

Journal of Mechanical Science and Technology 34 (2) 2020

Original Article

DOI 10.1007/s12206-020-0129-0

Keywords:

- · Angular contact ball bearing
- · Contact angles · The triangular geometric theorem
- · Vector diagram method

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Citation:

Zhang, J., Fang, B., Yan, K., Hong, J. (2020). A novel model for high-speed angular contact ball bearing by considering variable contact angles. Journal of Mechanical Science and Technology 34 (2) (2020) 809~816. http://doi.org/10.1007/s12206-020-0129-0

† Recommended by Editor Hyung Wook Park

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A novel model for high-speed angular contact ball bearing by considering variable contact angles

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Abstract In this study, on the basis of the various contact angles and the hybrid theory, a new mathematic model without the raceway control hypothesis is proposed for the analysis of ball bearing under the combined axial, radial and moment loads. Instead of the orthogonal decomposition method, the triangular geometric theorem and vector diagram method have been used in the force analysis of local ball to improve the computation efficiency of bearing analysis. For validation purpose, the comparative analysis of the ball-raceway contact angles and loads of ball bearing under different operation conditions obtained by the proposed model and the Jones' model with different raceway control hypotheses has been conducted. The results show that the proposed model has a higher applicability and rationality compared to the Jones' model, and a proper moment load can be used to improve the load distribution and service performance of ball bearing subjected to the radial load.

1. Introduction

Ball bearings as the core supporting and motion transmission components, are widely used in numerous rotating machinery. The dynamic behaviors of ball bearings have a significant effect on the service performance of the rotor-bearing system.

In order to accurately predict the dynamic behaviors of ball bearing, a large number of studies on ball bearing modeling have been presented in the past decades. In the earlier researches conducted by Stribeck [1], Sjovall [2], Lundberg and Palmgren [3, 4], the load distribution, deformation and various service parameters of ball bearing under different load conditions are determined by some empirical formulations. However, due to the ignorance of the inertia forces of balls, these models cannot be used in the accurate calculation of ball bearing at high speed range. Based on the raceway control hypothesis, Jones [5] firstly proposed a theoretical model for high-speed ball bearing under the combined axial, radial and moment loads, and the effect of ball centrifugal force and gyroscopic moment has been fully considered. As the most classical theoretical model of ball bearing, Jones' model provided important guidance for the presentation and improvement of the subsequent models. Based on the results of ball bearing under different operation conditions calculated by the Jones' model, Harris [6] found that the raceway control hypothesis has many limitations and is only suitable for a few working conditions of ball bearing. To overcome the defects of the raceway control hypothesis, several improved models of ball bearing without raceway control hypothesis have been proposed in recent years [7-9]. In addition, Harris [10] proposed a new analytical model based on the lubrication analysis of ballraceway contacts to investigate the skidding behaviors of the axially loaded ball bearing at high speed range. Above all, an effective and accurate mathematic model is the foundation for the preformation prediction and analysis of ball bearing.

Based on the summary of literatures for ball bearing modeling, it can be found that the dynamic parameters of ball bearing including the ball-raceway contact angles and contact loads et al. can be obtained by solving the nonlinear equations, and the nonlinear equations in ball bearing modeling can be divided into two categories: The local equilibrium equations of balls and the global equilibrium equations of ring. For example, in the Jones' model, 4**Z* local variables of balls (*Z* denotes the number of balls) and 3 or 5 global variables of ring have been used in the bearing modeling. It means that the number of nonlinear equations increases rapidly with the increase of ball number *^Z*, and it also causes a sharply increase in the computation load and time consuming for ball bearing analysis. In order to improve the computational efficiency of ball bearing, Liao and Lin [11-13] developed a new method for analyzing the deformations and loads of ball bearing with variable contact angles. However, this model not only neglected the effect of the gyroscopic moment of ball, but also adopt the assumption that the osculation of the inner and outer raceways are the same, it may cause a large error for ball bearing under some specific conditions [14]. Based on the Jones model and outer raceway control hypothesis, Antoine [15] presented a new method for the explicit contact angle calculation for ball bearing under the pure axial load, and then the model has been further extended to deal with ball bearing under the combined loads [16]. Above all, the above simplified models were mostly based on some certain assumptions. Although the calculation of the bearing model can be reduced to some extent, the calculation error of the bearing analysis is more or less increased. In addition, compared to the Jones' model, the use of variable contact angles in the local analysis of balls is more conducive to the selection of the initial value in the iterative calculation of ball bearing.

In this paper, in order to improve the calculation accuracy and efficiency of ball bearing analysis, a novel model for highspeed ball bearing under the combined axial, radial and moment loads is presented based on the various contact angles and the hybrid theory without raceway control hypothesis. Instead of the orthogonal decomposition method, the triangular geometric theorem and vector diagram method have been used in the local force analysis of ball to improve the computation efficiency of the proposed model. On this basis, the contact angles, contact loads and displacements of ball bearing under different working conditions are calculated and discussed. Besides, the results are compared with those from Jones' model with different raceway control hypotheses to verify the correctness of the proposed model.

2. Theoretical analysis

As shown in Fig. 1(a), in order to study the dynamic performance of ball bearing under the combined axial, radial and moment loads. It is assumed that the outer ring is fixed and and the inner ring rotates with the shaft, and the relative displacement vector of the inner ring with respect to outer ring of ball bearing is $\mathbf{d} = \{\delta_x, \delta_y, \delta_z, \theta_y, \theta_z\}$ under the action of the external load vector **F** = {*F^x* , *F^y* , *F^z* , *M^y* , *Mz*}. On this basis, for the ball at any position angular *ψ^k* , the geometry and me-

Fig. 1. The geometry and mechanical relations of ball and rings: (a) The mechanical state; (b) the geometry state.

chanical relations among the ball and rings are presented in Fig. 1, with the increase of rotating speed, the center of ball moves upward from the point O_b to point O'_b under the action of centrifugal force, and the curvature center of inner raceway also shifts from the point O_i to point O_i' under the action of external loads. Therefore, the ball-inner raceway contact angle α_{ik} increases, while the ball-outer raceway contact angle *αok* decreases. () cos ìï = + - + ^í .1. The geometry and mechanical relations of ball and rings: (a) The
chancel state; (b) the geometry state.
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hifts from the point O_i to point O'_i under the action of

loads. Therefore, the ball-inner raceway contact

increases.

shown in Fig. 1(b), the final distances b (a) (b)

1. The geometry and mechanical relations of ball and rings: (a) The

nanical state; (b) the geometry state.

