

Frequency-domain approach for the parametric identification of structures with modal interference†

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Abstract

This study aims to improve the accuracy of the parametric estimation of systems with modal interference in the frequency domain. The theory of modal identification states that the frequency response function can be expressed as a rational function form by using the curve fitting technique, and the modal parameters can then be estimated from rational fractional coefficients. The conventional common denominator model only indicates the frequency response function of a single-degree-of-freedom system; thus, it cannot acquire the mode shape information. In this paper, we propose the matrix-fractional coefficient model constructed by the frequency response functions of a multiple-degree-of-freedom system for modal identification. To avoid the phenomenon of omitted modes caused by the distortion from modal interference among the vibration modes of systems during modal estimation, we use a system model with higher-order matrixfractional coefficients, but fictitious modes may be caused by numerical computation. Structural and fictitious modes can be effectively separated by using a different-order constructed stabilization diagram. Modal identification can be implemented by solving the eigenproblem of the companion matrix yielded from least-square estimation. Numerical simulations, including a full model of sedan and onehalf railway vehicle in the form of a linear 2D model, as well as the experimental testing of an actual plate, confirm the validity and robustness of the proposed parametric-estimation method for a system with modal interference under noisy conditions.

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Keywords: Parametric identification; Modal interference; Frequency response function; Matrix fraction; Stabilization diagram

1. Introduction

In the system identification of structures, the physical parameter of the stiffness matrix may not be identifiable due to the practical limitation on the number of measurement channels. However, the modal estimation of a linear system is generally implemented from input and output data provided that the system (or the model) is controllable and observable. During modal identification, the content of interference among structural modes often affects the accuracy of modal estimation [1, 2]. Modal interference caused by the close frequency and high damping ratio means that the vibration energy of each mode of the system may overlap with other modes in a certain frequency range. The serious problems of modal interference may lead to difficulties in modal estimation, especially for identifying damping ratios. Accordingly, effective techniques for modal identification must be developed under the distortion from severe modal interference among the vibration modes of systems.

In the past, among many modal estimation techniques, the

frequency-domain methods deal with the frequency response functions of a structure under consideration from which modal parameters are estimated. These methods have been used extensively because the frequency response functions are readily available from input and output data [3]. Once the frequency response functions have been obtained, we can implement the modal estimation of a structural system. Fast Fourier transform has been extensively applied to the vibration testing of structures [4]; then, modal estimation in the frequency domain can be developed through frequency response function and spectrum analysis. Spitznogle and Quazi [5] proposed a complex exponential algorithm from the timelimited output data only. In 1981, by using a squared output matric-degace-of-Irreadom system; thus, it cannot acquire the mode

thic-int model constructed by the frequency response functions of a

the phenomenon of omitted modes caused by the distortion from

the phenomenon of omit functions [6], the least-square complex exponential (LSCE) algorithm was proposed to yield the global estimation of residues and poles. A poly-reference version of the LSCE method, that is, poly-reference complex exponential (PRCE) method [7], was subsequently proposed to implement modal estimation when one of the modes may not be present in structural vibration data. The closely spaced modes (even repeated modes with the same eigenvalues) are resolved using the PRCE method, but the determination of the proper

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model order and modal properties remains subjective through the construction of the stabilization diagram. Richardson and Formenti [8] accomplished a parametric identification from frequency response measurements in the form of rational fraction polynomials by using $\Omega_k = (j\omega)^k$ as a basis func- quency resp tion in conjunction with an orthogonal polynomial function. This function may produce ill-conditioned problems in system identification when using the excessive order of basis function $\Omega_k = (j\omega)^k$. Selecting the orthogonal poly *C.-S. Lin/Journal of Mechanical Science and Technology 33 (9) (2019) 40*
 er and modal properties remains subjective through
 2. Poly-reference least-

[8] accomplished a parametric identification from
 response mea function in parametric estimation can not only reduce the ill conditioned problems but also increase relatively many calculations. In 2001, Auweraer et al. [9] proposed a fast stabilizing parametric estimation method in the frequency domain (LSCF) by using the frequency response function $C.S. Lin/Journal of Mechanical Science and Technology 33 (9) (2019) 4081$

model order and modal properties remains subjective through

the construction of the stabilization diagram. Richardson and

Formenti [8] accomplished a parametric identification from

free Guillanume et al. [10] introduced the concept of matrix fraction description to extend LSCF for a poly-reference case and proposed the poly-reference least-square complex frequency-
sponse function matrix $[H(\omega)]$ with symmetric form as domain (PolyMAX) method. The main advantages of the PolyMAX method are its computation speed and clear stabilization diagrams [11] even with highly noise-contaminated measurement data. However, this method may yield poor tion between $[H(\omega)]$ and the matrix rational mathematical estimates in damping ratios especially for a system with heavy damping and insufficient modes to be completely ex cited under noisy conditions. In addition, by using the stabilization diagram in conjunction with the PolyMAX method, the accuracy of the identification results of structural modes is relatively consistent due to the sufficient order of the model to be estimated; therefore, the system and fictitious modes model of the frequency response function matrix $[H(\omega)]$ is a
can be effectively separated [12]. In recent years, the applica-
right matrix-fraction model as $[H(\omega)]$ can be effectively separated [12]. In recent years, the application of the PolyMAX method for modal estimation has been where $\left[A(\omega) \right]$ and $\left[B(\omega) \right]$ are the common denominator extensively considered [13] and investigated [14]. This method has been adopted for experimental modal analysis and parametric estimation in flight testing of large-scale flutter analysis [15] and can effectively identify the damping ratios of offshore wind turbine on a monopole foundation [16] and estimate the modal parameters of transformer coils [17] in the electric power system. Furthermore, the PolyMAX method has been extended to estimate parameters of a localized frame and implement the damage detection and assessment of large-scale structures [18].

In this paper, we propose the matrix-fractional coefficient model constructed by the frequency response functions of a multiple-degree-of-freedom (MDOF) system to perform the parametric estimation of structures with modal interference. By introducing a higher-order system model in conjunction with the different-order constructed stabilization diagram during modal estimation, identification results are sorted as either structural or fictitious parameters caused by numerical computation. Thus, we can further determine the number of structural modes to be identified. Additionally, the modal parameters of a system, including natural frequencies, damping ratios, and mode shapes, can be obtained by directly solving the eisquare estimation.

2. Poly-reference least-square complex frequency-domain method

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ion diagram. Richardson and
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 a a ba The theory of structural dynamics indicates that the frequency response function can be expressed as a rational function form. Through the curve fitting technique, the response data can be expressed in rational fraction form, and the modal parameters can be obtained from rational fractional coefficients. The conventional frequency-domain method using the common denominator model only indicates the single-degree of-freedom (SDOF) frequency response function; thus, it can not acquire the complete modal information. In this study, we use the matrix-fractional coefficients model constructed by MDOF frequency response functions and introduce this model in the estimation procedures of the PolyMAX method for the system identification of structures with modal interference. **2. Poly-reference least-square complex frequency-domain method**
The theory of structural dynamics indicates that the fre-
quency response function can be expressed as a rational function form, Through the curve fitting t The mooty or stunctural dynamics inferances that the re-
nency response function can be expressed as a rational func-
tion form. Through the curve fitting technique, the response
data can be expressed in rational fraction

The PolyMAX method [10] is based on a frequency reprimary data containing the FRFs between all inputs and outputs. The coefficients of numerator and denominator matrix polynomials can be identified through the least-square estimamodel. The modal parameters of a system can then be estimated from the coefficients of denominator matrix polynomials. The higher the constructed order mathematical model, the more complete the modal information that can be obtained. The sensitivity of polynomial coefficients is, however, affected by high-order polynomial curve fitting. A mathematical of-rrecosom (SDOP) rrequency esponse tunction; thus, tr can-
not acquire the omplete modal information. In this study, we
use the matrix-fractional coefficients model constructed by
MDOF frequency response functions and in not acquire in complete model and information. Im ins study, we
use the matrix-fractional coefficients model constructed by
MDOF frequency response functions and introduce this model
in the estimation procedures of the Po and numerator polynomials between the output and input degrees of freedom (DOFs). Any row in the frequency response system inentimentation or structures win modial interterence.
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sponse function matrix $[H(\omega)]$ with symmetric form as
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 $[\mathcal{H}(\omega)]$ and $[\mathcal{B}(\omega)]$ are the common denominator
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 by high-order polynomial curve fitting. A mathematical
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 $[A(\omega)]$ and $[B(\omega)]$ are the common denominator
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ht matrix-fraction model as $[H(\omega)] = B(\omega)$ is

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\underline{H}_j(\omega) = \underline{B}_j(\omega) \Big[A(\omega) \Big]^{-1}, \tag{1}
$$

