KSME

Springer

Journal of Mechanical Science and Technology 37 (3) 2023

Original Article
DOI 10.1007/s12206-023-0209-z

Keywords:
· Complementary functions method
· Cylindrical hollow body
· Functionally graded material
· Hyperbolic heat conduction
· Laplace transform
· Spherical hollow

Correspondence to:
Nihat Can
nihatcan1919@gmail.com

Citation:
Can, N., Keles, I. (2023). A practical jointed approach to transient hyperbolic heat conduction of FGM cylinders and spheres. Journal of Mechanical Science and Technology 37 (3) (2023) 1223~1231.
http://doi.org/10.1007/s12206-023-0209-z

Received March 16th, 2022
Revised December 5th, 2022
Accepted December 8th, 2022

† Recommended by Editor Chongdu Cho

A practical jointed approach to transient hyperbolic heat conduction of FGM cylinders and spheres

Nihat Can[1] and Ibrahim Keles[2]

[1]Department of Mechanical Engineering, Ondokuz Mayis University, Samsun 55139, Turkey, [2]Department of Mechanical Engineering, Samsun University, Samsun 55139, Turkey

Abstract In this study, a practical jointed approach of hollow cylinder and sphere shape functionally graded material (FGM) is considered. Non-Fourier hyperbolic heat conduction method is used. The FGM materials consists of a mixture of ceramic and metallic materials that shows exponential variation in the radial direction. First, the problems are transformed to the Laplace domain form. After that complementary functions method (CFM) is preferred, yielding the solution appropriately that the Durbin's numerical inverse Laplace transform method is used to transform to the real space. The transient dynamic responses of temperature and heat flux are examined with respect to specific inhomogeneity parameters and various relative temperature changes. Different plots are produced in order to emphasize the correlation of the temperature distribution and beside this heat flux between changing time and also material properties. In order to verify the results in this study, a comparison was made with the existing solutions obtained in the literatures.

1. Introduction

There has been a growing interest in the field of FGM cylindrical materials of variable thickness and the effect of high temperatures on these structures. The applications of cylindrical FGM structures include aerospace/nuclear industries, power plants, aero-nautical and chemical plants where metals and metal alloys utilized exhibit a behavior of elastic nature. FGMs are composed of different materials and therefore the elastic and thermal characteristics continuously change throughout the material. Constituent materials of various attributes are constructed by changing the volume fraction of materials methodically. The materials utilized in FGM applications are heat affection, against to metal corrosion problems and high metal fracture values. The material requirements are therefore quite advanced as these structures are usually exposed to high intensity heat fluxes and subject to significant shifts. Thus, an immaculate and accurate heat transfer analysis of thick-walled cylindrical FGM structures is a necessity for engineering design and manufacture.

There are various theories regarding the heat conduction within solid bodies. Theory of Fourier's heat conduction bases on the temperature gradient, hence a heat flux between two bodies due to direct thermal conductivity. Heat conduction by using Fourier's law is largely sufficient for most of the engineering applications. Yet there are shortcomings to this theory such as the example of highly varying thermal loading in the case of pulsed laser heating. A study conducted by Babaei and Chen [1] presents that the surface temperature of a slab sample taken promptly above after a sudden thermal shock is 300 °C higher than that show us out of bounds by Fourier's law. Another problematic aspect to the Fourier's theory related heat conduction is that the theory is prone to fail through the low temperatures or heat influx of application is exceedingly large. However, Fourier's law requires a physically unrealistic rate of thermal wave propagation. There have been different theories about non-Fourier heat conduction established to explain the heat conduction in solids. Hyperbolic heat conduction theory is one of the non-

Fourier theories as allowable. Hyperbolic heat conduction theory imposes a thermal relaxation time that can be defined as the time in which the temperature field needs to adjust itself to thermal disturbances. As the name suggests, hyperbolic heat conduction theory yields to a hyperbolic differential equation for temperature contrary to Fourier's law in which the equation is parabolic. In recent years, there has been a few studies concerning the heat transfer analysis of homogeneous and FGM hollow cylinders based on non-Fourier (hyperbolic) heat conduction law.

Babaei and Chen [1, 2] presented the analysis of transient hyperbolic heat conduction to examine the temperature distribution in heterogeneous spherical and cylindrical shells. Various non-homogeneity parameters and normalized thermal relaxation constants are applied in terms of temperature and heat flux by the transient responses. There are also numerous studies relating to the field of propagation of thermal waves in cylindrical domains that include studies by Wilhelm and Choi [3], Barletta and Zanchini [4]; Barletta [5]; Barletta and Zanchini [6]; Barletta and Pulvirenti [7]; Zanchini and Pulvirenti [8] and Sadd and Cha [9]. In these studies, the axisymmetric one-dimensional conduction regions are examined using Laplace transformation and asymptotic analysis. As another approach, linear hyperbolic heat conduction problems in the cylinder form and the hollow sphere form are investigated by Lin and Chen [10] under a constant boundary temperature. Darabseh et al. [11] examines the transient thermal stresses in an orthotropic cylinder by employing the model of hyperbolic heat conduction. One dimensional heat conduction behavior of an infinite FGM cylinder is studied by Hosseini et al. [12] using analytical methods. During the temperature distribution of the infinite cylinder is obtained via Bessel functions. A combination of numerical methods that includes Laplace transforms and control-volume formulations is utilized by Liu and Chen [13] to investigate heat conduction problem in a finite planar plate, cylinder form and sphere form with reference to the model of hyperbolic heat conduction theory. Interface resistance is another aspect of that effects the thermal wave propagation through a solid body. Liu et al. [14] investigated the effect of the interface resistance and the behavior wave of thermal propagation in a layered solid cylinder employing the control-volume method. An analytical approach is applied to composite cylinders with time dependent temperature changes for multi-dimensional heat conduction by Lu et al. [15]. Keles and Conker [16] are investigated of the non-Fourier hyperbolic heat conduction analysis for heterogeneous hollow functionally graded (FG) cylinders and spheres, whose material properties vary exponentially across the thickness.

