KSME

Springer

Journal of Mechanical Science and Technology 34 (9) 2020

Original Article
DOI 10.1007/s12206-020-0810-3

Keywords:
· Harmonic line loads
· Infinite Bernoulli-Euler beam
· Iterative solution procedure
· Viscoelastic foundation

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Citation:
Syngellakis, S., Park, J., Cho, D. S., Jang, T. S. (2020). A numerical study on an infinite linear elastic Bernoulli-Euler beam on a viscoelastic foundation subjected to harmonic line loads. Journal of Mechanical Science and Technology 34 (9) (2020) 3587~3595.
http://doi.org/10.1007/s12206-020-0810-3

Received February 29th, 2020
Revised June 15th, 2020
Accepted July 8th, 2020

† Recommended by Editor No-cheol Park

A numerical study on an infinite linear elastic Bernoulli-Euler beam on a viscoelastic foundation subjected to harmonic line loads

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Abstract This paper presents a numerical study on the low-amplitude responses of an infinite Bernoulli-Euler beam resting on a viscoelastic foundation subjected to harmonic line loads. To simulate the linear response, a semi-analytical solution procedure that was theoretically proposed by Jang (2016) is utilized and several numerical experiments are conducted to investigate the influence of key model parameters characterizing stiffness and damping. The properties of the viscoelastic foundation are based on theoretical and empirical values for cohesionless sand type foundation. According to the numerical experiments, the obtained responses are compared with those from the closed-form solution and found to have a good agreement with them.

1. Introduction

The behavior of an infinite beam resting on a flexible foundation subjected to dynamic loads has attracted much interest among both researchers and engineers. It has been widely investigated for its technological importance especially in various branches of civil engineering, for instance, geotechnical engineering, railway engineering, high-way, tunneling engineering and bridge engineering, among others. There are two basic approaches, that is, analytical and numerical methods, for the investigation of the dynamic response of an infinite beam resting on a flexible foundation.

Among the analytic approaches, there exists a closed-form solution of steady-state vibrations of an infinite Bernoulli-Euler beam on Winkler foundation for moving load first proposed by Kenney [1]; Mathews [2, 3] also carried out similar analytical studies. Stadler and Shreeves [4] obtained a solution for the transient and steady-state response of an infinite Bernoulli-Euler beam with damping resting on an elastic foundation; this was further developed by Sheehan and Debnath [5]. Closed-form, transient and steady-state solutions for an infinite Bernoulli-Euler beam on viscoelastic foundation subjected to harmonic line loads [6], moving loads [7-9], moving line loads [10] and arbitrary dynamic loads [11, 12] were also proposed.

Concerning numerical approaches, Andersen et al. [13] suggested a finite element solution for the response of an infinite beam subjected to moving loads and supported by a linear elastic Kelvin foundation with linear viscous damping. Nguyen and Duhamel [14, 15] proposed finite element procedures for the solution of infinite Bernoulli-Euler beams resting on Winkler foundations under moving axial and harmonic loads. Koh et al. [16] studied the train - track interaction problems based on the idea of a moving coordinate system implemented through the moving element method (MEM).

There are many valuable articles addressing potential applications. Lee [17] investigated the free vibration analysis of circularly curved multi-span beams using a pseudo-spectral method for various boundary conditions. Lee [18, 19] also analyzed the free vibration of the Bernoulli-Euler and the Timoshenko beams with non-ideal clamped boundary conditions. Akgöz and

Civalek [20, 21] showed the behavior of a size-dependent micro-beam model on the basis of hyperbolic shear deformation and modified strain gradient theorem. Numanoğlu et al. [22] presented the longitudinal free vibration behavior of one-dimensional nanostructures based on Eringen's nonlocal theory. Naghinejad and Ovesy [23] investigated the nano-scaled viscoelastic Bernoulli-Euler beam via the finite element method using the principle of total potential energy and nonlocal integral theory.

Recently, Jang [24] proposed a new semi-analytic procedure for a nonlinear infinite Bernoulli-Euler beam loaded by lateral nonlinear force (and its nonlinear reaction force) and examined both the convergence and uniqueness of the solution by showing the contraction of the nonlinear operator with an appropriate function space. In the procedure, a pseudo-stiffness parameter plays a crucial role in constructing the integrated integral equation that may be directly linked to the iterative solution method under the generalized external loads. In addition, the utilized semi-analytic solution procedure may be contrasted with the static analysis of a nonlinear beam to solve a general form of 4th order nonlinear ODE [25-29]. The present study can be regarded as an extension of the previously proposed semi-analytic procedure for water wave problems of 4th order nonlinear PDE [30-34].