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\underline{H}_{j}(\omega) = \underline{B}_{j}(\omega) \Big[A(\omega) \Big]^{-1},
$$
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$$
\text{there } j = 1, 2 \cdots n \text{ . The denominator coefficient polynomial}
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H_{j}(\omega) = \frac{1}{2} \left[A(\omega) \right]^{-1},
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\text{there } j = 1, 2 \cdots n \text{ . The denominator coefficient polynomial}
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$$
\Delta_{j}(\omega) = \sum_{k=0}^{m} \left[\Delta_{k} \right] \Omega_{k}(\omega),
$$
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$$
\text{there } \Omega_{k}(\omega) = e^{-i\omega_{k}T_{s}} \text{ and } T_{s} \text{ is the sampling period. } \underline{B}_{jk}
$$

genproblem of the companion matrix yielded from least-
order of the mathematical model, and $\Omega_k(\omega)$ is the polynowhere $\left[A(\omega) \right]$ and $\left[B(\omega) \right]$ are the common denominator
and numerator polynomials between the output and input de-
grees of freedom (DOFs). Any row in the frequency response
function matrix $\left[H(\omega) \right]$ can be expr $k(\omega) = e^{-i\omega_k T_s}$ and T_s is the sampling period. β_{jk} T_a (*x*(*a*)] and $[B(\omega)]$ are the common denominator
 $[A(\omega)]$ and $[B(\omega)]$ are the common denominator

nerator polynomials between the output and input de-

freedom (DOFs). Any row in the frequency response

matrix $[H(\omega)]$ ca and numerator polynomials between the output and input de-
grees of freedom (DOFs). Any row in the frequency response
function matrix $[H(\omega)]$ can be expressed as follows:
 $H_j(\omega) = \underline{B}_j(\omega)[A(\omega)]^{-1}$, (1)
where $j = 1, 2...n$. Th order of the numerator polynomial is, in general, not the same as that of the denominator polynomial. *m* is the polynomial $H_1(\omega) = \underline{B}_j(\omega) \left[A(\omega) \right]^{-1}$, (1)
where $j = 1, 2...n$. The denominator coefficient polynomial $\left[A(\omega) \right]$ and numerator coefficient polynomial $B_j(\omega)$ are
respectively defined as
 $B_j(\omega) = \sum_{k=0}^m \underline{B}_{jk} \Omega_k(\omega)$, (2)
 $\left[$ mial basic function, i.e., a frequency-domain model is derived

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from a discrete-time model. $[H(\omega)]$ can be written for all error $e^{LS}(\beta,\alpha)$ in least squares as follows:
values of the frequency axis of the values of the frequency axis of the FRF data. The unknown polynomial coefficients for the numerator vector and denominator matrix polynomials are then found using the least-square method after linearization. The constructed numerator and denominator matrix polynomial model can be viewed as a Given the complex numbers in conjugate pairs by solving function of β , α , and arbitrary ω . The prediction errors the roots of denominator coefficient polynom function of β _i, α , and arbitrary ω _i. The prediction errors t C-S. Lin / Journal of Mechanical Science and Technology 33 (9) (2019)

from a discrete-time model. $[H(\omega)]$ can be written for all error $t^{LS}(\beta, \alpha)$ in la

values of the frequency axis of the FRF data. The unknown

polynom matrix polynomial model $\underline{B}_{j}(\beta_{j}, \omega_{i})$ $[A(\alpha, \omega_{i})]^{-1}$ and fre- considered, and t C.-S. *Lin* / Journal of Mechanical Science and Technology 33 (9) (2019) 4081-4091

from a discrete-time model. [*H*(co]] can be written for all error $t^{18}(\beta, \alpha)$ in least squares as follows:

values of the frequency ax the numerator vector and denomi-

the numerator vector and denomi-

The constructed numerator and

omial model can be viewed as a

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m a discrete-time model. [*H*(ω)] can be written for all

tures of the frequency axis of the FRF data. The unknown

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 $\begin{aligned}\n &\text{dots} \quad \text{for all} \quad \text{$ values of the frequency axis of the FRF data. The unknown

polynomial coefficients for the numerator vector and denomi-

nator matrix polynomials are then found is one because

method after linearization. The constructed polynomial coefficients for the numerator vector and denomi-
polynomial coefficients for the numerator vector and denomi-
nator matrix polynomials are then found using the least-square
method after linearization. The cons *j j i* ^e ^b ^a ^w as follows: ^ˆ nod after linearization. The constructed numerator and

winniator matrix polynomial model can be viewed as a

tion of β , α , and arbitrary ω . The prediction errors
 β , α , ω) between the constructed numera *j j i j j i j i B H A ⁱ* ^e ^b ^a ^w ^b ^w ^w ^a ^w = -é ù ë û . (4)

$$
\underline{\varepsilon}_{j}\left(\underline{\beta}_{j},\alpha,\omega_{i}\right)=\underline{B}_{j}\left(\underline{\beta}_{j},\omega_{i}\right)\left[A(\alpha,\omega_{i})\right]^{-1}-\underline{\hat{H}}_{j}(\omega_{i}).
$$
\n(3)

Directly solving Eq. (3) in the form of simultaneous nonlin ear equations may be difficult. To obtain the linearization of

$$
\underline{\varepsilon}_{j}^{new}(\underline{\beta}_{j},\alpha,\omega_{i})=B_{j}(\underline{\beta}_{j},\omega_{i})-\underline{\hat{H}}_{j}(\omega_{i})[A(\alpha,\omega_{i})].
$$
\n(4)

The prediction error matrix consisting of the prediction- error vectors associated with all values of ω _i of the frequency axis of the FRF data is constructed as follows:

$$
\mathcal{E}_{J}(\underline{\beta},\alpha,\omega_{i}) = \underline{B}_{J}(\underline{\beta},\omega_{i}) \Big[A(\alpha,\omega_{i}) \Big]^{-1} - \hat{H}_{J}(\omega_{i}).
$$
\n
$$
\mathcal{E}_{J}(\underline{\beta},\alpha,\omega_{i}) = \underline{B}_{J}(\underline{\beta},\omega_{i}) \Big[A(\alpha,\omega_{i}) \Big]^{-1} - \hat{H}_{J}(\omega_{i}).
$$
\nDirectly solving Eq. (3) in the form of simultaneous nonlinear equations may be difficult. To obtain the linearization of Eq. (3), we postmultiply Eq. (3) by $[A(\alpha,\omega_{i})]$ and redefine the prediction errors $\underline{\mathcal{E}}_{j}^{new}(\underline{\beta},\alpha,\omega_{i})$ as follows:
\n
$$
\underline{\mathcal{E}}_{j}^{new}(\underline{\beta},\alpha,\omega_{i}) = B_{J}(\underline{\beta},\omega_{i}) - \hat{H}_{J}(\omega_{i}) [A(\alpha,\omega_{i})].
$$
\n
$$
\mathcal{E}_{j}^{new}(\underline{\beta},\alpha,\omega_{i}) = \frac{\sum_{i=1}^{n}(\alpha_{i})\sum_{i=1}^{n}(\alpha_{i}) \Big[A(\alpha,\omega_{i}) \Big] \cdot \Big[A(\alpha,\omega_{i}) \Big] \cdot \Big[B(\alpha_{j}) \Big]^{-1}}{\Big[B(\alpha_{j}) \Big[B(\alpha_{j}) \Big]^{-1}} = \begin{bmatrix} \text{Re}(J^{H}J) \end{bmatrix} = \begin{bmatrix} \text{Re}(J^{H}J) \end{bmatrix} = \begin{bmatrix} \text{Re}(J^{H}J) \end{bmatrix}
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\nThe prediction error matrix consisting of the prediction- error vectors associated with all values of ω_{i} of the frequency axis of the FRF data is constructed as follows:
\naxis of the FRF data is constructed as follows:
\n
$$
[E_{J}(\beta, \alpha, \omega_{i})] = \begin{bmatrix} \underline{\mathcal{E}}_{J}^{new}(\underline{\beta}, \alpha, \omega_{i}) \end{bmatrix} = \Big[X_{J} \Big[Y_{J} \Big] \Big[\begin{bmatrix} \beta_{J} \end{bmatrix},
$$
\n
$$
\begin{bmatrix} \text{Im} \underline{\mathcal{E}}_{J}(\alpha) \end{bmatrix} \text{ Then, we will select the value of the function $\mathcal{E}_{J}(\alpha, \beta, \alpha) = \frac{\sum_{i=$
$$

$$
\int_{\mathcal{E}_{\rho}^{\text{new}}(\beta_{\rho},\alpha,\omega_{\rho})=B_{\rho}(\beta_{\rho},\omega_{\rho})-\hat{H}_{\rho}(\omega_{\rho})[A(\alpha,\omega_{\rho})].
$$
\n
$$
\int_{\mathcal{E}_{\rho}^{\text{new}}(\beta_{\rho},\alpha,\omega_{\rho})}^{\text{new}}=B_{\rho}(\beta_{\rho},\omega_{\rho})-\hat{H}_{\rho}(\omega_{\rho})[A(\alpha,\omega_{\rho})].
$$
\n
$$
\int_{\mathcal{E}_{\rho}^{\text{new}}(\beta_{\rho},\alpha,\alpha_{\rho})}^{\text{new}}=B_{\rho}(\beta_{\rho},\omega_{\rho})+\hat{H}_{\rho}(\omega_{\rho})[A(\alpha,\omega_{\rho})]
$$
\n
$$
= \int_{\mathcal{E}_{\rho}^{\text{new}}(\beta_{\rho},\alpha,\alpha_{\rho})}^{\text{new}}=B_{\rho}(\beta_{\rho},\alpha_{\rho},\alpha_{\rho})+B_{\rho}(\alpha_{\rho},\alpha_{\
$$