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 A 7. The geometry state.
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 A with the increase of rotating speed, the center of ball
 A with the i

As shown in Fig. 1(b), the final distances between the curvature centers of inner and outer raceways in the horizon and vertical directions can be expressed as:

$$
\begin{cases}\nA_{1k} = (r_i + r_o - D)\sin\alpha^0 + \delta_{ak} \\
A_{2k} = (r_i + r_o - D)\cos\alpha^0 + \delta_{rk}\n\end{cases}
$$
\n(1)

where r_i and r_o are the curvature radii of inner and outer raceways, *D* is the diameter of ball, and α^0 is the initial contact angle of ball bearing. In addition, δ_{ab} and δ_{ab} are the relative displacements of inner ring with respect to outer ring at any angular position $\mathbf{\psi}_{\scriptscriptstyle{k}}$: m the point O_i to point O_i' under the action of ex-
Therefore, the ball-inner raceway contact angless,
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 α_{ik} increases, while the ball-outer raceway contact an-
 α_{ik} increases, while the ball-outer raceway contact angle

decreases.

As shown in Fig. nal loads. Therefore, the ball-inner raceway contact an-
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 a_n decreases.

As shown in Fig. 1(b), the (1)
 $a_{1k} = (r_i + r_o - D)\cos \alpha^0 + \delta_{rk}$ (1)
 $a_{2k} = (r_i + r_o - D)\cos \alpha^0 + \delta_{rk}$ (1)
 a_r *and* r_o are the curvature radii of inner and outer race-
 n, *D* is the diameter of ball, and α^0 is the initial contact
 acements of *s* shifts from the point *i*, to point *i*, to ly ander the action of ex-
 r_k increases, while the ball-outer raceway contact an-

decreases. While the ball-outer raceway contact angle

decreases.

s shown in Fig. 1(*al* loads. Therefore, the ball-inner raceway contact an-
 $\frac{x_k}{a_k}$ increases, while the ball-outer raceway contact angle

decreases.

s shown in Fig. 1(b), the final distances between the curva-

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3, *D* is the diameter of ball, and α^0 is the initial contact

acements of inner ring with respect to outer ring at any

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 $\delta_{ab} =$ the r_i and r_o are the curvature radii of inner and outer race-

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gle of ball bearing. In addition, δ_{ak} and δ_{ik} are the relative
placements of inne ere r_i and r_o are the curvature radii of inner and outer race-
ys, D is the diameter of ball, and α^o is the initial contact
gle of ball bearing. In addition, δ_{ab} and δ_{bc} are the relative
placements of inne *i a a a a a a <i>a* **a** *a* **a** *a α α_c* are the curvature radii of inner and outer race-
 α a α^{*α*} is the initial contact

all bearing. In addition, δ_{ak} and δ_{ak} are the relative

ents of inner ring with respect to outer ring at any

sit

$$
\begin{cases} \delta_{ak} = \delta_x + 0.5 d_m \sin \psi_k \theta_y - 0.5 d_m \cos \psi_k \theta_z \\ \delta_{rk} = \cos \psi_k \delta_y + \sin \psi_k \delta_z \end{cases}
$$
 (2)

where d_m denotes the pitch diameter of ball bearing.

As shown in Fig. 1(b), the final distances between the curvature centers of inner and outer raceways A_{1k} and A_{2k} can also be expressed by the final contact angles α_{ik} , α_{ok} and the final distances between the centers of ball and inner/outer raceway curvature Δ*ik* , Δ*ok* : d_m denotes the pitch diameter of ball bearing.
 d_m denotes the pitch diameter of ball bearing.

shown in Fig. 1(b), the final distances between the curva-

enters of inner and outer raceways A_{1k} and A_{2k} can al acements of inner ring with respect to outer ring at any

alar position ψ_k :
 $\delta_{ak} = \delta_x + 0.5d_m \sin \psi_k \theta_y - 0.5d_m \cos \psi_k \theta_z$ (2)
 $\delta_{rk} = \cos \psi_k \delta_y + \sin \psi_k \delta_z$ (2)
 $\epsilon_R = \cos \psi_k \delta_y + \sin \psi_k \delta_z$ (2)
 $\epsilon_R = \epsilon_R$ (2) and $\epsilon_R = \epsilon_R$ (3) th Ular position $ψ_k$:
 $\delta_{ab} = \delta_x + 0.5d_m \sin \psi_x \theta_y - 0.5d_m \cos \psi_x \theta_z$ (2)
 $\delta_A = \cos \psi_x \delta_y + \sin \psi_x \delta_z$ (2)
 $\sin \theta_x$ denotes the pitch diameter of ball bearing.

s fohown in Fig. (10), the final distances between the curva-

center placements of inner ring with respect to outer ring at any

gular position ψ_4 :
 $\int \delta_{ak} = \delta_x + 0.5d_m \sin \psi_x \theta_y - 0.5d_m \cos \psi_x \theta_z$ (2)
 $\delta_A = \cos \psi_x \delta_y + \sin \psi_x \delta_z$ (2)

ere d_m denotes the pitch diameter of ball bearing.

As sh gular position ψ_k :
 $\int_{\delta_{ak}} = \delta_x + 0.5d_m \sin \psi_x \theta_y - 0.5d_n \cos \psi_x \theta_z$ (2)
 $\delta_x = \cos \psi_x \delta_y + \sin \psi_x \delta_z$ (2)

ere d_m denotes the pitch diameter of ball bearing.

ere ϕ_m denotes the pitch diameter of ball bearing.

exenses fo *i i a* $\frac{1}{2}$ *i a* $\frac{1}{2}$ *i i i s* $\frac{1}{2}$ *i a* $\frac{1}{2}$ *i i a* $\frac{1}{2}$ *i s* $\frac{1}{2}$ *i s* Sosition ψ_k :
 $\bar{D}_x + 0.5d_m \sin \psi_k \theta_y - 0.5d_m \cos \psi_k \theta_z$ (2)
 $\cos \psi_k \delta_y + \sin \psi_k \delta_z$ (2)

denotes the pitch diameter of ball bearing.

