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Influence of Soft Filler on Stress Concentration Factor of Elliptic Holes in a Rectangular Plate*

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Abstract: Finite element models were established to analyze the influence of soft filler on stress concentration for a rectangular plate with an elliptic hole in the center. The influence was quantified by means of stress concentration factor (SCF). Seven shape factors of the elliptic hole and three levels of elasticity modulus of the soft filler were considered. The reduction coefficient and sensitivity index of SCF are the two indicators in evaluating the influence of soft filler. It was found that the reduction coefficient of SCF increases significantly as the shape factor and the elasticity modulus of the filler increase, indicating that soft filler can reduce the concentrated stress effectively, especially when the shape factor is great. Analysis for the sensitivity index of SCF indicates that SCF is more sensitive to materials with small elasticity modulus than to materials with large one.
Keywords: stress concentration factor; elliptic hole; soft filler; shape factor; elasticity modulus

Stress concentration takes place at discontinuities of materials, and the degree of stress concentration is highly affected by the shape of the discontinuities and the properties of the filled material at discontinuities.

Much work has been done on the stress concentration of a rectangular plate with a hole[1-5]. Simha and Mohapatra explored the stress concentration around irregular holes using complex variable method[6]. Babu *et al* determined stress concentration factors (SCFs) of the blades of steam turbine rotor with finite element analysis[7]. But work on the influence of soft filler is limited. The early work by the authors reveals that soft filler can shorten the radius of the affected surrounding stress field significantly[8]. Turatsinze *et al* pointed out that soft filler can reduce the stress singularity at the first crack tips running into the rubber/cement-matrix interface[9]. This study is to analyze the influence of soft filler on stress concentration of a rectangular plate with an elliptic hole in the center using finite element method. SCF is introduced in the analysis. The work is conducted with an assumption that the elliptic hole can be fully or partially filled with soft filler, and that the bonding strength between the soft filler and the matrix is adequate.

1 Definition and model

1.1 Definition

Fig. 1 shows the stress distribution of the cross-section of a rectangular plate with an elliptic hole in the center. The maximum stress σ_{max} takes place at points *A* and *C*.

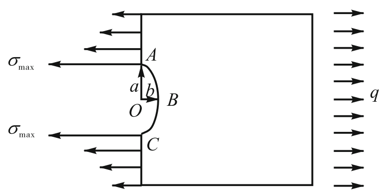

Fig.1 Stress distribution of the cross-section of a rectangular plate with an elliptic hole in the center

Theoretically, under unixial tension stress the relationship between SCF and the shape factor of the elliptic hole follows Eq. (1) [10], while in numerical analysis, SCF is obtained as the ratio of the maximum stress σ_{max} to the tensile loading *q*, which is shown in Eq. (2).

$$\mathrm{SCF} = 1 + \frac{2a}{b} = 1 + 2\lambda \tag{1}$$

$$\mathrm{SCF} = \frac{\sigma_{max}}{q} \tag{2}$$

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where a is the major axis length, b the minor axis length, λ the shape factor of the elliptic hole, and $\lambda = a/b$.

1.2 Model description

As shown in Fig. 2, a total of 7 different types of elliptic holes are considered with λ being 1, 2, 4, 8, 16, 32, and 64. Three types of finite element models are established and they are unfilled model Ⅰ, fully-filled model Ⅱ, and half-filled model Ⅲ.

Due to the symmetries of the models in geometry, loading, and boundary, a quarter of them were used in modeling. Shown in Fig.3 is the unfilled model Ⅰ. Point O is the coordinate origin. Rectangle $OCDE$ is 400 mm × 200 mm. Seven different elliptic holes are obtained by fixing the length of the line segment OA as a constant value of 20 mm, and regulating the length of OB to be 20 mm, 10 mm, 5 mm, 2.5 mm, 1.25 mm, 0.625 mm, and 0.312 5 mm, respectively. The fully-filled model Ⅱ is exactly the same as model Ⅰ, except that area OAB is filled with soft filler. For the half-filled model Ⅲ, as shown in Fig.4, M is the midpoint of OA, N is a point located on arc AB, and MN is parallel to OB. Area $OMNB$ is the part to be filled with soft filler.