where \otimes denotes the Kronecker product, which is an operation on two matrices of arbitrary size resulting in a block matrix. The Kronecker product is a generalization of the outer product from vectors to matrices and produces the matrix of the tensor product with respect to a standard choice of basis function. To obtain the optimum solution of α and β through the least-square method, we define the prediction

d Technology 33 (9) (2019) 4081~4091 4083
\nerror
$$
\ell^{LS}(\beta, \alpha)
$$
 in least squares as follows:
\n
$$
\ell^{LS}(\beta, \alpha) = \sum_{j=1}^{n} tr \left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^N \left[E_j(\underline{\beta}_j, \alpha) \right] \right\}.
$$
\n(7)
\nGiven the complex numbers in conjugate pairs by solving

nology 33 (9) (2019) 4081~4091 4083
 $\ell^{LS}(\beta, \alpha)$ in least squares as follows:
 $(\beta, \alpha) = \sum_{j=1}^{n} tr \{ [E_j(\underline{\beta}_j, \alpha)]^H [E_j(\underline{\beta}_j, \alpha)] \}$. (7)

en the complex numbers in conjugate pairs by solving

ots of denominator coeffi *p*¹⁵ (β , α) in least squares as follows:
 β , α) = $\sum_{j=1}^{n} tr \left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^n \left[E_j(\underline{\beta}_j, \alpha) \right] \right\}$. (7)

in the complex numbers in conjugate pairs by solving

that β , and $\left[\alpha \right]$ are constra Example 19 $\ell^{LS}(\beta, \alpha)$ in least squares as follows:
 $\ell^{LS}(\beta, \alpha) = \sum_{j=1}^{n} tr \Big\{ \Big[E_j(\underline{\beta}_j, \alpha) \Big]^n \Big[E_j(\underline{\beta}_j, \alpha) \Big] \Big\}.$ (7)

The single of the complex numbers in conjugate pairs by solving

roots of denominator coef Given the complex numbers in conjugate pairs by solving *d Technology 33 (9) (2019) 4081-4091* 4083

error $t^{LS}(\beta, \alpha)$ in least squares as follows:
 $t^{LS}(\beta, \alpha) = \sum_{j=1}^{n} tr \left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^n \left[E_j(\underline{\beta}_j, \alpha) \right] \right\}.$ (7)

Given the complex numbers in conjugate pairs by solv assume that β_i and $\lceil \alpha \rceil$ are constrained to real-valued co-2019) 4081-4091

in least squares as follows:
 $r\left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^u \left[E_j(\underline{\beta}_j, \alpha) \right] \right\}$. (7)

plex numbers in conjugate pairs by solving

minator coefficient polynomial $\left[A(\omega) \right]$, we

and $\left[\alpha \right]$ are constra efficients and matrix. Only the real part in Eq. (7) would be d Technology 33 (9) (2019) 4081-4091

error $t^{LS}(\beta, \alpha)$ in least squares as follows:
 $t^{LS}(\beta, \alpha) = \sum_{j=1}^{n} tr \Biggl\{ \Bigl[E_j(\beta_j, \alpha) \Bigr]^n \Bigl[E_j(\beta_j, \alpha) \Bigr] \Biggr\}.$ (7)

Given the complex numbers in conjugate pairs by solving

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 $\int f^{LS}(B, \alpha) = \sum_{j=1}^{n} tr \Biggl\{ \Bigl[E_j(\underline{B}_j, \alpha) \Bigr]^H \Bigl[E_j(\underline{B}_j, \alpha) \Bigr] \Biggr\}.$ (7)

iven the complex numbers in conjugate pairs by solving

iven the complex numbers in conjugate pair blogy 33 (9) (2019) 4081-4091
 $\ell^{1S}(\beta, \alpha)$ in least squares as follows:
 β, α = $\sum_{j=1}^{n} tr \{ [E_j(\beta_j, \alpha)]^N [E_j(\beta_j, \alpha)] \}$. (7)

en the complex numbers in conjugate pairs by solving

that β_j and $[\alpha]$ are constraine 3 (9) (2019) 4081-4091 4083

(a) in least squares as follows:
 $= \sum_{j=1}^{n} tr \left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^n \left[E_j(\underline{\beta}_j, \alpha) \right] \right\}.$ (7)

complex numbers in conjugate pairs by solving

denominator coefficient polynomial $[A(\omega)]$, we
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in least squares as follows:
 $\left\{ tr \left\{ \left[E_j(\underline{\beta}_j, \alpha) \right]^n \right[E_j(\underline{\beta}_j, \alpha) \right] \right\}.$ (7)

mplex numbers in conjugate pairs by solving

cominator coefficient polynomial $[A(\omega)]$, we

and $[\alpha]$ are constr

$$
\ell^{LS}(\beta,\alpha) = \text{tr}\left\{ \left[\begin{bmatrix} \beta \end{bmatrix}^\top \begin{bmatrix} \alpha \end{bmatrix}^\top \right] \begin{bmatrix} \text{Re}(J^H J) \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \beta \end{bmatrix} \\ \begin{bmatrix} \alpha \end{bmatrix} \end{bmatrix} \right\},\tag{8}
$$

where

$$
\begin{aligned}\n\text{where } \text{the number, more order, } \{1, 0, 0\} \text{ and } \text{the number of matrix, } \{1, 0, 0\} \text{ are } \text{the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ and } \{1, 0\} \text{ is the number of matrix, } \{1, 0, 0\} \text{ are } \{1, 0, 0\} \text{ and } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{ are the number of matrix, } \{1, 0, 0\} \text{
$$

and

$$
\begin{bmatrix} R_j \end{bmatrix} = \begin{bmatrix} \text{Re}(X_j^H X_j) \end{bmatrix} \n\alpha \end{bmatrix}, \qquad \begin{bmatrix} S_j \end{bmatrix} = \begin{bmatrix} \text{Re}(X_j^H Y_j) \end{bmatrix} ,
$$
\n
$$
\begin{bmatrix} T_j \end{bmatrix} = \begin{bmatrix} \text{Re}(Y_j^H Y_j) \end{bmatrix}
$$
\n(10)

Then, we will select β_j and $[\alpha]$ such that the measure
of fit $\ell^{LS}(\beta,\alpha)$ is minimized. $\ell^{LS}(\beta,\alpha)$ is partially differequated to zero:

$$
A(\alpha, \omega_1)
$$
 = $\begin{bmatrix} 4 & \mathbf{Re}(J^H J) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & [R_n] & [S_n] \\ 0 & 0 & \cdots & [R_n] & [S_n] \end{bmatrix}$ (9)
g of the prediction-er-
allows:
and
 $\begin{bmatrix} R_i \end{bmatrix} = [\mathbf{Re}(X_i^H X_i)]$
 $\begin{bmatrix} S_i \end{bmatrix} = [\mathbf{Re}(X_i^H Y_i)]$,
 $\begin{bmatrix} S_i \end{bmatrix} = [\mathbf{Re}(X_i^H Y_i)]$ (10)
 $\begin{bmatrix} T_i \end{bmatrix} = [\mathbf{Re}(Y_i^H Y_i)]$
(5) Then, we will select β_i and $\begin{bmatrix} \alpha_i \end{bmatrix}$ such that the measure
of fit $I^{LS}(B, \alpha)$ is minimized, $I^{TS}(B, \alpha)$ is partially different
entiated with respect to β_i and $\begin{bmatrix} \alpha \end{bmatrix}$, and the result is
equated to zero:
 $\frac{\partial I^{NS}(B, \alpha)}{\partial \beta} = 2(\begin{bmatrix} R_i \end{bmatrix} \underline{B} + [\underline{S}_i] \underline{I} \alpha \underline{I} = 0$,
 $\hat{H}_i(\omega_i)$
 $\hat{H}_i(\omega_i)$
 $\hat{H}_i(\omega_i)$
 $\hat{H}_i(\omega_{N_i})$,
Then normal equation is obtained from Eq. (11) as follows:
uct, which is an opera-
realization of the outer
produces the matrix of
 $\begin{bmatrix} 2\sum_{j=1}^{n} (T_j) - [S_j]^T [R_j]^{-1} [S_j] \end{bmatrix} \underline{I} \alpha \underline{I} = 0$. (12)
resulting in a block man-
ralization of the outer
undard choice of basis (6) into Eq. (10). To avoid a relatively high polynomial order
and reduce the matrix of
that choice of basis (6) into Eq. (10). To avoid a relatively high polynomial order
defined the prediction
and β_i to be chosen for obtaining numerous zero coefficients of poly-
define the prediction
and β_i to be chosen for obtaining numerous zero coefficients of poly-

$$
\left\{2\sum_{j=1}^{n}\left(\left[T_{j}\right]-\left[S_{j}\right]\right)^{T}\left[R_{j}\right]\right\}^{T}\left[S_{j}\right]\right\}[\alpha]=0.
$$
\n(12)

(6) into Eq. (10). To avoid a relatively high polynomial order to be chosen for obtaining numerous zero coefficients of polynomials, we set $\left[\alpha_m\right] = \left[I\right]$, where $\left[\alpha_m\right]$ is the highest polynomial order of the denominator matrix polynomial model.