m in Fig. 1(b), the final distances between the curva-

seed by the final contact angles

$$
\begin{cases} \Delta_{ik} \sin \alpha_{ik} + \Delta_{ok} \sin \alpha_{ok} - A_{1k} = 0 \\ \Delta_{ik} \cos \alpha_{ik} + \Delta_{ok} \cos \alpha_{ok} - A_{2k} = 0 \end{cases}
$$
 (3)

where Δ*ik* and Δ*ok* can be written as:

$$
\begin{cases}\n\Delta_{ik} = r_i - 0.5D + \delta_{ik} \\
\Delta_{ok} = r_o - 0.5D + \delta_{ok}\n\end{cases}
$$
\n(4)

Fig. 2. The force vector diagrams of ball: (a) Without the gyroscopic moment of ball; (b) with the gyroscopic moment of ball.

where δ_{ik} and δ_{ik} are the contact deformations of ball-inner raceway and ball-outer raceway, respectively. According to the

Q_{ok} (a)
\nFig. 2. The force vector diagrams of ball: (a) Without the gyroscope moment of ball; (b) with the gyroscope moment of ball.
\nwhere
$$
\delta_{ik}
$$
 and δ_{ok} are the contact deformations of ball-inner
\nraceway and ball-outer raceway, respectively. According to the
\nHertz theory of point contact, δ_{ik} and δ_{ok} can be expressed as:
\nIn O
\n
$$
\begin{cases}\n\delta_{ik} = \left(\frac{Q_{ik}}{K_{ik}}\right)^{2/3} & \text{if } i \text{ are given by the equation of the system.} \\
\delta_{ik} = \left(\frac{Q_{ik}}{K_{ok}}\right)^{2/3} & \text{if } i \text{ are given by the equation of the system.} \\
\delta_{ik} = \left(\frac{Q_{ok}}{K_{ok}}\right)^{2/3} & \text{or } i \text{ and } i \text{ and } j \text{ are given by the equation of the system.} \\
\delta_{ik} = \left(\frac{Q_{ok}}{K_{ok}}\right)^{2/3} & \text{or } i \text{ and } j \text{ is even.}\n\end{cases}
$$

where *Qik* and *Qok* denote the contact loads of ball-inner raceway and ball-outer raceway, respectively, and the *Kik* and *Kok* are the load-deformation coefficients of ball-inner raceway and ball-outer raceway, respectively.

In order to obtain the explicit expressions of contact loads *Qik* and *Qok*, the vector diagram method is applied in the force analysis of ball instead of the orthogonal decomposition method. For comparison, the mechanical state of ball without considering the gyroscopic moment is given first. As shown in Fig. 2(a), the vector equilibrium equation of ball can be written as: $\frac{Q_x}{K_{ab}}$ and Q_{ax} denote the contact loads of ball-inner race-
 $\frac{Q_x}{K_{ab}}$ are given by Jones based or
 Q_k and Q_{ax} denote the contact loads of ball-inner race-
 Q_k and Q_{ax} denote the contact loads of bal It wis the standard of the contact, σ_n and σ_n can be expressed as.

In different allocation scheme
 $\begin{vmatrix} \delta_a = \left(\frac{Q_a}{K_a}\right)^{2/3} & & & & \text{if iteratures, and the
\n one given by Jones base of
\n $\delta_a = \left(\frac{Q_a}{K_a}\right)^{2/3} & & & & \text{if iteratures, and the
\n one given b and doubler are decay, respectively, and the $K$$$ sing of $\left(\frac{Q_0}{R_a}\right)^{3/2}$. (5) there Recovery Control Hypothesis *i*₄ = 0.5 and *λ*₃ = 1.

inter Recovery Control Hypothesis *i*₄ = 0.5 and *λ*₃ = 1.

interval the control Hypothesis *i*₄ = 0.4 and *λ*₃ *ok* Q_a and Q_{ab} denote the contact loads of ball-inner race-

sented an obvious speed-discontinuit

and bal-outer raceway, respectively, and the K_{ba} and K_{cav} and improve allocated scheme of the

pouter raceway, **and ball-outer raceway, respectively, and the** K_a **and** K_{ca} **and improve allocation scheme of the front contracts was given based on the equal detormation coefficients of ball-inner raceway and contacts was given based** (a)
 $\frac{1}{4}$
 $\frac{1}{4$ Q_{d} denote the contact loads of ball-line race-

sented an obvious speed discontinuity. To overcome this contact

deformation coefficients of ball-liner raceway and

deformation coefficients of ball-liner raceway an **Example of the explicit expressions of contact loads** Q_k **

a** Q_{ab} , the vector diagram method is applied in the force
 A Q_{ab} , the vector diagram method is applied in the force
 A Q_{ab} **F** Q_a **E** Q_a **E** Q **a** all instead of the orthogonal decomposition $u = \frac{u}{C_a} = \frac{u_{ca}}{C_a} = \frac{u_{ca}}{C_a} = \frac{u_{ca}}{C_a} = \frac{u_{cd}}{C_a} = \frac{u_{$

$$
\overrightarrow{F_{ck}} + \overrightarrow{Q_{ik}} + \overrightarrow{Q_{ok}} = 0
$$
\n(6)

where \overrightarrow{F}_{ck} is the vector of ball centrifugal force, according to the sine theorem of plane triangle, one can obtain:

$$
\frac{Q_{ik}}{\sin \alpha_{ok}} = \frac{Q_{ok}}{\sin \alpha_{ik}} = \frac{F_{ck}}{\sin (\alpha_{ik} - \alpha_{ok})}
$$
(7)

$$
\overline{F}_{ck} + \overline{Q}_{ik} + \overline{Q}_{ok} = 0
$$
\n(6)
\nthere \overline{F}_{ck} is the vector of ball centrifugal force, according to the
\ne theorem of plane triangle, one can obtain:
\n
$$
\frac{Q_{ik}}{\sin \alpha_{ak}} = \frac{Q_{ok}}{\sin \alpha_{ak}} = \frac{F_{ck}}{\sin (\alpha_{ik} - \alpha_{ok})}
$$
\n(7)
\n
$$
\left\{\n\begin{aligned}\nQ_{ik} &= \frac{\sin \alpha_{ok}}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck} & B \\
Q_{ok} &= \frac{\sin \alpha_{ik}}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck} & B \\
Q_{ok} &= \frac{\sin \alpha_{ik}}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck} & F\n\end{aligned}\n\right.
$$
\n(8)
\n6)
\nWhen the gyroscope moment of ball is taken into account,
\nshown in Fig. 2(b) the vector equilibrium equation of ball