This work explores the potential of a numerical solution to compute the time-dependent displacement and, without additional computational effort, the velocity response of a Bernoulli-Euler beam resting on a viscoelastic foundation under harmonic line loads using the semi-analytic solution procedure proposed by Jang [24]. The numerical solutions can be obtained by combining well-known integration methods, that is, Simpson's rule and Trapezoidal rule, implemented through a computer software program, such as MATLAB ver. 9.6.0 [35]. The performance of the solution procedure is also assessed by the comparison of their predictions with those obtained from the conventional closed-form solution [6, 11].

2. Method

2.1 Statement of the problem

An infinite Bernoulli-Euler beam of uniform cross-section resting on viscoelastic foundation undergoes a transverse displacement $u(x,t)$ when subjected to an external load $W(x,t)$. The resisting forces against the beam's transverse deflection due to the spring, damper characteristics of the foundation and its own mass are assumed to be proportional to displacement u, velocity $\partial u/\partial t$ and acceleration $\partial^2 u/\partial t^2$, respectively. Then the transverse vibration of an infinite beam is governed by 4th order partial differential equation [4, 5, 9, 11, 12, 36]:

$$EI\frac{\partial^4 u}{\partial x^4} = W(x,t) - \rho A\frac{\partial^2 u}{\partial t^2} - C\frac{\partial u}{\partial t} - Ku .$$
$$-\infty < x < \infty , \quad t > 0 . \tag{1}$$

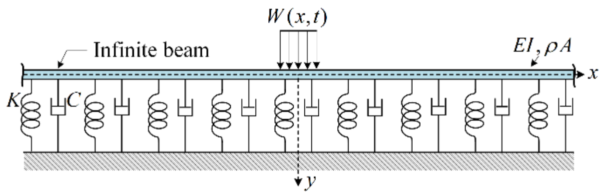

Fig. 1. An infinite Bernoulli-Euler beam on a viscoelastic foundation subjected to harmonic line loads $W(x,t)$.

Here, $EI > 0$, $\rho A > 0$, C and K are the flexural rigidity and the mass per unit length of the beam, the damper and the spring coefficients of the viscoelastic foundation, respectively, as shown in Fig. 1. The beam is assumed to be at rest at $t = 0$ s, then the dynamic loads are sufficiently localized and are applied over time $t > 0$ s, i.e., the initial displacement and velocity are null. So the initial-boundary conditions are stated as below,

$$u(x,t) \to 0 \text{ as } |x| \to \infty , \tag{2}$$

$$\frac{\partial^n u}{\partial x^n} \to 0 \text{ as } |x| \to \infty \text{ for } n = 1,2,3 , \tag{3}$$

$$u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0 . \tag{4}$$

2.2 Solution procedure

By employing the pseudo-parameter technique of Jang [24], a pseudo spring coefficient $K_p > 0$ is introduced and it will help transforming the original equation into equivalent integral equations. The pseudo spring force term, $K_p \cdot u$, is added to both sides of Eq. (1), which then can be modified as follows:

$$EI\frac{\partial^4 u}{\partial x^4} + \rho A\frac{\partial^2 u}{\partial t^2} + K_p u = \Phi(x,t) \tag{5}$$

with a new loading function $\Phi(x,t)$ defined by

$$\Phi(x,t) \equiv W(x,t) - C\frac{\partial u}{\partial t} + (K_p - K)\cdot u . \tag{6}$$

Here, Eq. (5) describes a system of an infinite Bernoulli-Euler beam on linear elastic foundation of pseudo spring stiffness K_p subjected to the external dynamic load $\Phi(x,t)$ which incorporates the damping and nonlinear stiffness characteristics of the foundation. Although it seems to be quite different from Eq. (1) governing the original system, Eq. (5) is still equivalent to Eq. (1). The incorporated pseudo parameter K_p will play an important role in the integral representation of the solution obtained from the PDE system Eq. (5) and directly leads to the iterative solution procedure presented by Jang [24].