The main idea behind the modelling of FGM structural elements is to create subdivisions of material that are homogeneous in themselves with different properties as is the case with graded behavior. Although some analytical solutions (see, e.g., Keles and Conker [16]) for this type of the problem is available in the literature, they either are restricted to one inhomogeneity parameter for all material properties that is not the case irreal or contain complex solutions such that usually it is necessary to solve for each parameter separately, which is not practical for parametric analysis. From a parametric analysis point of view, for this type of problems numerical solution is becoming essential. This study shows a practical and very effective method by combining Laplace transform and complementary functions method (CFM). CFM can be utilized in the hyperbolic heat conduction problems in FG cylinders and spheres that include a composite of ceramic and metallic materials changing exponentially in the direction of radial. The results are effective that achieved using Durbins inversion method are monitored in the time internal and proven to be exactly (see, e.g., Temel [17] and Temel et al. [18]). Again, the method is employed in FGM cylinders form with simple power-law properties that can also effortlessly be analyzed by a simple inversion (Pekel et al. [19]). Obtained differential equations are in spatial coordinates and have different coefficients. These yield to a boundary problem for the between two-point. Choosing CFM fort initial-value problems treating of that can promptly be solved by any one of the standard methods available in the literature. This study utilizes the fifth order of Runga-Kutta method. The method has been applied to different systems under static pressure conditions such as cylinders, spheres and disks forms and the boundary condition steady-state thermal loads as presented by Tutuncu and Temel [20, 21]. This study is the continuation to previous studies of hyperbolic heat conduction problems in the Laplace domain. This work asserts that introduction of Laplace trans- formation bears a unified procedure for transient analysis with a high degree of simplicity. The solution procedure presented here starts with the development of benchmark solutions for cylindrical and spherical bodies as these are then validated using the available in the literature. Upon the obtained variables non-dimensional from, similar results obtained with the numerical analysis. The overall procedure is very well established, very simple and effective beside this it has the capacity to be promptly applied to cylinders and spheres.

2. The governing equations

Functionally graded hollow cylinder and same as sphere is selected heat conduction problem one-dimensional radially. FG hollow cylinder form and sphere form have dimensions especially outer radius r_o and inner radius r_i as shown in Fig. 1. Heat conduction through the system and the boundary conditions can be expressed as

$$T(r,t)\Big|_{r=r_i} = T_{\omega_i}, T(r,t)\Big|_{r=r_0} = T_{\omega_0}, \quad (1)$$

Respectively, r is direction of radially. T is temperature, t is time. Here, T_{w_i} inner and T_{ω_0} outer temperature of the corresponding hollow body. The heat conduction is form of the hyperbolic as,

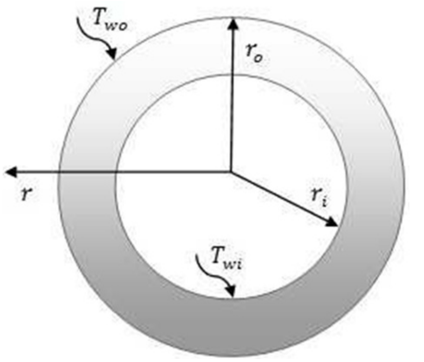


Fig. 1. Cross-section of the hollow bodies.

$$\vec{q} + \tau \frac{\partial q}{\partial t} = -K \nabla T \ . \tag{2}$$

Hence ∇ is the vector of heat flux. In addition, the symbols ($\vec{q}$, τ , K) are, respectively, thermal relaxation time, thermal conductivity, and the gradient operator, respectively. The governing energy balance equation is

$$\rho c_p \frac{\partial T}{\partial t} = S - \nabla \vec{q} \ . \tag{3}$$

Symbols ρ , c_p, and S are specific heat, density, and internal heat generation, respectively. Properties of the hollow bodies are assumed to be vary exponentially in radial direction as a mixture of the metal (with subscript, *m*) in the base and ceramic (with subscript, *c*) in the outer surface as:

$$\begin{aligned} K_{(r)} &= K_m e^{p(r-r_i)} , \\ \rho_{(r)} &= \rho_m e^{v(r-r_i)} , \\ c_{p(r)} &= c_{pm} e^{w(r-r_i)} , \end{aligned} \tag{4}$$

in which p , v and w are the dimensionless nonhomogeneity parameters.

$$\begin{aligned} p &= \frac{1}{r_0 - r_i} ln \left[\frac{K_c}{K_m} \right], \\ v &= \frac{1}{r_0 - r_i} ln \left[\frac{\rho_c}{\rho_m} \right], \\ w &= \frac{1}{r_0 - r_i} ln \left[\frac{c_{pc}}{c_{pm}} \right]. \end{aligned} \tag{5}$$

