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A robust-based fatigue optimization method for systems subject to uncertainty

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Abstract A robust design method based on fatigue for stochastic problems is proposed. In most cases, fatigue analyses of structures are performed by assuming the nominal values of their parameters in a deterministic way. However, since the systems are frequently subject to parametric uncertainty and random forces simultaneously, it does not provide a representative fatigue index. Thus, it becomes essential to consider these uncertainties on the fatigue models. Here, a plate stochastic finite element is used to formulate the probabilistic Sines' index in frequency-domain, where the random fields are discretized using Karhunen-Loève expansion with hypercube sampling as stochastic solver. To maximize fatigue performance, a robust optimization combined with a robust reduction is proposed to increase the efficiency of the method. The envelopes of stresses and the Sines' coefficients for optimized solutions confirm the importance of considering uncertainty on fatigue models for more realistic situations.

1. Introduction

In the last decades, the demand for reliability of industrial products has increased significantly, especially when the structural components are frequently subjected to undesirable vibrations and uncertainties [1]. These factors can lead to unexpected failures of engineering systems in service or even a catastrophe. It is observed that, some models have been developed to account for the uncertainties in the context of fatigue analyses [1-7]. However, in the context of fatigue damage [8-13], few works [14, 15] have proposed a robust-based fatigue optimization methodology for dealing with systems subject to uncertainties and random forces, simultaneously, with the aim of estimating their optimal and robust fatigue reliability, which motivates the present study.

In the quest for multiaxial fatigue analyses, it is not usual to predict the fatigue life of a system by its failure mode, which involves the study of microstructural defects, slip bands and dislocations propagation [8]. In this case, the practical way to predict the fatigue damage coefficients consists in applying some methods based on experimental data, as discussed in Ref. [11]. Among the available methods, the Sines' global criterion [9] has been selected herein due to its simple formulation and the fact that it requires only two fatigue material properties of the material. Additionally, based on the works by Weber [8] and Papadopoulos et al. [16], the fatigue methods based on the second invariant of the stress deviatoric tensor and the endurance limit [12], such as Sines' method, are the most adequate to estimate the fatigue damage index of structures subjected to multiaxial loads. Moreover, de Lima et al. [1] found that, the Sines' criterion is well adapted to deal with non-proportional deterministic or random loadings. In particular, Lambert et al. [3] have proposed a probabilistic approach of Sines' formulation to estimate the fatigue damage of structures subjected to stationary random loadings. However, nothing was reported regarding the possibility of considering the inherent parametric uncertainties of the structure on its fatigue behavior. Clearly, in the context of the finite element (FE) modelling of the fatigue problem [5], it requires an adequate modification of Sines' equation to account for these uncertainties, which is performed in this study.

Here, parametric uncertainties are introduced into the fatigue model by using the stochastic FE method [6, 17], where the design variables to be considered as random fields are discretized according to the Karhunen-Loève (KL) expansion method [17]. It is important to emphasized that it is a very special manner to decompose a stochastic process into a finite (or truncated) sum of spatial terms forming an orthogonal basis and a sequency of a Gaussian random fields [4]. In the context of the FE method, the integration of matrices must be modified to account for these spatial terms to produce the random matrices, accounting for the uncertain physical and/or geometrical parameters. Thus, Sines' criterion combined with this stochastic formulation results in a very useful design tool to predict the fatigue reliability of engineering structures for dealing with more realistic scenarios of industrial interest.

Since this work proposes a robust fatigue-based optimization method for stochastic problems, another important aspect to be considered is the use of robust optimization tools for the search of optimal-robust solutions to maximize the fatigue life. It can be done by using the concept of vulnerability functions [18], which must be optimized in conjunction with the original cost functions, simultaneouly, using a multiobjective optimization algorithm [19]. However, for the purposes of this work, it leads to costly computation, since it involves a large number of function evaluations during the robust optimization. The proposition of an efficient reduction method to approximate the random responses of the stochastic problem has been motivated.

After the theoretical aspects, numerical applications have been performed with an academic example composed of a thin rectangular aluminum plate subjected to uncertainty on its thickness and fatigue properties and external loading. Through the numerical results, it becomes evident the importance of considering the uncertainties in fatigue models. Also, the efficiency of the method is shown to predict the optimal and robust fatigue reliability index of stochastic structures.

2. Background on the stochastic FE formulation of a thin rectangular plate

To perform the stochastic modeling of a thin plate element, it is interesting to perform a parameterization process where the uncertain parameters of interest are factored-out of the elementary matrices for the membrane and bending effects [4]. It is a straightforward way to introduce further the uncertainties on the model and provides computational cost savings in the estimation of the random fatigue coefficients during the robust optimization. In this study, the thickness, h, Young's modulus, E, and mass density, ρ, can be assumed as uncertain variables. Also, due to the inherent errors associated with the experimental estimation of the torsion fatigue strength, t_{-1} [3], it will be later considered as uncertain, since it appears only in Sines' equation.

Thus, starting from the classical deterministic model of a thin rectangular plate [20], the parameterized mass and stiffness matrices are given as:

$$\boldsymbol{M} = \rho h \bar{\boldsymbol{M}} \tag{1a}$$

$$\boldsymbol{K} = Eh\bar{\boldsymbol{K}}_m + Eh^3\bar{\boldsymbol{K}}_b \tag{1b}$$

where $\bar{\boldsymbol{K}}_{m/b} = \int_x\int_y \boldsymbol{B}^T_{m/b}(x,y)\,\boldsymbol{C}\,\boldsymbol{B}_{m/b}(x,y)\,dydx$ and $\bar{\boldsymbol{M}} = \int_x\int_y \boldsymbol{N}^T(x,y)\boldsymbol{N}(x,y)\,dydx$ are the stiffness and mass matrices, respectively, related to the membrane (subscript m) or bending (subscript b) effects. The shape functions appear in $\boldsymbol{N}(x,y)$ and the differential operators of the strain-displacements are defined in matrix, $\boldsymbol{B}(x,y)$, for the plane stress-state and $\boldsymbol{C} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$ is the material properties matrix, with E Young's module and ν the Poisson ratio. Details of these parameterized matrices can be found in Ref. [4].

For instance, uncertainties are introduced on the thickness, since it appears as a cubic variable in the bending stiffness matrix. For a random process, θ, it can be modelled using the following KL expansion:

$$h(x,y,\theta) = h + \alpha(x,y,\theta) \tag{2}$$

where h is the mean value of the random thickness, $\alpha(x,y,\theta) = \sum_{r=1}^{nkl}\sqrt{\lambda_r}f_r(x,y)\xi_r(\theta)$ is a set of nkl orthogonal random variables, $\xi_r(\theta)$, and the eigenvalues, $\lambda_r = \lambda_x\lambda_y$, and deterministic space eigenfunctions, $f_r(x,y) = f(x)f(y)$, are obtained for the covariance function, $C(x,y) = \exp\left(-|\Delta x|/L_{cor_x} - |\Delta y|/L_{cor_y}\right)$, for the 2D plate, where L_{cor_x} and L_{cor_y} are the correlation lengths in directions x and y. The following analytical solutions of the eigenproblem for a rectangular plate FE of dimensions, $a \times b \times h$, are given as [17]:

$$\lambda_{r_\bullet} = \frac{2c_\bullet^2}{\omega_{r_\bullet}^2 + c_\bullet^2},\ f_r(\bullet) = \beta_{r_\bullet} g_r(\bullet). \tag{3}$$

