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On the Kond style of Islamic tiling: a study in practical Islamic geometry

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Abstract This paper is a contribution to the studies of practical Islamic geometry. The Kond style is one of the principal Islamic tiling styles in Anatolia, Iran and Central Asia. Its fundamental features are described and discussed as a basis of the decagonal quasiperiodic tiling in Maragha (Iran) and its follow-ups. It was richly employed in the stone carved reliefs of central Anatolia. Construction principles of non-decagonal Kond tiling are derived in the paper and examples from Iran and Uzbekistan described and illustrated. Interpretation of the Persian construction principles for the Kond patterns as well as a tentative explanation of rare curvilinear Kond patterns is given.

Keywords Islamic tiling · Pattern geometry · Kond style · Decagonal quasiperiodic tiling · Non-decagonal Kond tilings · Turkey · Iran · Uzbekistan

1 Opening: outline of the problem

Every Persian book on ornamentation and architecture starts with the definition of three fundamental styles of ornamentation, called Kond, Tond and Shol in professional Farsi. Examination of old buildings shows that these styles date back centuries—they were already used by the Seljuk artisans both in the Persian and the Turkish cultural areas, known as the Great Seljuks of Iran and the Seljuks of Rum, as early as before approximately the year 1200.

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The current author feels a special attraction to one of these styles, the Kond style with a bit unusual ornament elements and interesting achievements in pattern geometry, the other two styles being a bit more 'conventional' although some great works were produced in them and, especially in the Ottoman Turkish circles, they were combined with the Kond style.

All references start with a presentation of Kond tiles with pentagonal to decagonal symmetry (Fig. 1). The fundamental tile is a small regular pentagon, with radial organization based on multiples of 72° and angle between adjacent straight sections of the perimeter equal to 36°. In the Kond tiling, they are joined by twice-constrained, concave decagons ('butterflies', Makovicky 1992; 'bow-ties', Rigby 2005) and by lozenges truncated at the acute ends, which actually are two partly overlapping pentagons restored to a full lozenge shape by adding two small deltoids. The deltoids ('kites') fit into the recesses of the latter tiles, and into recesses of the last and largest element of the Kond tiling, the 10-fold rosettes. Here, the deltoids 'decorate' the zig-zagging perimeter of the rosette. This is the mode of description Makovicky (1992) chose for the quasiperiodic tiling at Maragha, NW Iran. All these tiles have angles adjusted to fit those of the fundamental pentagons (traditionally given as multiples of 36°).

The funeral tomb-tower of Maragha in NW Iran is one of the earliest known monuments of the Kond style. It turned out to be the first known quasiperiodic tiling (Makovicky 1992) which was then copied, with modifications, again and again in the Iranian–Turkish cultural area (Makovicky 2008, 2015). Further references on this subject are (Rigby 2005; Lu and Steinhardt 2007; Makovicky 2007, among others). In the case of Maragha (Fig. 2), the large tiles are not colored so that colors cannot influence the interpretation as it happens for nearly all later Kond

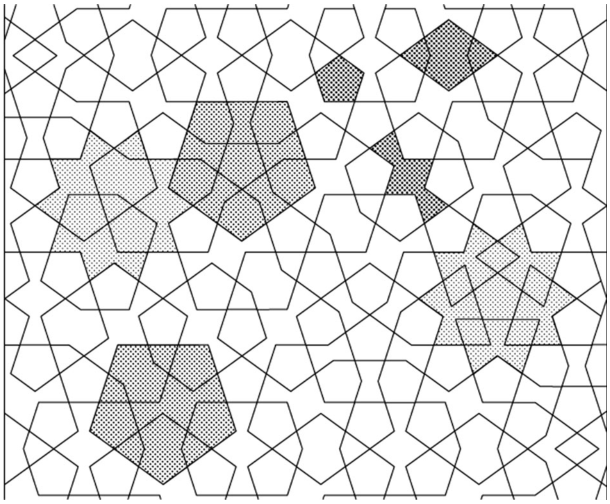

Fig. 1 Overview of fundamental Kond tiles (pentagons, butterflies and composed lozenges *shaded*) and composite Kond tiles (composite pentagon and composite 10-fold star in original *m*—symmetrical version and in the symmetrized 5*m* version). Redrawn from Makovicky (1992) with modifications

Fig. 2 A detail of large-scale wall tiling on the walls of the funeral tower Gunbad-e-Qabud at Maragha, NW Iran

occurrences of the mosaic kind. That is why Makovicky (1992, 2008) joined the truncated lozenges and the two fitted deltoids into one decorated tile. This amalgamation expresses the function of the lozenge tile in the pattern, and the lozenge tile occurs only in this form. The same holds for the 10-fold rosettes decorated on the perimeter by the small deltoids in order to fit in the pattern—these rosettes never occur without this wreath. In the pattern from Darb-e-Imam shrine and the Friday Mosque (Esfahan) (Fig. 3), the small deltoids are black while the larger tiles are light (white and yellow), thus breaking up the just described units.

The Maragha experience brought about one more truth—the 10-fold stars, employed in later applications and accepted by the Persian canon as primary elements, actually represent secondarily emptied low-symmetry assemblages which originally filled the visually inconspicuous 10-fold star-like outlines in the pattern. The rotational fills with fivefold-symmetry, which can replace these assemblages in the stars (intensely used in, e.g., the Karatay Medresa in Konya, Turkey (Rigby 2005; Makovicky 2015), and the fivefold rotational fills of composite pentagons in the Maragha-type tiling (departing from the classical Kond set, in which the fill of these pentagons is only mirror-symmetrical) *are secondary elements*, based on a fact that in the quasiperiodic designs both elements with the original mirror-symmetrical fill are freely rotatable without destroying the pattern continuity. The freely rotatable composite elements may be still present in a periodic design, as recently illustrated by Castéra (2016) for some patterns from Isfahan, but my experience suggests that this can happen only when a full set of (filled) Kond tiles is present. Thus, a full Kond set is dynamic but for that we need the original, non-symmetrized composite pentagons and 10-fold stars to be present (in sufficient frequency).

The objections and doubts raised concerning the quasiperiodic character of the Maragha tiling have following problems: they forget (a) that even in a large tile patch, it is fully convertible to Penrose tilings of at least two different kinds (Makovicky 1992; Saltzman 2008),

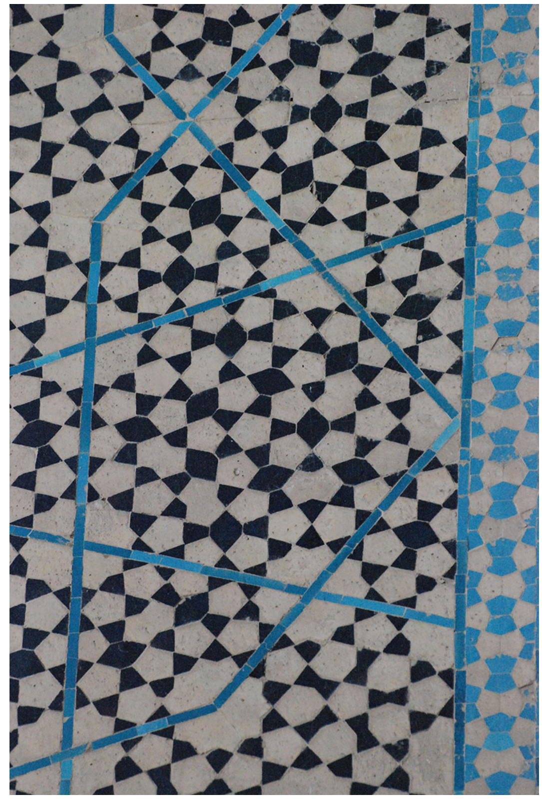

Fig. 3 A detail of large-scale wall tiling from the iwan area of the Friday Mosque in Isfahan, Iran

(b) that the tiling is dynamic and, contrary to usual periodic tilings, it tacitly includes (or at the same time contains) *all rotations* of the rotatable elements (for such analysis, all pentagons and all 10-fold stars should be back-substituted by the original tile assemblages with low local symmetry), and (c) the rotation-symmetric fills of composite pentagons and of 10-fold stars, as well as the void 10-fold stars, are *secondary ornamentations* which either symbolize free rotations or should be back-substituted, according to the investigator's taste.

As already mentioned, the Kond tiles had a number of names given to them by non-Iranian authors. Obviously, Makovicky (1992) calls them a Maragha tile set (with the above specified little amalgamations of minor tiles), the other colorful names were given by Rigby (2005) to the tiles from the Karatay Medresa, whereas terms as 'bow-tie' relate more to the culture circle of the investigator than to the culture of the country. There is a fundamental difference between the true Maragha tile set from a Penrose-type quasiperiodic tiling and the general Kond set: the Maragha tile set has corner markings derived from Penrose tile set markings in order to induce and maintain quasiperiodicity (Makovicky 1992). Of course, the constructors did not know it and we do not know their way of construction but the Maragha tiling faithfully reproduces a cartwheel pattern of Penrose tiling, with only one or two places where it was purposefully adjusted to the architecture of the tomb-tower. Makovicky (1992, 2008) prefers a model of direct tile composition, whereas Lu and Steinhardt (2007) preferred an indirect way, via 'girih' tiles. All known later applications are copies of the Maragha tiling with the ornamental modifications (symmetrization or omission of some elements) (Makovicky 2008) and with a full set of tile types present.

What is the origin of these remarkable and, let us say, unique Kond tile set? After having demonstrated in 1992 that there is a 1:1 correspondence between the original Penrose tiling of pentagons and stars, respectively fragments of stars, and the Maragha tiling (Fig. 4), I cannot escape a feeling that the Seljuk ornamentalists originally explored the agglomeration of pentagons, creating Penrose stars (like also Kepler and others did) and they did not find it exciting enough until somebody got the idea to inscribe smaller pentagons into Penrose's edge-sharing pentagons. The resulting tiling of corner-sharing pentagons turned out more exciting and full of new interesting polygons—the Maragha (Kond) tiling was born. And this process depended directly on the pentagonal/decagonal geometry of the tiling.

