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3D Identification and Stability Analysis of Key Surface Blocks of Rock Slope*

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Abstract: Complicated geological structures make it difficult to analyze the stability of rock slopes, such as faults, weak intercalated layers or joint fissures. Based on 3D geological modeling and surface block identifying methods, an integrated methodology framework was proposed and realized to analyze the stability of surface blocks in rock slopes. The surface blocks cut by geological structures, fissures or free faces could be identified subjected to the four principles of closure, completeness, uniqueness and validity. The factor of safety (FOS) of single key block was calculated by the limit equilibrium method. If there were two or more connected blocks, they were defined as a block-group. The FOS of a block-group was computed by the Sarma method. The proposed approach was applied to an actual rock slope of a hydropower project, and some possible instable blocks were demonstrated and analyzed visually. The obtained results on the key blocks or block-groups provide essential information for determining potential instable region of rock slopes and designing effective support scheme in advance.

Keywords: rock slope; 3D model; surface block; block-group; stability; factor of safety (FOS)

Numerous practices have revealed that the failures of rock slopes are caused by cutting rock blocks from structural surfaces and free faces[1, 2]. Some rock surfaces from major geological structures can be determined, such as faults and weak intercalated layers, while more minor structural surfaces from joint fissures are stochastic. Besides, the structural surfaces and free faces may be planes, surfaces or multiple surfaces. These make it more difficult to analyze the stability of rock slopes. Norris *et al*[3] recorded the first direct scientific observation of rock blocks in motion using GPS-instrumented rocks and photography. Since the rock block theory was put forward by Warburton[4] and Goodman and Shi[5], it offers a powerful method for the stability analysis of rock slopes and underground tunnels in fractured rocks[6-8].

Based on the traditional 2D block theory, the modeling, identifying and stability analysis of rock blocks have been developed to 3D in recent years. Turanboy[9] proposed new geometrical classifications of rock blocks according to the spatial orientations of the discontinuities and their locations relative to each other. Kulatilake *et al*[10] evaluated the stability of a rock slope in the dam site based on kinematic and block theory analyses. Brideau *et al*[11] used block theory to evaluate the finiteness and removability of blocks in the rock mass. Li *et al*[12] proposed an identification approach of 3D surface blocks for rock mass structures. Jafari *et al*[13] described a new method to reconstruct polyhedral rock blocks created by the intersection of planar discontinuities in a rock mass. Zhang and Lei[14] presented an object-oriented computer model for 3D multi-block system. Wang and Ni[15] developed a computer program GeoSMA-3D to perform stability analysis of rock slope based on the topological identification techniques of spatial block with stochastic discontinuities cutting. Zhang[16] proposed a method for block progressive failure analysis and applied it to two engineering case studies. Therefore, the 3D stability analysis for 3D rock blocks can be better in accordance with actual rock engineering.

Generally, conventional stability analysis methods

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of blocks in rock slopes assumed that the structural surfaces and slope surfaces are planes and the interaction of the adjacent blocks was rarely considered, which may lead to a few limitations. In this study, an integrated method was used to build more actual models of rock blocks and the interaction of the adjacent blocks was computed by the Sarma method. The modeling, computation and evaluation of the stability of rock slopes cover the following issues: (1) building the overall methodology framework including geological modeling, block identification and stability computation; (2) calculating the stability coefficients of both single surface block and block-groups; and (3) assessing the stability of surface blocks in an actual rock slope.

1 Methodology framework

Fig. 1 shows the overall framework for the stability assessment of rock slopes based on 3D geological modeling and surface block identifying methods. The framework consists of three parts: 3D multi-scale integrated modeling of complex rock mass structures, the identification of 3D surface blocks and the computation of stability coefficient of key blocks and the whole slope.