For r odd, $\beta_{r_\bullet} = 1\Big/\sqrt{L_{cor_\bullet} + \sin\left(2\omega_{r_\bullet}L_{cor_\bullet}\right)/2\omega_{r_\bullet}}$ and $g_r = \cos(\omega_{r_\bullet}\bullet)$, with $\omega_{r_\bullet}$ being the solutions of the following equation, $c_\bullet - \omega_{r_\bullet}\tan\left(\omega_{r_\bullet}L_{cor_\bullet}\right) = 0$, into the domain, $\left[\frac{(r-1)\pi}{L_{cor_\bullet}}; \frac{(r-0.5)\pi}{L_{cor_\bullet}}\right]$. But, for r even, $\beta_{r_\bullet} = 1\Big/\sqrt{L_{cor_\bullet} - \sin\left(2\omega_{r_\bullet}L_{cor_\bullet}\right)/2\omega_{r_\bullet}}$, $\omega_{r_\bullet}$ being the solutions of, $\omega_{r_\bullet} + c_\bullet\tan\left(\omega_{r_\bullet}L_{cor_\bullet}\right) = 0$, into the domain,

$\left[\frac{(r-0.5)\pi}{L_{cor_\bullet}};\frac{r\pi}{L_{cor_\bullet}}\right]$, $g_r=\sin(\omega_{r_\bullet}\bullet)$, $c_\bullet=1/L_{cor_\bullet}$, where $\bullet=x$ or $\bullet=y$.

Hence, by combining Eqs. (2) and (1) taking into account Eq. (3) and assuming the approximation of the random field, $h(\theta)^3\approx h^3+3h^2\alpha(\theta)$, it results in the following stochastic matrices:

$$\boldsymbol{M}(\theta)=\boldsymbol{M}+\sum_{r=1}^{nkl}\boldsymbol{M}_r\xi_r(\theta) \tag{4a}$$

$$\boldsymbol{K}(\theta)=\boldsymbol{K}+\sum_{r=1}^{nkl}\boldsymbol{K}_{m_r}\xi_r(\theta)+3h^2\sum_{r=1}^{nkl}\boldsymbol{K}_{b_r}\xi_r(\theta) \tag{4b}$$

where the elementary random mass and stiffness matrices are defined as:

$$\boldsymbol{M}_r=\delta_{xy}\int_x g_r(x)\int_y g_r(y)\boldsymbol{N}^T\boldsymbol{N}dydx \tag{5a}$$

$$\boldsymbol{K}_{(m/b)_r}=\delta_{xy}\int_x g_r(x)\int_y g_r(y)\boldsymbol{B}^T_{(m/b)}\boldsymbol{C}\boldsymbol{B}_{(m/b)}dydx \tag{5b}$$

where $\delta_{xy}=\beta_{r_x}\beta_{r_y}\sqrt{\lambda_{r_x}}\sqrt{\lambda_{r_y}}$, $\boldsymbol{B}(x,y)$ and $\boldsymbol{N}(x,y)$.

The elementary random matrices can be assembled to construct the random equations of motion for an N DOFs system. Then, the random frequency response functions (FRFs) matrix, $\boldsymbol{G}(\omega,\theta)=\left[\boldsymbol{K}(\theta)-\omega^2\boldsymbol{M}(\theta)\right]^{-1}$, can be obtained, where ω is the oscillation frequency.

If the stochastic system is also subjected to a stationary random force, $\boldsymbol{f}(t,\theta)$, and based on the strain-displacement relations, $\boldsymbol{\varepsilon}(x,y,t,\theta)=\boldsymbol{B}(x,y)\boldsymbol{u}(t,\theta)$, its random stresses are:

$$\boldsymbol{s}(x,y,t,\theta)=\boldsymbol{CB}(x,y)\boldsymbol{u}(t,\theta) \tag{6a}$$

$$\boldsymbol{u}(t,\theta)=\sum_{-\infty}^{+\infty}\boldsymbol{h}(t,\theta)\boldsymbol{f}(t-\tau,\theta)d\tau \tag{6b}$$

where $\boldsymbol{h}(t,\theta)=\frac{1}{2\pi}\sum_{-\infty}^{+\infty}\boldsymbol{G}(\omega,\theta)e^{j\omega t}d\omega$ is the response of the system due to an impulse-type forcing.

By applying the Fourier transform on Eq. (6b) for ω_0 = 0, the mean value of the stresses is given as:

$$\bar{\boldsymbol{s}}(x,y,t,\theta)=\boldsymbol{CB}(x,y)\boldsymbol{K}^{-1}(\theta)\bar{\boldsymbol{f}}(t). \tag{7}$$

It can be perceived that the mathematical expectation of the random stresses is proportional to the mean value of the stationary random loading, $\bar{\boldsymbol{f}}(t)$, and the random stiffness. To characterize it in the frequency domain, power spectral density (PSD) [16] is used here for a Gaussian random force with its mean, $\bar{\boldsymbol{f}}(t)$, and its PSD, $\boldsymbol{\Phi}_f(\omega)$. Based on Eq. (6b), the PSD of the response, $\boldsymbol{\Phi}_u(\omega,\theta)=\boldsymbol{G}(\omega,\theta)\boldsymbol{\Phi}_f(\omega)\boldsymbol{G}(\omega,\theta)^T$, is obtained and from Eq. (7), the stress response functions (SRFs) matrix can be computed as:

$$\boldsymbol{\Phi}_s(\omega,\theta)=\boldsymbol{\Psi}\boldsymbol{\Phi}_u(\omega,\theta)\boldsymbol{\Psi}^T. \tag{8}$$

For example, for a random stress-state of the form, $\boldsymbol{s}(t,\theta)=\left[s_{xx}(t,\theta)\ \ s_{yy}(t,\theta)\ \ s_{xy}(t,\theta)\right]^T$, the stress PSD is $\boldsymbol{\Phi}_s(\omega,\theta)=\begin{bmatrix}\boldsymbol{\Phi}_{xx,xx}(\omega,\theta) & \boldsymbol{\Phi}_{xx,yy}(\omega,\theta) & \boldsymbol{\Phi}_{xx,xy}(\omega,\theta)\\ & \boldsymbol{\Phi}_{yy,yy}(\omega,\theta) & \boldsymbol{\Phi}_{yy,xy}(\omega,\theta)\\ sym & & \boldsymbol{\Phi}_{xy,xy}(\omega,\theta)\end{bmatrix}$ where, $\boldsymbol{\Psi}=\boldsymbol{C}\,\boldsymbol{B}(x,y)$.

3. Estimation of sines' fatigue index for the stochastic system

Among the multiaxial fatigue criteria available in the open literature [9-13, 21] that should be used in this study, those that are based on the second invariant of the deviatoric stress tensor are preferable here. In fact, Weber [8] concluded that the best results are obtained with Fogue and Sines methods. Fogue's criterion showed slightly better results than Sines, but this later had a simpler formulation, requiring only two fatigue material properties, which facilitates its extension for dealing with the present stochastic problem.

In his work, Sines [9] established a fatigue damage coefficient, D_s, for deterministic problems. However, for systems subjected to uncertainties, as it is the case of interest here, it must be statistically estimated:

$$E\left[D_s(\theta)\right]=\frac{E\left[\sqrt{J_{2a}(\theta)}\right]}{E\left[t_{-1}(\theta)\right]} \tag{9}$$

where $J_{2a}(\theta)$ is the amplitude of the second invariant of the random deviatoric stress tensor and $t_{-1}(\theta)$ is the random torsion endurance limit. In this case, $E\left[D_s(\theta)\right]>1$ indicates fatigue failure for a sample, θ, with $E[\bullet]$ the expected value. Thus, for stationary random loadings, the estimation of Sines' index relies on the estimation of the square root of, $J_{2a}(\theta)$, accounting for the random nature of the SRFs, $\boldsymbol{s}(t,\theta)$, and, $t_{-1}(\theta)$, of the material.