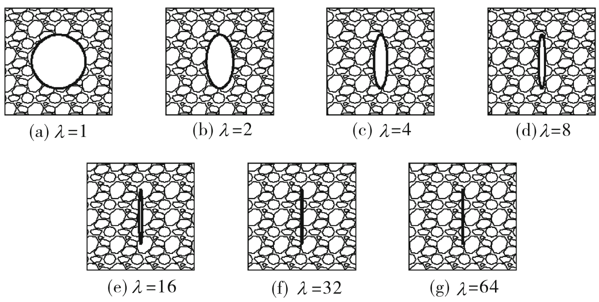

Fig.2 Seven cases of the elliptic hole

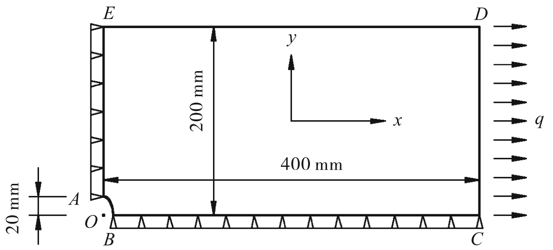

Fig.3 Unfilled model Ⅰ

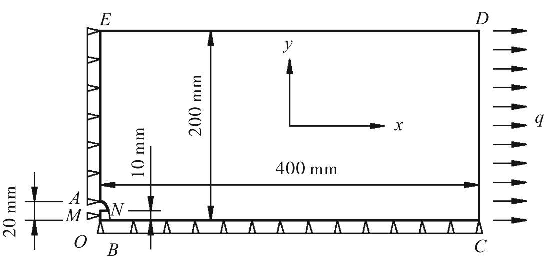

Fig.4 Half-filled model Ⅲ

Meshing plan was conducted as follows. In area ⅰ, which is far from the elliptic hole, the element size is approximately 4 mm; in area ⅱ, which is near line segment AB, the element size is shortened to 1 mm; and in area ⅲ, which is in the vicinity of point A, the element size is further shortened and gets to 0.001 mm as it reaches point A. From area ⅰ to area ⅲ, element size reduces gradually. Plane element 183 is applied in the analysis.

The boundaries of the models are as follows:

(1) For Edge OE, all nodes are constrained, $UX = 0$;

(2) For Edge OC, all nodes are constrained, $UY = 0$;

(3) For Edge CD, tensile loading $q = 100\ \text{N/mm}^2$ is applied;

(4) The other edges are free.

The elasticity modulus of the matrix is $E_0 = 2.5\times10^4$ MPa. Poisson's ratio is 0.25. The three levels of the elasticity modulus of the filler are taken by setting E/E_0 to be 0.1, 0.01, and 0.001, where E is the elasticity modulus of the filler and its Poisson's ratio is taken as 0.30.

In addition, an assumption should be made before the calculation proceeds: the interface between the soft filler and the cement matrix is a perfect bond and will not be separated under tension. This assumption is incorporated in the finite element model by sharing nodes between the two materials at the interface.

1.3 Error evaluation

Error evaluation is conducted by comparing the theoretical values of SCF (k_0), which is figured out with Eq.(1), and numerical values of SCF (k_1), which is calculated through model Ⅰ. The two sets of values for SCF are listed in Tab.1.

Tab.1 Theoretical values and numerical values of SCF

λ	$a\times b$ /(mm×mm)	k_0	k_1
1	20×20	3	3.037
2	20×10	5	5.044
4	20×5	9	9.066
8	20×2.5	17	17.110
16	20×1.25	33	33.170
32	20×0.625	65	65.630
64	20×0.312 5	129	129.200

Error evaluation of the model is obtained through Eq.(3), and the result shows that the numerical values agree well with the theoretical ones.

$$\text{Error} = \frac{1}{7}\sum\frac{|k_1 - k_0|}{k_0}\times100\% = 0.73\% \tag{3}$$

Fig.5 is a fringe plot for area ⅲ in the vicinity of point A of the unfilled model Ⅰ when $\lambda = 64$ and magnified 1 300 times.

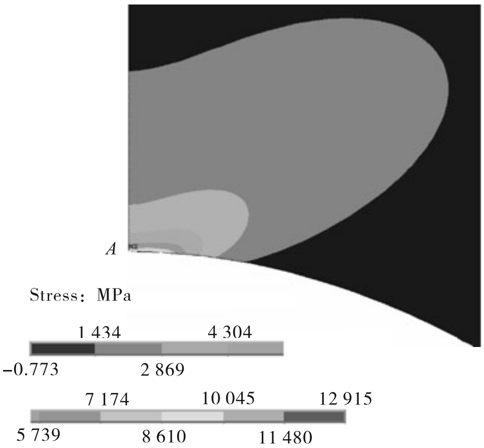

Fig.5 Stress contour of area iii in model Ⅰ ($\lambda = 64$)

It can be seen that the contour curve is of great smoothness, which indicates that the meshing plan used above is rational.