To simplify the procedure of the PolyMAX method, we directly estimate the modal parameters of a structure by solving the eigenvalue problem associated with the companion matrix constructed by denominator polynomial coefficients. On the basis of the property of companion matrix, modal identification can be implemented by directly solving the eigenproblem of the companion matrix instead of solving the coefficients of the numerator and denominator matrix polynomials in rational 4084
 C.S. Lin / Journal of Mechanical Science and Technology 33 (9) (2019) 4081-4091

To simplify the procedure of the PolyMAX method, we di-

response function (and the property estimate the modal parameters of a struc the relatively many computations required in the conventional PolyMAX method. After deriving the denominator coefficient, the poles (indicating the information of natural frequencies and damping ratios) and mode shape vectors of a system are directly related to the eigenvalues and eigenvectors of their companion matrix. One can derive the following: order of the denominator matrix polynomial model.

structural modes to be identified, the phase of the fit

implify the procedure of the PolyMAX method, we di-

response function will be used in modal identificat

structu implify the procedure of the PolyMAX method, we di-

expirime implify the procedure of the PolyMAX method, we di-

expirime the modal parameters of a structure by solving basis of the theory of structured tymes, the phase envalue problem associated with the companion matrix

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ticted by denominator polynomial coefficients. On the

solid denoting the approblem antix, modal identifica-

when natural frequencies

o Let by denominator polymomial coetinesies. On the SDU's system will vary instantaneously from the inference of a structure are equal to the implemented by directly solving the eigenproblem boding frequencies of a structur mial order of the denominator matrix polynomial model.

in structural modes to be identified, the phase of the free free free to the PolyMAX method, we di-

in separate the PolyMAX method, we di-

in separate the polymany mial order of the denominator matrix polynomial model. Structural modes to be identified, the phase of the from infilial the procedure of the PolyMAX method, we di-
notification will be used in modal dientification of the simplify the procedure of the PolyMAX method, we di-
esponse function will be used in modal identification
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is of the property of companion matrix, modal iden otation in action
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implify the procedure of the PolyMAX method, we di-

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implify the procedure of the PolyMAX method, we di-

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\end{bmatrix}\n\begin{bmatrix}\nV\n\end{bmatrix} = \begin{bmatrix}\nV\n\end{bmatrix} \begin{bmatrix}\n\Lambda\n\end{bmatrix},
$$
\nwhere $\overline{\omega}_r$ and ξ_r denote the f of the *r*th mode of a system. The defined as ω_r is the eigenvector matrix and $\begin{bmatrix} \Lambda \end{bmatrix}$ is the eigenvalue matrix\n
$$
\omega_r = \frac{1}{2} \left[\frac{1}{2} \oint_C \left[I \right] \oint_C e^{j\omega \Delta t} \left[I \right] \oint_C e^{j\omega \Delta t} \right] \therefore \left[I \right] \oint_C e^{j\omega (m-1)\Delta t} \right]^T
$$
\n
$$
\theta_H = \tan^{-1} \left(\frac{(1 - \overline{\omega}_1^2)(-2\xi_2\overline{\omega}_2) + (\overline{\omega}_1^2)(-2\xi_2\overline{\omega}_1)(1 - \overline{\omega}_2^2)\right) = 0
$$
\n
$$
\omega = \frac{1}{2} \left[\frac{1}{2} \oint_C \left[I \right] \oint_C e^{j\omega \Delta t} \left[I \right] \oint_C e^{j\omega \Delta t} \right] \therefore \left[I
$$

incomo omno in $[10]$ is approach symmetric value of the eigenvalue matrix and planet in the eigenvalue matrix (i) and α_{μ} is the eigenvalue matrix (i) and α_{μ} is the eigenvalue matrix and $[\alpha_{\mu}]$ if α_{μ} i consisting of the diagonal element $e^{\lambda A}$. λ , 's are the complex numbers in conjugate pairs and the roots of denominator and damping ratios) and mode shape vectors of a system are closely spaced modes in the case of the two-DOF system, the

directly related to the eigenvalues and eigenvectors of their corresponding frequency of response fun depends on the order of a system to be identified, which can be estimated from the rich frequency content around the structural modes of interest through the stabilization diagram with a different order or frequency response function of the structural system. λ , 's are related to the natural frequencies ω , problem w and damping ratios ζ of the system as follows: $[-\lfloor a_0 \rfloor - \lfloor a_1 \rfloor - \lfloor a_4 \rfloor - \lfloor a_4 \rfloor - \lfloor a_5 \rfloor - \lfloor a_6 \rfloor - \lfloor a_7 \rfloor - \lfloor a_8 \rfloor - \lfloor a_7 \rfloor - \lfloor a_8 \rfloor$ * = - ± - . (14)

$$
\lambda_r, \lambda_r^* = -\xi_r \omega_r \pm j \sqrt{1 - {\xi_r}^2} \omega_r. \tag{14}
$$

The mode shape vectors of a system correspond to the eigenvectors related to the eigenvalues λ ², ²s.

3. Estimation of identified modes from the phase of frequency response function

In general, by examining the Fourier spectrum associated with each of the response channels, one can estimate the num ber of structural modes to be identified. However, such approach may lead to a distortion in the quantity estimation of identified modes due to the modal interference among the modes with relatively heavy damping and closely spaced modes. To estimate the natural frequencies and number of

ture by solving basis of the theory of structural dynamics, the phase φ_n of
manion matrix the frequency of response function $H(\vec{\omega})$ associated with an
icinesis. On the SDOF system will vary instantaneously from -90° ture by solving basis of the theory of structural dynamics, the phase φ_n of
mpanion matrix the frequency of response function $H(\bar{\omega})$ associated with an
ficients. On the SDOF system will vary instantancously from -9 erator and denominator matrix polynomials in rational

form of $Lf(\omega)$]. This approach significantly reduces However, for most MDGF systems in practic, the number of
 V Member of *V* (*N*) in supportional structural m encator and denominator matrix polynomials in rational

phase φ_n of the frequency of response function $H(\overline{\omega})$ [19

if form of [$H(\omega)$]. This approach significantly reduces: However, for most MDOF systems in practic ë û ë û ë û ë û ^ë ^L ë û ^û structural modes to be identified, the phase of the frequency of response function will be used in modal identification. On the basis of the theory of structural dynamics, the phase φ _{*H*} of the frequency of response function $H(\bar{\omega})$ associated with an SDOF system will vary instantaneously from -90° to 90° when natural frequencies of a structure are equal to the applied loading frequency. Hence, we can estimate the number of structural modes to be identified by roughly examining the phase φ _{*H*} of the frequency of response function *H*($\bar{\omega}$) [19]. However, for most MDOF systems in practice, the number of structural modes to be identified may be erroneously deter mined due to the distortion in modal-identification information among the modes with relatively heavy damping and closely spaced modes. In the case of the two-DOF system, the corresponding frequency of response function $H(\bar{\omega}_r)$ can be expressed as follows: DF systems in practice, the number of
identified may be erroneously deter-
tion in modal-identification informa-
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n the case of the two-DOF system, the
y of response function $H(\bar{\omega}_r)$ ca des to be identified, the phase of the frequency of
ction will be used in modal identification. On the
theory of structural dynamics, the phase φ_n of
y of response function $H(\overline{\omega})$ associated with an
m will vary ins be vacuated, our phase or are reducting or

will be used in modal identification. On the

voltarian dynamics, the phase φ_n of

sponse function $H(\overline{\omega})$ associated with an

1 vary instantaneously from -90° to 90° *r* **r r** *r <i>r r <i>r r <i>r <i>r r <i>r <i><i>r <i>r <i><i>r <i><i>r <i>r <i><i>r <i>r* wever, for most MDOF systems in practice, the number of

actural modes to be identified may be erroneously deter-

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n among the modes with relatively heavy dampi es to be identified, the phase of the frequency of
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function will be used in modal identification. On the
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ency of response function $H(\bar{\omega})$ associated with an
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es to be identified may be erroneously deter-