To estimate, $E\left[\sqrt{J_{2a}(\theta)}\right]$, the *prismatic hull* method, proposed by Khalij et al. [22] for deterministic problems, is used herein. It is also based on the developments by Lambert et al. [3] and de Lima et al. [1] for deterministic structures under stationary random forces, resulting in the following estimation, where the random nature has been omitted for simplicity:

$$E\left[\sqrt{J_{2a}}\right]\approx\sqrt{\sum_{i=1}^{5}E\left[R_i\right]^2} \tag{10}$$

where the first two statistical moments of the random semi-axes, R_i, of the *prismatic hull* circumscribing the loading path are given as [3]:

$$E[R_i] = \sqrt{\lambda_0}\left(\mu_R + \gamma\beta_R\right) \tag{11a}$$
$$V[R_i] = \lambda_0 \pi^2 \beta_R^2 / 6 \tag{11b}$$

where $\mu_R = \sqrt{2\ln(\kappa_u N_p)}$ and $\beta_R = 1/\sqrt{2\ln(\kappa_a N_p)}$ are the mode and dispersion of the semi-axes, R_i, $N_p = \frac{T_p}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$ is the number of maxima for Gaussian random stresses, $\boldsymbol{s}(t,\theta)$, in a period of time, T_p, and λ_0, λ_1 and λ_2 are the zero, first and second orders spectral moments of, R_i, extracted from the PSD of the SRFs. γ = 0.5772 is the Euler-Mascheroni constant, and $\kappa_u = 1.5(1-\exp(-1.8\delta))$ and $\kappa_a = 7\delta$ for $\delta <$ 0.5, and κ_u = 0.94 and κ_a = 4.05 for $\delta \geq$ 0.5, with $\delta = \sqrt{1-\lambda_1^2/(\lambda_0\lambda_2)}$ known as irregularity factor, as detailed in Refs. [14, 22].

Based on Eq. (11) and moments definition for uncorrelated stationary Gaussian random processes, R_i, the mean and variance of, R_i^2, can be found as:

$$E\left[R_i^2\right] = E\left[R_i\right]^2 + V\left[R_i\right] \tag{12a}$$

$$V\left[R_i^2\right] = \overbrace{E[R_i]^2 V[R_i] + \frac{22}{5}V[R_i]^2 + \frac{48\zeta\sqrt{6}}{\pi^3}E[R_i]V[R_i]^{3/2}}^{E[R_i]^4} - E[R_i]^2 \tag{12b}$$

where ζ = 1.20206 is Apery's constant.

Finally, to estimate, $E\left[\sqrt{J_{2a}}\right]$, it is assumed a Gumbel distribution [5], and the term, $E\left[J_{2a}^2\right]$, is written similarly to, $E\left[R_i^4\right]$, in Eq. (12b). Thus, using the variance definitions, $E\left[J_{2,a}^2\right] = E\left[J_{2,a}\right]^2 - V\left[J_{2,a}\right]$ and $V\left[J_{2,a}\right] = E\left[J_{2,a}\right] - E\left[J_{2,a}\right]^2$, Eq. (13) is found with the unknown, $E\left[\sqrt{J_{2,a}}\right]$, to be predicted by using the Newton Rapson method.

$$\begin{aligned} &E[J_{2a}]^2 + V[J_{2a}] - 4E[J_{2a}]^2\left(E[J_{2a}] - E[J_{2a}]^2\right)\dots \\ &-\frac{22}{5}\left(E[J_{2a}] - E[J_{2a}]^2\right)^2\dots \\ &-\frac{48\zeta\sqrt{6}}{\pi^3}E[J_{2a}]\left(E[J_{2a}] - E[J_{2a}]^2\right)^{3/2} = 0. \end{aligned} \tag{13}$$

4. Reduction method applied in the study

It is not difficult to see that the fatigue analysis of stochastic systems leads to costly computations compared with deterministic cases, especially for situations in which optimization tools must be further used to maximize the fatigue reliability index. Thus, to increase the efficiency of the fatigue robust-based optimization method proposed here, a robust reduction basis, $\boldsymbol{T} \in C^{N\times Nr}$, is suggested with the aim of approximating the exact random responses, $\boldsymbol{u}(t,\theta) = \boldsymbol{T}\hat{\boldsymbol{u}}(t,\theta)$, where Nr is the number of vibration modes retained in $\boldsymbol{T}$, with $Nr << N$, and $\hat{\boldsymbol{u}}(t,\theta) \in C^{Nr}$ are the generalized coordinates.

Hence, the reduced FRFs and SRFs matrices are given, respectively, as:

$$\hat{\boldsymbol{G}}(\omega,\theta) = \left[\hat{\boldsymbol{K}}(\theta) - \omega^2\hat{\boldsymbol{M}}(\theta)\right]^{-1} \tag{14a}$$
$$\hat{s}(x,y,t,\theta) = \hat{\boldsymbol{\Psi}}(x,y)\hat{\boldsymbol{u}}(t,\theta) \tag{14b}$$

where $\hat{\boldsymbol{K}}(\theta) = \boldsymbol{T}^T\boldsymbol{K}(\theta)\boldsymbol{T}$, $\hat{\boldsymbol{M}}(\theta) = \boldsymbol{T}^T\boldsymbol{M}(\theta)\boldsymbol{T}$ are the stiffness and mass matrices of the reduced model and $\hat{\boldsymbol{\Psi}}(x,y) = \boldsymbol{CB}(x,y)\boldsymbol{T}$ is the reduced stress matrix.

Consequently, the reduced SRFs of the stochastic system is given as:

$$\hat{\boldsymbol{\Phi}}_s(\omega,\theta) = \hat{\boldsymbol{\Psi}}\hat{\boldsymbol{G}}(\omega,\theta)\hat{\boldsymbol{\Phi}}_f(\omega)\hat{\boldsymbol{G}}(\omega,\theta)^T\hat{\boldsymbol{\Psi}}^T \tag{15}$$

where $\hat{\boldsymbol{\Phi}}_f(\omega) = \mathbf{T}^T\boldsymbol{\Phi}_f(\omega)\mathbf{T}$.

However, the construction of a robust basis accounting for the modifications induced by the uncertainties is not easy, as discussed in Ref. [4]. For deterministic damped systems, a constant enriched Ritz basis should be used, $\boldsymbol{T}_0 = \left[\boldsymbol{\phi}_0 \ \ \boldsymbol{R}\right]$, formed by the vibration modes, $\boldsymbol{\phi}_0 = \left[\boldsymbol{\phi}_1,\dots,\boldsymbol{\phi}_{Nr}\right]$, of the associated conservative system, enriched by static residues, $\boldsymbol{R} = \boldsymbol{K}^{-1}\boldsymbol{f}$, due to the applied forcing, $\boldsymbol{f}$. But, for the stochastic system addressed herein, it is necessary to solve the eigenproblem Eq. (16) to update the vibration modes, $\boldsymbol{\phi}_0$, of the modified structure at each sample, since this basis does not represent with reasonable accuracy the modifications provoked by the uncertainties. Clearly, it increases strongly the computational burden, especially for real-word complex models requiring a large number of samples for the convergence of the stochastic responses.

$$\left[\boldsymbol{K}(\theta) - \lambda_i\boldsymbol{M}(\theta)\right]\boldsymbol{\phi}_i = 0 \qquad i = 1\,to\,Nr \tag{16}$$

where $\boldsymbol{\phi}_i$ is the vector containing the vibration modes.