Geometry, rather than symmetry: if we do not empty or symmetrize the infill of 10-fold stars or composite pentagons in the Maragha tiling (as discussed above), there is very little fivefold or 10-fold symmetry present in this tiling. Fivefold and 10-fold outlines are there but some of the 'symmetry' observed may disappear when we alter one type of Penrose tiling into another (e.g., pentagons and stars into kites and darts), and other type of 'symmetry' may appear instead. And the Kond tiling is the best illustration of it. Penrose's star alters into just one orientation of the 'butterfly' surrounded by pentagons (we can do it in any of the five possible orientations but only one at a time). Furthermore, a composite lozenge arises in the Maragha (Kond) tiling between two stars, or star fragments, of Penrose tiling and it actually overlaps two alternative and potentially flipping positions of Penrose's pentagon. The geometric origin of the decagonal cartwheel Maragha tiling hosts a decagonal disc with only mirror-symmetrical inner symmetry, and all Maragha pentagons have internally only mirror symmetry, unless artificially altered into a rotation-symmetrical version. Symmetry of a cartwheel form of quasiperiodic tiling (Grünbaum and Shephard 1987) is a rotation symmetry around its center. So, the 'symmetry' is a matter of bulk, as witnessed by diffraction on corresponding quasiperiodic crystal structures and almost never a matter of a small patch.

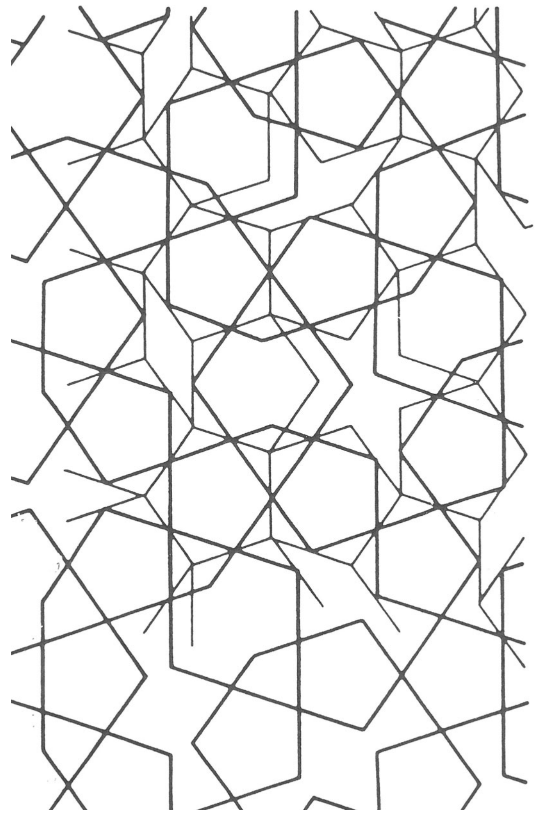

Fig. 4 Correspondence between the first form of Penrose tiling and the Maragha tiling. Modified from Makovicky (1992)

2 Kond tiling applied to other symmetries

The decagonal patterns prevail among the Kond patterns. However, travelling among the architectural localities with the Kond tiling, soon we observe that especially the emptied stars of the Kond-style patterns have rotation symmetries stretching from fourfold or fivefold up to the maximum observed, that of 12-fold. This modifies geometry of the pattern and shape of individual tiles. Based on observations of patterns with these symmetries, a following construction model is suggested, which follows closest the principles of the observed patterns.

The net of edge-sharing polygons which will be decorated with pentagons in their corners is drawn, including the 'hour-glass' gaps between the polygons. According to well-defined examples from architecture (illustrated in the next sections), the adjacent pentagons which decorate the net are vertex-interconnected, with the common pentagon vertices lying on the join of the adjacent net vertices (Fig. 5). This divides the perimeter of each decorating pentagon into in-polygon and out-of-polygon portions, the inner one consisting of two perimeter segments, the outer portion consisting of three segments. The only regular pentagons are those decorating vertices of large decagons in the derivatives of Maragha tiling. All other cases have segments of the inner and of the other portion different to an increasing degree when the multiplicity of large polygons increasingly differs from ten.

To quantify the geometry of pentagons decorating the vertices of large polygons, each pentagon is considered as inscribed in a circle which runs through all its vertices. Again, inspection of real examples shows that it is a close approximation to real patterns, if not an exact approach. If the inner angle of a large polygon is $2\alpha_{inner}$ (for 2 segments of the pentagon), the pentagon (segment) edge has length $s_{inner} = 2r \sin (\alpha/2)$; r is the radius of a circle circumscribed to the pentagon. Remembering that there are three edges of pentagon which are outside the large N-polygon, so that the outer angle of the large polygon is $3\alpha_{outer}$ and keeping a constant r value ($2r$ is the edge of the large polygon) but using $\alpha_{outer}/3$ we obtain Table 1 as a measure of regularity of decorating pentagons.

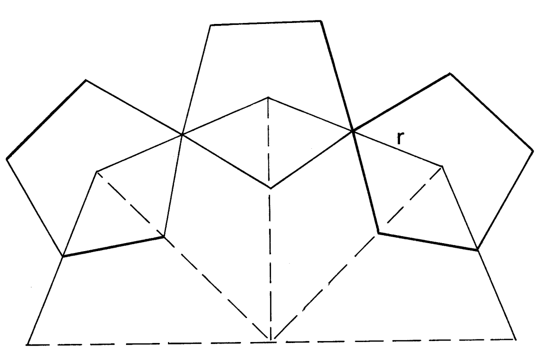

Fig. 5 Construction of Kond tiling based on large octagons as an example of the non-decagonal case. Details in the text

We can see that multiplicities N of the large polygons at, or within, the $N = 10 \pm 2$ (octagon–dodecagon) limits yield tolerable distortions of the ornamental pentagons, below 10 % of edge length. When two different polygons are combined, however, a compromise is required. For example, the very exact uncolored dodecagon–octagon (quoted as 12–8) pattern from the front wall of the Kalyan mosque (Fig. 6) appears to obey the $N = 8$ rules, whereas the $N = 12$ geometry had to be adjusted to them.

A recurrent configuration in these Kond patterns with different symmetries is a rectangular group of four pentagons enveloping a composite rhomb starting from its acute corners (Fig. 7). This group *joins* two N-stars of the pattern (those at which the acute corners of the central lozenge point). In the decagonal patterns, the pentagon edges which are parallel to the elongation of the rectangular group are strictly parallel, and the short side of the rectangle consists of oblique edges separated by an indentation between two pentagons (Fig. 7). This configuration interconnects two (filled or empty) 10-fold stars. When we look at the 12–8 combination from the Kalyan mosque perimeter (Fig. 6), the rectangular configuration is inflated on the long edges and the protrusion of pentagon edges along the short side was reduced. Although this rectangular configuration lies squeezed between two 12-fold stars in this pattern, *it joins* two eightfold stars. On the contrary, such a configuration squeezed between two ninefold stars in the 12–9 tympanum pattern of the Kalyan Mosque (and also of the Nadir Divanbegi Khanaka) in central Bukhara (Fig. 8) joins two 12-fold stars and has its width reduced in the waist portion and the protrusion of pentagon edges along the short side of the rectangle is enhanced.

Table 1 Geometry of pentagons decorating large polygons of a *Kond*-style tiling

N	Central angle	α_{inner}	α_{outer}	s_{inner}	s_{outer}
4	45	45	90	0.7654 r	1.4142 r
5	36	54	84	0.9080 r	1.3383 r
6	30	60	80	1.0000 r	1.2856 r
7	25.7	64.3	77.13	1.0643 r	1.2468 r
8	22.5	67.5	75	1.1111 r	1.2175 r
9	20	70	73.33	1.1472 r	1.1943 r
10	18	72	72	1.1756 r	1.1756 r
11	16.35	73.65	70.9	1.1987 r	1.1600 r
12	15	75	70	1.2175 r	1.1472 r

'Central angle' from the center of the N-polygon is the angle subtended by a radius of one of the pentagons, whereas the 's' lengths are those of the pentagon's perimeter edge, respectively inside and outside of the polygon on which the pentagon resides

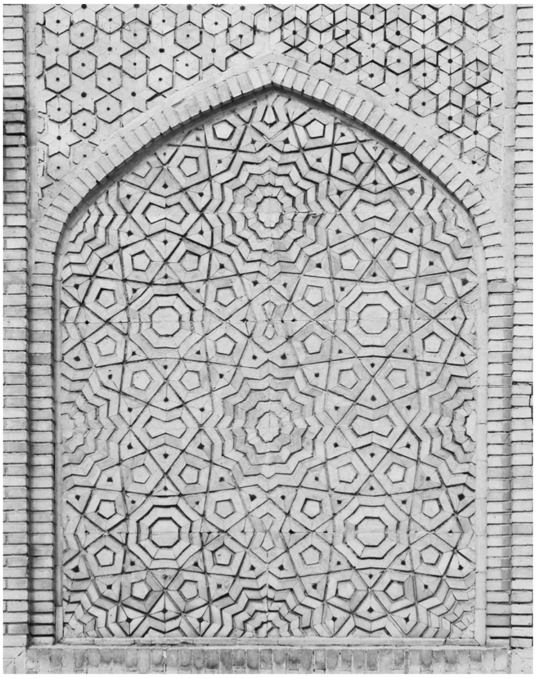

Fig. 6 A dodecagon–octagon '12–8' pattern of uncolored formatted tiles from the front wall of the Kalyan mosque in Bukhara, Uzbekistan. Details are in the text