Dividing Eq. (5) by mass per unit length ρA, results in

$$\alpha^4 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} + k_p u = \phi(x,t) \tag{7}$$

where $\phi(x,t) \equiv \Phi(x,t)/\rho A$ and, from Eq. (6),

$$\phi(x,t) = w(x,t) - c\frac{\partial u}{\partial t} + (k_p - k)\cdot u \tag{8}$$

while $\alpha \equiv \sqrt[4]{EI/\rho A}$ and the other variables are defined as follows:

$$(w,c,k,k_p) \equiv \frac{1}{\rho A}(W,C,K,K_p)\ . \tag{9}$$

Following the integral formalism with the zero initial displacement and velocity as expressed by Eq. (4) leads to an integral equation equivalent to the PDE (7) [24]:

$$u(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G(x,t,\xi,\tau,\omega;k_p)\cdot\phi(\xi,\tau)\,d\xi\, d\omega\, d\tau \tag{10}$$

where the kernel function, G, is defined by

$$G(x,t,\xi,\tau,\omega;k_p) \equiv \frac{1}{\pi}\frac{\sin\left[\beta_p\cdot(t-\tau)\right]}{\beta_p}\cdot\cos\left[\omega\cdot(\xi-x)\right] \tag{11}$$

where $\beta_p(\omega) \equiv \sqrt{k_p + (\alpha\omega)^4}$ with $G:\mathbb{R}\times\mathbb{R}^+\times\mathbb{R}\times\mathbb{R}^+\times\mathbb{R}^+ \to \mathbb{R}$, $\mathbb{R}$ is the set of real numbers.

Substituting the load $\phi(x,t)$ given by Eq. (8) to the equivalent integral Eq. (10) yields the integral

$$u(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G(x,t,\xi,\tau,\omega;k_p) \cdot\left[w(\xi,\tau) - c\cdot\frac{\partial u}{\partial t} + (k_p - k)\cdot u\right] d\xi\, d\omega\, d\tau. \tag{12}$$

Letting the time derivative of $u(x,t)$ be denoted by a new variable $v(x,t)$ ($\equiv \partial u/\partial t$), the displacement $u(x,t)$ is expressed as

$$u(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G(x,t,\xi,\tau,\omega;k_p) \cdot\left[w(\xi,\tau) - c\cdot v + (k_p - k)\cdot u\right] d\xi\, d\omega\, d\tau. \tag{13}$$

The velocity $v(x,t)$ can be obtained from

$$v(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G_t(x,t,\xi,\tau,\omega;k_p) \cdot\left[w(\xi,\tau) - c\cdot v + (k_p - k)\cdot u\right] d\xi\, d\omega\, d\tau \tag{14}$$

where the subscript t denotes differentiation of G (x, t, ξ, τ, ω; k_p) with respect to time, thus G_t is defined by

$$G_t \equiv \frac{1}{\pi}\cos\left[\beta_p\cdot(t-\tau)\right]\cdot\cos\left[\omega\cdot(\xi - x)\right]. \tag{15}$$

Now, the modified iterative solution procedure for the coupled integral Eqs. (13) and (14) yields new estimates u_{n+1} and v_{n+1} from u_n and v_n, where $n = 0,1,2,...$ is the iteration number according to the iterative solution procedure [24]:

$$u_{n+1}(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G(x,t,\xi,\tau,\omega;k_p)\cdot[w(\xi,\tau) - c\cdot v_n + (k_p - k)\cdot u_n]\,d\xi\, d\omega\, d\tau, \tag{16}$$

$$v_{n+1}(x,t) = \int_0^t\int_0^\infty\int_{-\infty}^\infty G_t(x,t,\xi,\tau,\omega;k_p)\cdot[w(\xi,\tau) - c\cdot v_n + (k_p - k)\cdot u_n]\,d\xi\, d\omega\, d\tau. \tag{17}$$

The iteration begins with initial guesses u_0 and v_0 which satisfy the initial and boundary condition Eqs. (2)-(4).

3. Numerical experiments

3.1 Elastic foundation

The relations developed by Vlasov & Leont'ev [37] and Lysmer [38] provide the means to estimate two parameters of the elastic foundation. The (Winkler) spring coefficient *K* (in N/m^2), which was derived by using the principle of virtual displacement under the assumption of the foundation being an elastic continuum [9, 37], is given by

$$K = \frac{E_s b\gamma(1-\nu_s)}{2(1+\nu_s)(1-2\nu_s)} \tag{18}$$

where E_s, b, γ and ν_s are the Young's modulus (MPa) of the foundation (i.e., soil), the width of the beam cross-section (m), the rate of the vertical displacement in the ground decreasing with depth which has a typical range of 1.0-2.0 (in m^{-1}) [9] and the Poisson's ratio of the foundation, respectively.