The strategy proposed here is to solve Eq. (16) only to determine the nominal basis, $\boldsymbol{T}_0$, which must be further enriched by *robust static residues* to account for the uncertainties. Based on Eq. (4) and considering small perturbations, the stochastic matrices can be written as follow, $\boldsymbol{M}(\theta) = \boldsymbol{M} + \Delta\boldsymbol{M}(\theta)$ and $\boldsymbol{K}(\theta) = \boldsymbol{K} + \Delta\boldsymbol{K}(\theta)$, in such a way that, the time-domain equilibrium equation is rewritten as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} - \boldsymbol{f}_{\Delta M}(\theta) - \boldsymbol{f}_{\Delta K}(\theta)\,. \tag{17}$$

Eq. (17) can be interpreted as being the equation of motion of the nominal problem subjected to an external load and forces of modifications $\boldsymbol{f}_{\Delta M}(\theta) = \Delta\boldsymbol{M}(\theta)\ddot{\boldsymbol{u}}$ and $\boldsymbol{f}_{\Delta K}(\theta) =$

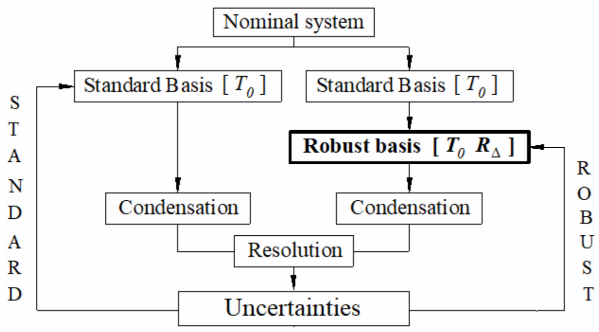

Fig. 1. Comparison between the standard and robust bases.

$\Delta \boldsymbol{K}(\theta)\boldsymbol{u}$ due to the uncertainties [23]. However, since the response, $\boldsymbol{u}$, of the modified system is unknown *a priori*, these forces cannot be computed. In this case, the responses of the nominal problem are used. Clearly, it is an approximated method to compute the forces of modifications, but it represents, at least, a subspace of the perturbed system. For an uncertain parameter, $\Delta p(\theta)$, the modified forces are given as:

$$\boldsymbol{f}_{\Delta p} = \left[\boldsymbol{f}^p_{\Delta M} \ \boldsymbol{f}^p_{\Delta K} \right] \tag{18}$$

where $\boldsymbol{f}^p_{\Delta M} = \Delta \boldsymbol{M}(\theta)\boldsymbol{\phi}_0\boldsymbol{\Lambda}_0$, $\boldsymbol{f}^p_{\Delta K} = \Delta \boldsymbol{K}(\theta)\boldsymbol{\phi}_0$, $\boldsymbol{\Lambda}_0 = diag \ (\lambda_1, \ldots, \lambda_{Nr})$ and $\Delta \boldsymbol{M}(\theta)$ and $\Delta \boldsymbol{K}(\theta)$ are the random matrices obtained by KL, as defined in Eq. (4), having the uncertain parameter, $p(\theta)$. Thus, based on the static residues associated to the forces of modifications, $\boldsymbol{R}_\Delta = \boldsymbol{K}^{-1}\boldsymbol{f}_{\Delta p}$, the robust basis is:

$$\boldsymbol{T} = [\boldsymbol{T}_0 \ \boldsymbol{R}_\Delta]. \tag{19}$$

It is important to emphasize this robust method is based on the model updating technique [4, 6], where it avoids the resolution of costly eigenproblems at each sample to update the basis, resulting in a significant reduction of the computational burden. Fig. 1 compares the standard and robust reduction methods.

5. Robust multiobjective optimization

In the contest of the present fatigue reliability study, the interest in using robust optimization tools is to find a set of optimal and robust solutions to optimize the fatigue behavior of the system subject to uncertainties. It involves simultaneous optimization of the cost functions with their corresponding vulnerabilities using the concept of robust multiobjective optimization (RMO) [18], as defined by:

$$\begin{cases} \min_x \boldsymbol{f}(\boldsymbol{x}) = \left(f_1(\boldsymbol{x}), f_{v_1}(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x}), f_{v_n}(\boldsymbol{x})\right) \\ \qquad \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \quad \boldsymbol{x} \in C \end{cases} \tag{20}$$

where $f_{v_i}(\boldsymbol{x}) = \mu_{f_i} / \sigma_{f_i}$ is the vulnerability related to the cost function, $f_i(\boldsymbol{x})$, and μ_{f_i} and σ_{f_i} are, respectively, the mean and standard deviation computed for a number of samples. $C \subset R^n$ is the design space limited by lateral constraints, $[\boldsymbol{x}_L, \boldsymbol{x}_U]$.

Since, the vulnerability is used to evaluate the impact of the uncertainties on the robustness of the cost functions during the optimization, they must be optimized simultaneously. However, the interaction between them in the RMO gives rises to a set of Pareto solutions [19], where the user can choose the best solution to be implemented based on a specific design criterion.

To solve the optimization problem Eq. (20), we used the well-known non-dominated sorting genetic algorithm (NSGA), proposed by Srinivas and Deb [24] and available in MATLAB® toolbox [25]. Its choice is based on its powerful and robust adaptive search mechanism for the present study, when compared with other evolutionary algorithms (EAs) available in the Ref. [19].

6. Numerical applications

To highlight the capability and main features of the proposed methodology, an academic example is used formed by a clamped-clamped aluminum plate, as depicted in Fig. 2, where the point *P* is used to compute the responses due to a transverse forcing applied on it. The nominal properties are: thickness, 1.8 mm; Young's modulus, 70 GPa; mass density, 2700 Kg/m^3; Poisson's ratio, 0.33; and alternate torsion limit, 92, obtained for 2×10^6 cycles. After performing a mesh refinement, the 8×8 FE mesh was considered to be adequate in this study.

To ensure a coherent model, a mesh convergence analysis was performed taking into account the first three natural frequencies. The 8×8 FE mesh adopted presented satisfactory results with maximum variations of order of 0.5 % for the third natural frequency, if compared with higher density element meshes, as noted in Fig. 3.

6.1 Fatigue analysis of the mean model under random loading

First, we studied the failure condition of the mean FE model by using the nominal values of its parameters. To consider the random nature of the forcing in the estimation of the SRFs and Sines' index of each element (by computing the mean of the four nodal stresses), the plate was subjected to a random loading applied at point *P*, defined by its uniform PSD, $\Phi(\omega)$ = 8500 N^2/Hz (white noise). Thus, Eq. (8) is used to find the SRF matrix.

To provide a sense of the stresses acting on the mean model due to the random forcing, Fig. 4 shows the SRFs for the most critical element at the center of the plate. Based on it, the spectral moments involved in the estimation of Sines' index can be extracted.