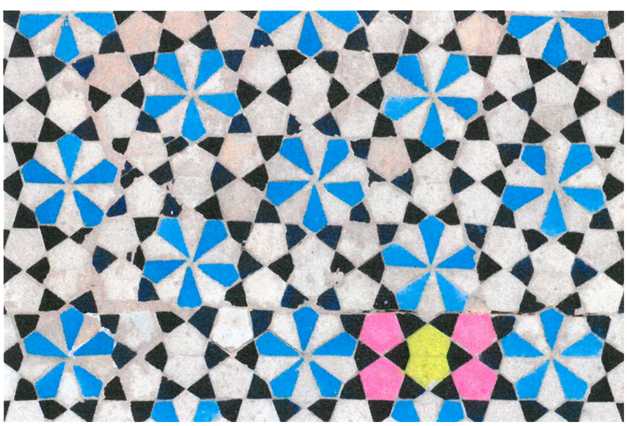

Fig. 7 A decagonal Kond *cmm* pattern from Isfahan with the configuration of 'four pentagons and a lozenge' colored

These conspicuous configuration variations are caused by rotation of pentagons. On the short side of the 'four pentagons and a lozenge' rectangle the two adjacent pentagons belong to the wreath which surrounds an *N*-fold star situated beyond the rectangular configuration in the direction of the long axis of the lozenge (rhomb). The pentagons of this wreath are oriented radially around this star and their angular deviation from one another is determined by two times the 'central angle' from Table 1. Only for $N = 10$ (central angle 36°) the adjacent pentagons have one edge each parallel to the long side of the above rectangle. For higher *N* values the central angle is smaller and the orientations of somewhat

Fig. 8 Tympanum of the Kalyan Mosque in central Bukhara, Uzbekistan, with a combination of large 12-fold and 9-fold stars surrounded by vertex-sharing pentagons. Details are in the text

distorted pentagons are less divergent so that the edges exposed on the long side of the rectangle comprise a concave corner. For the *N* values smaller than 10, divergence of pentagon orientations is larger and the long edges of the rectangular configuration bulge out.

For the 12–8 panel from the Kalyan Mosque, Bukhara (Fig. 6), there are eightfold stars in the extension of the rectangle and the divergence between the two halves of the bulging long edge of the rectangular configuration is about 14°. The pentagons are rotated by 4° more than in a decagonal pattern so that their exposed edges should comprise 8°. However, the above described distortion for $N = 8$ changes the shape of the pentagon and the ideal angle of 72° between two adjacent edges is altered because of the edge modifications to (measured) 76°, producing additional 8° of edge rotation. The result, calculated 16° of convex divergence is in good agreement with the measured 14°, considering that it is a tiling of independently fired and subsequently arranged tiles. In the 12–9 tympanum of the Kalyan Mosque in Bukhara, there are 12-fold stars at the short edges of the rectangular four pentagon + rhomb configuration and consequently the long edges are slightly concave (Fig. 8). The scheme fails for the depicted 8–4 pattern (Fig. 9) because the pentagons are 180° rotated against the present scheme, giving a strong constriction of the 'four pentagons and a lozenge' rectangle between two adjacent fourfold stars.

If we redraw the large polygons of the existing combined tilings, using the pentagon centers as vertices (try it yourself in Fig. 8), there are hourglass 'leftovers' between edge-sharing polygons. They are centered on the lozenges of the original pattern—a scheme very much like the M2 tiling of Makovicky (1992, his Fig. 10), equal to the 'girih

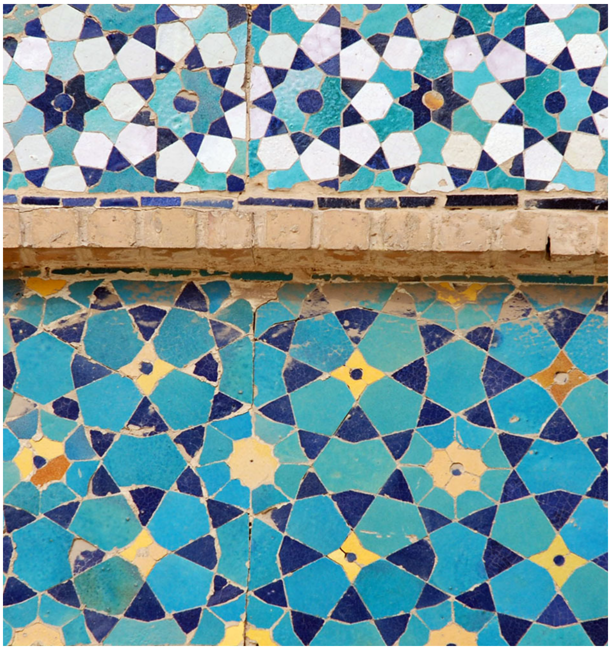

Fig. 9 A strip of pseudo-Kond tiling, a '8–4' pattern from the external wall of the Kalyan Mosque, Bukhara. Analysis is in the text

tiling' of Lu and Steinhardt (2007, their Fig. S6). The presence or absence of the hourglass interspaces in the packing of two types of polygons sharing common edges is a quick check whether a Kond pattern of the type described here can be constructed by a given combination of N_1 and N_2 polygons. This explains immediately the failure of the 8–4 scheme and the clustering of distorted pentagons in the 12–12 scheme described in more detail further below. In the latter, the hourglass interspaces are divided into separate triangular interspaces.

These patterns lack the 'butterflies' because the butterflies lie over the overlapping portions of two partly overlapping M2 decagons of the Maragha tiling and in the presently discussed constructions, these overlaps are absent. Such overlaps, however, were frequent in the quasiperiodic Maragha tiling and its close derivatives (illustrated, e.g., in Makovicky 2008, 2015).

The approach and method described here differs from the method given by Bonner (2014a) in which a ring of *edge-sharing* pentagons is constructed as a basis for Tond patterns or to be transcribed into Kond patterns in a way analogous to the above mentioned transcription of the Penrose tiling into the Maragha-style tiling.

3 A Persian compass-straightedge method

There are several methods of constructing Islamic ornamental tilings (presented, e.g., by Bonner 2014b; Castéra 2014; Thalal et al. 2014; Benslimane 2014) but here I

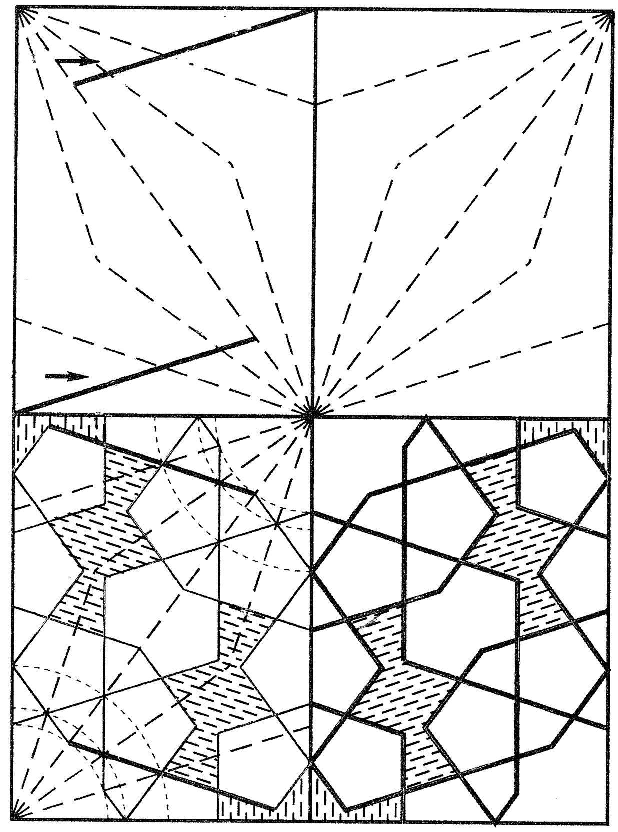

Fig. 10 Illustration of the Persian method of Kond pattern construction (redrawn with changes from Mofid and Raeeszadeh (1995)). *Dashed lines* a bunch of leading lines spaced 18° apart; *solid lines* marked by *arrows*: a selected common edge of adjacent polygons; its intersections with *dashed lines* define stippled *circles*; their intersections with *dashed lines* define corners and edges of polygons. More details in the text

would like to present a single example of a traditional Persian method specified in the subtitle because it differs a lot from other approaches. A more complete overview of variations of the method is given by Mofid and Raeeszadeh (1995), and unpublished conference contributions by Ghari (2015). The example I use has been redrawn from the quoted sources because it results in a complete spectrum of all Kond tiles. What is added by me are the calculations and considerations, trying to explain the substance of the technique. Such explanation appears to be absent in accessible literature.

Let us start with a rectangle which dimensionally corresponds to a rectangular *cmm* unit cell with 10-fold rosettes in the corners and the center. The diagonals, which connect the rosettes comprise 72° and the ideal side-length ratio is 1.3764. We shall construct an asymmetric unit (1/4 of the cell) with this ratio. The traditional start is to draw a bunch of rays spaced by 18° from two opposing corners of this asymmetric rectangle (Fig. 10). The next stage are two parallel lines (or one in the median line of the rectangle) which are drawn either from alternate corners, or they are parallel to two sides of the rectangle and situated at a specified distance from them. They intersect the rays of the bunches and the length of segments cut out on the rays defines radii of circles to be drawn around both opposing bunch origins (Fig. 10). Intersections of the rays (including the rectangle edges) with these circles create a net of nodes on which the corners of tiles are positioned.