When the gyroscopic moment of ball is taken into account, as shown in Fig. 2(b), the vector equilibrium equation of ball can be rewritten as:

$$
\overrightarrow{F_{ck}} + \overrightarrow{Q_{ik}} + \overrightarrow{T_{ik}} + \overrightarrow{Q_{ok}} + \overrightarrow{T_{ok}} = 0
$$
\n(9)

thrology 34 (2) 2020 DOI 10.1007/s12206-020-0129-0

E rewritten as:
 $+\overline{Q_{ik}} + \overline{T_{ik}} + \overline{Q_{ok}} + \overline{T_{ck}} = 0$ (9)
 $\overline{T_{ik}}$ and $\overline{T_{ok}}$ are the friction forces in ball-inner race-

individual outer raceway contacts, resp Technology 34 (2) 2020 DOI 10.1007/s12206-020-0129-0

n be rewritten as:
 $\overline{F_{ck}} + \overline{Q_{ik}} + \overline{T_{ik}} + \overline{Q_{ok}} + \overline{T_{ck}} = 0$ (9)

ere $\overline{T_{ik}}$ and $\overline{T_{ok}}$ are the friction forces in ball-inner race-

y and ball-outer race where \overrightarrow{T}_{ik} and \overrightarrow{T}_{ok} are the friction forces in ball-inner raceway and ball-outer raceway contacts, respectively. Since the friction forces are used to offset the action of ball gyroscopic moment, the follow expressions can be obtained:

Technology 34 (2) 2020 DOI 10.1007/s12206-020-0129-0
\nno be rewritten as:
\n
$$
\overrightarrow{F_{ck}} + \overrightarrow{Q_{ik}} + \overrightarrow{T_{ik}} + \overrightarrow{Q_{ok}} + \overrightarrow{T_{ok}} = 0
$$
\n(9)
\nhere $\overrightarrow{T_{ik}}$ and $\overrightarrow{T_{ok}}$ are the friction forces in ball-inner race-
\nay and ball-outer raceway contacts, respectively. Since the
\nction forces are used to offset the action of ball gyroscopic
\noment, the follow expressions can be obtained:
\n
$$
\begin{cases}\nT_{ik} = \lambda_{ik} \frac{2M_{gk}}{D} & (10) \\
T_{ok} = \lambda_{ok} \frac{2M_{gk}}{D}\n\end{cases}
$$
\n(10)
\n $\lambda_{ik} + \lambda_{ok} = 1$ (11)
\nIn order to further determine the sizes of T_{ik} and T_{ok} , several
\nfferent allocation schemes of the frictions were given in previ-
\ns literatures, and the most representative allocation methods
\ngiven by Jones based on the Raceway Control Hypothesis:

where M_{qk} is the gyroscopic moment of ball.

$$
\lambda_{ik} + \lambda_{ok} = 1 \tag{11}
$$

In order to further determine the sizes of T_{ik} and T_{ok} , several different allocation schemes of the frictions were given in previous literatures, and the most representative allocation methods are given by Jones based on the Raceway Control Hypothesis:

Inner Raceway Control Hypothesis: *λik* = 0.5 and *λok* = 0.5;

Outer Raceway Control Hypothesis: *λik* = 0 and *λok* = 1.

However, the follow up studies showed that the above allocation schemes based on the raceway control hypothesis presented an obvious speed-discontinuity. To overcome this defect, an improve allocation scheme of the frictions in ball-raceway contacts was given based on the equal friction coefficient assumption [8]. e M_{gk} is the gyroscopic moment of ball.
 $+\lambda_{ck} = 1$ (11)

order to further determine the sizes of T_{ik} and T_{ok} , several

entrallocation schemes of the frictions were given in previ-

tieratures, and the most repr Raceway Control Hypothesis: $\lambda_{ik} = 0.5$ and $\lambda_{ok} = 0.5$ and $\lambda_{ok} = 7$

Raceway Control Hypothesis: $\lambda_{ik} = 0$ and $\lambda_{ok} = 7$

ver, the follow up studies showed that the abov

chemes based on the raceway control hypothe *i*_{sex} is the gyroscopic moment of ball.
 $\lambda_{ak} = 1$ (11)

der to further determine the sizes of T_{ik} and T_{obs} several

tallocation schemes of the frictions were given in previ-

ratures, and the most representative *μ*_{α_k} + λ_{α_k} = 1 (11)
 n order to further determine the sizes of T_{ik} and T_{α_k} , several

erent allocation schemes of the frictions were given in previ-

illeratures, and the most representative allocati *i* data obvious speed-discontinuity. To overcome this deprove allocation scheme of the frictions in ball-race
approve allocation scheme of the frictions in ball-race
acts was given based on the equal friction coefficient is to further determine the sizes of T_{ik} and T_{ok} , several
allocation schemes of the frictions were given in previ-
tures, and the most representative allocation methods
by Jones based on the Raceway Control Hypothes allocation schemes of the frictions were given in previ-
tures, and the most representative allocation methods
by Jones based on the Raceway Control Hypothesis:
faceway Control Hypothesis: $\lambda_{\rm at} = 0.5$ and $\lambda_{\rm at} = 0.5$ below the below the mean of the raceway control hypothesis pre-
 Phemes based on the raceway control hypothesis pre-
 D overcome this defect,

we allocation scheme of the frictions in ball-raceway

was given based on

$$
\mu = \frac{T_{ik}}{Q_{ik}} = \frac{T_{ok}}{Q_{ok}} = \frac{1}{Q_{ik} + Q_{ok}} \frac{2M_{gk}}{D}
$$
(12)

ontacts was given based on the equal friction coefficient assumption [8].