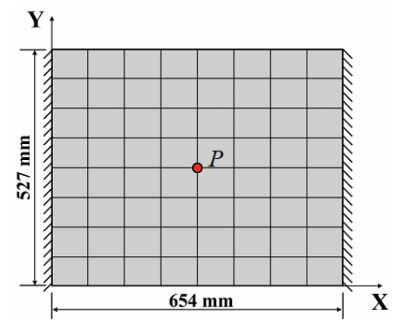

Fig. 2. Illustration of the FE mesh of the plate.

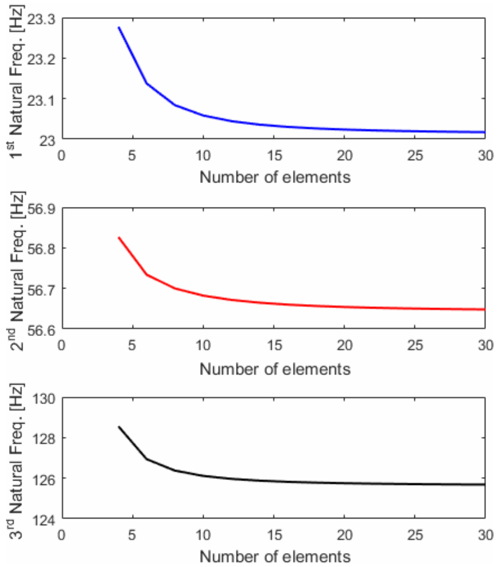

Fig. 3. Mesh convergence analysis of the plate.

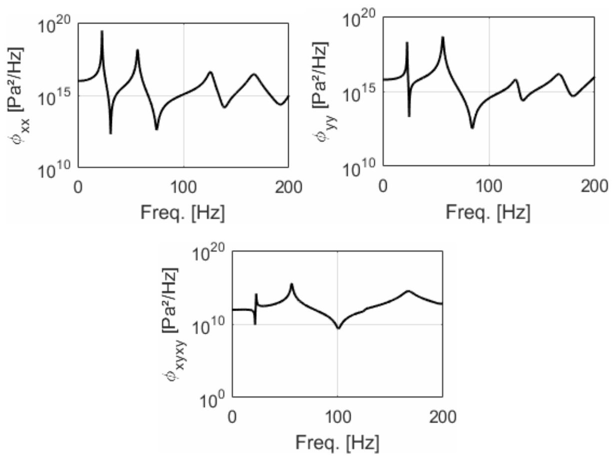

Fig. 4. SRFs for the most critical element of the mean model.

Fig. 5 shows the distribution of Sines' coefficients, where the critical values are related to the elements of the border and central parts o the plate. It is due to the symmetry of the system with the boundary conditions and the nature of the applied forcing. It leads to a maximum value of the fatigue index of, 0.73, which does not indicates fatigue failure of the

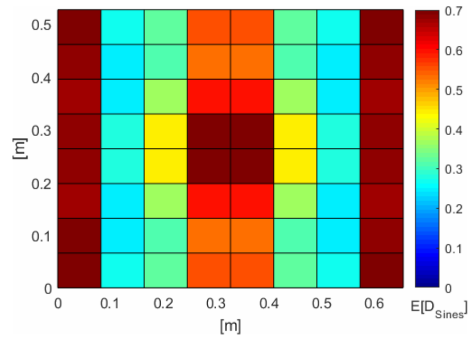

Fig. 5. Distribution of the Sines' coefficients for the mean model.

mean model before 2×10^6 cycles.

6.2 Accuracy of the reduced stochastic model

To reduce the computational cost needed to perform the robust fatigue-based optimization, the robust basis is used here, but its accuracy in approximating the random FRFs must be verified first. Within this aim, the FRFs of the full stochastic FE model are compared with the corresponding obtained by using the standard, $\boldsymbol{T}_0$, and robust, $\boldsymbol{T}$, bases. Here, the thickness of the plate is assumed as uncertain with a dispersion of 5 % around its nominal value. The Latin hypercube (LHC) sampling method [26] was used to generate 500 samples of it by assuming a Gaussian distribution. To construct the random FE matrices, as defined by Eq. (5), the correlation lengths were assumed to be equal to the dimensions of the elements in the FE mesh. Also, based on previous studies [5], the number of terms used in the KL was assumed as, *nkl* = 10.

At this time, it is important to know the number of samples, n_s, needed to assure the convergence of the FRFs' variabilities. It can be done by the root-mean square deviation, $RMSD=\frac{1}{n_s}\sqrt{\sum_{i=1}^{n_s}\left|\boldsymbol{G}(\omega,\theta_i)-\boldsymbol{G}(\omega)\right|^2}$, where $\boldsymbol{G}(\omega,\theta_i)$ designate the amplitudes of the FRFs of the stochastic system for a sample, θ_i, and $\boldsymbol{G}(\omega)$ are the responses of the mean model. Fig. 6 shows the RMSD as function of the samples for the uncertain thickness, showing that the random responses converge for, n_s = 300. Thus, it is the number of samples used in all stochastic simulations that follow.

Fig. 7 demonstrates the accuracy of the reduction bases in approximating the envelopes of random FRFs for a confidence level of, 95 %, implying that the responses of the system are inside the maximum and minimum envelopes with a probability of 95 %. Also, the significant influence of the uncertain thickness on the response variability can be perceived, especially for high frequencies.

The accuracy of the robust basis can be confirmed by analyzing the relative errors between the approximated and exact random FRFs, as shown in Fig. 8, with a maximum error of 0.016 % for the Ritz basis and 0.025 % for the robust case. However, the advantage in using the robust basis is the reduc-

Table 1. Computational times for random FRFs predictions.

FE model	Time [s]	Gain [s]
Full	68.4	---
Ritz basis	202.6	70

Computer: Intel (R) Core (TM) i7-9700KF CPU 3.60 GHz, 16.0 GB RAM

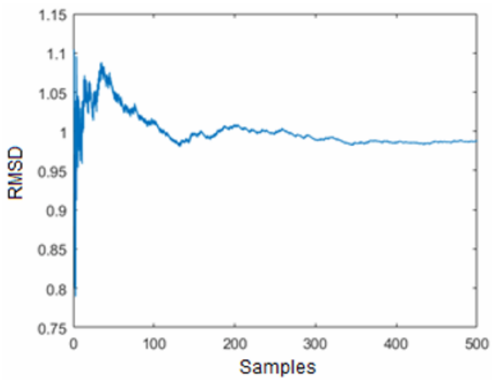

Fig. 6. Convergence analysis of the random FRFs for uncertainties on the thickness.

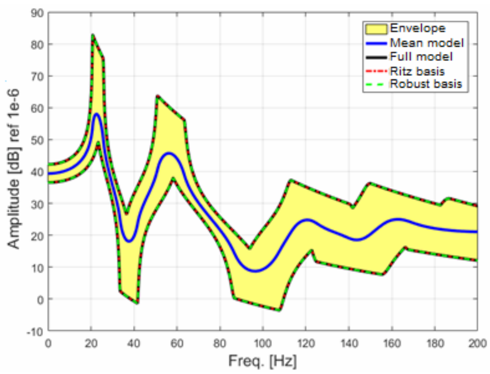

Fig. 7. Envelopes of random FRFs of the full and reduced models.

tion of 96 % on the computational time required to compute the random FRFs compared with the 70 % for the Ritz basis, as given in Table 1. Clearly, for the purposes of robust optimization based on fatigue, in which multiple functions are evaluated, the use of the robust basis is more attractive.