In the example illustrated, the two intersecting lines are drawn at 18° from the shorter edge of the rectangle. If the short edge of the rectangle is defined as a unit length, then the three segments, which respectively, express the radius of the 10-fold star with deltoids included (see above), and the longer and the shorter radii of the star alone (without deltoids), have relative lengths 0.5257, 0.3820, and 0.3249 (the latter length being a diagonal of a pentagon), and intersections of the line with two adjacent rays, respectively define the edge of a pentagon (with a relative length 0.2008 in terms of the short side of the fundamental rectangle of the asymmetric portion), and the short edge of the deltoid (relative length 0.1241). Joining up to three intersection points at a time by a line defines lines which represent *edge orientations and positions of tile groups which share the given edge*, and allows interpolations further away from the origin (which could also be constructed directly by inserting and using additional rays at 9°). Triangulation of this set of points and lines allows one to draw the entire pattern with compasses and straightedge.

Different sets of starting lines give different Kond tilings. For example (Mofid and Raeeszadeh 1995), drawing lines at 18° to the long edge of the starting rectangle, eliminate the presence of the 'butterfly-*sormedan*' tiles, and the pentagon tiles will have the relative edge length of 0.2716 of the short rectangle side, i.e., a larger motif is formed. This traditional working procedure serves also the other ornament styles and it may become quite complicated and variable as the pattern complexity increases.

What is the nature of this procedure? Consecutive rays in the bunch alternatively are parallel and perpendicular to the expected (and for the given construction the only possible) *tile edges* of the decagonal Kond tiling. Taking two adjacent asymmetric rectangles, the extended bunch contains a ray parallel to a given edge and, 90° away from it, a ray perpendicular to this edge (Fig. 10). Thus, all *edge directions* possible in the pattern are concentrated around the bunch origin, a bit like a Patterson function in crystallography, in which directions of all edges of all coordination polyhedra, i.e., of all point-to-point (=interatomic) vectors are concentrated around the unit cell origin. To resolve the positioning and dimensioning of the polygons, a line parallel to a selected polygonal edge (exemplified by one of the rays of the extended bunch) is drawn from the adjacent corner (or parallel to a rectangle edge), knowing that its intersections with rays are equal to, and coincide with, edges of several adjacent polygons which share this particular edge orientation. This will position the first set of polygon edges. The obtained dimensions (lengths of selected edges and polygon-diagonals) are extended by means of concentric circles; intersections of the latter with the ray bunch define position of further edges (only every second ray evokes parallelity!), which allows to construct the rest of the decagonal pattern. For a defining line which is parallel to a rectangle edge, the starting line has/lines have to be positioned in such a way that the pattern constructed fits the area of the rectangle, for example by positioning it at a midpoint of the 36° ray.

This is my understanding of the construction principles. Strangely, I did not find this explanation in any of the available sources; they just offer traditional construction 'recipes'. Have the ingenious origins of this procedure been lost?

4 Some examples

4.1 The Anatolian realm

The Kond tiling produced by Turcoman Seljuks can be divided into two main groups: (a) outstanding two-, and one-dimensional designs in high-quality ceramics and/or brickwork, and (b) ornamental carving in stone.

The monumental ceiling of the Karatay Medrese in Konya (Fig. 11) (constructed in 1251–1252) combines the Kond tiles with rosettes placed in the pattern origin and the corresponding nodes of the *cmm* tiling. It is distinguished by a rich use of fivefold rotation-symmetric fill of the 10-fold stars and a frequent rotation-symmetric fill of selected pentagons. It was first analyzed by Rigby (2005) and further commented by Makovicky (2015). They find it a creative but still pretty close copy of the Maragha pattern itself. Kond tiles were used for ceramic panels in the Kaykavus Hospital in Sivas, Turkey (1217–1218), dealt with by Schneider (1980) and Makovicky (2015), among others; blue-and-black ceramic Kond friezes from the Karatay Medrese are of outstanding quality as well but they do not directly follow the Kond style.

Central Turkey, occupied by Seljuks, is rich in young, basaltic rocks. In spite of technical difficulties presented by this grainy and partly porous material of rocks and tuffs, in approximately the years 1220–1280 it became the principal raw material for the ornamental art of Seljuk Turkey. Outstanding are especially the complicated friezes of variable width, whereas the carved panels are mostly in form of tympana. The geometry of the carving is closely related to the two-dimensional panels of the Blue Tower of Maragha in NW Iran and of the Karatay Medresa in Konya,

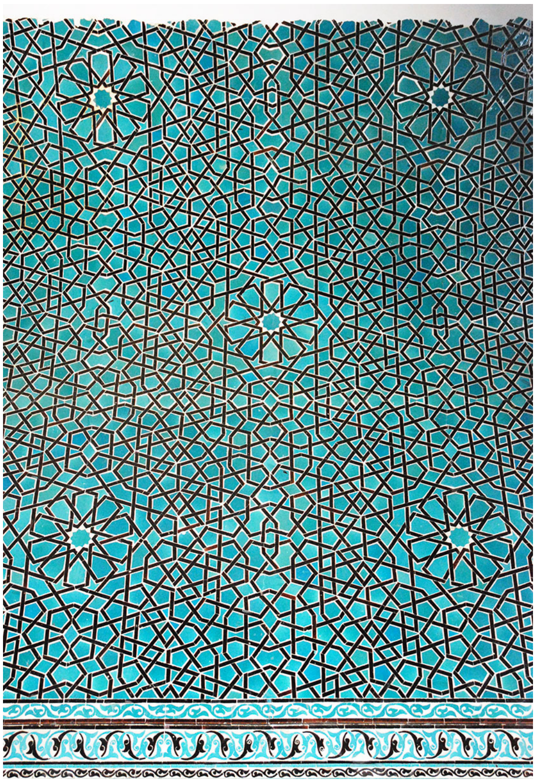

Fig. 11 A part of the ceramic coating of the iwan vault in the Karatay Medrese in Konya, Turkey. A Kond tiling, largely copying the quasiperiodic Maragha motif (Makovicky 2015)

Fig. 12 A solitary 10-fold star with a symmetric fill of a pentagon with lozenges, a typical element of ornamented Kond tilings. Kayseri, Turkey

central Turkey (see Figs. 2, 11). They use a set of '*Kond tiles*' (Mofid and Raeeszadeh 1995)/'*Maragha-type tiles'* of Makovicky (1992), here sculpted as outlines in solid rock, with a small pentagon as a fundamental element, and with a composite lozenge and a butterfly-shaped element as the other fundamental elements. Interlacing of boundaries is predominant, turning plane patterns into two-sided layers.

The extremely complicated friezes of the rock carving style appear unintelligible when they are not interpreted in the Kond-Maragha tiling. The two derived elements, a complex larger pentagon (which contains two lozenges, one butterfly and three small pentagons) and a filled 10-fold star, are often replaced by their symmetrized versions or by the 'emptied' version of the latter. The symmetrized versions were very much used elements of this style, sometimes suppressing the fundamental elements. To remind the reader, the symmetrized larger pentagons contain one central small pentagon and five kite-like small hexagons, and are rather conspicuous in a pattern when used. The core pentagons of the symmetrized version of 10-fold rosettes, with small lozenges attached to the apices, are even more conspicuous as long as they are not condensed into ribbons (Fig. 12). Encatenation of them into narrow strips by sharing small apical lozenges obscures their individual identity and results in rafts of small lozenges. The resulting rod group of such, *always interlaced* patterns is *p*222 (Fig. 13).

A number of other narrow strips are present, based on combinations of the basic tile types. They can be separated into a group with, and a group without emptied 10-fold stars. The first type is illustrated by Fig. 14 (right). The strips have lateral stars with rings of pentagons around them, followed by perfect to rudimentary rings of lozenges and butterflies; these strips can have different widths. The other group of patterns shows more variability. Combinations of pentagons, lozenges and butterflies occur with the lozenges oriented horizontally (Fig. 14, left), vertically or in an alternating fashion. The latter frieze (Fig. 15) is based on an array of overlapping composite pentagons, which overlap with both (by 180° rotated) neighbours via common lozenges.

An exceptionally broad strip has symmetrical borders based on alternation of the filled and symmetrized pentagons with the pentagons which have apical attachments; the latter placed in 10-fold stars (Fig. 16). Its central strip, however, is asymmetric. The stars with ornamented attachment-bearing pentagons, all pointed to one side, alternate in this strip with groups of four-pentagons-and-rhomb configurations. The interior of the central symmetrized stars, however, can be reoriented freely by 180° rotation. This reverses the orientation of the frieze but does not change the configuration of the frieze itself.

Friezes/mosaics in the Anatolian Kond style are common in, and typical for, a unified artistic region between (at least) Maragha to the east and Konya to the west. It was a

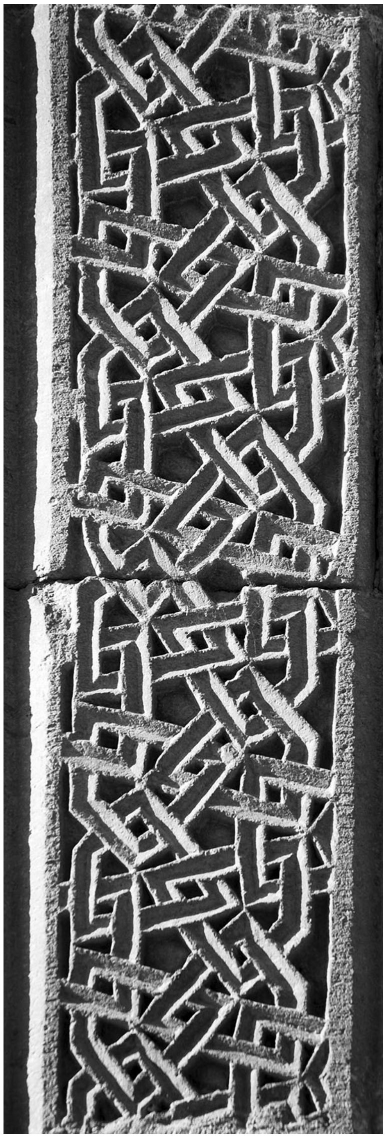

Fig. 13 Encatenation of winged pentagons into a narrow strip. Haci Kiliç Cami in Kayseri, Turkey (thirteenth century)

definite style type in the mid-years of the thirteenth century. Among his examples of Kond tilings, Schneider (1980) quotes 1205 as the earliest, lonely date, 1213/17 being more acceptable and his count ends in principle in 1289, with 1421 as the last of the rare mentions of dating in post-1300s. Examples of the *Tond* and *Shol* tilings, with small stars instead of the small pentagons (Mofid and Raeeszadeh 1995) are interspersed with the Kond products among the ornaments of Konya, Kayseri and beyond. They coexisted with the Kond-style tiles, sometimes producing mixed panels (for example the Maragha-like vault pattern from Karatay Medrese in Konya). They appeared to be used and to coexist in artistically outstanding examples over several hundreds of years (e.g., Makovicky 2015; Castéra 2016).