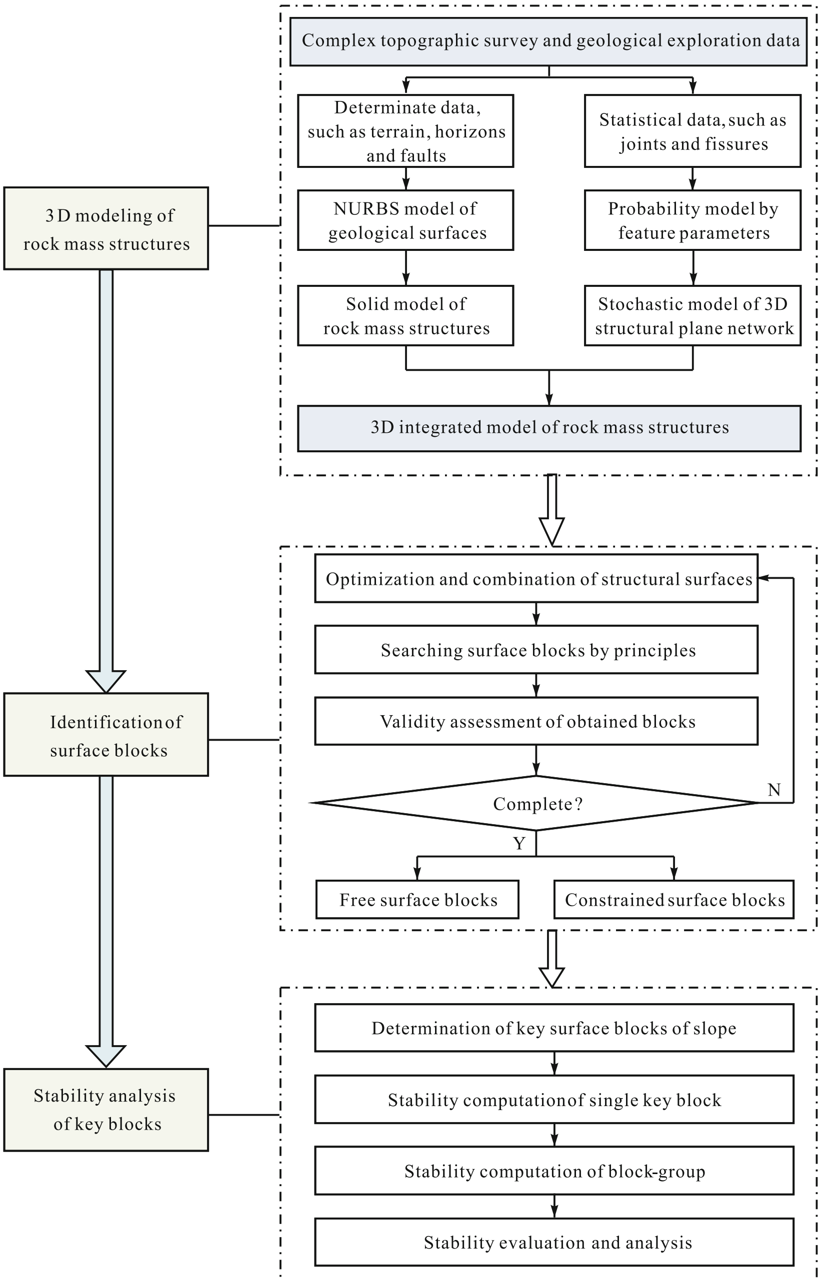


Fig. 1 Overall framework for stability assessment of rock slopes

1.1 3D geological modeling of rock mass structure

Firstly, the original geological data were obtained by geological mapping, remote sensing, and geological exploration. They were transformed into NURBS data by spatial interpretation and multi-source data coupling methods. Then, based on these data the solid model of major rock mass structures could be reconstructed. On the other hand, the statistical data were obtained by geological survey of the traces of structural planes. The probability model was built by analyzing the distribution law of the statistical data. Then the stochastic model of 3D structural planes network was built. Finally, the refined 3D integrated model of rock mass structures was established by coupling the solid model and the stochastic model. These two parts can be realized by our previous work[12, 17].

1.2 Identification of 3D surface block

A 3D engineering-scale and statistical-scale integrated modeling approach was used to reconstruct the multi-scale model considering complex geological structures and discontinuities[17], and the surface blocks method was focused on defining, searching and identifying the 3D surface blocks of rock mass structures subjected to the four principles of closure, completeness, uniqueness and validity[12].

A surface block is defined by the following mathematical model[12]:

$$\begin{cases} B_n = (\bigcup_{i=1}^{n_1} S_i) \cup (\bigcup_{j=1}^{n_2} C_j) \\ S_i = s(P_i), i = 1, 2, \cdots, n_1 \\ C_j = c(O_j, \boldsymbol{V}_j, R_j), j = 1, 2, \cdots, n_2 \\ n = n_1 + n_2 \end{cases} \tag{1}$$

where B_n is the surface block with n boundary surfaces; S_i is the geological surface with the data set P_i, e.g., joint, bedding plane or fault surface; C_j is the simulated joint plane with the central point O_j, the normal vector $\boldsymbol{V}_j$ and the radius R_j.