According to the material properties Eq. (4), through the surface of the cylinder is metal rich and outside of surface is ceramic rich. It is obvious that selecting the parameters p = 0, v = 0 and w = 0 results in homogeneous material, which is pure metal. In the numerical method path the following dimensionless parameters used,

$$\begin{aligned} & n = \frac{r}{r_0} , \ Q_0 = \frac{{}_m \tau}{r_0^2} , \ \theta = \frac{T - T_i}{T_{w0} - T_i} , \ \xi = \frac{{}_m t}{r_0^2} , \ r_\gamma = \frac{r_i}{r_0} , \\ & T_\gamma = \frac{T_{wi} - T_i}{T_{w0} - T_i} , \ Q = \frac{r_0}{K_m T_i} q_r , \end{aligned} \tag{6}$$

are introduced. Here, κ_m and T_i are, respectively, the thermal diffusivity of metal (κ_m = $K_m / \rho_m c_{pm}$) initial temperature inner surface of the body.

2.1 Cylindrical hollow body

This study overcome the properties of cylindrical symmetry and isotropy of the specimen. It should be noted that heat flux cannot vanish due to the part of the radial component. Additionally, all other derivatives equal to 0 whereas the derivative about the radial coordinate seems to be non-zero. By excluding the heat generation in internal terms, Eqs. (2) and (3) shows,

$$-\left(1 + \tau \frac{\partial}{\partial t}\right) q_r = K \frac{\partial T}{\partial r} , \tag{7}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(r q_r) = \rho c_p \frac{\partial T}{\partial t} \tag{8}$$

where q_r is includes radial component of the heat flux. If exponentially changing material properties Eq. (4) are substituted into Eqs. (7) and (8), the following linear differential equations are obtained:

$$-\left(1 + \tau \frac{\partial}{\partial t}\right) q_r = K_m e^{p(r-r_i)} \frac{\partial T}{\partial r} , \tag{9}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(r q_r) = \rho_m e^{v(r-r_i)} c_{pm} e^{w(r-r_i)} \frac{\partial T}{\partial t} \ . \tag{10}$$

Laplace transformation is taken of the normalized versions Eqs. (9) and (10) with Eq. (6) and taking an initial temperature distribution as uniform in the cylinder before thermal effect formulated as, $T(r, t)|_{t=0} = T_i$, managed equations are obtained Laplace form so domain that yields the results;

$$-(1 + Q_0 s)\tilde{Q} = e^{p(\eta r_0 - r_i)} \left(\frac{T_{w0} - T_i}{T_i}\right) \frac{\partial \tilde{\theta}}{\partial \eta} , \tag{11}$$

$$-\frac{1}{\eta}\frac{\partial}{\partial \eta}(\eta \tilde{Q}) = e^{(\eta r_0 - r_i)(w+v)} \left(\frac{T_{w0} - T_i}{T_i}\right) s \tilde{\theta} \tag{12}$$

where $\tilde{Q}$ and $\tilde{\theta}$ are the nondimensional heat flux and temperature in Laplace domain, Q_0 nondimensional thermal relaxation time ($Q_0 = {}_m \tau / r_0^2$) and s is the Laplace parameter. On can get the ordinary differential equation in terms of nondimensional temperature in Laplace domain $\tilde{Q}$ by simply eliminating $\tilde{Q}$ between Eqs. (11) and (12) as:

$$\eta^2\tilde{\theta}_{,\eta\eta} + (1 + p\eta r_0)\eta\tilde{Q}_{,\eta} - \eta^2 e^{(\eta r_0 - r_i)(w+v-p)} s(1 + Q_0 s)\tilde{\theta} = 0\,, \quad (13)$$

with the transformed boundary conditions Eq. (1) in the form of Laplace domain,

$$\tilde{\theta}_{(\eta,s)}\Big|_{\eta=1} = \frac{1}{s}\,,\ \tilde{\theta}_{(\eta,s)}\Big|_{\eta=r_\gamma} = \frac{T_\gamma}{s}\,. \quad (14)$$

2.2 Spherical hollow body

The spherical object can be obtained from Eqs. (2) and (3) condition for the absence of internal heat generation.

That is:

$$-\left(1+\tau\frac{\partial}{\partial t}\right)q_r = K\frac{\partial T}{\partial r}\,, \quad (15)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 q_r\right) = \rho cp\frac{\partial T}{\partial t}\,. \quad (16)$$

Substituting Eq. (4) into Eqs. (15) and (16), one gets the following differential equations:

$$-\left(1+\tau\frac{\partial}{\partial t}\right)q_r = K_m e^{p(r-r_i)}\frac{\partial T}{\partial r}\,, \quad (17)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 q_r\right) = \rho_m e^{v(r-r_i)} c_{pm} e^{w(r-r_i)}\frac{\partial T}{\partial t}\,. \quad (18)$$