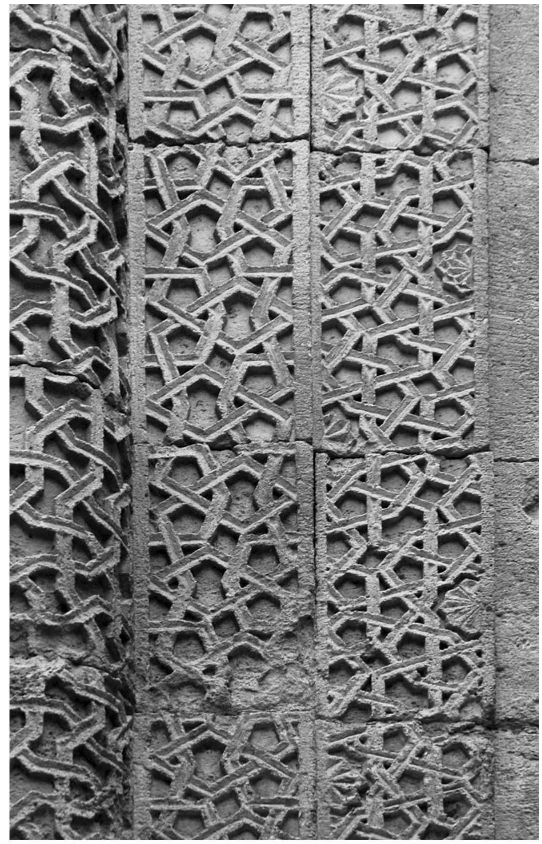

Fig. 14 Friezes composed of fundamental Kond tiles (details in the text). Mahperi Hunat Hatun Complex, Kayseri, Turkey (about 1238)

4.2 Iran, Central Asia and some comparisons

The variously broad decagonal Kond stripe patterns in Isfahan contain imperfectly ordered (or partly disordered) arrays of tile sets with a complete Kond spectrum. The disordered portions of the pattern, however, contain regularly spaced (emptied or simply ornamented) decagonal stars, furthermore only approximately ordered, complexly filled decagonal stars and pentagons (as described above). An example is shown in Fig. 17. Figure 18a, b shows two analysis methods for such patterns: (a) a more usual mapping of the distribution and orientation of filled 10-fold stars (e.g., Castéra 2016) and pentagons (only some of the latter are indicated) and (b) a hitherto nearly unused method of mapping the distribution and aggregation of complex lozenges in the pattern (Fig. 18b). This method was used only by Makovicky, in several oral presentations and now in press (Makovicky 2016) to analyze the nature of a tympanum pattern in the Darb-e-Imam shrine in Isfahan in order to confirm a clear separation of quasiperiodic and of partly-disordered, but in general, periodic portions of the pattern. In the present case, the latter analysis gives a wonderful insight into especially the construction methods used by the mosaicist. In Fig. 18b,

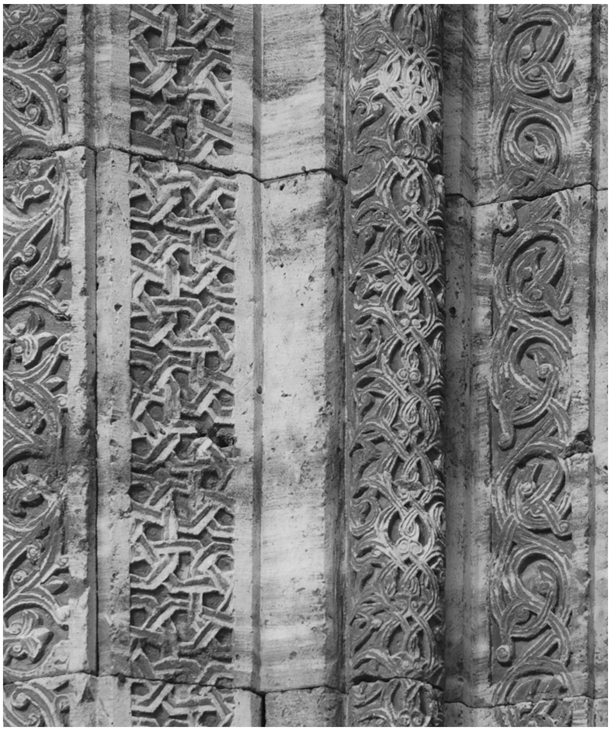

Fig. 15 Friezes framing the ornamental gate. Central Sivas, Turkey. The pentagon frieze is on the *left*

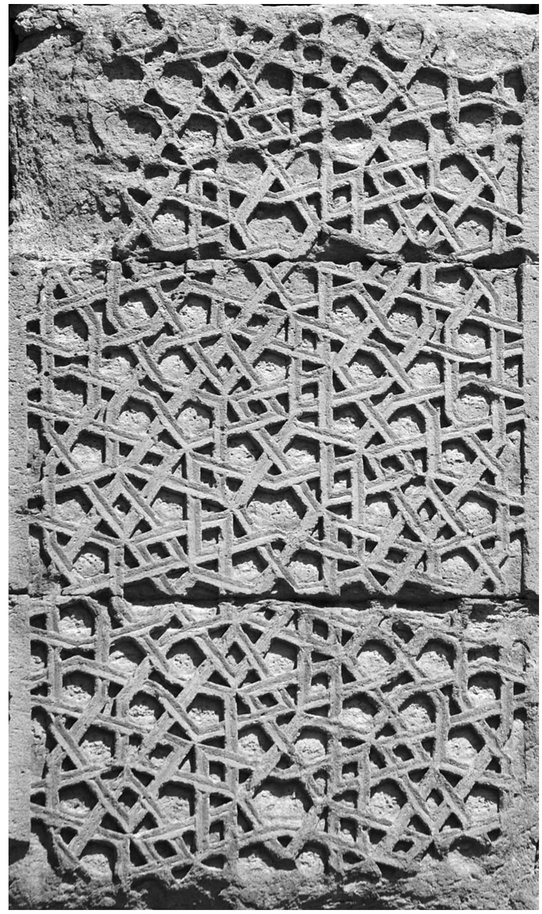

Fig. 16 A broad complex Kond frieze; details in the text. Yeni Cami, Kayseri, Turkey

we can see the zig-zag lines of wobbling lozenges, separated by straight-to-wavy double rows of pentagons and butterflies. The horizontal rows of empty 10-fold stars are the levels at which the line direction changes (Fig. 18b). A narrower pattern of this kind from a small medresa in the quarter north of Meidan Imam Khomeini in Isfahan has a much more disorganized distribution of lozenges, with a number of imperfect and 'forbidden' contacts between them. Number of filled 10-fold stars in this pattern is low (most of such round stars have a segment 'bitten off' by an invading tile or tiles) but entire blocks seem to have been copied at least twice in the same pattern.

Patterns of Kond tiles with two pattern levels automatically raise a question whether there is an independent substructure onto which the superstructure has been grafted or the substructure has been created as a pendant of the large motif. We can state the latter for the large mosaic panel from Darb-e-Imam in Isfahan shown in Fig. 19. It is a large *cmm* pattern with large decagonal *Shol* stars in the unit cell origin and centration point, and large fivefold *Tond* stars and *m*-symmetric hexagons forming respectively centers and corners of pentagons which surround the decagonal elements and are marked in aggregation of small tiles of the substructure. Boundaries of these pentagons are decorated by typical star-to-star connections via small pentagons, butterflies and sometimes lozenges (Fig. 19) and the internal substructure in the pentagons obeys the 5*m* point group. Thus, the panel does not contain an independent substructure, all of it is a large-scale motif.

As already described in the geometric section above, the *Kond style* appears to be designed with the decagonal style in mind. As we have seen, designs with the *Kond style* (i.e., the *Maragha style* of Makovicky 1992) of tiles did not remain limited to the decagonal sets of tiles. The most interesting examples of this generalization appear in Iran and Central Asia. And, as also mentioned, the tiles of these tilings have apex angles adjusted to those of the modified and partly rotated pentagons. The system of close-to-regular pentagons, stars and lozenges in the pattern from the '12–8' tiling from the Kalyan Mosque in Bukhara (Fig. 6) has already been discussed. A prominent 12–8 pattern (Fig. 20), mentioned briefly in Makovicky (2015) is on one of the tympana in the Masjid-e-Hakim courtyard which also houses prominent and compositionally similar patterns based on 10-fold stars, at least one of which is directly related to the Maragha composition (Makovicky 2015). The 12–8 pattern obeys demonstratively the geometry of the four-pentagons-and-a lozenge rectangle which connects adjacent eightfold stars. A pristine example of the 12–8

Fig. 17 A band of partly ordered Kond pattern from the courtyard of the Darb-e-Imam shrine in Isfahan. Three levels of order–disorder in this pattern are analyzed in Fig. 18

pattern is the ceramic band with a white lace from the upper reaches of the Shah-i-Zinda mausoleum row in Samarkand (fourteenth century) (Fig. 21).