Typically, for a European rail track to be laid, the foundation is required to be sufficiently stiff. So a reasonable range for the modulus of the foundation (E_s) is 0.1 MPa - 100 MPa while for the Poisson's ratio (ν_s), it is 0.1 - 0.4 [39]. The range of the two parameters for specific foundations is listed in Refs. [40-42]. Finaly, a typical range of the parameter γ is 1.0 - 2.0 m^{-1} [37].

The damping coefficient C (in N·s/m^2) may be estimated from the following relation proposed by Lysmer [38] as in Refs. [9, 42]:

$$C = \frac{0.765b\sqrt{E_s\gamma_s/g}}{(1-\nu_s)(1+\nu_s)^{0.5}}\ . \tag{19}$$

Here, γ_s is the unit weight of soil (N/m^3) and g the acceleration due to gravity (taken equal to 9.81 m/s^2).

3.2 Harmonic line load

It is assumed that the beam is at rest prior to the external loads being applied (i.e., initial condition Eq. (4)). The harmonic line load is expressed as follows [6, 11, 12]:

$$w(x,t) = \frac{P}{\rho A}\frac{H(r_0^2 - x^2)}{2r_0}\cdot e^{i\kappa t} \tag{20}$$

where r_0, κ, P and H are, respectively, the half-width of the line load (in m), the loading frequency (in rad/s), the amplitude of the applied load (in N) and the Heaviside step function using the half-maximum convention which is defined by

$$H(x-x_0) = \begin{cases} 0 & \text{for} \quad x < x_0 \\ 1/2 & \text{for} \quad x = x_0 \\ 1 & \text{for} \quad x > x_0 \end{cases}. \tag{21}$$

Since the load is applied symmetrically with respect to mean position ($x = 0$) with a span range $[-r_0, r_0]$, the response profile of beam will also be symmetric with respect to $x = 0$, i.e., the response profiles of u in Eq. (13) and v in Eq. (14) are even functions in the space domain [-∞, ∞]. Then, the iterations in Eqs. (16) and (17) can be eventually written with modified kernel functions as follows

$$\begin{aligned} &u_{n+1}(x,t) \\ &= \frac{2}{\pi}\int_0^t\int_0^\infty\int_0^\infty \frac{\sin\left[\beta_p\cdot(t-\tau)\right]}{\beta_p}\cdot\cos\omega x\cdot\cos\omega\xi \\ &\qquad \cdot\left[w(\xi,\tau) - c\cdot v_n + (k_p - k)\cdot u_n\right]d\xi d\omega\delta\tau, \end{aligned} \tag{22}$$

$$\begin{aligned} &v_{n+1}(x,t) \\ &= \frac{2}{\pi}\int_0^t\int_0^\infty\int_0^\infty \cos\left[\beta_p\cdot(t-\tau)\right]\cdot\cos\omega x\cdot\cos\omega\xi \\ &\qquad \cdot\left[w(\xi,\tau) - c\cdot v_n + (k_p - k)\cdot u_n\right]d\xi d\omega\delta\tau, \end{aligned} \tag{23}$$

due to the relation $\cos[\omega(x-\xi)] = \cos\omega x\cdot\cos\omega\xi + \sin\omega x\cdot\sin\omega\xi$ and the evenness of the integrands. In Eqs. (22) and (23), the integration domain with respect to ξ is thus shortened from [-∞, ∞] to [0, ∞].

The closed-form solution, $u_{\text{cf,H}}(x,t)$, of the initial-boundary value problem governed by Eqs. (1)-(4) is given as follows [6, 11, 12]:

$$\begin{aligned} &u_{\text{cf,H}}(x,t) \\ &= \frac{2}{\pi}\int_0^t\int_0^\infty\int_0^\infty \frac{\sin\left[\beta(\omega)\cdot(t-\tau)\right]}{\beta(\omega)}\cdot\cos\omega x \\ &\qquad \cdot\cos\omega\xi\cdot e^{-c(t-\tau)/2}\cdot w(\xi,\tau)d\xi d\omega d\tau \end{aligned} \tag{24}$$

where $\beta(\omega) = \sqrt{(\alpha\omega)^4 + k - c^2/4}$. The exact beam response is calculated from the closed-form solution Eq. (24) by using general numerical integration methods, such as Simpson's rule, Newton-Cotes formulas and Gaussian integration. In this work, for every calculation, the Simpson's rule will be utilized for space and frequency domain integrations and the Trapezoidal rule for time domain.