There are three general assumptions for surface blocks: (1) Structural surfaces consist of major geological surfaces or minor joint planes. The joint planes are built using Baecher disk model and the geometric parameters conform to special probability distributions. (2) The geological surfaces and the joint planes cut the rock masses to form the surface blocks. (3) The surface blocks are uniform rigid bodies.

Based on the reconstructed refined rock mass model, the surface blocks can be identified, and then they are divided into free blocks and constrained blocks according to their space constraints. A free surface block has free space and may be the key block if its factor of safety (FOS) is lower than the minimum designed FOS. A constrained surface block is surrounded by the rock mass and may convert into a free block after external blocks fall. Therefore, the next section will present the computation method of FOS for a single block and block-group in detail.

2 Stability coefficients of surface blocks

2.1 Stability of single block

The main forces of a block include the active force R, the reaction force N vertical on the sliding surface, the friction force f and the cohesion force c along the sliding surface direction. The active force R consists of the gravity, the external water pressure, the anchoring force, and some forces transmitted by other rock masses. In this paper, only the gravity is considered as the active force. Restricting to these forces and sliding surfaces, there are three motion modes for a potential key block as follows[18].

(1) Falling mode. As shown in Fig. 2(a), the block will detach from the rock mass and move if the following equation is satisfied:

$$\hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{V}}_{Li} > 0, \quad i = n-1 \tag{2}$$

where $\hat{\boldsymbol{S}}$ is the motion direction vector of the block with n boundary surfaces; $\hat{\boldsymbol{V}}_{Li}$ is the normal vector pointing to the interior of the ith boundary surface.

If a block would fall, its factor of safety is 0 (FOS = 0).

(2) Sliding mode along a single surface. As shown in Fig. 2(b), the block will slide along a single surface if the following equation is satisfied:

$$\begin{cases} \hat{\boldsymbol{R}} \cdot \hat{\boldsymbol{V}}_S \leqslant 0 \\ \hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{V}}_{Li} > 0, \quad i = n-2 \end{cases} \tag{3}$$

where $\hat{\boldsymbol{R}}$ is the direction vector of the active force R; $\hat{\boldsymbol{V}}_S$ and $\hat{\boldsymbol{V}}_{Li}$ are the normal vectors of the sliding surface and the ith boundary surface, respectively.

If a block is limited to a single sliding surface, its factor of safety can be calculated as

$$\text{FOS} = \frac{N \tan\varphi + cA}{T} \tag{4}$$

where N and T are the normal force and the tangential force of the active force R on the sliding surface; φ and c are the internal friction angle and cohesion of the sliding surface; A is the contact area between the block and the sliding surface.

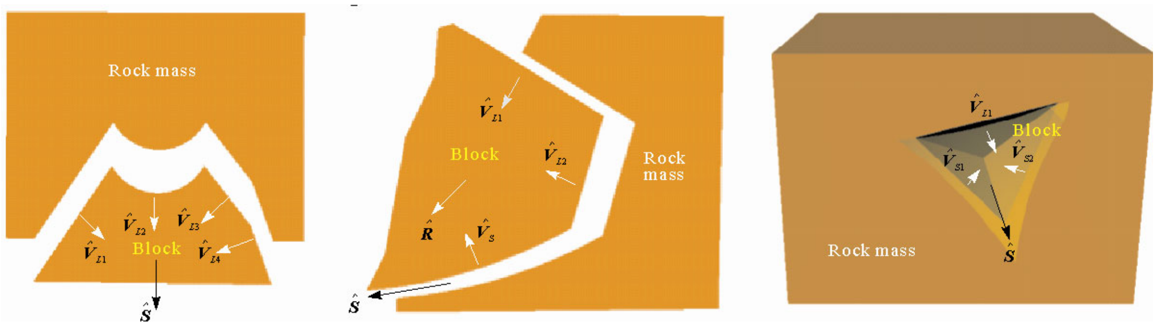
(3) Sliding mode along two surfaces. As shown in Fig. 2(c), when the block slides along two sliding surfaces, i.e., their intersecting line, the following conditions are