Laplace transformation is taken of the normalized versions Eqs. (17) and (18) with Eq. (6) and as beside this taking an initial temperature uniformly in the sphere specimen before heat impact shows, $T(r, t)|_{t=0} = T_i$, the managed equations are obtained in the form of Laplace that yields the results;

$$-(1+Q_0 s)\tilde{Q} = e^{p(\eta r_0 - r_i)}\left(\frac{T_{w0}-T_i}{T_i}\right)\frac{\partial\tilde{\theta}}{\partial\eta}\,, \quad (19)$$

$$-\frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left(\eta^2\tilde{Q}\right) = e^{(\eta r_0 - r_i)(w+v)}\left(\frac{T_{w0}-T_i}{T_i}\right)s\tilde{\theta} \quad (20)$$

where $\tilde{Q}$ and $\tilde{\theta}$ are the normalized heat flux and temperature in Laplace domain. On can get the ordinary differential equation in terms of nondimensional temperature in Laplace domain $\tilde{\theta}$ by simply eliminating $\tilde{Q}$ between Eqs. (19) and (20) as

$$\eta^2\tilde{\theta}_{,\eta\eta} + (2 + p\eta r_0)\eta\tilde{\theta}_{,\eta} - \eta^2 e^{(\eta r_0 - r_i)(w+v-p)} s(1 + Q_0 s)\tilde{\theta} = 0\,. \quad (21)$$

3. Solution procedure

CFM is used to solve two-point boundary values problems as a system of initial-value problems is applied to the present problem. Using numerical method fifth order Runge-Kutta method (RK5) is utilized to solve the system of initial conditions through the problems of concern. It is created at 21 collocations points through the thickness (20 intervals) in the interval $0 \le \eta \le 1$. The solution procedure of these systems can be followed from the study of Tutuncu and Temel [20, 21]. By using this procedure both homogeneous solutions $\tilde{\theta}_i$ and its derivative $\tilde{\theta}'_i$ are calculated at the same time.

The non-dimensional temperature values, $\theta(\eta, \xi)$ and its derivative, $\theta'(\eta, \xi)$ in the time domain can be obtained numerically from inverse Laplace transform.

The non-dimensional temperature values, $\theta(\eta, \xi)$ and its derivative, $\theta'(\eta, \xi)$ in the time domain can be obtained numerically from inverse Laplace transform. For this purpose, the modified Durbin's inverse Laplace transform technique based on fast Fourier transform is used, see, e.g., Durbin [22]; Narayan [23]; Keles and Conker [16].

Table 1. Properties of ZrO_2 and T_i-6Al-4V at room temperature [24].

	k W/mK	E GPa	α 1/K	ν	ρ kg/m^3	cp J/kg.K
T_i-6Al-4V	7.5	116.7	9.5×10^{-6}	0.3	4.43×10^{-3}	560
ZrO_2	2.09	151.0	10×10^{-6}	0.3	5.5×10^{-3}	418

4. Numerical results and discussion

Numerical experiments are performed on the FG spherical and cylindrical body. The material properties of these hollow bodies are ceramic ZrO_2 in the heated outer surface ($r = r_o$) and alloy Ti-6A1-4V in the inner surface ($r = r_i$). These material compositions vary exponentially in the radial direction of the cylinder and the porosity is negligible. The thermomechanical properties of these materials at room temperature are given in Table 1. The inhomogeneity constants for this FGM are $p = -3.194$, $v = 0.541$ and $w = -0.731$.

In this research, heat propagation formed hyperbolic behavior is examined for determined inhomogeneity constants and different connected temperature changes (T_γ). For these each study, the inner and outer radii of objects are taken between 0.6 and 1. The surface conditions for the cylinder and sphere specimens are assumed to be equivalent. Initial temperature is 300 K taken for both spherical and cylindrical examples. Whereas the outer surface temperature is assumed to be 500 K. The propagation of thermal waves occurs from the outer surface to inner part of the specimen body that is due to the rapid increase in the outer surface temperature. Results obtained by Keles and Conker [16] are used for validation purposes, numerically. The comparing will be illustrated in the Tables 2-5. When viewed from Tables, overlapping results are seen agreement with the same results from Keles and Conker [16]. It is proven that upon a screening result given in Tables 2-5, a substantial amount of accuracy and efficiency is achieved using the CFM method. Furthermore, examined at only 21 points through the thickness yielded six-digit accuracy.

Figs. 2 and 3 illustrate the effect of inhomogeneity parameters (p, v, w) thermal waves have at three-time levels.

Table 2. Comparison of CFM results with Keles and Conker [16] for the temperature distribution (θ) of the cylinder with constant material properties at different time levels (β = 0.5, T_γ = 1, Q_0 = 0.35).

	ξ = 0.12		ξ = 0.36		ξ = 3.48	
η	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]
0.60	1.0013845	1.0013845	1.0007273	1.0007272	1.0006955	1.0006953
0.76	0.4535170	0.4535171	1.7403899	1.7403897	0.9941948	0.9941941
0.84	0.4935337	0.4935336	1.6326057	1.6326058	0.9941706	0.9941707
0.96	0.9941838	0.9941833	1.0472096	1.0472091	0.9947993	0.9947992
1.00	1.0013845	1.0013842	1.0007273	1.0007275	1.0006955	1.0006951

Table 3. Comparison of CFM results with Keles and Conker [16] for the heat flux (Q) of the cylinder with constant material properties at different time levels (β = 0.5, T_γ = 1, Q_0 = 0.35).

	ξ = 0.12		ξ = 0.36		ξ = 3.48	
η	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]
0.60	1.8173984	1.8173983	-1.4234832	-1.4234835	-0.0019621	-0.0019627
0.76	0.9323270	0.9323272	0.2665509	0.2665507	-0.0012374	-0.0012372
0.84	-1.0286423	-1.0286425	0.2062872	0.2062871	0.0018227	0.0018226
0.96	-1.8151240	-1.8151249	1.0897209	1.0897204	0.0042336	0.0042338
1.00	-1.7527049	-1.7527041	1.0349070	1.0349075	0.0008690	0.0008698

Table 4. Comparison of CFM results with Keles and Conker [16] for the temperature distribution (θ) of the sphere with constant material properties at different time levels (β = 0.5, T_γ = 1, Q_0 = 0.35).