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es. In the case of the two-DOF system, the
lency of respon φ_n of the frequency of response function $H(\overline{\omega})$ [19].
ver, for most MDOF systems in practice, the number of
raral modes to be identified may be erroncously deter-
due to the distortion in modal-identification infor

$$
H(\overline{\omega}_{r}) = \sum_{r=1}^{2} \frac{1}{K_{r}} \left[\frac{1 - \overline{\omega}_{r}^{2}}{(1 - \overline{\omega}_{r}^{2})^{2} + (2\xi_{r}\overline{\omega}_{r})^{2}} + i \frac{-2\xi_{r}\overline{\omega}_{r}}{(1 - \overline{\omega}_{r}^{2})^{2} + (2\xi_{r}\overline{\omega}_{r})^{2}} \right],
$$
\n(15)
\nhere $\overline{\omega}_{r}$ and ξ_{r} denote the frequency and damping ratios
\nthe *r*th mode of a system. The phase θ_{H} of $H(\overline{\omega}_{r})$ can
\ndefined as
\n
$$
\theta_{H} = \tan^{-1} \left(\frac{(1 - \overline{\omega}_{1}^{2})(-2\xi_{2}\overline{\omega}_{2}) + (1 - \overline{\omega}_{2}^{2})(-2\xi_{1}\overline{\omega}_{1})}{(1 - \overline{\omega}_{1}^{2})(1 - \overline{\omega}_{2}^{2}) - (2\xi_{1}\overline{\omega}_{1})(2\xi_{2}\overline{\omega}_{2})} \right),
$$
\n(16)

 $\left[\alpha_{m-2}\right]^{n}$ $-\left[\alpha_{m-1}\right]^{n}$ of the *r*th mode of a system. The phase θ_{H} of $H(\bar{\omega}_{r})$ can where $\overline{\omega}_r$ and ξ_r denote the frequency and damping ratios be defined as

$$
\theta_{H} = \tan^{-1}\left(\frac{\left(1-\overline{\omega}_{1}^{2}\right)\left(-2\xi_{2}\overline{\omega}_{2}\right)+\left(1-\overline{\omega}_{2}^{2}\right)\left(-2\xi_{1}\overline{\omega}_{1}\right)}{\left(1-\overline{\omega}_{1}^{2}\right)\left(1-\overline{\omega}_{2}^{2}\right)-\left(2\xi_{1}\overline{\omega}_{1}\right)\left(2\xi_{2}\overline{\omega}_{2}\right)}\right),\qquad(16)
$$

mind due to the ustation in modal-identification information
tion among the modes with relatively heavy damping and
closely spaced modes. In the case of the two-DOF system, the
corresponding frequency of response function where $\overline{\omega}_1 = \omega/\omega_1$ and $\overline{\omega}_2 = \omega/\omega_2$, ω_2 , ω_1 , and ω_2 are the applied loading, first natural free-vibration, and second natural free-vibration frequencies, respectively. In Eq. (16), the phase θ_{H} of the frequency of response function $H(\bar{\omega}_{r})$ will vary instantaneously from -90° to 90° when either the first or second natural free-vibration frequency of a structure equals the applied loading frequency. If the system has a serious problem with modal interference induced by close (even re-(15)

where $\overline{\omega}_z$ and ξ_z denote the frequency and damping ratios

of the *r*th mode of a system. The phase θ_n of $H(\overline{\omega}_z)$ can

be defined as
 $\theta_n = \tan^{-1} \left(\frac{(1-\overline{\omega}_1^2)(-2\xi_2\overline{\omega}_2) + (1-\overline{\omega}_2^2)(-2\xi_1\overline{\omega}_$ damping [i.e., ξ \geq 10%], then we cannot relatively accurately determine the natural frequencies and number of structural modes by examining the phase θ_{H} of the frequency of response function. This case due to the distortion the modalidentification information among the modes with relatively heavy damping and closely spaced modes. The more serious the problem of modal interference is, the more distortion the information of modal estimation has.

The modal interference among modes will be considerable due to the closely coupled modes and may lead to a curvefitting problem associated with the frequency response function that involves the estimation of the appropriate model size. All MDOF curve-fitting methods assume that interference exists among all modes. The frequency response function data at any frequency is a summation of contributions from all modes. To avoid the phenomenon of omitted modes during the modal identification of systems with modal interference,

we use a higher-order model, but fictitious modes may be caused by numerical computation. The system and fictitious modes can be separated by using the different-order constructed stabilization diagram. This diagram is a powerful tool for effectively estimating the accurate number of poles, i.e., polynomial model order, for the PolyMAX method. The idea behind the stabilization diagram is to repeat the estimation process with a different polynomial model order each time. The stable poles should remain constant for all or most of the iterations, and then the polynomial model order can be determined. By using a stabilization diagram to estimate the stable poles, the curve-fitting problem involved in the modal interference can be solved through different model sizes.

4. Numerical simulations

The modal identification can be performed from the excitation and response data of a structural system under external force excitation in dynamic tests. However, obtaining the exact modal information in the practical dynamic testing of large-scale structure is difficult. Consequently, the effectiveness of the present method must be verified in advance $H_{16}(a)$ of the system showing modal interference among the structhrough the numerical simulations. To demonstrate the effectiveness of the proposed method, we consider a six-DOF chain-model system with a pair of closely spaced modes (frequency separation smaller than 0.42 rad/sec) and high damping ratios (whose values are above 10 % except for the first mode). Fig. 1 shows a schematic representation of this model. The mass matrix M , stiffness matrix K , and damping matrix C of the system are given as follows: Fical simulations

Fical dietrification can be performed from the excita-

response data of a structural system under external

response data of a structural system under external

liation in dynamic tests. However, obtai **Example 12**
 Example 12
 Codal identification can be performed from the excita-
 Codal identification can be performed from the excita-
 Consequently, the effective-
 Consequently, the effective-
 Consequently dal identification can be performed from the excita-

response data of a structural system under external

it in dynamic tests. However, obtaining the ex-

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all information in the practical dynamic testing of

la information in the practical dynamic testing of

le structure is difficult. Consequently, the effective-

the pr itation in dynamic tests. However, obtaining the ex-

la information in the practical dynamic testing of

le structure is difficult. Consequently, the effective-

the present method must be verified in advance
 $H_u(\omega)$ of

$$
M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ kg} \text{ system}
$$

\n
$$
K = 600 * \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} N/m
$$

\n
$$
K = 600 * \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} N/m
$$

\n
$$
C = 0.05M + 0.01K N \cdot \text{sec } / m.
$$

\n
$$
C = 0.05M + 0.01K N \cdot \text{sec } / m.
$$

The chain-model system has proportional damping, because the damping matrix C can be expressed as a linear combination of M and K . The simulated impulse function serves as the excitation input acting on each mass point of the system. pling period is $T = N_t \cdot \Delta t = 81.92$ s. Assuming that the system is initially at rest, the displacement responses of the system can be obtained using Newmark's method [20]. To consider the measurement noise in practice, we perform modal

Fig. 1. Schematic plot of the six-DOF chain system.

Fig. 2. Typical plot of the amplitude frequency response function tural modes except for the first mode.

identification from the simulated impulse response data contaminated with 5 % white noise.

al information in the practical dynamic testing of

le structure is difficult. Consequently, the effective-

the present method must be verified in advance

the present simulations. To demonstrate the effec-

lead on the Lear is outbure in the experiment of the system and interfering a multiple of the system with a parameterizal simulations. To demonstrate the effec-

transmost secure for the first model interferience are proposed method, Exam incursion with a pair of closely space method interferience and interfer the proposed method, we consider a six-DOF

in modes except for the inst mode.

the proposed method, we consider a six-DOF

system with a pair of closely spaced modes (fre-

identification from the simulated impulse respo s of the proposed method, we consider a six-DOF
solution from the simulated impulse response date separation smaller than 0.42 rad/sec) and high damp-
smaller dimensions of every for the first increase that is the space o ystem with a pair of closely spaced modes (fre-

identification from the simulated impulse respo

cons value are above 10 % except for the first \mathbf{M} , and displananty 5 % white noise.

Shows a schematic representation tion smaller than 0.42 rad/sec) and high damp-

sons values are above 10 % except for the first in a oriental order phenomenon of omitted modes,

shows a schematic representation of this model. In al order m in the PohyMA lose values are above 10 % except for the first

shows a schematic representation of this model.

mail order m in the PolyMAX method must

shows:

shows a schematic representation of this model.

The Muscul muscules By ex Figure 1 and the proposed method mass to eventual in avantate effect

intention and the proposed method, we consider a six-DOF

system with a pair of closely spaced modes (fre-

identification from the simulated with pair From a sinual one of closely spaced median of the sinus and the sinus or
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m with a pair of closely spaced modes (fre-
identification from the simulated impurement of M , and M , stif ith a pair of closely spaced modes (fre-

identification from the simulated impulse

leler than 0.42 rad/sec) and high damp-

arminated with 5 % white noise.

See a above 10 % except for the first indical mail order m in than 0.42 rad/sec) and high damp-

taminated with 5 % white noise.