3.3 Numerical setup

To simulate the response of a beam under harmonic line loads by the iterative solution procedure based on relation Eqs. (22) and (23), the space and time domains are selected as $0\text{ m} < x < 70$ m (increment $\Delta x = 0.2$ m) and $0\text{ s} < t < 0.1$ s ($\Delta t = 0.002$ s), respectively. The spatial domain is considered sufficiently long relative to the adopted value of $r_0 = 1$ m. The results from both the proposed iterative method in Eqs. (22) and (23) and the closed-form solution Eq. (24) are computed using the software package MATLAB ver. 9.6.0. The magnitude of the initial guess entered into the iterative scheme is assumed to be $u_0 = v_0 = 0.1\exp(-0.1x^2)$ and the pseudo spring stiffness K_p is taken as $1.2K$ where K is the spring coefficient.

An infinite beam of rectangular uniform cross-section with breadth b = 3 m and height h = 0.2 m is considered. The beam is assumed linear elastic, homogeneous and isotropic with the physical parameters: density $\rho = 7850$ kg/m^3 and Young's modulus $E = 210$ GPa. Next, the viscoelastic foundation is assumed cohesionless sand [40] with Young's modulus of soil E_s, the rate of the displacement in the ground decreasing with depth γ, the Poisson's ratio ν_s and an average value of soil unit weight γ_s. The harmonic line load is applied with amplitude P and frequency $\kappa = 40\pi$ rad/s. Table 1 shows the range of the principal parameters with which the beam response will be simulated in the following subsections.

Fig. 2(a) shows results from the exact solution Eq. (24) using the parameters listed in Table 1 (case 2) which is the response of the beam resting on a viscoelastic foundation subjected to a harmonic line load given by Eq. (20). Fig. 2(b) presents two beam profiles at times $t = 0.06$ s and 0.1 s in the space domain $x \in$ [0 m, 20 m] and Fig. 2(c) shows two responses at locations $x = 0$ m and 4 m in the time domain $t \in$ [0 s, 0.1 s].

3.4 Effect of spring coefficient

As shown in relation Eq. (18), the spring coefficient K depends on four parameters. Among them, in the present subsection, the rate of the vertical displacement decreasing with soil depth (γ) is varied while fixing all other parameters to investigate only the effect of spring coefficient. For that, we compare the simulation results using two sets (cases 1-4) of parameters from Table 1. These values are empirical ones taken from Ref. [40]. For instance, for the empirical properties of loose sand, the unit weight of soil (γ_s) ranges from 14 to 18 kN/m^3, the Poisson's ratio from 0.2 to 0.35 and the Young's modulus of foundation (E_s) from 10 to 25 MPa. Moreover, for

Table 1. Properties of a beam rectangular uniform cross-section with breadth b = 3.0 m and depth h = 0.2 m.

Item	Notation	Cases							
		1	2	3	4	5	6	7	8
Beam									
Young's modulus (GPa)	E	210							
Mass density (kg/m^3)	ρ	7850							
Second moment of area (m^4)	I	0.002 (b = 3 m, h = 0.2 m)							
Foundation									
Young's modulus (MPa)	E_s	25	25	50	50	25	50	25	50
Rate of the vertical displacement decreasing with depth (m^{-1})	γ	1.0	1.5	1.0	1.5	1.5	1.5	1.5	1.5
Unit weight of soil (kN/m^3)	γ_s	18	18	17	17	14	22	18	17
Poisson's ratio	ν_s	0.25	0.25	0.30	0.30	0.25	0.30	0.25	0.30
Spring coefficient (MN/m^2)	K	45.0	67.5	101.0	151.4	67.5	151.4	67.5	151.4
Damping coefficient (kN · s/m^2)	C	586.2	586.2	846.4	846.4	517.0	962.9	586.2	846.4
Harmonic line load									
Load (kN)	P	21	21	21	21	21	21	105	105
Half-width of the line load (m)	r_0	1.0							
Frequency (rad/s)	κ	40π							