6.3 Fatigue analysis of the stochastic system

Now, the uncertainties of 5 % and 10 % on the thickness and endurance limit are considered, respectively. Also, the system is subjected to the same random force given in Sec. 6.1. Since, the torsion endurance limit affects only the estimation of Sines' index, the random FRF for this scenario is similar to that depicted in Fig. 7.

For instance, we studied the influence of the uncertain thickness on the fatigue index by assuming the nominal value of the torsion endurance limit. Fig. 9 shows the random SRFs for the most critical element, and Fig. 10 represents the

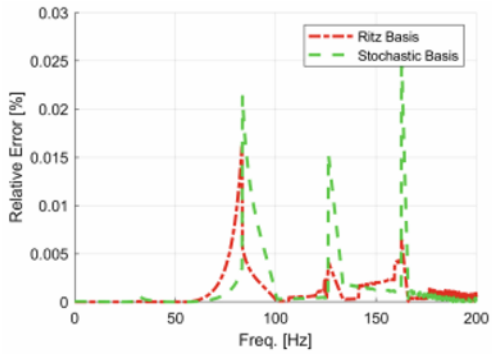

Fig. 8. Relative errors for the full and reduced models.

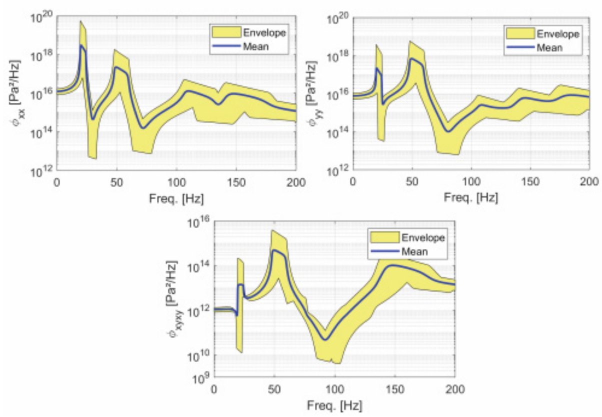

Fig. 9. Envelopes of SRFs for the most critical element of the stochastic system for uncertainties introduced on its thickness.

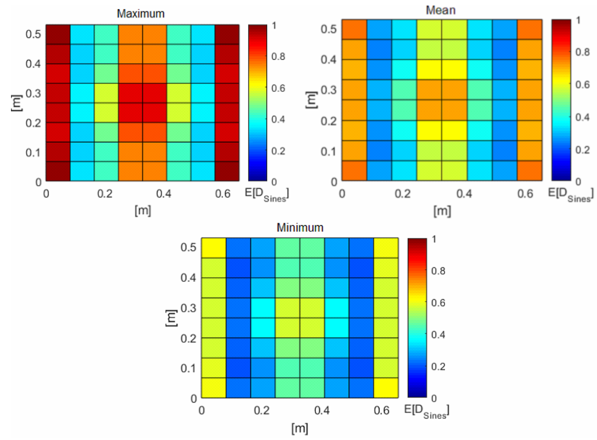

Fig. 10. Distributions of sines' coefficients for the stochastic system for uncertainties introduced on its thickness.

distributions of the maximum, mean and minimum values of Sines' coefficients, with a maximum value of, 0.98. Thus, it does not indicate fatigue failure before 2×10^6 cycles, but it demonstrates the influence of the uncertainties on its fatigue condition, since the fatigue index was increased from 0.73 for the mean model (see Fig. 5), to 0.98 with uncertainties.

These results demonstrate the importance of considering the presence of parametric uncertainties during the fatigue analysis of engineering systems, since they can affect significantly their fatigue failure condition.

At this time, the uncertainties on the endurance limit are also considered. Based on the work by Lambert et al. [3], it is reasonable to consider a dispersion of 10 % on its nominal value due to the experimental acquisition errors. Similarly to the results of Fig. 6, here, the responses always converge for n_s = 300. Since $t_{-1}(\theta)$ does not influence directly either the random FRFs or the SRFs, similar results of those given in Figs. 6 and 8 for FRFs and SRFs are obtained, respectively. However, based on Fig. 11, a Sines' index of, 1.28 has been found, for the elements located on the border of the plate. It indicates a fatigue failure of it due to the uncertainties, compared with the nominal system.

6.4 Robust-optimal design based on fatigue

After having studied the influence of the uncertainties on the fatigue behavior of the system, the interest now is to use optimization tools combined with the concept of robustness to increase its fatigue life accounting for these uncertainties. The main goal is to minimize the Sines' coefficients of the critical elements depicted in Fig. 11 to avoid a fatigue failure and the total mass of the system. It leads to the following RMOP problem to be solved:

$$minimize \begin{cases} f_1(\boldsymbol{x}) = \max\left(E\left[D_s\right]\right) & f_{v_1}(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) = total\ mass & f_{v_2}(\boldsymbol{x}) \end{cases}. \tag{21}$$

To perform the multiobjective optimization problem Eq. (21) using the NSGA-II available in MATLAB® toolbox [24], the following parameters were defined: the same probability of 0.25 for the selection, crossover and mutation phases; 30 individuals per generation and 100 generations with a sharing coefficient of 0.2. The uncertain parameters considered in this study with their admissible variations, taken as constraints in the RMOP, are defined in Table 2.

At each generation, 300 LHC samples for each uncertain variable are generated to compute the vulnerability functions, $f_{v_1}(\boldsymbol{x})$ and $f_{v_2}(\boldsymbol{x})$, associated to the objective functions. As a result, it requires a total number of functions evaluations of 9×10^5, which justifies the use of the robust basis proposed herein.

Fig. 12 shows the NSGA solutions where the non-dominated points indicated in this figure are compromise optimal solutions known as Pareto front. However, since none of them can be considered as better than others, the final decision of the best solution to be chosen must be made by the user's preferences, such as the fatigue failure condition, mass restrictions and vulnerabilities. However, sometimes it is not easy to perform this choice due to the large number of solutions appearing in the Pareto front. Clearly, in the search for the robust solutions, it is interesting to choose the points which minimize the dispersions around the objective functions based on the best compromise between the maximum performance in terms of the fatigue life and total mass.

By analyzing the vulnerabilities shown in Fig. 13, the following intervals of dispersions can be clearly perceived: [0.014-0.026] for the Sines' index and [0.09-0.18] for the total mass of the system. For the present study, the point A indicated in Fig. 13 has been chosen to be investigated, accounting for its fatigue failure condition and vulnerability. It is related to the mass of 1.743 kg, which corresponds to a thickness of approximately, 1.9 mm.

To verify the failure condition of the optimized stochastic system for the optimal solutions related to point A, fatigue analysis was performed by applying the same dispersions of 5 % and 10 % on the thickness and endurance limit, respectively, as

Table 2. Variables with their admissible variation and uncertainty.

Variable	Nominal values	Variations	Uncertainty
$h[mm]$	1.8	± 20 %	5 %
t_{-1}	92	---	10 %

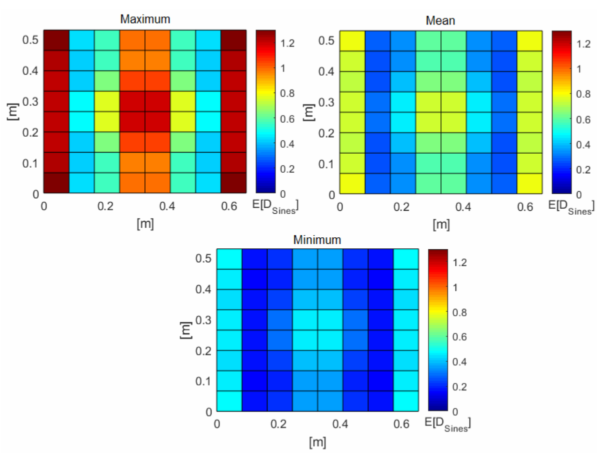

Fig. 11. Distributions of sines' coefficients for the stochastic system for uncertainties introduced on its thickness and endurance limit.