$$\begin{cases} \hat{\boldsymbol{S}}_1 \cdot \hat{\boldsymbol{V}}_{S2} \leqslant 0 \\ \hat{\boldsymbol{S}}_2 \cdot \hat{\boldsymbol{V}}_{S1} \leqslant 0 \\ \hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{V}}_{Li} > 0, \quad i = n-3 \end{cases} \tag{5}$$

where $\hat{\boldsymbol{S}}_1$ and $\hat{\boldsymbol{S}}_2$ are the motion direction vectors of the block along the two sliding surfaces; $\hat{\boldsymbol{S}}$ is the motion direction vector along the intersecting line; $\hat{\boldsymbol{V}}_{S1}$, $\hat{\boldsymbol{V}}_{S2}$ and $\hat{\boldsymbol{V}}_{Li}$ are the normal vectors of the two sliding surfaces and the ith boundary surface, respectively.

If a block is limited to two sliding surfaces, its factor of safety can be calculated as

$$\text{FOS} = \frac{N_1 \tan\varphi_1 + N_2 \tan\varphi_2 + c_1 A_1 + c_2 A_2}{T} \tag{6}$$



(a) Falling (b) Sliding along a single surface (c) Sliding along two surfaces

Fig. 2 Motion modes of single surface block

2.2 Stability of block-group

When the structural surfaces are relatively dense, several connected blocks can form a block-group. The block-group usually controls the stability of rock slope on a large scale. Here Sarma method[19-21] is used to calculate the FOS of a block-group. Sarma method considers the interaction of the adjacent blocks, and the contact surfaces need not be vertical, which is consistent with the actual situation of multiple structural surfaces. Here, only the gravity is considered as the active force. The gravity is projected into two component forces, one along the sliding direction and the other perpendicular to the sliding surface. Besides, we assume that all the blocks in a block-group slide along the same motion direction. It means that the resultant force on the sliding surface and perpendicular to the motion direction equals zero. In this case, the 3D calculation of the block-group can be simplified to a 2D calculation. For instance, Fig. 3 gives the forces of a block-group.

First, a recursion formula is obtained based on the static equilibrium equations and Mohr-Coulomb failure criterion, it is expressed as

$$E_{i+1} = A_i - p_i K_{\mathrm{C}} + E_i e_i \tag{7}$$

Then by uniting the recursion formula and the boundary conditions, the FOS of the block-group can be calculated by using the following iterative equations:

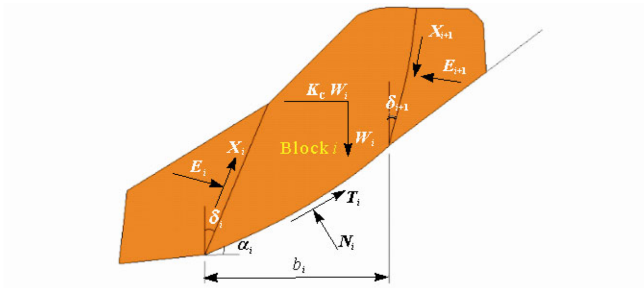


Fig. 3 Schematic diagram with forces of a block-group

$$K_{\mathrm{C}} = \frac{a_n + a_{n-1}e_n + a_{n-2}e_n e_{n-1} + \cdots + a_1 e_n e_{n-1} \cdots e_3 e_2}{p_n + p_{n-1}e_n + p_{n-2}e_n e_{n-1} + \cdots + p_1 e_n e_{n-1} \cdots e_3 e_2} \tag{8}$$

$$\begin{cases} a_i = [W_i \sin(\varphi_i - \alpha_i) + R_i \cos\varphi_i + S_{i+1}\sin(\varphi_i - \alpha_i - \delta_{i+1}) - \\ \quad S_i \sin(\varphi_i - \alpha_i - \delta_i)]/g_i \\ e_i = \cos(\varphi_i - \alpha_i + \varphi_i' - \delta_i)\sec\varphi_i'/g_i \\ p_i = W_i \cos(\varphi_i - \alpha_i)/g_i \\ R_i = c_i b_i \sec\alpha_i \\ S_i = c_i' d_i \\ g_i = \cos(\varphi_i - \alpha_i + \varphi_{i+1}' - \delta_{i+1})\sec\varphi_{i+1}' \end{cases} \tag{9}$$

where K_{C} is the horizontal critical acceleration coefficient; W_i is the weight of block i; E_i and E_{i+1} are the normal forces on the left and right surfaces of block i, and $E_{n+1} = E_1 = 0$; X_i and X_{i+1} are the shear forces on the left and right surfaces of block i, and $X_{n+1} = X_1 = 0$; N_i is the normal force on the bottom surface of block i; T_i is the shear force on the bottom surface of block i; α_i is the angle between the bottom surface of block i and the horizontal plane; φ_i, φ'_i and φ'_{i+1} are the average friction angles on the bottom surface and the sides of block i; δ_i and δ_{i+1} are the angles between the vertical surface and the left and right sides of block i; b_i is the horizontal distance of the bottom surface of block i; d_i is the length of the side of block i; c_i and c'_i are the average cohesions on the bottom surface and the left side of block i.