\n
$$
\mu = \frac{T_{ik}}{Q_{ik}} = \frac{T_{ok}}{Q_{ok}} = \frac{1}{Q_{ik} + Q_{ok}} \frac{2M_{sk}}{D}
$$
\n(12)

\n
$$
\begin{cases}\nT_{ik} = \frac{Q_{ik}}{Q_{ik} + Q_{ok}} \frac{2M_{sk}}{D} \\
T_{ok} = \frac{Q_{ok}}{Q_{ik} + Q_{ok}} \frac{2M_{sk}}{D}\n\end{cases}
$$
\n(13)

\nus

thus

inimprove allocation scheme of the frictions in ball-raceway
intacts was given based on the equal friction coefficient as-
imption [8].

$$
\mu = \frac{T_{ik}}{Q_{ik}} = \frac{T_{ok}}{Q_{ik}} = \frac{1}{Q_{ik}} = \frac{2M_{sk}}{Q_{ik} + Q_{ok}} = \frac{2M_{sk}}{D}
$$
(12)

$$
\begin{cases}\nT_{ik} = \frac{Q_{ik}}{Q_{ik}} = \frac{2M_{sk}}{Q_{ik}} = \frac{2M_{sk}}{Q_{ik} + Q_{ok}} = \frac{2M_{sk}}{D}\n\end{cases}
$$
(13)
us

$$
\begin{cases}\n\lambda_{ik} = \frac{Q_{ik}}{Q_{ik} + Q_{ok}} \\
\lambda_{ok} = \frac{Q_{ok}}{Q_{ik} + Q_{ok}} \\
\lambda_{ok} = \frac{Q_{ok}}{Q_{ik} + Q_{ok}} \\
\overline{F_{ik}} + \overline{F_{ik}} + \overline{F_{ik}} = 0\n\end{cases}
$$
(14)
Based on the above analysis, the vector equilibrium equation
ball can be further simplified as:

$$
\overline{F_{ik}} + \overline{F_{ik}} + \overline{F_{ik}} = 0
$$
(15)

$$
\overline{F_{ik}} = \overline{Q_{ik}} + \overline{T_{ik}} \\
\overline{F_{ck}} = \overline{Q_{ik}} + \overline{T_{ok}} \\
\overline{G_{ik}} = \overline{Q_{ik}} + \overline{T_{ik}} \\
\overline{G_{
$$

 (8) of ball can be further simplified as: Based on the above analysis, the vector equilibrium equation

$$
\overrightarrow{F_{ck}} + \overrightarrow{F_{ik}} + \overrightarrow{F_{ok}} = 0 \tag{15}
$$

$$
\begin{cases}\nF_{ik} = Q_{ik} + T_{ik} \\
\overline{F} = \overline{O} + \overline{T}\n\end{cases}
$$
\n(16)

As shown in Fig. 2(b), according to the sine theorem of plane triangle:

$$
\frac{F_{ik}}{\sin \theta_{ok}} = \frac{F_{ok}}{\sin \theta_{ik}} = \frac{F_{ck}}{\sin (\theta_{ik} - \theta_{ok})}
$$
 the

Journal of Mechanical Science and
\nAs shown in Fig. 2(b), according to the sine theorem of plane
\nangle:
\n
$$
\frac{F_{ik}}{\sin \theta_{ok}} = \frac{F_{ok}}{\sin \theta_{ik}} = \frac{F_{ck}}{\sin(\theta_{ik} - \theta_{ok})}
$$
\n $\begin{cases}\n\theta_{ik} = \alpha_{ik} - \Delta \alpha_{ik} \\
\theta_{ok} = \alpha_{ok} - \Delta \alpha_{ik}\n\end{cases}$ \n(18)

\nAccording to Eqs. (12) and (13), one can obtain:

According to Eqs. (12) and (13), one can obtain:

$$
\frac{F_{ik}}{\sin \theta_{ok}} = \frac{F_{ok}}{\sin \theta_{ik}} = \frac{F_{ck}}{\sin(\theta_{ik} - \theta_{ok})}
$$
\n
$$
\begin{cases}\n\theta_{ik} = \alpha_{ik} - \Delta \alpha_{ik} & \text{that occurs} \\
\theta_{ok} = \alpha_{ok} - \Delta \alpha_{ok} & \text{equation [7]}\n\end{cases}
$$
\n
$$
\Delta \alpha = \Delta \alpha_{ik} = \Delta \alpha_{ok} = \arctan\left(\frac{1}{Q_{ik} + Q_{ok}} \frac{2M_{sk}}{D}\right)
$$
\n
$$
\theta_{ik} - \theta_{ok} = \alpha_{ik} - \alpha_{ok}
$$
\n(18)\n
$$
\begin{cases}\nC = \frac{Q}{Q_{ck}} \\
\theta_{ik} - \theta_{ok} = \alpha_{ik} - \alpha_{ok}\n\end{cases}
$$
\n(19)\n
$$
\begin{cases}\nC = \frac{D}{Q_{ck}} \\
G = \frac{D}{d_{ik}} \\
S = \frac{1 + \frac{1}{2}}{Q_{kc}}\n\end{cases}
$$

$$
\theta_{ik} - \theta_{ok} = \alpha_{ik} - \alpha_{ok} \tag{20}
$$

therefore

According to Eqs. (12) and (13), one can obtain:
\n
$$
\Delta \alpha = \Delta \alpha_{ik} = \Delta \alpha_{ok} = \arctan \left(\frac{1}{Q_{ik} + Q_{ok}} \frac{2M_{sk}}{D} \right)
$$
\n
$$
\theta_{ik} - \theta_{ok} = \alpha_{ik} - \alpha_{ok}
$$
\n
$$
\theta_{ik} - \theta_{ok} = \alpha_{ik} - \alpha_{ok}
$$
\n
$$
\theta_{ik} = \frac{\sin (\alpha_{ok} - \Delta \alpha)}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck}
$$
\n
$$
\begin{cases}\nF_{ik} = \frac{\sin (\alpha_{ak} - \Delta \alpha)}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck} \\
F_{ck} = \frac{\sin (\alpha_{ak} - \Delta \alpha)}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck}\n\end{cases}
$$
\n
$$
\begin{cases}\nF_{ik} = \frac{\sin (\alpha_{ak} - \Delta \alpha)}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck} \\
F_{ck} = \frac{\sin (\alpha_{ik} - \Delta \alpha)}{\sin (\alpha_{ik} - \alpha_{ok})} F_{ck}\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \text{ and } \alpha_{ik} \text{ the solution that } \alpha_{ik} \text{ the above} \\
Q_{ik} = F_{ik} \cos \Delta \alpha \\
Q_{ik} = F_{ck} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = F_{ck} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = H_{ck} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = H_{ck} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = H_{ik} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = H_{ik} \cos \Delta \alpha\n\end{cases}
$$
\n
$$
\begin{cases}\nQ_{ik} = H_{ik} \cos \Delta \alpha \\
Q_{ik} = H
$$

In addition, the ball-raceway contact loads *Qik* and *Qok* can be written as:

$$
\begin{cases}\nQ_{ik} = F_{ik} \cos \Delta \alpha \\
Q_{ok} = F_{ok} \cos \Delta \alpha\n\end{cases}
$$
\n(22) $\frac{\text{kr}}{\text{im}}$