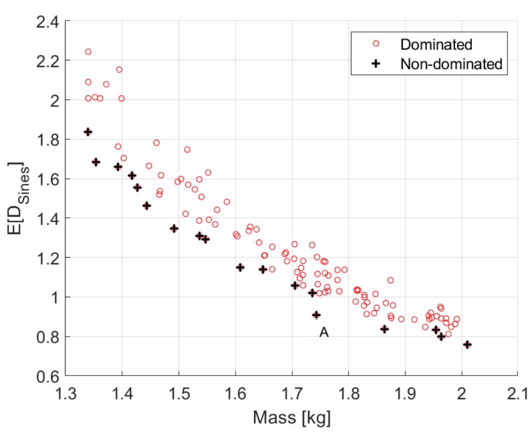

Fig. 12. NSGA solutions and pareto front for the cost functions.

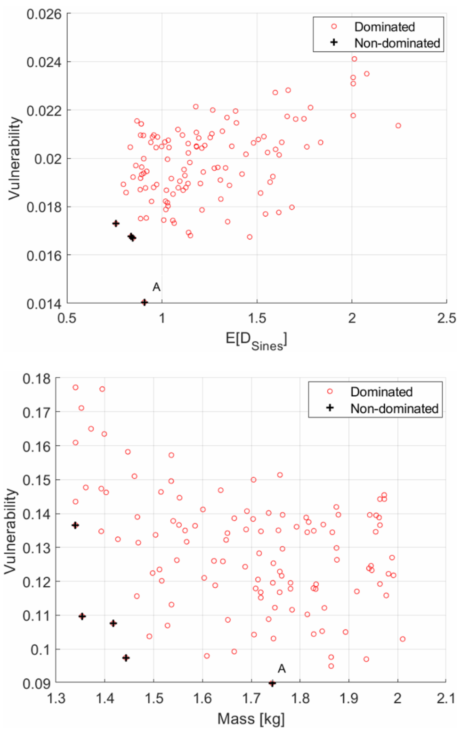

Fig. 13. NSGA solutions and Pareto front for the vulnerabilities.

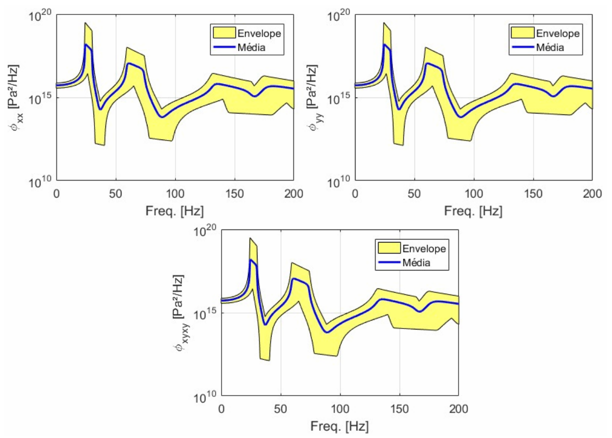

Fig. 14. SRFs for the most critical element of the optimized system.

performed in Sec. 6.3. For this application, it has been found that the stochastic responses always converge for 300 samples.

Fig. 14 shows the envelopes of PSDs of stresses for the most critical element of the optimized stochastic system. By comparing these results with those of Fig. 9 for the non-optimized stochastic system, a reduction of the stress responses can be clearly perceived and, consequently, a reduction of the Sines' coefficients. It can be confirmed by analyzing the distributions of the Sines' coefficients in Fig. 15, with a maximum value of, 0.89, for the most critical elements. It indicates a non-failure condition, even for an increase in the total mass of the system of approximately, 4 %.

Hence, the robust-based optimization method suggested

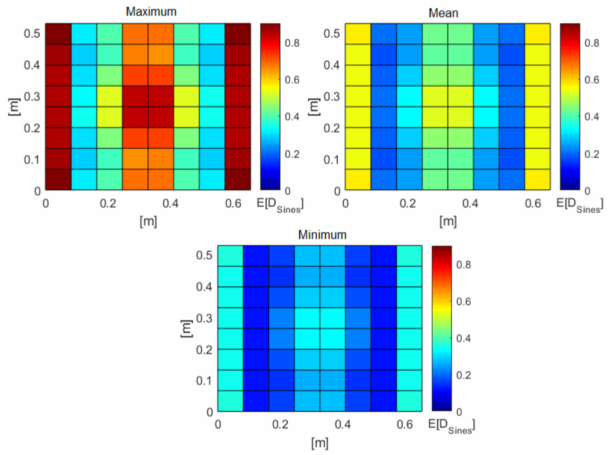

Fig. 15. Distributions of sines' coefficients for the optimized system.

herein is considered as an interesting tool to perform fatigue analyses of systems subject to uncertainties, since it has been shown that it can affect significantly their failure condition.

Clearly, the choice of the best solution on the first Pareto front having a small vulnerability must be not disregarded in this kind of problem. It is based on the fact that it has a lower vulnerability compared with other points in the non-dominated solution space. As a result, it is expected that the presence of small fluctuations on the optimized variables will not affect significantly the distribtuion of the Sines' fatigue coefficients.

7. Concluding remarks

The main goal of this work was to propose a robust design method to perform fatigue analyses of stochastic systems in the frequency-domain, where the uncertainties can be introduced on their physical and/or geometrical parameters and external forcing, simultaneously. Within this aim, a new probabilistic approach of the Sines' criterion was suggested to account for these uncertainties based on the formulation of the random FRFs and SRFs matrices. Also, a robust multiobjective optimization method was used, where vulnerability functions associated with the cost functions have been evaluated for the HCL samples. Clearly, due to the high computational cost required to evaluate the vulnerability functions, even for the present academic stochastic fatigue problem, an efficient robust reduction basis was proposed to approximate the random FRFs and SRFs. For the academic example used, the efficiency of the robust basis was of 96. Clearly, the proposed strategy can be extended with advantage for dealing with more complex systems composed by large number of DOFs or samples.

Based on this study, it is evident the importance of considering not only the uncertainties acting on the external loadings, as investigated before by some authors in the open literature [1-3], but also the inherent uncertainties acting on the physical and geometrical parameters of the system. Clearly, it requires an adequate stochastic structural formulation and, here, the KL expansion has been combined with the FE method, in the context of the stochastic FE method, to generate the random FE matrices of the stochastic structure subjected to random loads. To estimate the Sines' fatigue coefficients of the stochastic system, the *prismatic hull* method, initially proposed for deterministic cases, was modified to consider the random nature of the stress responses in the frequency domain.

For the mean model subjected only to a random forcing, a maximum value of the Sines' index has been found of 0.7130, for the most critical elements in the FE mesh, while for the system subjected to uncertainties on its thickness, the fatigue index was of 1.19. It indicates a fatigue failure condition before 2×10^6 cycles and makes evident the importance of considering the presence of parametric uncertainties on fatigue failure analyses, since a non-fatigue failure condition for the mean model can become a fatigue failure for the system with uncertainties. Based on this fact and with the aim of optimizing the fatigue performance of the stochastic problem accounting for these uncertainties, a robust multiobjective optimization strategy using the NSGA was performed with the robust reduced-order stochastic model. It reduces significantly the computational cost of the vulnerability functions evaluations for the samples during the robust optimization.