2 Calculations and discussion

Seven shape factors, two kinds of filling cases, and three levels of E/E_0 are considered in the following analysis. Another two sets of values for SCF are obtained through the fully-filled model Ⅱ and the half-filled model Ⅲ and the results are tabulated in Tab. 2 and Tab. 3, respectively.

Tab.2 SCF for model Ⅱ

λ	k_2		
	$E/E_0 = 0.001$	$E/E_0 = 0.01$	$E/E_0 = 0.1$
1	3.030	2.976	2.520
2	5.024	4.847	3.589
4	8.990	8.367	4.958
8	16.830	14.610	6.357
16	32.050	24.620	7.490
32	61.250	38.320	8.203
64	112.050	52.750	8.601

Tab.3 SCF for model Ⅲ

λ	k_3		
	$E/E_0 = 0.001$	$E/E_0 = 0.01$	$E/E_0 = 0.1$
1	3.034	3.016	2.854
2	5.037	4.976	4.501
4	9.039	8.823	7.450
8	16.960	16.190	12.590
16	32.450	29.180	21.610
32	63.530	54.510	38.460
64	113.780	90.130	65.660

Tabs.1—3 reveal that SCF is reduced significantly by the soft filler in both fully-filled and half-filled cases. The reduction coefficients for the two cases are determined by Eqs.(4) and (5), respectively.

$$r_1 = \frac{k_1 - k_2}{k_1} \times 100\% \tag{4}$$

$$r_2 = \frac{k_1 - k_3}{k_1} \times 100\% \tag{5}$$

where r_1 and r_2 represent the reduction coefficients of SCF in fully-filled and half-filled cases, respectively and the results are shown in Figs.6 and 7, respectively.

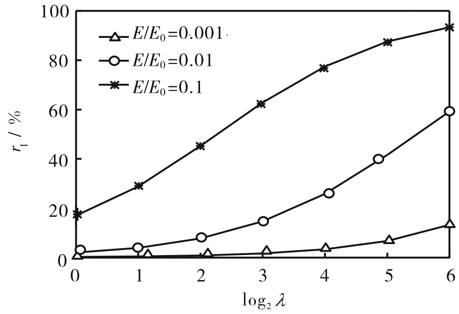

Fig.6 Reduction coefficient of SCF in fully-filled case

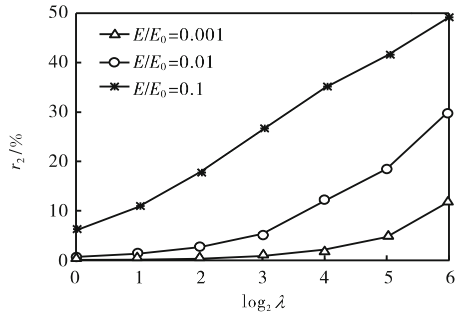

Fig.7 Reduction coefficient of SCF in half-filled case

It can be seen from Figs. 6 and 7 that the reduction coefficient of SCF increases with λ and E/E_0 in both cases. In the fully-filled case, SCF is reduced approximately by 93.3% with E/E_0 being 0.1 and λ being 64, and this value is 49.2% in the half-filled case. Among the three levels of E/E_0, the reduction coefficient of SCF reaches the highest value when E/E_0 is 0.1 and reaches the lowest value with E/E_0 being 0.001.

Another concern about the influence of soft filler is the sensitivity index of SCF, which refers to the reduction rate of SCF per unit E/E_0. The sensitivity indexes in both cases are obtained according to Eqs.(6) and (7) and the calculated results are shown in Fig.8 and Fig.9, respectively.

$$s_1 = \frac{k_1 - k_2}{k_1} \bigg/ \frac{E}{E_0} = \frac{E_0(k_1 - k_2)}{E k_1} \tag{6}$$

$$s_2 = \frac{k_1 - k_3}{k_1} \bigg/ \frac{E}{E_0} = \frac{E_0(k_1 - k_3)}{Ek_1} \tag{7}$$

where s_1 and s_2 represent the sensitivity indexes in fully-filled and half-filled cases, respectively.