By solving Eq.(8), the FOS is found by reducing the shear strength values $\tan\varphi$ and c to $\tan\varphi$ /FOS and c/FOS until K_C is reduced to zero.

3 Case study

3.1 Geological conditions

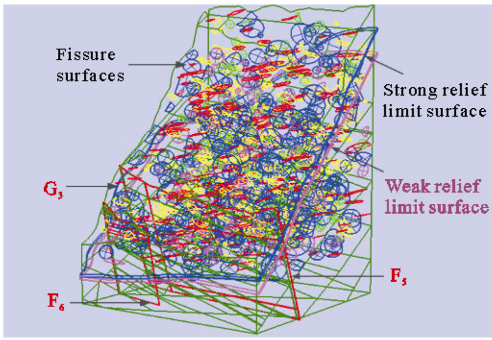
An actual slope is located on the right bank of a hydropower facility being constructed in the Yalong River, China, which trends in the N35°—40°E direction, as shown in Fig. 4. It is an intake slope located in the upstream of the dam. The rock mass of the slope mainly consists of marbles, and the rock mass quality classification includes III_2 and IV_1. The slope angle is about from 40° to 50° below the elevation of 1 850 m, and above the elevation the angle is over 65° like a cliff. The height of the cliff is about 60—100 m. The overall occurrence of the rock strata is N30°—35°E/NW∠30°—35°. There are two large faults of F_5 and F_6 in the slope. F_5 runs through the whole slope, and its occurrence is N50°—60°E/SE∠75°—90° with about 10—20 cm wide fracture zone. The occurrence of F_6 is N40°—50°E/SE∠70°—75°, with about 5—20 cm wide fracture zone. While a large interlayer compressive belt of G_3 is exposed in the elevation of 1 780 m, and it is the main structural surface that controls the slope stability. The joint fissures are divided into five groups by occurrence: N30°—60°E/NW∠30°—50°, N40°—60°E/SE∠60°—80°, N10°—30°E/NW∠40°—60°, N40°—70°W/NE∠70°—85°, and N10°—30°W/NE∠60°—75°.

(a) Overall aspect (b) Local crack

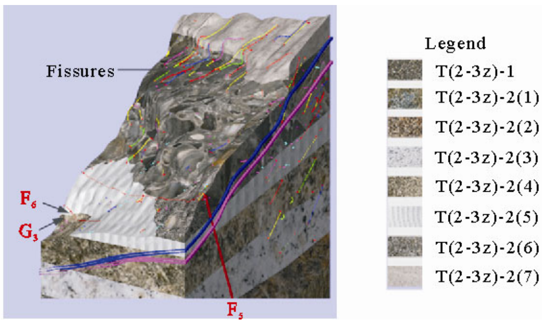
Fig. 4 Photographs of slope

3.2 Identification of key blocks

The proposed method was programmed based on the VisualGeo platform[22] by Visual C++, OpenGL, and OpenNURBS graphics library. The geological model of the slope was reconstructed as shown in Fig. 5, integrated with several rock strata, two faults, an interlayer compressive belt, 1 137 fissures, and the strong and weak relief limit surfaces. Based on the integrated model, the slope was cut into many surface blocks and block-groups by the structural surfaces through Boolean operation. As shown in Fig. 6, 6 surface blocks and 3 block-groups over 200 m^3 were identified in the studied region.