From the Pareto solutions, the user can choose an optimal point to be implemented accounting for its robustness and fatigue failure condition. Here, the interest was to maximize the fatigue reliability and minimize the total mass of the system. For a thickness value of 1.9 mm, it leads to an increase in the total mass of 4 %, but a fatigue failure with a maximum value of the fatigue index of 0.9 was not observed.

Finally, the proposed method can be applied with advantage for fatigue analyses of existing structures or during the preliminary design phases of mechanical systems subjected to undesirable vibrations and uncertainties.

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Nomenclature

$\bar{\boldsymbol{M}}$	: Parameterized mass matrix
$\bar{\boldsymbol{K}}$	: Parameterized stiffness matrix
$\bar{K}_{m/b}$	: Stiffness matrix related to the membrane (m) or bending (b) effects
$\boldsymbol{B}(x,y)$	: Strain-displacement differential operators
C	: Isotropic material properties matrix
$h(x,y,\theta)$	: Bidimensional random thickness
$\xi_r(\theta)$	: Set of random variables
$E[\bullet]$	: Mathematical expectation
$\boldsymbol{M}(\theta)$	: Stochastic mass matrix
$\boldsymbol{K}(\theta)$	: Stochastic stiffness matrix
$G(\omega,\theta)$	: Frequency response function of the stochastic system
$\varepsilon(x,y,t,\theta)$	: Time-domain random strains
$s(x,y,t,\theta)$	: Time-domain random stresses
$\bar{s}(x,y,t,\theta)$	: Mean value of the random stresses
$\Phi_s(\omega,\theta)$	: Stress response function (SRF) of the stochastic system
$\Phi_f(\omega)$	: PSD of random loading
$s(t,\theta)$	: Time-domain random stresses
D_s	: Sines' index
R_i	: Semi-axes of the *prismatic hull*
$J_{2a}(\theta)$	: Second invariant of the random

References

[1] A. M. G. de Lima, S. Lambert, D. A. Rade, E. Pagnacco and L. Khalij, Fatigue reliability analysis of viscoelastic structures subjected to random loads, *Mechanical Systems and Signal Processing,* 43 (1-2) (2014) 305-318.

[2] E. Pagnacco, S. Lambert, L. Khalij and D. A. Rade, Design optimisation of linear structures subjected to dynamic random loads with respect to fatigue life, *International J. of Fatigue*, 43 (2012) 168-177.

[3] S. Lambert, E. Pagnacco and L. Khalij, A probabilistic model for the fatigue reliability of structures under random loadings with phase shift effects, *International J. of Fatigue,* 32 (2) (2010) 463-474.

[4] A. M. G. de Lima, D. A. Rade and N. Bouhaddi, Stochastic modeling of surface viscoelastic treatments combined with model condensation procedures, *Shock and Vibration,* 17 (4, 5) (2010) 429-444.

[5] L. K. S. Gonçalves, U. L. Rosa and A. M. G. de Lima, Fatigue damage investigation and optimization of a viscoelastically damped system with uncertainties, *J. of the Brazilian Society of Mechanical Sciences and Engineering,* 41 (9) (2019) 1-15.

[6] A. M. G. de Lima, N. Bouhaddi, D. A. Rade and M. Belonsi, A time-domain finite element model reduction method for viscoelastic linear and nonlinear systems, *Latin American J. of Solids and Structures,* 12 (6) (2015) 1182-1201.

[7] Z. Chen, T. Li, X. Xue, Y. Zhou and S. Jing, Fatigue reliability analysis and optimization of vibrator baseplate based on fuzzy comprehensive evaluation method, *Engineering Failure Analysis*, 127 (2021) 105357.

[8] B. Weber, Fatigue multiaxiale des structures industrielles sous chargement quelconque, *Doctoral Dissertation*, Lyon, INSA (1999).

[9] G. Sines, Behavior of metals under complex static and alternating stresses, *Metal Fatigue,* 1 (1959) 145-169.

[10] B. Crossland, Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel, *Proc. Int. Conf. on Fatigue of Metals*, Institution of Mechanical Engineers London, 138 (1956) 12.

[11] B. R. You and S. B. Lee, A critical review on multiaxial fatigue assessments of metals, *International J. of Fatigue,* 18 (4) (1996) 235-244.

[12] X. Pitoiset and A. Preumont, Spectral methods for multiaxial random fatigue analysis of metallic structures, *International J.*

of Fatigue, 22 (7) (2000) 541-550.
[13] K. Dang-Van, Macro-micro approach in high-cycle multiaxial fatigue, *Advances in Multiaxial Fatigue*, ASTM International (1993).
[14] A. Preumont, On the peak factor of stationary gaussian processes, *J. of Sound and Vibration,* 100 (1) (1985) 15-34.
[15] Z. Chen, Z. Huang, S. Jing, Z. Tao and Q. Zhao, The fatigue behavior and fatigue reliability analysis of vibroseis baseplates based on fracture mechanics, *Proceedings of the Institution of Mechanical Engineers, Part O: J. of Risk and Reliability*, 231 (6) (2017) 732-749.
[16] I. V. Papadopoulos, P. Davoli, C. Gorla, M. Filippini and A. Bernasconi, A comparative study of multiaxial high-cycle fatigue criteria for metals, *International J. of Fatigue*, 19 (3) (1997) 219-235.
[17] R. G. Ghanem and P. D. Spanos, *Stochastic Finite Elements: A Spectral Approach*, Courier Corporation (2003).
[18] R. A. Borges, A. M. G. de Lima and V. Steffen Jr., Robust optimal design of a nonlinear dynamic vibration absorber combining sensitivity analysis, *Shock and Vibration*, 17 (4, 5) (2010) 507-520.
[19] H. Eschenauer, J. Koski and A. Osyczka, *Multicriteria Design Optimization: Procedures and Applications*, Springer Science & Business Media (2012).
[20] S. S. Rao, *The Finite Element Method in Engineering*, Butterworth-Heinemann (2017).
[21] B. Li and M. de Freitas, A procedure for fast evaluation of high-cycle fatigue under multiaxial random loading, *J. Mech. Des.,* 124 (3) (2002) 558-563.
[22] L. Khalij, E. Pagnacco and S. Lambert, A measure of the equivalent shear stress amplitude from a prismatic hull in the principal coordinate system, *International J. of Fatigue*, 32 (12) (2010) 1977-1984.
[23] M. H. Belonsi, A. M. G. de Lima, T. Trevilato and R. A. Borges, An efficient iterative model reduction method for stochastic systems having geometric nonlinearities, *J. of the Brazilian Society of Mechanical Sciences and Engineering*, 42 (11) (2020) 1-16.
[24] N. Srinivas and K. Deb, *Multiobjective Using Nondominated Sorting in Genetic Algorithms*, Tech. Rep., Department of Mechanical Engineering, Institute of Technology, New Delhi, India (1993).
[25] MathWorks, *MATLAB User Manuals*, http://www.mathworks.com/, Accessed 01 October (2020).
[26] A. Haldar and S. Mahadevan, *Probability, Reliability and Statistical Methods in Engineering Design*, Wiley (1992) 123-130.

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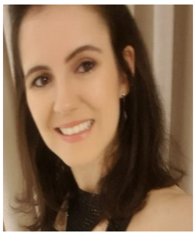

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