Besides, the centrifugal force and gyroscopic moment of ball can be expressed:

$$
\begin{cases}\nF_{ck} = 0.5md_m\omega_{mk}^2 \\
M_{gk} = J\omega_{mk}\omega_{bk}\sin\beta_k\n\end{cases}
$$
\n(23)

where the angular speeds *ωmk* and *ωbk* denote the revolution speed and spinning speed of ball, respectively. *m* and *J* denote the mass and mass moment of ball. And the angular speeds $\omega_{\mu k}$ and $\omega_{\mu k}$ can be obtained according to the assumption that no macroscopic sliding phenomenon occurs in ballraceway contact zones:

() () () () () () ()() () () () () ï = í = *i ik ok mk ok ik k ik ok k ^m i ik ok bk ok ik k ik ok k ^ω ^γ αβ ^ω γ α β γ α β ^d ^ω ^γ ^γ ^ω ^D ^γ ^α ^β ^γ ^α ^β* (24) cos cos ^ì ⁼ í î = *ik ik m ok ok m ^γ ^D ^α ^d γ D α d* (25)

Usually, the ball pitch angle β_k is also calculated by the raceway control hypothesis that the ball has no spinning motion on the "control raceway". However, it has been proved that

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As shown in Fig. 2(b), according to the sine theorem of plane the spinning speed between based

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and light load conditions. In order
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As shown in Fig. 2(b), according to the sine theorem of plane the spinning speed between the spinning speed between the spinning speed between the spinning speed b Journal of Mechanical Science and Technology 34 (2) 2020

As shown in Fig. 2(b), according to the sine theorem of plane the spinning speed between the small ight load conditions

and light load conditions
 $\frac{F_a}{\sin \theta_{\alpha}}$ Journal of Mechanical Science and Technology 34 (2) 2020

Brown in Fig. 2(b), according to the sine theorem of plane the spinning speed between b.

e:

ward zero only when ball best

and light load conditions I. In ord
 ^α ^α ^α Q Q D b
 b - *b b according to the sine theorem of plane*
 As shown in Fig. 2(b), according to the sine theorem of plane
 of $\frac{F_a}{\sin \theta_{a}} = \frac{F_{ab}}{\sin \theta_{a}} = \frac{F_a}{\sin (\theta_a - \theta_{aa})}$ **

(17) the raceway control in youth band
** the spinning speed between ball and outer raceway tends toward zero only when ball bearing is operated at high-speed and light load conditions. In order to overcome the limitations of the raceway control hypothesis, the d'Alembert principle without raceway control hypothesis is used for ball pitch angle calculation [7]: DOI 10.1007/s12206-020-0129-0

een ball and outer raceway tends to-

all bearing is operated at high-speed

In order to overcome the limitations of

othesis, the d'Alembert principle with-

thesis is used for ball pitch a 2) 2020 DOI 10.1007/s12206-020-0129-0

d between ball and outer raceway tends to-

then ball bearing is operated at high-speed

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rol hypothesis, the d'Alembert principle wi 0 DOI 10.1007/s12206-020-0129-0

ween ball and outer raceway tends to-

ball bearing is operated at high-speed

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pothesis, the d'Alembert principle with-

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eed between ball and outer raceway tends to-

when ball bearing is operated at high-speed

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ntrol hypothesis is used for ball pitch *i* **o** *i* **o** *c c c <i>c ^β C S ^α ^α D d G* nology 34 (2) 2020 DOI 10.1007/s12206-020-0129-0

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spinning speed between ball and outer raceway tends to-

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gy speed between ball and outer raceway tends to-

only when ball bearing is operated at high-speed

ad conditions. In order to overcome the limitations of

y control hypothe

$$
\tan \beta_k = \frac{C(S+1)\sin \alpha_{ik} + 2\sin \alpha_{ok}}{C(S+1)\cos \alpha_{ik} + 2(\cos \alpha_{ok} + D/d_m) + G}
$$
(26)

(17) the recovery control hypothesis, the d'Alembert principle with-
\nout raceway control hypothesis is used for ball pitch angle cal-
\nculation [7]:
\n(18)
\n
$$
\tan \beta_k = \frac{C(S+1)\sin \alpha_{ik} + 2\sin \alpha_{ok}}{C(S+1)\cos \alpha_{ik} + 2(\cos \alpha_{ok} + D/d_m) + G}
$$
\n(26)
\n3), one can obtain:
\n
$$
\frac{1}{\alpha_{ik} + Q_{ok}} \frac{2M_{gk}}{D}
$$
\n(19)
\n(19)
\n
$$
\begin{cases}\nC = \frac{Q_{ik}a_{ik}L_{ik}}{Q_{ok}a_{ok}L_{ok}} \\
G = \frac{D}{d_m}C[\cos(\alpha_{ik} - \alpha_{ok}) - S] \\
S = \frac{1 + D/d_m \cos \alpha_{ok}}{1 - \cos \alpha_{ik}}\n\end{cases}
$$
\n(27)
\nwhere a_{ik} and a_{ik} the semi-major axis of ball-inner raceway and

 (21) the second kind elliptic integral of ball-inner raceway and ball where a_{ik} and a_{ok} the semi-major axis of ball-inner raceway and ball-outer raceway contacts, respectively, and *Lik* and *Lok* are outer raceway contacts, respectively.

ck $F_a = \frac{\sin(\alpha_a - \Delta \alpha)}{\sin(\alpha_a - \alpha_a)} F_a$
 F_{at} $= \frac{\sin(\alpha_a - \Delta \alpha)}{\sin(\alpha_a - \alpha_a)} F_a$
 F_{at} $= \frac{\sin(\alpha_a - \Delta \alpha)}{\sin(\alpha_a - \alpha_a)} F_a$
 CAL **are** $\frac{\sin(\alpha_a - \Delta \alpha)}{\sin(\alpha_a - \alpha_a)} F_a$ **
** *CAL**CAL are