To avoid the phenomenon of omitted mode

main cropescentition of this model.

mail order m in the PolyMAX method must n

mess matrix **K**, and damping ma- $\begin{bmatrix} 0 & 0 & 0 & -1 & 2 \end{bmatrix}$ frequency response function exists. Only upon the phase angle de). Fig. 1 shows a schematic representation of this model. man order *m* in the PolyMAX method music response that is emiss of the system are syen as follows:
 $M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 &$ $M = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \ 0 & 0 & 2 & 0 & 0 & 0 \ 0 & 0 & 0 & 1 & 0 & 0 \ 0 & 0 & 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ Eig. 2 shows the typical plots of the amplitude frequence $\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 2 &$ pling period is 81.92 s *T N t* = × D = *^t* To avoid the phenomenon of omitted modes, the polynomial order *m* in the PolyMAX method must not be less than the number of modes to be identified. However, a continuum structure theoretically has an infinite number of DOFs and modes. By examining the Fourier spectra associated with the measured vibration response histories, the important modes of a system under consideration could not be roughly found because of a distortion in the modal-estimation resulting from a system with modal interference among the vibration modes. Fig. 2 shows the typical plots of the amplitude frequency response functions of the system, wherein the serious problems of modal interference exist in most structural modes except for the first mode. The theory presented in the previous sections indicates that the phase angle diagram of frequency response function, as shown in Fig. 3, cannot be used to determine the number of structural modes influenced by serious modal interference relatively accurately. Thus, the phenomenon of omitted modes from the distortion of system order estimated by diagram of frequency response function may lead to an erroneous estimation of the modal frequencies to be identified for the case of a MDOF system with modal interference. A typimoaes. by examming the Fourer spectra associated with the
measured vibration response histories, the important modes of
a system under consideration could not be roughly found be-
cause of a distortion in the modal-estima shows at least five excited modes of this six-DOF system with modal interference. However, corresponding modal frequencies could not be accurately estimated except the first three structural modes.

Table 1 shows the results of modal estimation when selecting the polynomial order $m = 2$ in the PolyMAX method, indicating that the errors in natural frequencies are less than

Table 1. Results of the modal estimation of a six-DOF chain system with a pair of closely spaced modes and high damping ratios through the PolyMAX method with polynomial order $m = 2$ (contaminated with 5 % white noise).

Mode		Natural frequency (rad/s)		Damping ratio $(\%)$	MAC		
	Exact	PolyMAX	Error $(\%)$	Exact	PolyMAX	Error $(\%)$	
	8.72	8.79	0.81	4.65	5.42	16.65	0.91
2	19.89	19.98	0.45	10.07	11.84	17.57	0.90
3	27.63	27.68	0.17	13.91	16.34	17.50	0.92
4	31.74	31.78	0.13	15.95	18.74	17.50	0.89
5	43.13	43.23	0.23	21.62	25.20	16.54	0.98
6	43.55	43.62	0.15	21.83	25.37	16.19	0.96

Table 2. Results of the modal estimation of a six-DOF chain system with a pair of closely spaced modes and high damping ratios through the PolyMAX method with polynomial order $m = 4$ (contaminated with 5 % white noise).

Fig. 3. Typical plot of the phases associated with the frequency remodes and high damping ratios.

1 % and those in damping ratios are around 17 %. The errors of identified damping ratios are higher than those of natural frequencies, which may be due to the system response generally having lower sensitivity to damping ratios. To obtain improved estimation results of damping ratios, we further ferent polynomial-order $m=1 \sim 8$. Furthermore, the stabilizaincrease the polynomial order *m* to reach 4. Table 2 shows the corresponding results of modal estimation when selecting $m = 4$ in the PolyMAX method, indicating that the errors in damping ratios significantly reduce to less than 2 %. The identified mode shapes are compared with the exact values in Fig. 4, in which we observe good agreement with the value of the

Fig. 4. Comparison between the identified and exact mode shapes of the six-DOF system with a pair of closely spaced modes and high damping ratios.

modal assurance criterion (MAC) [21] of 0.93 on average.

In the proceeding, to avoid the phenomenon of omitted modes during the modal identification of systems with modal interference, we use a higher-order system model through PolyMAX, but fictitious modes may be caused by numerical computation. However, as the correct order of a model to be estimated is often unknown a priori, different model orders are postulated and then the "best" one is selected in accordance with a certain criterion, such as singular value analysis [19] or stabilization diagram method [11]. A stabilization diagram is used to display stable pole groups consisting of natural frequency and damping ratio pairs that exist when applying curve fitting to the data of frequency response function obtained from the measured response histories with different model sizes. This diagram is utilized to determine the number of modes to be identified in structural response data. Extra computational modes are always used with a stabilization diagram to account for the residual effects of additional "out of band modes" in the data and then are ignored in the final modal-estimation results. The system and fictitious modes can be separated by using the constructed stabilization diagram with different-polynomial order, as shown in Fig. 5. The number of structural modes to be identified is six, which is obtained from the stabilization diagram associated with the difessumate is orien unknown a priori, durient model orders are
postulated and then the "best" one is selected in accordance
with a certain criterion, such as singular value analysis [19] or
stabilization diagram method [11] tion diagram shows a relatively evident "location" of natural frequencies of the system, with no clear peak in the amplitude of frequency response function due to the modal interference caused by heavy damping.

To demonstrate the effectiveness of the present method for relatively complicated structural systems, we further consider

system with a pair of closely spaced modes and high damping ratios: (a) Overall; (b) part of frequency information.

a full model of motor vehicle [19] with two pairs of closely spaced modes (frequency separation smaller than 0.2 rad/sec), as shown in Fig. 6. A full model of a motor vehicle can be generally viewed as a seven-DOF system, which includes the bounce, pitch, and roll motions for the body of a motor vehicle, and four bounce motions for the wheels. For further complex systems, the Lagrange equation can be used to derive the equation of motion whose advantage is that only displacement and velocity enter into the energy functions and can be written in a slightly simplified form without using free-body diagrams, as well as summing forces and moments; thus, intricate kinematic calculation of acceleration is avoided [22]. A model of the sedan under consideration, in this case, is a seven-DOF Fig. 5. Typical plot of the stabilization diagram and $[H_{\alpha}(p)]$ of the

system with a pair of closely spaced model of motor vehicle [19] with two pairs of closely

and (a) Overall; (b) part of frequency information.

Spac behavior of the motor vehicle, respectively, and the others are the vertical displacements of the bounce behavior of the motor vehicle and four wheels, as shown in Fig. 6. The mass matrix a rull moode of motor ventice [19] with two parts of colored to the seven-DOF

spaced modes (frequency separation smaller than 0.2 rad/sec),

as shown in Fig. 6. A full model of a motor vehicle can be

generally viewed as is diagonal, diag $M = [m_1, m_2, m_3, m_4, m_5, m_6, m_7]$, where the sprung mass m_1 represents the corresponding body mass of \blacksquare the motor vehicle to the wheels and the unsprung mass, generally viewed as a seven-DOF system, winch mendicates the

and four bounce, pitch, and roll motions for the body of a motor vehicle,

and four bounce motions for the wheels. For further complex

systems, the Lagrange e m_4 , m_5 , m_6 , and m_7 , represents the wheel and its associated $k_1 (= k_2)$, $k_3 (= k_4)$, $k_{11} (= k_{12})$, and $k_{13} (= k_{14})$ are the front source, pincing involtation of the occupy and both and relief $\begin{bmatrix}\n-L_1k_1 & -L_1k_2 & -L_1k_3 & -L_1k_4 & -L_1k_5 & -L_1k_6 \\
-L_1k_1 & -L_1k_2 & -L_1k_3 & -L_1k_5 & -L_1k_6 & -L_1k_6 \\
-L_2k_2 & -L_2k_3 & -L_2k_4 & -L_2k_5 & -L_2k_6 & -L_2k_6 \\
-L_3k_1 &$ moments of inertia of the motor vehicle, respectively. The stiffness matrix can be obtained as

Table 3. Results of the modal estimation of a seven-DOF system of a motor vehicle through the modified PolyMAX method with polyno-

mial order $m = 2$.									
		Natural frequency (rad/s)		Damping ratio $(\%)$					
Mode	Exact	PolyMAX	Error $(\%)$	Exact	PolyMAX	Error $(\%)$	MAC		
1	5.03	5.11	1.49	1.25	1.21	2.56	0.99		
2	7.82	7.89	0.92	1.03	1.01	1.64	0.95		
3	18.48	18.50	0.13	1.19	1.18	1.11	0.93		
$\overline{4}$	73.79	70.77	4.09	3.76	3.45	8.20	0.99		
5	73.87	70.85	4.10	3.76	3.45	8.22	0.97		
6	87.93	82.93	5.68	4.45	3.96	11.10	0.99		
7	88.07	83.05	5.70	4.46	3.96	11.14	0.96		

Fig. 6. Schematic plot of the seven-DOF system of a sedan.

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where L_1 , L_2 , L_3 , and L_4 are the half of the axle track of the front and rear wheels and the distances to the front and rear axles from the center of gravity of the motor, respectively. The summation of L_3 and L_4 is the wheelbase of a motor. and rear suspension spring stiffness and the front and rear tire stiffness, respectively. Throughout this numerical study,

Fig. 7. Comparison between the identified and exact mode shapes of the seven-DOF system of a motor vehicle with two pairs of closely spaced modes.