	ξ = 0.12		ξ = 0.36		ξ = 3.48	
η	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]
0.60	1.0013845	1.0013843	1.0007273	1.0007274	1.0006955	1.0006951
0.76	0.6764156	0.6764154	0.6745350	0.6745353	1.0157001	1.0157005
0.84	0.9452520	0.9452524	0.7591375	0.7591378	1.0100147	1.0100142
0.96	1.0728651	1.0728652	0.9373393	0.9373391	1.0055012	1.0055019
1.00	1.0013845	1.0013847	1.0007273	1.0007279	1.0006955	1.0006950

Table 5. Comparison of CFM results with Keles and Conker [16] for the heat flux (Q) of the sphere with constant material properties at different time levels (β = 0.5, T_γ = 1, Q_0 = 0.35).

	ξ = 0.12		ξ = 0.36		ξ = 3.48	
η	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]	CFM	Keles and Conker [16]
0.60	0.1352381	0.1352385	0.1841313	0.1841314	-0.5866499	-0.5866490
0.76	0.6833333	0.6833335	-0.1942857	-0.1942851	0.0755956	0.0755955
0.84	-0.2474782	-0.2474783	-0.2286756	-0.2286751	0.0806954	0.0806953
0.96	-0.6049047	-0.6049046	-0.3308378	-0.3308375	0.0633018	0.0633011
1.00	-0.4783696	-0.4783699	-0.3752729	-0.3752723	0.0503581	0.0503586

Respectively, ξ = 0.12, 0.36, 3.48. Figs. 2(a) and 3(a) propose that there is the finite amount needed to adjust to the thermal disturbances at dimensionless time 0.12 consequent to the application heat conduction theory as hyperbolic. It can be deducted that the temperature increases in the inner division of the cylinder and sphere has not reflected on the temperature gradient as the changing temperature and heat flux is seen as zero. As indicated in Figs. 2(a) and 3(a), the wave fronts of the homogeneous cases (p = 0, v = 0, w = 0) are ahead of the inhomogeneity cases. This indicates that for the homogeneous

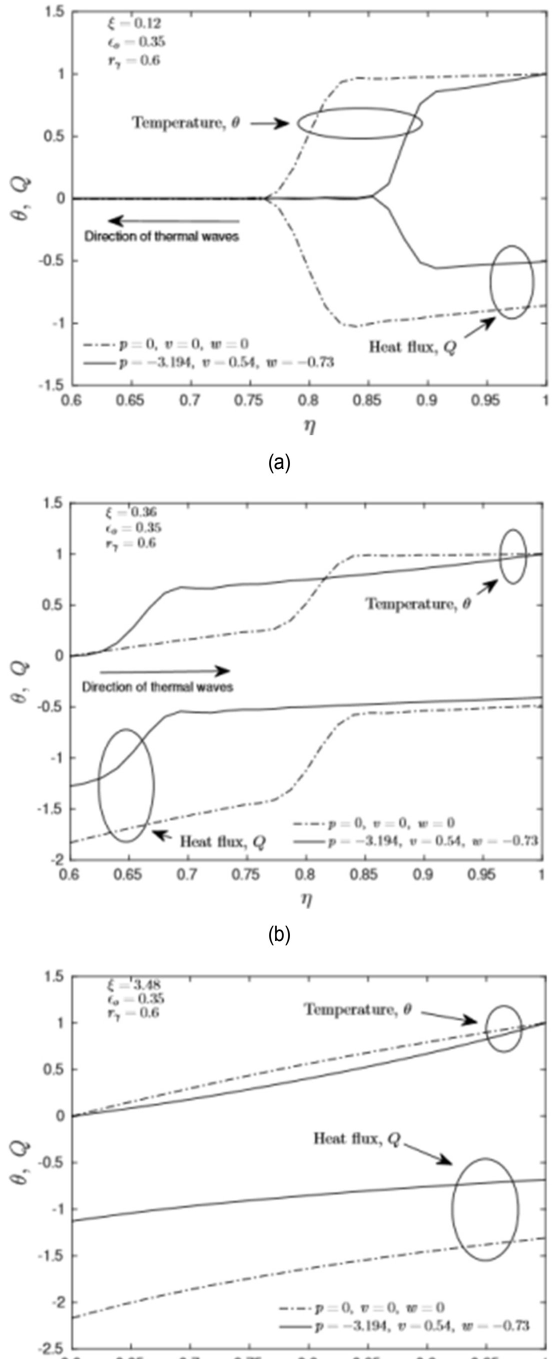


Fig. 2. Inhomogeneity parameters shows the distribution of temperature and heat flux parameters at different time levels for T_γ = 0 (hollow cylinder).