Patterns which are composed of small ceramic tesserae/tiles do not allow evaluation of tile adjustments because the tesserae have been hand-fashioned by clipping from large ceramic tiles. An example of such a pattern comes from the Friday Mosque in Isfahan (Fig. 22). Bulging of the rectangular bridge between adjacent eightfold stars is less prominent than in the above 12–8 examples. Similar bridges between adjacent 12-fold stars are absent. Deviations from the tetragonal pattern occur at the margins. The 8–4 pattern from Bukhara (Fig. 9) was already discussed.

The tympanum of the Kalyan Mosque in Bukhara, Uzbekistan, shows a 12–9 pattern with ornamented pentagons and lozenges (Fig. 8). Not only the rectangular bridge composed of four-pentagons-and-a lozenge which spans two adjacent 12-fold stars is constricted in the middle, but also the two 'inner' edges of the pentagons are a bit longer than the three 'outer edges' which actually face the ninefold stars. All this is as the theoretical derivation requires. Several other tympana in the region have the same 12–9 pattern.

If we return to Turkey for the good examples, the hexagonal schemes with 12-fold or 6-fold stars deviate most from the here-sketched model. Interestingly, an interlaced pattern from the roof lantern of the Kaykavus Hospital (Şifaye Medresa) in Sivas, Turkey (built in 1217–1218), is identical (except for unsubstantial details) with octagonal discs of a hexagonal pattern in the Divriği complex nearby (Fig. 23). The wreath of pentagons is ruled by a circumscribed hexagon and the three pentagons which are squeezed between three stars always form a (basically) 3*m* configuration around a small triangle. This arrangement is dictated by the lack of 'hourglass polygons' between the basic hexagons on which the pentagons are placed.

Adaptation of the carved Seljuk Kond patterns to other symmetries than decagonal is rare. In a frieze from Fig. 24 Sultanhani, Aksaray, with a hexagonal layer group symmetry, exceptionally it is the 12-fold and 9-fold stars which are interconnected by the above-discussed four-pentagons-and-a lozenge configuration, and the small pentagons surrounding the 12-fold star on the one hand, and those surrounding the ninefold star on the other hand, are oriented as the above-discussed model predicts. A hauntingly curvilinear pattern (Fig. 25) from Sivas, Turkey, with symmetry contradictions (*m* versus horizontal twofold rotation axes) simulates the same rectangular groups, with pentagons oriented properly and the simulated lozenges inflated, in agreement with the fact that they interconnect two adjacent eightfold stars of the tetragonal pattern.

An impressive, sinusoidally modified large-scale decagonal Kond pattern adorns walls of the entrance space of the Abdullah Khan Madrasa in Samarkand. It is composed of numerous tiles glazed in several colors (Fig. 26). It needs a bit of concentration before its classical decagonal Kond character emerges from the ornamental curves.

Thus, the patterns with symmetries other than decagonal are found in central Turkey, in Isfahan, Bukhara and Samarkand (Uzbekistan) and within a time span between early thirteenth century and late sixteenth century. Somewhat different character has the outer ornamentation of the domes which can be typified by the famous Aramgah-e-Shah Ne'matollah Vali shrine in Mahan (Iran) (Fig. 27). This ornamentation reminds of a 'classical' Kond style, but using a scheme which simulates *cmm* modified by changing the star multiplicity and adjusted to a curved surface. I

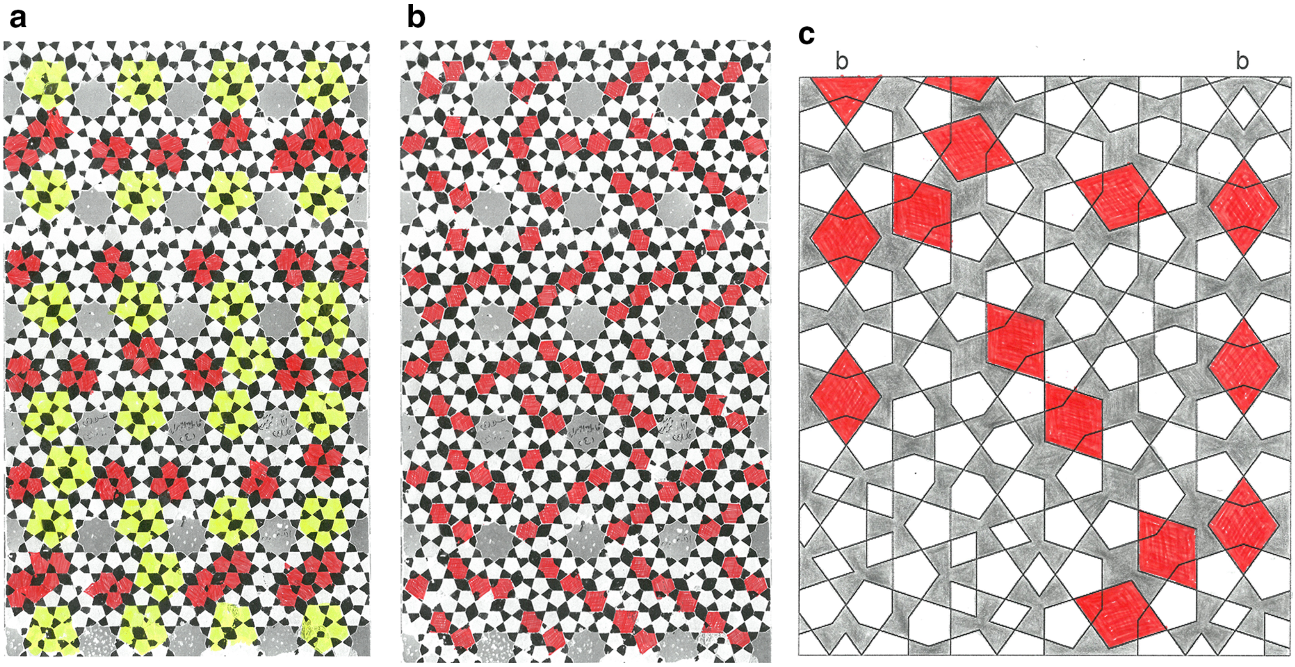

Fig. 18 Analysis of order–disorder in the band from this figure in terms of distribution of **a** respectively empty and filled *decagonal stars* and of *filled pentagons*, and **b** complex lozenges mostly interspaced by butterflies. When such configurations in **a** are disfigured by an odd tile intruding from the perimeter, they are left out of consideration in this figure, **c** tiling on a wall of the Maragha tower

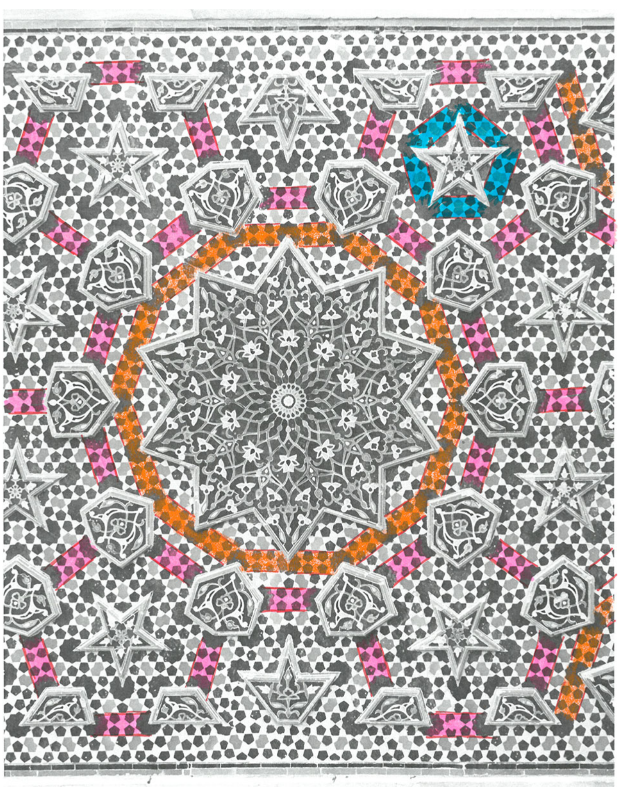

Fig. 19 A large two-level pattern with minute Kond tiles as a complex substructure, closely tied with the large-scale, raised combined Tond–Shol structure. Configurations of the substructure are accentuated by coloring. The Darb-e-Imam shrine complex, Isfahan

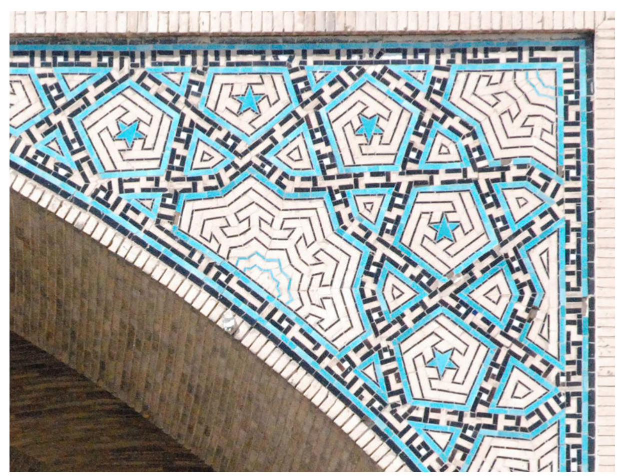

Fig. 20 An ornamented edition of the 12–8 Kond pattern from a tympanum in the courtyard of the Masjid-e-Hakim in Isfahan (built in 1656–1662). All pentagons exhibit the same chirality

suggest that it's a cleverly integrated sequence of a horizontal band with a decagonal Kond configuration at the bottom, transforming seamlessly into a band with a tetragonal configuration, then a narrower band resembling a hexagonal configuration, a band with 12–9 combination and, finally, an altered pattern closing the dome at the apex. The sequence of stars is: 10-fold at the bottom, 9-fold, 11-fold, 12-fold, 9-fold, distorted 7-fold and a smaller 5-fold; the mentioned stripe-like configurations always enclose one and a half of adjacent stars.