 $I_v = 1.831 \times 10^3$ kg·m², and m₃ = I_x = 4.98 × 10² kg·m²; 1.5415 m ; *C M K* = + × 0.1 0.001 N sec/ m . The system of a motor vehicle has proportional damping because the damping matrix *C* can be expressed as a linear combination of *M* and K . Table 3 summarizes the modal-estimation results, showing that the average errors in natural frequencies are less than 5 % and those in damping ratios are less than 10 %. The identified mode shapes are also compared with the exact values in Fig. 7, in which we observe good agreement with a minimum value of MAC [21] of 0.93. The first three mode merical study, $\lceil m_1, m_2, m_3, m_4 \rceil = \lceil 1200, 850, 4125, 850, 1220 \rceil$ shapes are modal behaviors with bounce, pitch, and roll kg, and $m_4 = I_B = 1.25 \times 10^5$ kg·m²; $k_1 = k_6 = 3.0 \times 10^7$ N/m, modes, respectively, of the global motor vehicle, whereas the $k_2 = k_3 = 1.0 \times 10^6$ N/m, and $k_3 = k_4 = 6.0 \times 10^6$ N/m; $c_1 =$ last four mode shapes are modal behaviors with bounce modes of the local left front, right front, left rear, and right rear $N \cdot \sec / m$; $L = 8.53$ m. The damping matrix C that cannot wheels, respectively.

We also consider a linear 2D model of one-half of a railway vehicle excited by a simulated impulse loading. This simulated system in the numerical study, as shown in a sketch in Fig. 8, is identical to that in Ref. [19] and has the features of the system. The sampling interval is set as $\Delta t = 0.01$ s, and

Fig. 8. Simplified 2D model of one-half railway vehicle.

heavy damping (damping ratio above 10 %) and closely spaced modes (frequency separation smaller than 1.33 rad/sec).
The system is a six-DOF system with $\mathbf{u} = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6]$, havior of the car body, and others are the vertical displacements of the bounce behavior of the car body, leading (trailing) bogies, and leading (trailing) wheelsets. The mass matrix Fig. 8. Simplified 2D model of one-half railway vehicle.
Fig. 8. Simplified 2D model of one-half railway vehicle.
Fig. 8. Simplified 2D model of one-half railway vehicle.
The system is a six-DOF system with $\mathbf{u} = [u_1, u$ Where \overline{m}
 $\overline{$ is the mass moment of inertia of the rigid body B at the top of the structure. The stiffness and damping matrices can be obtained as vior of the car body, and others are the vertical displace

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diag $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_6]$, where $\mathbf{m}_4 = \mathbf{I}_B$

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one-half railway vehicle.

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 $\frac{1}{k_2}$ plified 2D model of one-half railway vehicle.
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where *L* is the horizontal distance between the center of the rigid body B and the springs/dashpots. Throughout this nu $c_6 = 0$, $c_2 = c_5 = 6.0 \times 10^3$ N·sec / m, and $c_3 = c_4 = 1.8 \times 10^4$ 0 $-k_3$ $k_3 + k_4$ $k_3L - k_4L$ $-k_4$ 0

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 0 & 0 & 0 & 0 & -k_5 & k_5 + k_6\n \end{bmatrix}$
 $\begin{bmatrix}\n c_1 + c_2 & -c_2 & 0 & 0 & 0 & 0 \\
 -c_2 & c_2 + c_3 & -c_2 & -c_5 L & -c_4 L & 0 \\
 0 & -c_3 & c_3 + c_4 & c_3 L^2 - c_4 L & -c_4 & 0 \\
 0$ be expressed as a linear combination of *M* and *K* ; thus, this one-half of a railway vehicle is a six-DOF nonproportionally damped system. The simulated impulse function serves as the excitation input acting on each mass point of $C=\begin{bmatrix} -c_2 & c_2+c_3 & -c_3 & -c_1L & 0 & 0 \\ 0 & -c_3 & c_3+c_4 & c_4L-c_4L & -c_4 & 0 \\ 0 & -c_1L & c_3L-c_4L & c_1L^2+c_4L & 0 \\ 0 & 0 & -c_4 & c_4L & c_4+c_5 & -c_5 \\ 0 & 0 & 0 & 0 & -c_5 & c_5+c_6 \end{bmatrix}$
where L is the horizontal distance between the center of the

Table 4. Results of the modal estimation of a six-DOF system of a railway vehicle through the modified PolyMAX method with polyno-

					C.-S. Lin / Journal of Mechanical Science and Technology 33 (9) (2019) 4081~4091		
	mial order $m = 4$.	Table 4. Results of the modal estimation of a six-DOF system of a railway vehicle through the modified PolyMAX method with polyno-					
	Damping ratio $(\%)$ Natural frequency (rad/s)						
Mode		Exact PolyMAX	Error $(\%)$		Exact PolyMAX	Error $(\%)$	MAC
1	17.51	17.55	0.19	4.89	4.84	0.96	0.99
$\overline{2}$	23.31	23.28	0.12	6.62	6.53	1.31	0.98
3	103.96	96.27	7.40	16.65	14.24	14.45	0.93
$\overline{4}$	121.07	109.50	9.56	18.78	15.34	18.33	0.91
5	159.31	144.61	9.23	1.74	1.26	27.59	0.87
6	160.64	145.42	9.48	1.75	1.27	27.89	0.89
		the sampling period is $T = N$, $\Delta t = 81.92$ s. Table 4 summarizes the modal-estimation results obtained from the simulated impulse response data, showing that the errors in natural frequencies and damping ratios are less than 10 % and 30 %, respectively. By solving a simplified general- ized eigenvalue problem of the equation of motion in the state-					

the sampling period is $T = N_t \cdot \Delta t = 81.92$ s.
Table 4 summarizes the modal-estimation results obtained from the simulated impulse response data, showing that the errors in natural frequencies and damping ratios are less than 10 % and 30 %, respectively. By solving a simplified generalized eigenvalue problem of the equation of motion in the statespace form of the six-DOF non-proportionally damped system, the "exact" modal damping ratios, as listed in Table 4, are the equivalent values obtained from the free-vibration analysis of the structural system with nonproportional damping. In addition, the result is in good agreement with the minimum value of MAC [21] between the identified and exact mode shapes of approximately 0.87. The modal analysis of numerical simulation models of a sedan and one-half of a railway vehicle is relatively complicated, but such analysis remains applicable for an accurate numerical simulation to confirm the validity of the proposed modal-estimation method.

To demonstrate the effectiveness of the present method for relatively practical structural systems with the experimental point of view, we considered an actual plate example. An experimental testing is conducted on a plate, which is tested under a free-free boundary condition and suspended by simple strings. A Brüel & Kjær RT Pro Photon 7.0 data acquisition system, along with PCB piezoelectric accelerometer 352B10 (with 10.3 mV/g sensitivity and 10 kHz frequency range), and a PCB impulse hammer 086C03 (with 2.25 mV/N sensitivity and 2224 N measurement range) are used to measure the response of the structure.

This experimental testing consisted of a 150 mm \times 150 mm rectangular steel plate, which is studied in modal testing condition. The thickness of the plate is 3 mm. As the sides of the structure only slightly differ (0.25-5 mm), the plate will have closely spaced modes. To confirm this assumption, a simple finite element model of the plate is created, and the approximate natural frequencies and mode shapes of the structure are identified. This structure possesses at least one pair of closely spaced (even probably repeated) modes around 1160 Hz, as shown in Fig. 9 and Table 5. Currently, an FRF-based roving hammer testing is performed to measure the actual modal properties of the plate structure. Fig. 10 shows the experiment setup of this test. Sixteen points on the plate are marked and

Fig. 9. Typical plot of the amplitude frequency response function H₂₂(ω) of the system showing at least a pair of closely spaced (even repeated) modes around 1160 Hz.

Fig. 10. Schematics of experiment setup of hammer testing for actual plate.

hammer impacts are acted through these 16 locations of the plate. For the EMA testing, a simple hammer, to which the force sensor is attached, is used to excite the structure with a steel tip.

Through this simple hammer testing, the 256 acceleration responses are extracted by a sampling frequency of 12800 Hz and recording length of approximately 0.5 s. Then, a stabilization diagram with different model orders can be constructed using the PolyMAX method, as shown in Fig. 11. The natural frequencies and damping ratios of the plate are estimated directly by solving the eigenvalue problem associated with the companion matrix constructed by denominator polynomial coefficients, as listed in Table 5. The "exact" modal frequencies and damping ratios listed in Table 5, as well as the exact mode shapes, are the equivalent values obtained by using the ibrahim time-domain (ITD) method from the impulse acceleration response data of the practical plate structure. These results show that the accuracy of the modified PolyMAX method in calculating the natural frequencies of closely spaced modes of the plate is reasonable. In general, as the structural response is less sensitive to damping ratios than to the natural frequencies, the identification errors of damping ratios, as shown in Table 5, are relatively higher, but the accuracy is acceptable.