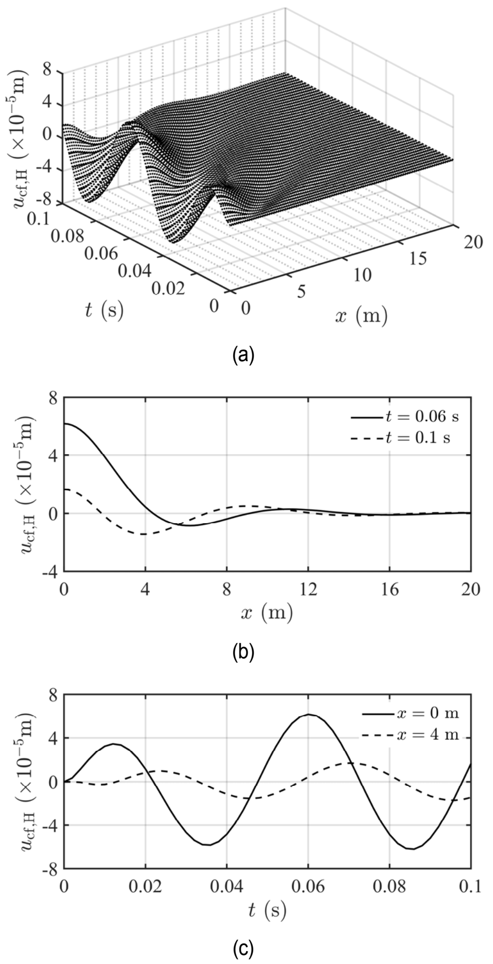

Fig. 2. Exact solution obtained from Eq. (24) using the material and loading data given in Table 1 (case 2).

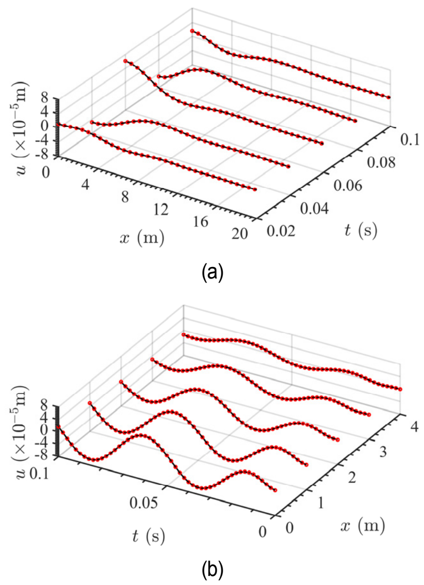

Fig. 3. Responses of the beam obtained from the closed-form solution (solid line) and the numerical one (circle) for case 2: (a) variations in space at time increments of 0.02 s; (b) variations in time at space increments of 1.0 m.

dense sand, $\gamma_s \in$ [17 kN/m^3, 22 kN/m^3], $\nu_s \in$ [0.3, 0.4] and $E_s \in$ [50 MPa, 81 MPa] (Tables 2-7, 2-8 and 3-4 presented in Ref. [40]).

Cases 1 and 2 show the effect of spring coefficient K for cohesionless loose sand type foundation and cases 3 and 4 for cohesionless dense foundation. Fig. 3 presents the numerical solution from case 2 compared to the corresponding closed-form solution given by Eq. (24). In addition, Fig. 4 demonstrates the converged space domain solution at times $t = 0.06$ s and 0.1 s compared to the closed-form solution Eq. (24). It is

