The horizontal solid wall impose constrains on the approaching the concentration front and it turns inside the cell, creating another flow organization, when the denser liquid is on top of the less dense, Fig. 3 ($t = 75$). This scenario is a kind of the Rayleigh-Taylor instability, which is observed in ground conditions (when heavy liquid is on the top). Further the denser liquid starts to descend, and it splits the region of low concentration in two zone, see Fig. 3 ($t = 200$). For this set of parameters the amplitudes of the velocity and of the concentration oscillations slowly decay with time. The flow is not exactly similar in the upper and lower fluids. The break of symmetry is related to the non-linear dependence of the viscosity and the diffusion coefficients on the concentration. Note, that during relatively long time over which Kelvin-Helmholtz and Rayleigh-Taylor instabilities develop, the role of diffusion is undetectable. The major part of the interface remains almost as sharp as in the initial stage.



(a) (b)

Fig. 2 The flow structure at the very beginning ($t = 1$); (a) Snapshot of the full flow; (b) Mean flow, $g = 0$

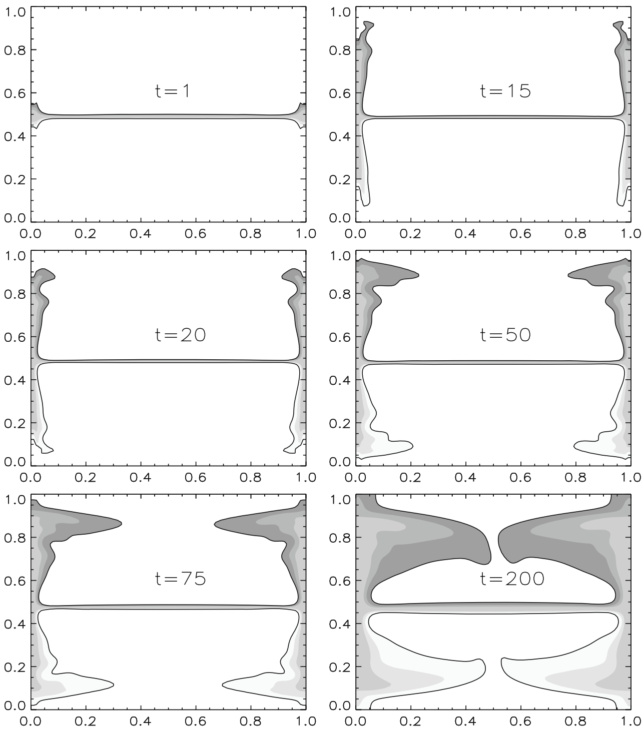


Fig. 3 Evolution of the concentration front; The leading profiles are $c = 0.3$ and $c = 0.7$ at upper and lower liquid correspondingly. Non-shadowed space corresponds to the concentration close to zero (one) in upper (lower) fluid

$Gr \neq 0$

In the presence of non-zero gravity, the flow dynamics is changing. The concentration front rolls up with a smaller velocity. Let us compare the top point of isolines $c = 0.3$ when $t = 50$: for $Gr = 0$, $z_c = 0.97$; for $Gr = 10^2$, $z_c = 0.96$, for $Gr = 10^3$, $z_c = 0.86$, and for $Gr = 10^4$, $z_c = 0.58$. In the latter case the Kelvin-Helmholtz instability does not set-in. For this case, $Gr = 10^4$, the ratio of Rayleigh numbers is about $Ra/Ra_{vib} = 2$, and buoyancy force is dominated. When the classical Rayleigh number becomes comparable with the vibrational one, the Kelvin-Helmholtz instability is not observed. Thus the vibrations appear to drive this instability.

We found that for $Ra_{vib} = 5.6 \cdot 10^7$ the Kelvin-Helmholtz and Rayleigh-Taylor instabilities exist for $Gr \leq 1500$. In the interval $1500 \leq Gr \leq 5000$ only Rayleigh-Taylor instabilities was observed. For $Gr \geq 5000$ mixing is controlled by diffusion.

To conclude, the effect of external vibrations on the flow organization in two immiscible fluids was investigated. For small Grashof numbers the Kelvin-Helmholtz and Rayleigh-Taylor instabilities were observed. The first one produces vortices near solid walls which can serve as a stirring mechanism to promote local mixing. The second one strongly affects on large scale mixing.

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