<i>CAL CAL CAL* **a** *CAL CAL CAL CAL C* (23) physical meaning as the iterative variables can facilitate the shown variables α_n can be obtained in vector

and force and gyroscopic moment of ball proved model has three obtained by implicit Eq. (3) based on the Newton-Ray

pared to the classical Jones' model of

of unknown vari $\int_{\alpha}^{x} \cos \Delta \alpha$ (22) when the centrifugal force and gyroscopic moment of ball weights Eq. (3) based on the Newton-Raphson met

the centrifugal force and gyroscopic moment of ball weight end to the Newton-Raphson met

th $\begin{bmatrix}\nQ_a = F_a \cos \Delta a & (22) \text{ known variables } a_a \text{ and } a_a \text{ can be obtained by}\\
Q_a = F_a \cos \Delta a & (22) \text{ important value of the classical Jones'}\\
\text{Besides, the centrifugal force and gyroscope moment of ball per two-
Besides, the centrifugal force and gyroscope moment of ball per two-
pleistics. (1) is a good value of the classical Jones' model is a fixed value of the characteristic.\n\end{bmatrix}\n\n\begin{bmatrix}\nF_a = 0.5md_a \omega_a^2 & (3) \text{ based on the Newton-Raphson model,}\\
F_a = J\omega_{aa}\omega_a \sin \beta_a & (23) \text{ physical meaning as the derivative in the direction of the derivative in$ cos $\Delta \alpha$ (22)

implicit Eq. (3) based on the Newton-Raphson method. C

has centrifugal force and gyroscopic moment of ball

the centrifugal force and gyroscopic moment of ball

implicit Eq. (3) based on the Newton-Raphs Through the above analysis of local ball, it can be found that only two unknown variables α_{ik} and α_{ok} have appeared in the ball modeling, and other intermediate variables can be explicitly expressed by the variables α_{ik} and α_{ak} . Therefore, for a given global displacement vector $\mathbf{d} = \{\delta_x, \delta_y, \delta_z, \theta_y, \theta_z\}$, the unknown variables α_{ik} and α_{ok} can be obtained by solving the implicit Eq. (3) based on the Newton-Raphson method. Compared to the classical Jones' model of ball bearing, the improved model has three obvious advantages: (1) The number of unknown variables for single ball reduces from four to two, thus greatly reduces the computation load in the single iterative calculation; (2) using the ball-raceway contact angles with clear selection of the iterative initial values; (3) due to the abandonment of raceway control hypothesis, the applicability and accuracy of the model are improved. ment vector **d** = { δ_x ,, δ_y ,, θ_z ,, θ_y ,, θ_z }, the un-
s α_{α} and $\alpha_{\alpha\alpha}$ can be obtained by solving the
based on the Newton-Raphson method. Com-
assical Jones' model of ball bearing, the im-
assical Jones' *x* $\int_{\alpha}^{x} f(x) dx$ *i x is* $\int_{\alpha}^{x} f(x) dx$ *is* $\int_{\alpha}^{x} f(x) dx$ *is* $\int_{\alpha}^{x} f(x) dx$ *is* $\int_{\alpha}^{x} f(x) dx$ *z* $\int_{\alpha}^{x} f(x) dx$ *is the iterative variables can facilitate the tion of the iterative initial values; (3)* modeling, and other intermediate variables can be explicitly
ressed by the variables α_a and α_{sa} . Therefore, for a given
al displacement vector **d** = { δ_a , δ_b , δ_a , θ_c , θ_c , θ_c , the un-
F n variab *F* **EXECT ASSAURE THE ASSAURE THE ASSAURE THE ASSAURE** α_{μ} **and** α_{μ} **can be obtained by solving the limit of Eq. (3) based on the Newton-Raphson method. Com-
Fig. it also the Newton-Raphson method. Com-
He can con** icit Eq. (3) based on the Newton-Raphson method. Com-
H to the classical Jones' model of ball bearing, the im-
H model has three obvious advantages: (1) The number
ed model has three obvious advantages: (1) The number

On the basis of local ball analysis, the global equilibrium equations of inner ring can be written as:

Exercise, the centrifugal force and gyroscopic moment of ball
\nbe expressed:

\nthe centrifugal force and gyroscopic moment of ball
\nthe expressed:

\n\n- the amplitudes for single tentile
\n- the expressed:
\n- the non-factors of the model has three obvious advantages:
\n- The number of unknown variables for single tentile
\n- the generalized reduction of the tietative calculation
\n- the same differential values
\n
\nand
$$
\omega_{\mu}
$$
 denote the revolution of the interative method

\n\n- the angular speeds ω_{μ} and ω_{μ} denote the revolution
\n
\nand ω_{μ} denote the revolution

\n\n- the angular speed of ball, respectively.
\n- the angular speed of ball, respectively.
\n- the angular field of the field is
\n- the velocity of the model are improved.
\n
\nand ω_{μ} denote the revolution

\n\n- the maximum of the field is
\n- the angular equation of the field is
\n- the maximum equation of the field is
\n
\nand ω_{μ} can be written as:

\n\n- $$
\omega_{\mu} = \frac{\omega_{\mu}}{(1 + \gamma_{\mu})\cos(\alpha_{\mu} - \beta_{\mu}) + (1 - \gamma_{\mu})\cos(\alpha_{\mu} - \beta_{\mu})}
$$
\n- $$
\omega_{\mu} = \frac{d_{\mu}}{2} \sum_{i}^{2} F_{\mu} \cos \theta_{\mu} \sin \psi_{\mu}
$$
\n- the ball of the angle β_{μ} is also calculated by the\n
	\n- the normal coordinates of the field is
	\n- the absolute direction of the above global equilibrium equations of the field is
	\n\nand ω_{μ} and ω_{μ} is

\n
	\n- $$
	\omega_{\mu} = \frac{d_{\mu}}{2} \sum_{i}^{2} F_{\mu} \sin \theta_{\mu} \cos \psi_{\mu}
	$$
	\n- the ball

In order to solve the above global equilibrium equations of ring and local equilibrium equations of balls, a nested-loop iteration algorithm is given in Fig. 3. Firstly, input the basic

Fig. 3. The detailed calculation flow of the proposed model.

structural parameters and operation condition of ball bearing and give the proper iterative initial values of the global displacement vector $\mathbf{d} = \{ \delta_x, \delta_y, \delta_z, \theta_z, \theta_z \}$ and ball-raceway contact angles α_{ik} and α_{ik} ; secondly, solve the local equilibrium equations of ball and update the values of ball-raceway contact angles α_{ik} and α_{ik} ; then, substitute the new local variables into the global equilibrium equations of ring and update the values of the global displacement vector **d** = $\{\delta_{x}, \delta_{y}, \delta_{z}, \theta_{y}, \theta_{z}\}$; repeat the above iteration until the calculation error is less than the set threshold; finally, output the final values of global displacement vector **d** and ball-raceway contact angles α_{ik} and α_{ik} .