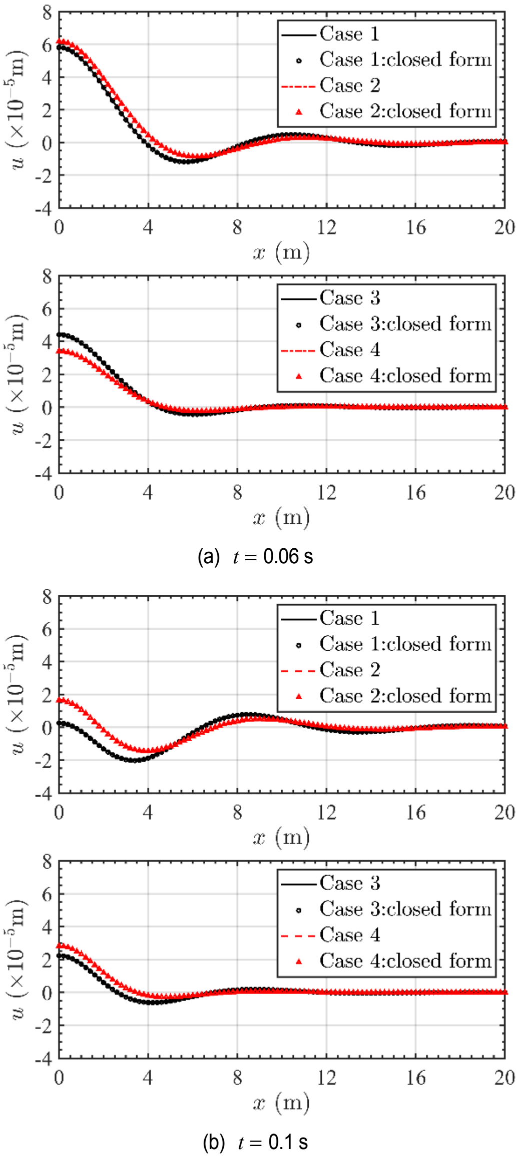

Fig. 4. The response profiles of cases 1-4 in Table 1 at times t = 0.06 s, 0.1 s in space domain $x\in$ [0 m, 20 m].

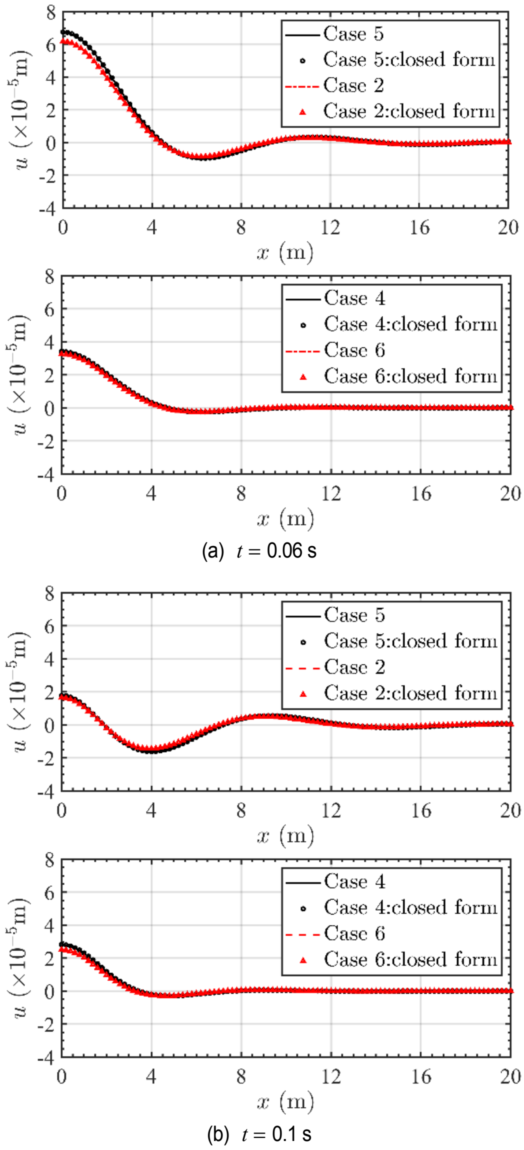

Fig. 5. The response profiles of cases 5 and 2, 4 and 6.

noted that higher stiffness of the foundation results in smaller oscillation amplitudes of the beam. Here, the numerical solutions have a good agreement with the closed-form solutions.

3.5 Effect of damping coefficient

As a second experiment, the effect of damping coefficient of the foundation, which, according to Eq. (19), is influenced by five parameters (i.e., b, E_s, γ_s, g and v_s), is investigated. Compared to the stiffness coefficient, the damping of the foundation is affected by the unit weight of soil. In Table 1, cases 5 and 2, the magnitude of the damping characteristic of soil foundation differs due to relation Eq. (19). For example, the magnitude of the damping coefficient C of case 2 whose unit weight of soil γ_s = 18 kN/m^3 is bigger than that of case 5 (γ_s = 14 kN/m^3). Cases 5 and 2 reflect the cohesionless loose sand type foundation. To take into account the cohesionless dense sand type foundation, cases 4 and 6 are also analysed. Fig. 5 shows the converged solutions compared to the corresponding closed-form solutions at each time 0.06 s and 0.1 s for two sets of parameters in Table 1 (cases 5 and 2, 4 and 6). Since the damping coefficient C, given by (19), is proportional to the square root of unit weigh of soil, the profiles in Fig. 5 do not show evident deviation, however, the numerical solutions converge to the closed-form ones well.