(a) Geometry model



(b) Geological model

Fig. 5 3D models of slope

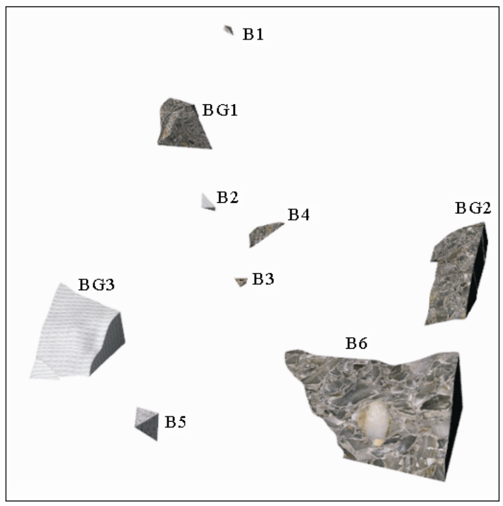


Fig. 6 Identified 6 blocks and 3 block-groups

3.3 Stability computation and analysis of blocks

The required physical and mechanical parameters of rock structures are listed in Tab. 1. The calculation results are shown in Tab. 2 and Tab. 3. The lithology of a block can be obtained from the reconstructed 3D geological model. The center of gravity is the coordinates of the block volume centroid, and the volume is calculated by the Gauss-Legendre quadrature formula based on NURBS structure. The sliding surfaces are determined by the occurrences of the cutting structural surfaces. Tab. 2 lists the attributes of 6 surface blocks, such as lithology, center of gravity, volume, sliding surface and factor of safety. Tab. 3 lists the results of 3 block-groups, including the center of gravity, block number, total volume and factor of safety.

Considering the engineering requirement, the minimum FOS of the slope is 1.2. From Tab. 1, the results indicate that the FOS values of 6 surface blocks are greater than 1.2 and they can meet the demand. From Tab. 2, the factors of safety for BG1 and BG3 block-groups are between 1.0 and 1.2, indicating that they may be unsafe. The block-group BG1 includes four blocks and its FOS is 1.019. The block-group BG3 consists of two blocks, which were cut by faults, the interlayer compressive belts, fissures and the free face. The total volume of BG3 is over 100 000 m^3 and its FOS is 1.073. These blocks are close to the upstream dam and a tunnel would pass through the slope, so they should be reinforced to avoid failure by necessary measures.

Tab. 1 Physical and mechanical parameters of rock structural surfaces

Structural surface	Cohesion c /MPa	Internal friction angle φ/ (°)	Density γ/ (kN·m^{-3})
Rock masses	0.35	38.7	27.0
Faults	0.02	22.0	27.0
Discontinuities	0.07	31.8	27.0
Interlayer belts	0.15	26.8	27.0

Tab. 2 Calculation results of surface blocks

No.	Center of gravity (x, y, z)	Volume/m^3	Sliding surface	FOS
B1	(916.72, 911.19, 2 046.18)	213.45	Single surface	3.475
B2	(814.79, 894.15, 1 849.31)	529.09	Single surface	1.812
B3	(794.42, 9 848.70, 1 769.20)	297.00	Two surfaces	4.527
B4	(704.43, 794.95, 1 821.65)	2 061.16	Single surface	1.792
B5	(690.33, 909.70, 1 635.88)	3 680.83	Two surfaces	2.654
B6	(600.38, 627.90, 1 680.98)	174 073.07	Single surface	1.411

Tab. 3 Calculation results of block-groups

No.	Block number	Center of gravity (x, y, z)	Total volume/ m^3	FOS
BG1	4	(802.66, 918.03, 1 935.77)	35 569.34	1.019
BG2	4	(688.67, 596.47, 1 793.79)	69 780.28	1.445
BG3	2	(652.83, 966.19, 1 725.95)	106 046.73	1.073

The results were verified by the practice. The sliding failure occurred in the rock mass about 15 000 m^3 after near blasting operation during the project construction, as shown in Fig. 7. It was mainly caused by F_5, G_3 and external explosion action. The block-group BG3 is the major part of the failure rock mass.

Fig.7 Instable rock mass in the studied region

4 Conclusions

This research identified and analyzed the stability of surface blocks of a rock slope under complex geological conditions. Based on the 3D multi-scale geological modeling method and surface block theory of rock mass structures, the integrated model was reconstructed including rock strata, faults, interlayer compressive belts, and fissures, and some key surface blocks were identified and obtained. The factor of safety for single key block was calculated by different equations under various motion modes. Two or more connected blocks were grouped together and they were defined as a block-group. The Sarma method was used for calculating the FOS of a block-group. The proposed approach was applied to an actual rock slope of a hydropower project. The potential instable blocks were presented visually. The results can provide useful information for determining possible instable region of rock slopes and designing effective support scheme in advance.

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