3. Results and discussion

Based on the model proposed above, the angular contact ball bearing B218 under different operation conditions was studied and analyzed, and the detailed structural parameters of B218 are listed in Table 1. To validate the correctness of the proposed model, the comparison results of ball-raceway contact angles calculated by the proposed model and Jones'

Table 1. The structural parameters of B218.

Parameters	B ₂₁₈
Bearing inner raceway curvature radius r_i (mm)	11.6281
Bearing outer raceway curvature radius r_0 (mm)	11.6281
Bearing inner raceway contact diameter d_i (mm)	102.7938
Bearing outer raceway contact diameter d_0 (mm)	147.7264
Number of balls Z	16
Ball diameter D (mm)	22.225
Pitch diameter d_m (mm)	125.26

Fig. 4. The comparison curves of speed-varying contact angles between the proposed model and the Jones' model.

Fig. 5. The comparison curves of speed-varying contact loads between the proposed model and the Jones' model.

model with two different raceway control hypotheses are given in Figs. 4-8.

As shown in Fig. 4, the results of the speed-varying contact angles of ball bearing under a constant axial load (*F^x* = 2000 N) obtained by the proposed model and Jones' model with the Inner and outer raceway control hypotheses (I/ORC Hypothesis) are presented. It can be found that, the change curves of ball-raceway contact angles obtained by three different methods have a similar law. By further comparing the three result curves, it also can be found that, the result curve by the proposed model is relatively close to that obtained by Jones' model with IRC hypothesis at low speed range and relatively close to that of Jones' model with ORC hypothesis at high speed range. Meanwhile, the similar law can also been found in the results of speed-varying ball-raceway contact loads and ball pitch angle as shown in Figs. 5 and 6. By reviewing the

Fig. 6. The comparison curves of speed-varying pitch angle between the proposed model and the Jones' model.

Fig. 7. The comparison results of the ball-raceway contact angles under the given displacement vector **d** = {10 um, 15 um, 0, 0, 0}.

Fig. 8. The comparison results of the ball-raceway contact loads under the given displacement vector **d** = {10 um, 15 um, 0, 0, 0}.

application conditions and scopes of two different raceway control hypotheses: The IRC Hypothesis is more applicable for ball bearing at low speed range and ORC hypothesis is more applicable for ball bearing at high speed range. It is indicated that the proposed model without raceway control hypothesis has a higher applicability and rationality compared to Jones' model with different raceway control hypotheses.

In order to further validate the correctness of the proposed model, the results of ball-raceway contact angles and loads of ball bearing under the given displacement vector $d = \{10 \text{ um},\}$ 15 um, 0, 0, 0} are shown in Figs. 7 and 8. It can be found that the results of the proposed model show a good agreement with those of Jones' model with two different raceway control hypotheses.

Based on the proposed model, the results of the ball-

Fig. 9. The results of the ball-raceway contact angles against the axial loads of ball bearing under different speeds.

Fig. 10. The results of the ball-raceway contact loads against the axial loads of ball bearing under different speeds.

raceway contact angles and loads varying with the axial loads for ball bearing under different speeds are given in Figs. 9 and 10. As shown in Fig. 9, for ball bearing under the constant axial load, with the increase of rotating speed, the ball-inner raceway contact angle *αⁱ* increases and ball-outer raceway contact angle *α^o* decreases. In addition, for ball bearing under the constant rotating speed, with the increase of axial load, the ballinner raceway contact angle *αⁱ* decreases and ball-outer raceway contact angleα_c increases. It means that axial load can effectively reduce the difference between contact angles *αⁱ* and *α^o* caused by the ball inertia forces.

Besides, it can be seen from Fig. 10, for ball bearing under the constant axial load, with the increase of rotating speed, the ball-outer raceway contact load *Qo* increases and ball-inner raceway contact load *Qi* basically remains unchanged. In addition, for ball bearing at the high speed range, with the increase of axial load, the ball-outer raceway contact load *Qo* first decreases and then increases and ball-outer raceway contact load *Qⁱ* increases linearly.

Fig. 11. The contact angles distribution of ball bearing under different combined loads conditions.

Fig. 12. The contact loads distribution of ball bearing under different combined loads conditions.

Different from the above analysis of ball bearing under the pure axial load, the studies of ball bearing under different combined load conditions and 10000 rpm have been conducted and the relevant results have been presented in Figs. 11 and 12. Without loss of generality, it is assumed that ball bearing is subjected to the constant axial load (*F^x* = 2000 N) and radial load (*F^y* = 500 N), then the coupling mechanism of radial and moment loads is studied by changing the size of moment load *M^z* . It can be seen from Figs. 11 and 12, when the moment load is equal to zero, the ball-raceway contact angles and loads at different angular positions present a distinct uneven distribution that is harmful for the fatigue life and dynamic performance of ball bearing. As a certain amount of reverse moment load (*M^z* = -30 Nm) applied, the ball-raceway contact angles and loads at different angular positions tend to be equal, it indicates that the uneven load distribution caused by the radial load can be improved by the moment load with a certain [7] direction and size. However, as the moment load further increases (*M^z* = -50 Nm), the distribution states of ball-raceway contact angles and loads are deteriorated again. Above all, for ball bearing subjected to the radial load, a proper moment load with the certain direction and size can be used to improve the load distribution and service performance of ball bearing. Meanwhile, the above analysis also gives a good explanation

of the action mechanism for the non-uniform preload mechanism and provides a theoretical guidance for the selection of the size and distribution scheme of non-uniform preload [17-19].

4. Conclusions

Based on the various contact angles and the hybrid theory without the raceway control hypothesis, this paper presents a new efficient and accurate algorithm for the performance prediction of ball bearing under the combined axial, radial and moment loads. In the proposed model, the triangular geometric theorem and vector diagram method are used in the force analysis of local ball instead of the orthogonal decomposition method, thus effectively reducing the computation load of bearing model. Then, the contact angles and loads of ball bearing under different operation conditions are calculated, and partial results are compared with the Jones' model with two different raceway control hypotheses (IRC/ORC) to verify the correctness of the present model. Besides, the results show that the axial load can effectively reduce the difference between ballraceway contact angles α_i and α_o , and a proper moment load can be used to improve the load distribution and service performance of ball bearing under the action radial load.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (51635010) and Henan Key Laboratory of High-performance Bearings, Luo Yang 471003, China (2016 ZCKF01).

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