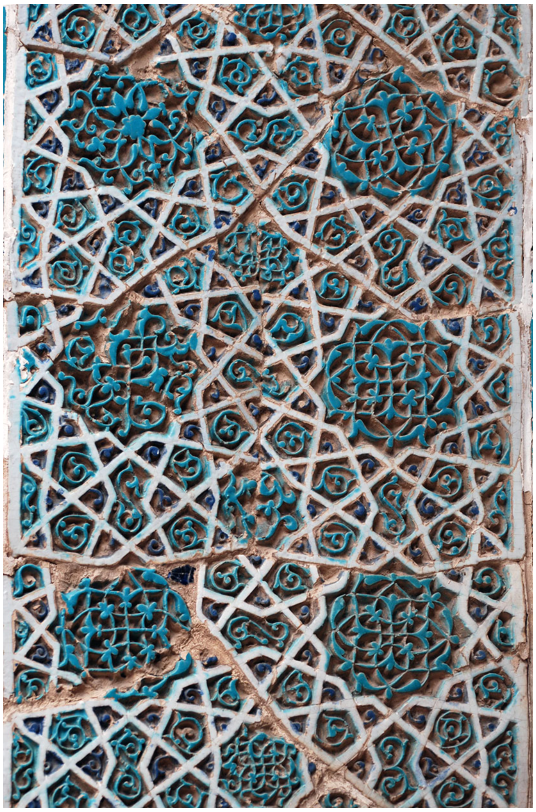

Fig. 21 A lace-like 12–8 Kond pattern of exceptional clarity with the fill ornamented in relief. Uniform chirality of pentagon fill in one wreath is mostly maintained. Upper parts of the Shah-i-Zinda mausoleum row in Samarkand

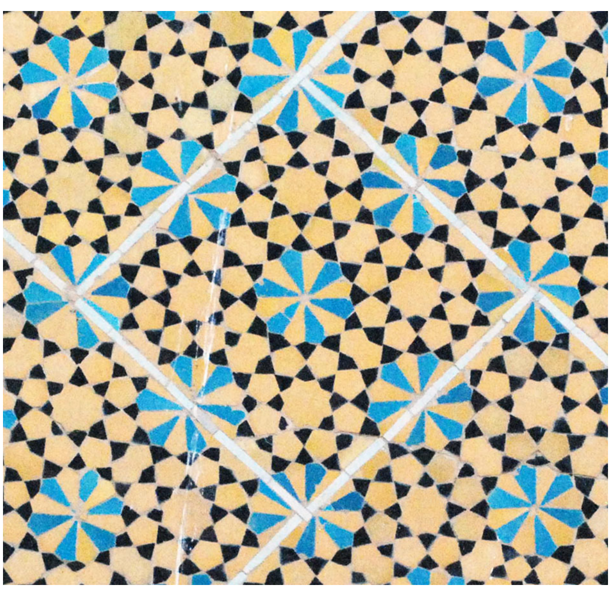

Fig. 22 Central portions of a Kond mosaic with 12-fold and 8-fold stars from the courtyard wall of the Friday Mosque in Isfahan

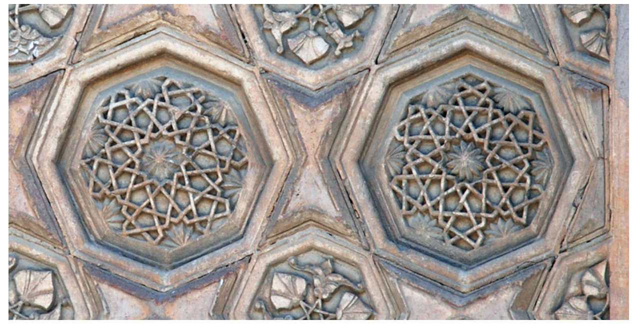

Fig. 23 A disc of hexagonal pattern from the Divriği complex, Turkey. Although related, it is not a true Kond tiling (reasoning details in the text)

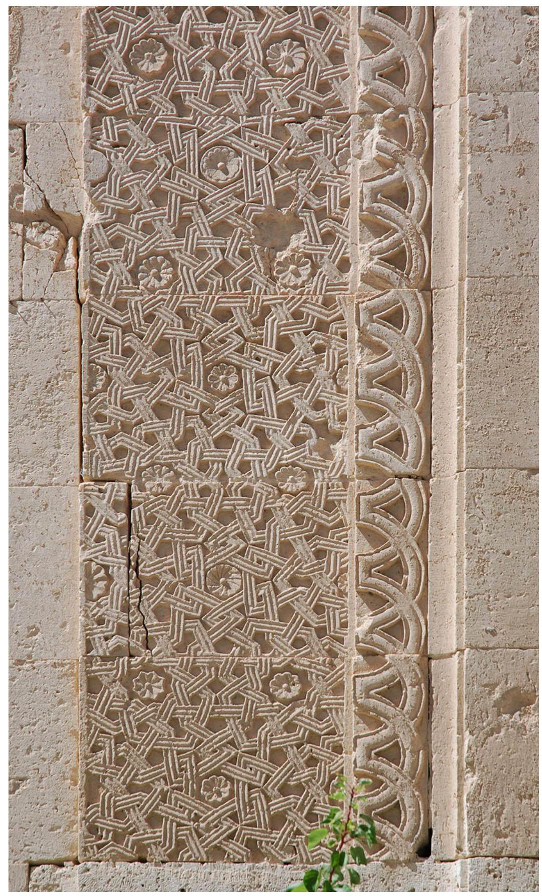

Fig. 24 A hexagonal carved and interlaced 9–12 pattern from Sultanhani, Aksaray, Turkey, in which the 'four pentagons and a lozenge' configuration interconnects the 12-fold and the 9-fold stars

5 The enigmatic curvilinear panels in the Ulug Beg Madrasa, Registan, Samarkand

The iwans of the Ulug Beg Madrasa, one of the three madrasas constituting the complex of Registan in central Samarkand (Uzbekistan) have paneled lower walls that

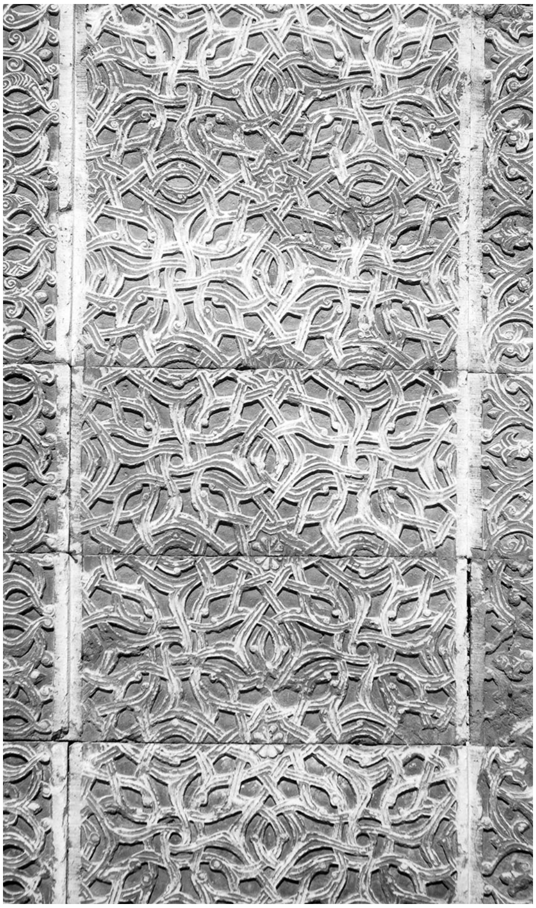

Fig. 25 A curvilinear, carved tetragonal pattern, which simulates the 'four pentagons and a lozenge' configurations between the eightfold stars. Central Sivas, Turkey

Fig. 26 A large scale decagonal Kond pattern to which a slight wavy modulation of boundary lines was applied. The especially powerful visual impression has been obtained by *color* enhancement in a computer. The original mainly buff mosaic is in the Abdullah Khan Madrasa in Samarkand

show small rectangular stone plates carved with geometric ornaments. The majority of them display a decagonal pattern of the Kond style, only some are hexagonal, etc.

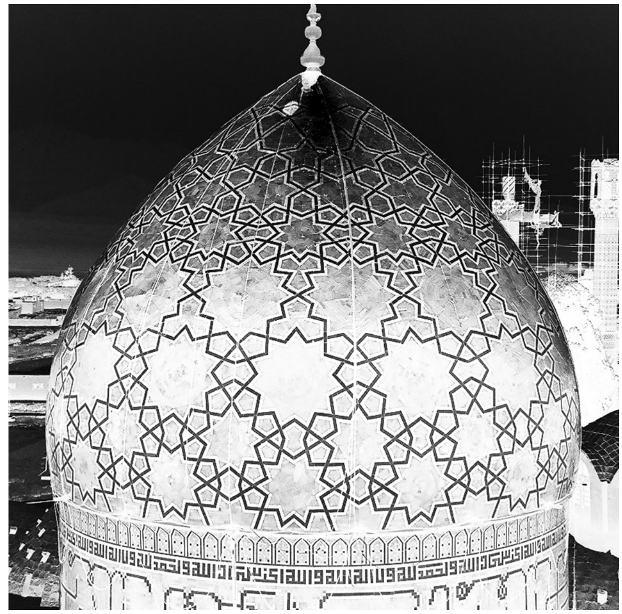

Fig. 27 The dome of the Aramgah-e-Shah Ne'matollah Vali shrine in Mahan (Iran) with smoothly transiting zones of adjusted Kond tiling: a decagonal zone for the 1½ large stars at the *bottom*, an (in construction principles) tetragonal zone for the following ½ and one stars, followed by hexagonal for the 12-fold star ring, with a 12–9 pattern afterwards. Black-white inversion was applied for clarity

The Kond patterns (Fig. 28) are all of the same type although the perfection of execution visibly varies.