Table 5 lists the results of MAC verification between the mode shapes identified by the modified PolyMAX method and by modal testing. In Table 5, the mode shapes of the closely spaced modes are reasonably consistent with those extracted by hammer testing. The MAC values for the mode

Table 5. Results of the modal estimation of a practical plate through the modified PolyMAX method with polynomial order $m = 100$.

		Natural frequency (Hz)			Damping ratio $(\%)$		
Mode	ITD	PolyMAX	Error $(\%)$	ITD	PolyMAX Error (%)		MAC
1	452.81	451.43	0.30	2.18	2.48	13.76	0.99
2	678.57	674.97	0.53	1.01	1.07	5.94	0.98
3	794.47	797.88	0.43	0.99	0.84	15.15	1.00
4	1156.76	1159.23	0.21	0.93	0.81	12.90	1.00
5	1167.37	1161.39	0.51	0.52	0.58	11.54	0.97
6	2015.91	2020.57	0.23	0.44	0.61	38.64	0.98
7	2055.59	2072.68	0.83	0.24	0.33	37.50	1.00
8	2315.27	2329.52	0.62	0.81	0.86	6.17	0.76
9	2542.59	2535.75	0.27	0.29	0.18	37.93	0.89
10	3466.09	3454.02	0.35	0.37	0.41	10.81	1.00
11	3889.63	3900.03	0.27	2.18	2.48	13.76	0.99

Fig. 11. Typical plot of the stabilization diagram and $H_{11}(\omega)$ of the actual plate with a pair of closely spaced modes.

shapes estimated by the ITD and modified PolyMAX methods show that each set of mode shapes is well correlated within itself. Therefore, both methods present great robustness in identifying well-correlated mode shapes for the plate.

In this study, the multi-reference identification of the Poly-MAX method is used to implement the parametric estimation $\begin{bmatrix} A(\omega) \end{bmatrix}$ of structures with modal interference. In the procedure of modal estimation, we propose a modification to replace the rational fraction in the form of conventional scalar coefficients \otimes by constructing the frequency response function matrix of the \boldsymbol{M} rational fraction of the matrix coefficients. We can thus prevent the incomplete modal information obtained from the C scalar-coefficient rational fraction from the single-frequency m response function only. A high polynomial order for the PolyMAX method can be significantly reduced when the rational fraction of the matrix coefficients is used, and further accuracy of modal estimation can be effectively performed. In addition, such approach may significantly improve the computation efficiency in that modal identification can be implemented by solving the eigenproblem of the companion matrix only. The proposed algorithm is applicable for improving the validity and accuracy of the modal estimation of the system with modal interference.

5. Conclusions

The extent of interference among structural modes may often affect the accuracy of parametric estimation during modal identification. In this paper, the matrix-fractional coefficient model constructed by MDOF frequency response functions in the modified PolyMAX method is presented to perform the system identification of structures with modal interference. By introducing a system model with higher-order matrixfractional coefficients in conjunction with the different-order constructed stabilization diagram during modal estimation, we can determine the number of modes to be identified. We can thus avoid the erroneous estimation of the modal frequencies to be identified for the case of an MDOF system with modal interference only upon the phase angle diagram of the frequency response function. Furthermore, the modal parameters of systems can be obtained by solving the eigenproblem of the companion matrix yielded from least-square estimation. Moreover, the computation required for the conventional PolyMAX method can be significantly reduced. Numerical simulations, including a full model of sedan and one-half railway vehicle in the form of a linear 2D model, as well as the experimental testing of an actual plate, confirm the validity and robustness of the proposed parametric-estimation method for a system with modal interference under noisy conditions. Example the manner of moster of modes of orelational interactions
and the erroneous estimation of the modal frequencies
be identified for the case of an MDOF system with modal
therefore colly upon the phase angle diagram final is a fill model of sedan and one-half

simulations, including a full model of sedan and one-half

railway vehicle in the form of a linear 2D model, as well as

a reading of an actual plate, confirm the validity

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Acknowledgments

This research was supported in part by the Ministry of Science and Technology of Taiwan under the Grant MOST 107- 2221-E-020-010-. The author wishes to thank anonymous reviewers for their valuable comments and suggestions in revising the paper. Frametal testing of an actual plate, confirm the valid with
the experimental testing of an actual plate, confirm the validity
and robustness of the proposed parametric-estimation method
for a system with modal interferenc

Nomenclature-

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-
- : Kronecker product
- *M* : Mass matrix
- *K* : Stiffness matrix
- *C* : Damping matrix
- *m* : Polynomial order

References

- [1] S. A. H. Kordkheili, S. H. M. Massouleh, M. J. Kokabi and H. Bahai, A modal coupling procedure to improve residual modal effects based on experimentally generated data, *Journal of Sound and Vibration*, 331 (1) (2012) 66-80.
- [2] S. A. H. Kordkheili, S. H. M. Massouleh, S. Hajirezayi and H. Bahai, Experimental identification of closely spaced modes using NExT-ERA, *Journal of Sound and Vibration*,

412 (6) (2018) 116-129.

- [3] J. O. Hougen and R. A. Walsh, Pulse testing method, *Chemical Engineering Progress*, 57 (3) (1961) 69-79.
- [4] E. O. Brigham, *The Fast Fourier Transform,* Prentice-Hall (1974).
- [5] F. R. Spitznogle and A. H. Quazi, Representation and analysis of time-limited signal using a complex exponential algorithm, *Journal of the Acoustical Society of America*, 47 (5) (1970) 1150-1155.
- [6] W. R. Smith, Least squares time-domain methods for simultaneous identification of vibration parameters from multiple free-response records, *AIAA/ASME/ASCE/AHS 22nd SDM Conference,* April (1981) 194-201.
- [7] H. Vold, J. Kundrat, G. T. Rocklin and R. Russell, A multiinput modal estimation algorithm for mini-computers, *SAE Technical Paper Series*, 91 (1) (1982) 815-821.
- [8] M. H. Richardson and D. L. Formenti, Parameter estimation from frequency response measurements using rational fraction polynomials, *Proceedings of the 1st International Modal Analysis Conference*, Orlando, Florida, November 8-10 (1982).
- [9] H. Van der Auweraer, P. Guillaume, P. Verboven and S. Vanlanduit, Application of a fast-stabilizing frequency domain parameter estimation method, *Journal of Dynamic Systems Measurement and Control - Transactions of the ASME* (2001) 651-658.
- [10] P. Guillaume, P. Verboven, S. Vanlanduit, H. Van Der Auweraer and B. Peeters, A poly-reference implementation of the least-squares complex frequency-domain estimator, *International Modal Analysis Conference (IMAC XXI)*, Kissimmee, Florida (2003).
- [11] H. Van der Auweraer and B. Peeters, Discriminating physical poles from mathematical poles in high order systems: Use and automation of the stabilization diagram, *Proceedings of the IEEE Instrumentation and Measurement Technology Conference*, Como, Italy, May (2004) 2193-2198.
- [12] B. Peeters, H. Van der Auweraer and P. Guillaume, The PolyMAX frequency-domain method: A new standard for modal parameter estimation, *Shock and Vibration*, 11 (3-4) (2004) 395-409.
- [13] X. Liu, Y. Luo, B. W. Karney, Z. Wang and L. Zhai, Virtual testing for modal and damping ratio identification of submerged structures using the PolyMAX algorithm with two-way fluid-structure interactions, *Journal of Fluids and Structures*, 54 (2015) 548-565.
- [14] P. Sitarz and B. Powalka, Modal parameters estimation using colony optimization algorithm, *Mechanical Systems and Signal Processing*, 76-77 (2016) 531-554.
- [15] T. D. Troyer, P. Guillaume and G. D. Sitter, Improved poly-reference frequency-domain modal estimators for flutter analysis, *14th IFAC Symposium on System Identification,* Newcastle, Australia (2006).
- [16] R. Shirzadeh, C. Devriendt, M. A. Bidakhvidi and P. Guillaume, Experimental and computational damping estimation of an offshore wind turbine on a monopile foundation, *Journal of Wind Engineering and Industrial Aerodynamics*, 120 (2013) 96-106.
- [17] C. Geng, F. Wang, J. Zhang and Z. Jin, Modal parameters identification of power transformer winding based on improved empirical mode decomposition method, *Electric Power Systems Research*, 108 (2014) 331-339.
- [18] W. J. Yi, Y. Zhou, S. Kunnath and B. Xu, Identification of localized frame parameters using higher natural modes, *Engineering Structures*, 30 (2008) 3082-3094.
- [19] C.-S. Lin, Parametric estimation of systems with modal interference, *Archive of Applied Mechanics*, 87 (2017) 1845- 1857.
- [20] N. M. Newmark, A method of computation for structural dynamics, *ASCE Journal of Engineering Mechanics Division*, 85 (1959) 67-74.
- [21] R. L. Allemang and D. L. Brown, A correlation coefficient for modal vector analysis, *Proceedings of the 1st International Modal Analysis Conference,* Society for Experiment Mechanics, Bethel, CT (1983) 110-116.
- [22] S. S. Rao, Coordinate coupling and principal coordinates, *Mechanical Vibrations*, 5th Ed., Person, United States (2011) 488-491.

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