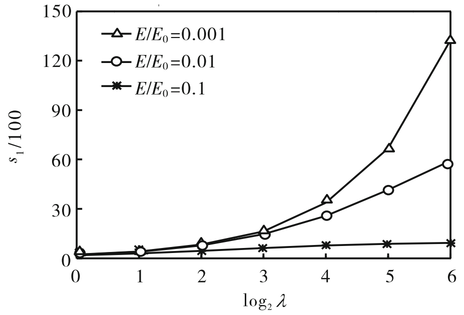

Fig.8 Sensitivity index of SCF in fully-filled case

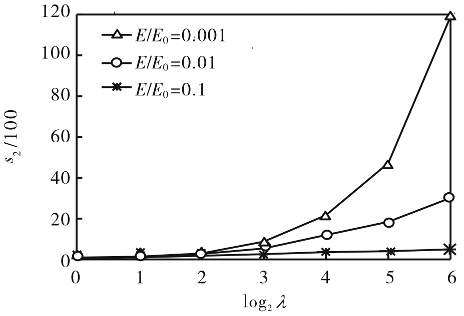

Fig.9 Sensitivity index of SCF in half-filled case

It can be seen that firstly the sensitivity index increases significantly with λ in both cases; secondly, within the range of E/E_0 being 0.001—0.1, the smaller the elasticity modulus of the filler, the larger the sensitivity index of SCF. It indicates that SCF is more sensitive to materials with small elasticity modulus than to materials with large elasticity modulus.

3 Conclusions

Finite element models were established to analyze the influence of soft filler on SCF. Four types of SCF were figured out with theoretical formula, unfilled model, fully-filled model and half-filled model. The first two types were used for error evaluation of the models and the last two were for the influence of soft filler. It was found that in half-filled and fully-filled cases, the reduction coefficient of SCF increases significantly with the shape factor of the elliptic hole and the elasticity modulus of the soft filler. It suggests that soft filler can reduce the concentrated stress effectively, especially when the shape factor is great. Within the range of E/E_0 being 0.001—0.1, SCF is more sensitive to materials with small elasticity modulus than to materials with large one.

References

[1] Dai Longchao, Guo Wanlin, Wang X. Stress concentration at an elliptic hole in transversely isotropic piezoelectric solids [J]. *International Journal of Solids and Structures*, 2006, 43(6): 1818-1831.

[2] Dumir P C. Stress concentration around elliptical holes in a rectangular plate [J]. *Arabian Journal for Science and Engineering*, 1988, 13(1): 109-112.

[3] Hanus J B, Burger C P. Stress-concentration factors for elliptical holes near an edge [J]. *Experimental Mechanics*, 1981, 21(9): 336-340.

[4] Kubair D V, Bhanu-Chandar B. Stress concentration factor due to a circular hole in functionally graded panels under uniaxial tension [J]. *International Journal of Mechanical Sciences*, 2008, 50(4): 732-742.

[5] Bakhshandeh K, Rajabi I. Orthotropy and geometry effects on stress concentration factors for short rectangular plates with a centred circular opening[J]. *Journal of Strain Analysis for Engineering Design*, 2007, 42(7): 551-556.

[6] Simha K R Y, Mohapatra S S. Stress concentration around irregular holes using complex variable method [J]. *Sadhana*, 1998, 23(4): 393-412.

[7] Babu R N, Ramana K V, Rao K M. Determination of stress concentration factors of a steam turbine rotor by FEA [J]. *Proceedings of World Academy of Science*, *Engineering and Technology*, 2008, 39: 302-306.

[8] Yang Linhu, Zhu Han, Liu Chunsheng. Two-dimensional analysis of the influence of soft material on stress field around an elliptic hole [J]. *Journal of Tianjin University*, 2010, 43(7): 601-605(in Chinese).

[9] Turatsinze A, Garros M. On the modulus of elasticity and strain capacity of self-compacting concrete incorporating rubber aggregates [J]. *Resources*, *Conservation and Recycling*, 2008, 52(10): 1209-1215.

[10] Saven Γ H. *Stress Concentration Around Holes* [M]. Lu Dinghuo trans. Science Press, Beijing, China, 1958 (in Chinese).