The pattern is of a fairly common type (Fig. 29, for derivation see Fig. 10), with a decagonal star in the center, surrounded by an incomplete wreath of composite lozenges alternating with 'butterflies'. Only at the top and bottom these elements are oriented radially. The innermost circle of pentagons is preserved but the waist portions of the pattern continue sideways in a way very different from the up-and-down directions. Corners of the rectangular field contain stars comparable to the central one.

The remarkable feature of all panels is their curvilinear character—the vertical lines are not rectilinear but they form a concave pattern (Fig. 28). Central portions of the panel are not substantially deformed but the corner areas have pentagons elongate and distorted, some lozenges drawn out, and some butterflies affinely distorted. All panels are about the same except for the variable quality of elaboration. There is a mysterious punch in the central star of all of the panels (Fig. 28) for which we have no explanation except for an attempt to pry something from the central star or to pry the panel out of the wall.

I attempted various approaches trying to understand a nature of the non-linear pattern. Finally, what remained are two possible explanations: the central star is 10-fold, whereas the corner stars are 12-fold which requires a change of direction and spacing causing the curvature. However, besides the conspicuous vertical boundaries, the

Fig. 28 A carved panel from the Ulug Beg Madrasa, The Registan, Samarkand, Uzbekistan with a curvilinear Kond pattern

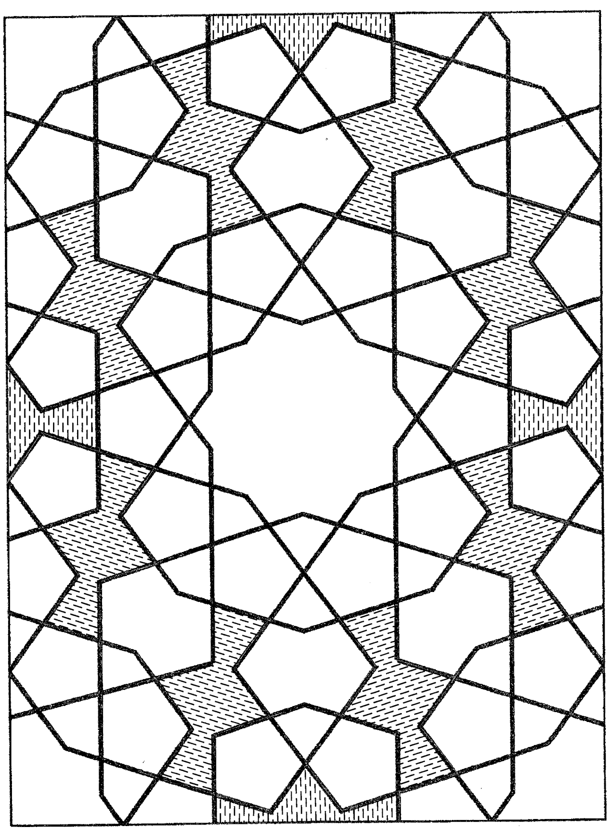

Fig. 29 The ideal *cmm* Kond pattern from which Fig. 28 was derived. Redrawn with changes from Mofid and Raeeszadeh (1995)

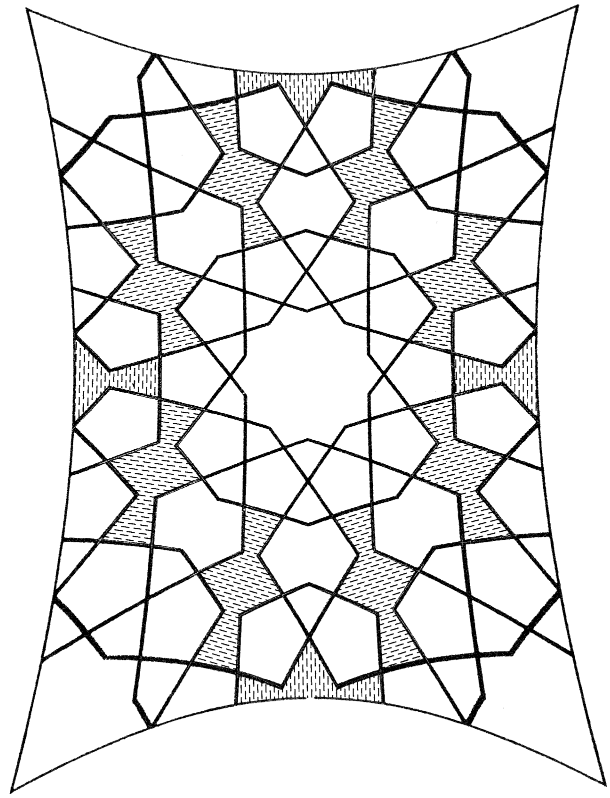

Fig. 30 A 'pinched' hyperbolic-rectangle version of the pattern from Fig. 29 with distortions simulating those in Fig. 28

panels contain line systems at 36° to the vertical one—and these are practically rectilinear, with constant spacing, maintaining the $\sim 72°$ divergence, and they embrace three 'teeth' of the deltoid-decorated star scheme. All of this is appropriate for the decagonal system, i.e., it practically excludes the just-sketched scheme of combined star multiplicities.

The alternative explanation is a non-linear stretching of the panel at the corners (Fig. 28), resulting in elongate distorted pentagons and with the visible bits of corner stars deformed. Boundary regions were subsequently truncated. The situation (in the best executed panel) has been approximated by taking the pattern created in the previous section (Fig. 10) and stretching it to approximately the observed degree (Fig. 30). Not all distortion details are reproduced but the pattern-stretching distortion program in the computer cannot be assumed to work in a way identical with the manual distortion of the old masters.

How did the old masters get this kind of distortion? It looks like a projection from a curved template onto a plane or from a planar template onto a curved, cushion-like application surface. Alternatively, it could be a pattern from Fig. 29, drawn on an stretchable material, or such a

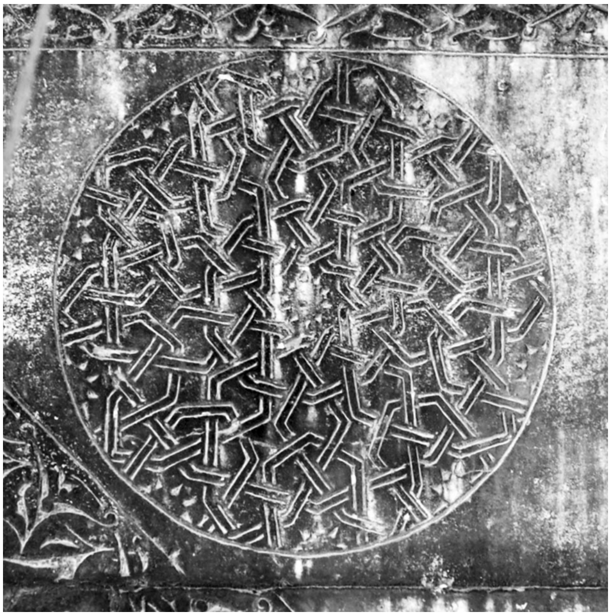

Fig. 31 A stone rondel with interlaced cartwheel Kond pattern that is slightly distorted by stretching in the peripheral parts. The Ulu Cami and Darüşşifa Complex in Divriği, central Anatolia, Turkey

stretchable net, and then stretched out in the corners when copied onto a stone plate. What practical procedure might have been behind these 'modern' concepts?

A more sober explanation is suggested by the cartwheel rondel from the Divriği complex (founded in 1228) in central Turkey. In this cut-out of interlaced decagonal Kond tiling (Fig. 31), all five directions of parallel tile edges form slightly concave lines, broadening towards the circle perimeter. The regular pentagons in the center become elongate towards the margin and the entire scheme appears to be a result of realization that the spacings carved in the central parts of the disc are in bad agreement with the planned pattern extent to the perimeter. The pattern had to be stretched out in order to fit. In the case of the Registan madrasa, the mismatch was even more critical (was it perhaps made by a beginner?).

6 Conclusions

The Kond style means abundance of small pentagons in corner contact, presence of composite lozenges and/or 'butterflies', a usual presence of larger star discs of different order, absence of rayed rosettes and small fivefold stars (except for mixed-style products), and rotational freedom of composite tiles observed in decagonal variants. This results in an impression that is different from that gained from the Iranian Tond and Shol styles which appear to be less widespread in the region. I think that the Kond-Maragha canon can be called an *art style*, comparable for example with European Rococo style and era. Similar to other art styles, it was not geographically limited, being spread over a considerable area of Turkey, Iran and Central Asia; for some uses it survived for a considerable time. The decagonal quasiperiodic patterns, which exist because of the special geometric properties of the decagonal tile set, are the most important subfamily of the Kond style set (Makovicky 1992, 2008, 2015 with references). Friezes of the Turkish Seljuk mosques and madrasas carved in volcanic rock or constructed from ceramic tiles form the other large and important group of examples from the early period of the Kond family. Periodic mosaic panels and wooden Kond ceilings were a widespread and constant feature of especially Iranian architecture over centuries, including the Safavid and Qajar monuments. In the present publication we concentrated on the geometrically challenging problems, not only by describing them but also attempting to explain them in a way as close to the original concept as possible.

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