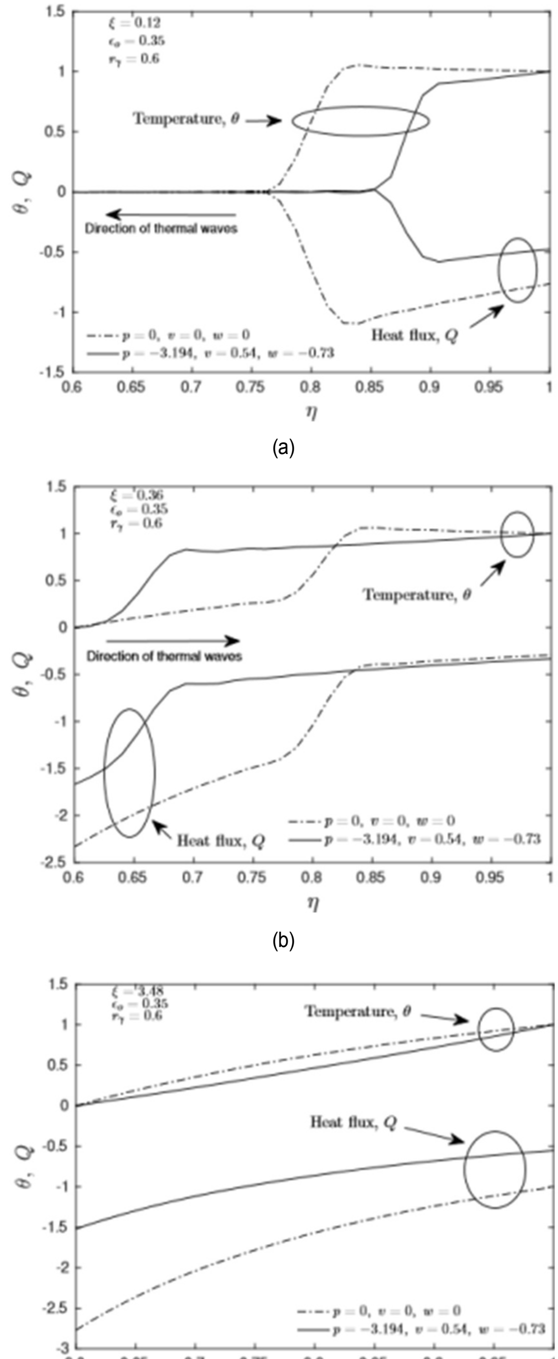
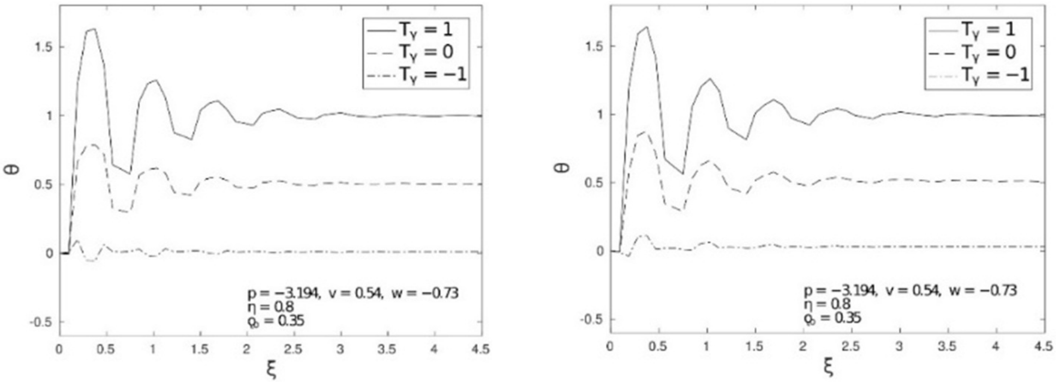


Fig. 3. Temperature and heat flux distribution fort the condition of inhomogeneity parameters at different time levels for T_γ = 0 (hollow sphere).

cases the thermal wave propagation is higher than inhomogeneity cases. A correlation is drawn between thermal conductivity and thermal wave dissipation speed in the form of direct proportionality, meaning that a bigger value of thermal conductivity yields a high value speed about the thermal wave distribution as seen in Figs. 2(b) and 3(b).

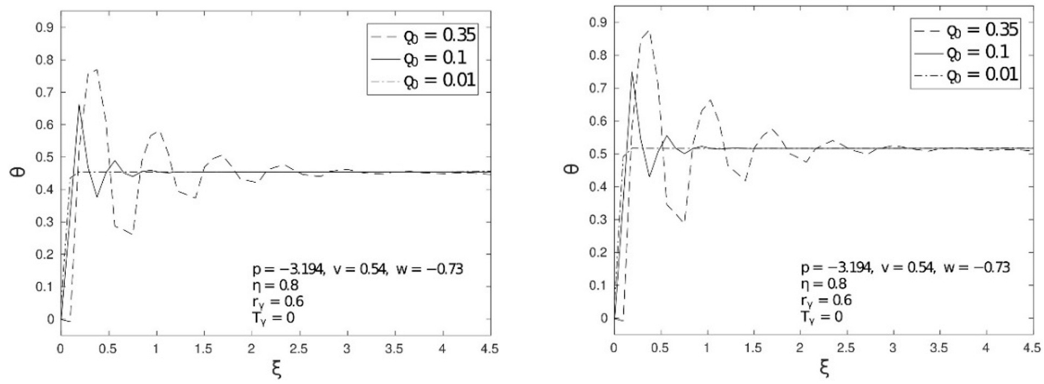
As suggested in Figs. 2(c) and 3(c), the thermal waves dispel that eventually leads to the full extermination, which indicates the condition steady-state and temperature distribution and heat flux distribution both for inhomogeneity and homogeneous cases.