3.6 Effect of the load amplitude

As a final experiment, the effect of amplitude of the external load on the solution is assessed. Actually, since the present work is for a linear problem and the excitation is steady state, the solution will be proportional to the amplitude of the load. Table 2 demonstrates the predicted maximum amplitude of the

Table 2. Amplitudes of the response profiles of cases 2 and 7, 4 and 8 at $x = 0$ m.

Time (s)	0.06			
Cases	2	7	4	8
Amplitude ($\times 10^5$ m)	6.1757	30.8785	3.4040	17.0201
Time (s)	0.1			
Cases	2	7	4	8
Amplitude ($\times 10^5$ m)	1.6468	8.2340	2.8214	14.1072

response profiles. Two different amplitudes are considered, that is, $P = 21$ kN, 105 kN while the properties for loose/dense cohesionless sand type foundation are given in Table 1. At the mean position, $x = 0$ m, the amplitude for cases 7 and 8, where the amplitude $P = 105$ kN, is exactly five times bigger than those of cases 2 and 4, respectively.

4. Concluding remarks

In this paper, several numerical experiments were conducted to evaluate the dynamic responses of an infinite Bernoulli-Euler beam resting on a viscoelastic foundation loaded by harmonic line loads. The applied numerical iterative procedure was based on the semi-analytic approach proposed by Jang [24] capable of yielding time-dependent displacement and, without additional computational effort, the velocity response of the beam. The numerical results could be obtained from the integral equation form of the solution by combining the well-known integration methods, e.g., Simpson's rule and Trapezoidal rule with the help of a computer software program, such as MATLAB ver. 9.6.0. The main aim of the paper was to assess the performance of the iterative solution procedure by comparison of its predictions with those of the conventional closed-form solution.

As a specific viscoelastic foundation model, a commonly used cohesionless sand type foundation was adopted. The viscoelasticity of the foundation was characterized by using a conventional empirical model of soil behavior. Using the closed-form solution, it was possible to assess the validity of the present numerical solution by comparing the respective predictions of the displacement profiles. Then, some parameters of the problem were allowed to vary to assess their influence on the solutions. Variations of the rate of the displacement in the ground decreasing with vertical depth and the unit weight of soil of the foundation as well as the amplitude of an excitation load were considered.

In conclusion, based on the original theoretical research presented in Ref. [24], the present study is its first numerical implementation applied to the investigation of the dynamic responses of an infinite Bernoulli-Euler beam resting on a viscoelastic foundation under harmonic line loads which were simulated successfully using realistic material, geometric and loading parameters. The obtained responses were compared with the closed-form solution. Finally, the good agreement between numerical and closed-form solution validated the rationality of the iterative procedure.

The presented work may be extended to further studies involving, e.g., nonlinear Bernoulli-Euler beam, Timoshenko beam and non-uniform beam on a general (visco-) elastic foundation with shear effect subjected to several cases of external loads, considering variations of physical variables.

Acknowledgments

The present study was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2018R1D1A1B06049813). And it was also supported by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MSIP) (NRF-2017R1A5A1015722 and 2011-0030013).

Nomenclature

A : Cross sectional area
b : Width of the beam cross-section
C : Damping coefficient
E : Young's modulus of beam
E_s : Young's modulus of foundation
g : Gravity acceleration
G, G_t : Kernel function
H : Half-maximum convention (Heaviside step function)
h : Height
I : Second moment of area
K : Spring coefficient
K_p : Pseudo-spring coefficient
P : Amplitude of the applied external load
r_0 : Half-width of the line load
u, v : Iterative (response) solutions
x_0 : Constant
W : External load
α, β, β_p : Parameters
γ : Rate of the vertical displacement decreasing with depth
γ_s : Unit weight of soil
κ : Loading frequency
ν_s : Poisson's ratio of foundation
ρ : Density

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