Figs. 4(a) and (b) show the temperature from at position of η



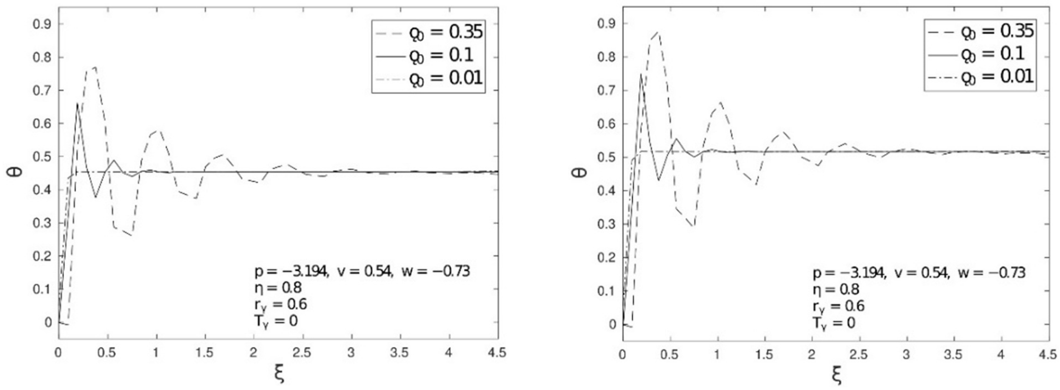
(a) Cylindrical hollow body (b) Spherical hollow body

Fig. 4. The relation temperature T_γ on the dimensionless temperature over the nondimensional time at the middle of FG object.



(a) Cylindrical hollow body (b) Spherical hollow body

Fig. 5. Distribution of the dimensionless temperature over time on variations of inhomogeneity parameters for T_γ = 0.



(a) Cylindrical hollow body (b) Spherical hollow body

Fig. 6. Examine between thermal relaxation time and the temperature at the middle of the FG object.

= 0.8 in the FG cylinder formed hollow and sphere specimen. The aim of relative temperature distribution (T_γ). It should be noted that there is proportional relationship between higher relative temperature and higher transient amplitude, hence longer duration of thermal waves. Therefore, it can be deduced that smaller relative temperature change yields to different diffusivity behavior throughout specimens as seen in Figs. 4(a) and (b).

Figs. 5(a) and (b) show us relation between temperature history and variations along with hollow cylinder as well as sphere specimens have non humogen parameters, respectively, when Q_0 = 0.35. For the steady-state boundary conditions final value of the temperature depends on the inhomogeneity parameters. Inhomogeneity cases results at the condition steady-state temperature yields higher amplitudes about the thermal waves.

Thermal relaxation time effect on the centerline temperature regime of functional graded cylinder and sphere is seen in Figs. 6(a) and (b). Much similar to relative temperature behavior, Figs. 6(a) and (b) illustrate that relation between higher thermal relaxation times and higher transient amplitudes. Beside this long duration the thermal waves. In the case of smaller relaxation time values, the diffusive behavior is observed. When Q_0 = 0. The use of theory Fourier's theory is considered more advantageous when we compare hyperbolic heat conduction theory.

5. Conclusions

In this research, hollow cylinder and sphere made of functionally graded materials are investigated jointed approach of non-Fourier hyperbolic heat conduction theory. The FGM materials consists of a mixture of ceramic and metallic materials that shows exponential variation through radial direction. First, the managed equations are used the Laplace form and then CFM is employed, beneficial the final data that are eventually the Durbin's numerical inverse Laplace transform method is used to form to the real space. The comparison of both temperature distributions and heat flux for the homogeneous material are given in the form of tables. For all cases, the numerical results are exactly agreed with this reported in Keles and Conker [16]. The influence of the inhomogeneity parameters and relative temperature changes on the temperature and heat flux distribution were analyzed. It is concluded that:

1) Inhomogeneity parameters are useful tools both for creating specific application models and controlling the temperature distribution and heat flux. They also have important role on the speed of thermal wave propagation and beside this the final temperature distribution.

2) Laplace, CFM and Durbin theories were successful in solving differential equations and as a result, the relationship between temperature distributions and heat flux was investigated.

Nomenclature

c_p : Specific heat
K : Thermal conductivity
Q : Dimensionless radial component of heat flux
$\vec{q}$: Heat flux vector
q_r : Radial component of heat flux
r : Radial coordinate
r_i : Inner radius
r_o : Outer radius
r_γ : Relative radius
S : Internal heat generation
s : Laplace parameter
T : Temperature field
T_{wi} : Inner temperature of the body
T_{wo} : Outer temperature of the body
T_γ : Relative temperature
T_i : Initial temperature
t : Time
p, w : Dimensionless nonhomogeneity parameters

Greek letters

η : Dimensionless radial coordinate
ξ : Dimensionless time
θ : Dimensionless temperature field
ρ : Density
τ : Thermal relaxation time
Q_o : Dimensionless thermal relaxation time
κ_m : Thermal diffusivity of metal ($\kappa_m = K_m/\rho_m c_{pm}$)
v : Dimensionless nonhomogeneity parameter

Subscript

m : Metal
c : Ceramic

References

[1] M. H. Babaei and Z. T. Chen, Transient hyperbolic heat conduction in a functionally graded hollow cylinder, *International Journal of Thermophysics*, 24 (2010) 325-330.

[2] M. H. Babaei and Z. T. Chen, Hyperbolic heat conduction in a functionally graded hollow sphere, *International Journal of Thermophysics*, 29 (2008) 1457-1469.

[3] H. E. Wilhelm and S. H. Choi, Nonlinear hyperbolic theory of thermal waves in metals, *Journal of Chemical Physics*, 63 (1975) 2119-2123.

[4] A. Barletta and E. Zanchini, Hyperbolic heat conduction and thermal resonances in a cylindrical solid carrying a steady-periodic electric field, *International Journal of Heat and Mass Transfer*, 39 (1996) 1307-1315.

[5] A. Barletta, Hyperbolic propagation of an axisymmetric thermal signal in an infinite solid medium, *International Journal of Heat and Mass Transfer*, 39 (1996) 3261-3271.

[6] A. Barletta and E. Zanchini, Thermal-wave heat conduction in a solid cylinder which undergoes a change of boundary temperature, *Heat and Mass Transfer*, 32 (1997) 285-291.

[7] A. Barletta and B. Pulvirenti, Hyperbolic thermal waves in a solid cylinder with a non-stationary boundary heat flux, *International Journal of Heat and Mass Transfer*, 41 (1998) 107-116.

[8] E. Zanchini and B. Pulvirenti, Periodic heat conduction with relaxation time in cylindrical geometry, *Heat and Mass Transfer*, 33 (1998) 319-326.

[9] M. H. Sadd and C. Y. Cha, Axisymmetric non-Fourier temperatures in cylindrically bounded domains, *International Journal of Non-Linear Mechanics*, 17 (3) (1982) 129-136.

[10] J. Y. Lin and H. T. Chen, Numerical solution of hyperbolic heat conduction in cylindrical and spherical systems, *Applied Mathematical Modelling*, 18 (7) (1994) 384-390.

[11] T. Darabseh, M. Naji and M. Al-Nimr, Transient thermal stresses in an orthotropic cylinder under the hyperbolic heat

conduction model, *Heat Transfer Engineering*, 29 (7) (2008) 632-642.

[12] S. M. Hosseini, M. Akhlaghi and M. Shakeri, Transient heat conduction in functionally graded thick hollow cylinders by analytical method, *Heat and Mass Transfer*, 43 (7) (2007) 669-675.

[13] K. C. Liu and H. T. Chen, Numerical analysis for the hyperbolic heat conduction problem under a pulsed surface disturbance, *Appl. Math. Comput.*, 159 (2004) 887-901.

[14] K. C. Liu, C. N. Lin and J. S. Wang, Numerical solutions for the hyperbolic heat conduction problems in a layered solid cylinder with radiation surface, *Appl. Math. Comput.*, 164 (2005) 805-820.

[15] X. Lu, P. Tervola and M. Viljanen, Transient analytical solution to heat conduction in composite circular cylinder, *Int. J. Heat Mass Transfer*, 49 (2006) 341-348.

[16] I. Keles and C. Conker, Transient hyperbolic heat conduction in thick-walled FGM cylinders and spheres with exponentially-varying properties, *Eur. J, Mech, A/Solids*, 30 (2011) 449-455.

[17] B. Temel, Transient analysis of viscoelastic helical rods subjected to time dependent loads, *Int. J. Solids Struct.*, 41 (5-6) (2004) 1605-1624.

[18] B. Temel, F. F. Calm and N. Tutuncu, Forced vibration of composite cylindrical helical rods, *Int. J. Mech. Sci.*, 47 (7) (2005) 998-1022.

[19] H. Pekel, I. Keles, B. Temel and N. Tutuncu, Transient response of FGM pressure vessels, In: J. Náprstek, J. Horáček, M. Okrouhlík, B. Marvalová, F. Verhulst and J. Sawicki (eds), *Vibration Problems ICOVP 2011, Springer Proceedings in Physics*, Springer, Dordrecht, 139 (2011) 315-320.

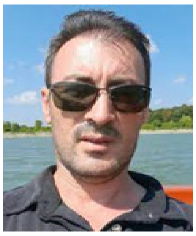
[20] N. Tutuncu and B. Temel, A novel approach to stress analysis of pressurized FGM cylinders, disks and spheres, *Compos. Struct.*, 91 (3) (2009) 385-390.

[21] N. Tutuncu and B. Temel, An efficient unified method for thermoe lastic analysis of functionally graded rotating disks of variable thickness, *Mech. Adv. Mater. Struct.*, 30 (1) (2013) 38-46.

[22] F. Durbin, Numerical inversion of laplace transforms: an efficient improvement to Dubner and abate's method, *Comput. J.*, 17 (1974) 371-376.

[23] G. V. Narayan, Numerical operational methods in structural dynamics, *Ph.D. Thesis*, University of Minnesota, USA (1979).

[24] T. Darabseh and K. B. Salameh, Numerical solution of transient thermal stresses in a functionally graded cylinder, *3rd WSEAS International Conference on Engineering Mechanics, Structures* (2010) 89-96.



Nihat Can completed his Master's degree at Çukurova University, Adana, Turkey. He received his Ph.D. in Mechanical Engineering from Ondokuzmayis University Samsun National University. His research interests include functionally graded materials and composite materials.

Ibrahim Keles is a Professor of Department of Mechanical Engineering, Samsun University. He received his Ph.D. in Mechanical Engineering from Çukurova University Adana, Turkey. His research interests include solid mechanics, composite materials, finite element analysis and functionally graded materials.