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Dynamic modeling and analysis of a quad horizontal damper system for transient vibration reduction in top loading washing machine †

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Abstract

The transient horizontal vibration of top loading washing machine with large capacity has been more important because of the radius of the inner tub in the washing machine increases proportionally. In this paper, we introduce a new vibration reduction technology, namely, a quad horizontal damper system for reduction of the transient horizontal vibration of top loading washing machine. And analyzes the vibration characteristics of a top loading washing machine with a quad horizontal damper system using Newtonian mechanics. For verification and validation of the established dynamic model, the displacement of an actual top loading washing machine are obtained from six laser sensors attached at the upper part and the lower part of outer tub respectively. Optimal design was carried out to ensure sufficient strength and reliability for each component of the horizontal damper. The damping force of the horizontal damper was also optimized so that the horizontal vibration in the transient state of top loading washing machine was reduced by about 35~45 % more than the conventional top loading washing machine and no abnormal noise was generated at high spin speed.

Keywords: Top loading washing machine; Quad horizontal damper system; Damping force; Unbalance; Transient vibration

1. Introduction

Generally, two vibration reduction technologies are utilized in the top loading washing machine: A hydraulic balancer and a suspension system. Usually, a hydraulic balancer is used to reduce the steady-state horizontal vibration of a top loading washing machine during the dehydration process at maximum spin speed [1-19]. Four suspensions are located on each side of outer tub in the top loading washing machine for reducing transient vertical vibration [20-22]. However, there are several limitations of using these conventional vibration reduction technologies for top loading washing machines with a higher capacity with a limited cabinet size. A hydraulic balancer, which is an important vibration reduction technology used in conventional top loading washing machine, is filled with high-density salt water. As shown in Fig. 1, in case of unbalanced loads in the washing machine, vibration is generated during the dehydration process at high speed. The hydraulic balancer is used to reduce the vibration due to the unbalanced loads. The location of salt water in the hydraulic balancer is the same phase with the unbalanced load at a low spinning RPM during initial dehydration process, so that the vibration of the washing machine tends to increase ($\omega << \omega_n$ in Fig. 2). As the rotational speed increases gradually, the salt water moves toward the location of the unbalanced loads. In the vertical transient vibrational resonance ($\omega = \omega_n$ in Fig. 2), which is determined by the suspension spring and the total mass of the driving part of the washing machine, the salt water in the hydraulic balancer has a phase difference of 90° with the unbalanced loads, which causes a large vertical vibration. At this time, the damping force of the suspension reduces the vertical vibration. If the spinning RPM increases sufficiently after passing through the vertical resonance region ($\omega >> \omega_n$ in Fig. 2), the salt water in the hydraulic balancer shifts in an opposite direction to the unbalanced loads, thereby reducing horizontal vibration at high speed. Studies on the optimization design of a hydraulic balancer has been carried out by several researchers. However, the hydraulic balancer is an effective technique for reducing vibrations only at high rotational speed. It does not reduce the transient horizontal vibration of the washing machine at a low rotational speed. In fact, it causes transient horizontal vibration, as shown in Fig. 2 ($\omega << \omega_n$). As mentioned in the introduction, as the capacity of the washing machine gradually increases, the rotating radius of the washing machine becomes larger and the diameter of the hydraulic balancer becomes larger. Therefore, the balancing force for reducing the vibration of the washing machine at a high rotational speed also

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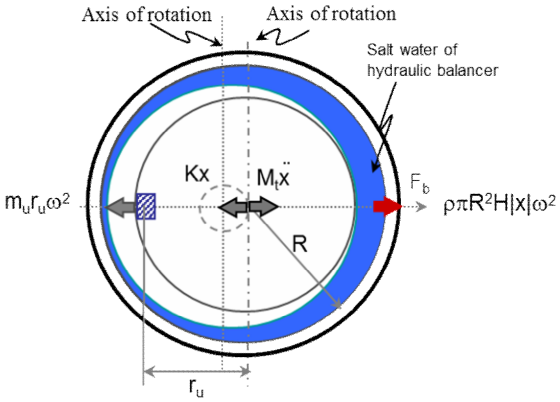


Fig. 1. Mechanical modeling of hydraulic balancer at the steady state rotation.

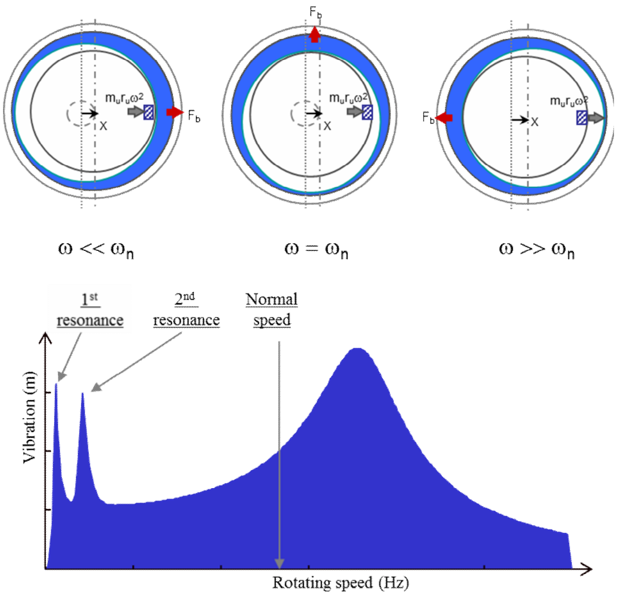


Fig. 2. Phase difference between salt water within the hydraulic balancer and the unbalanced loads within the driving part of the washing machine according to rotating speed. Each vibrational response of the top loading washing machine is also shown.

increases linearly. However, the transient horizontal vibration also increases at a low rotational speed according to the rotating radius of the washing machine; thus, the washing machine is forcibly stopped by the excessive horizontal vibration for the operator's safety.

In order to increase of the capacity of the washing machine, the width of the top loading washing machine cannot be easily increased because of installation constraints of the washing machine. If the width of the washing machine is not increased, the gap between the cabinet and outer tub of the washing machine decreases as increasing the diameter of the outer tub of the washing machine (Fig. 3). Further, the possibility of conflict between the cabinet and outer tub of the washing machine also increases during the low-RPM dehydration process.

As shown in Fig. 4, the transient horizontal vibration of the driving part may exceed the gap between the cabinet and driving part of the washing machine, therefore, the normal dehydration process cannot proceed.

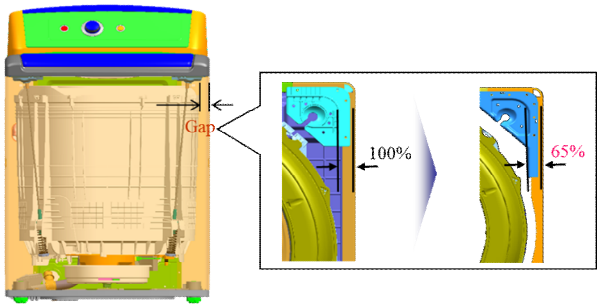


Fig. 3. Top loading washing machine with bigger capacity and a limited size cabinet.

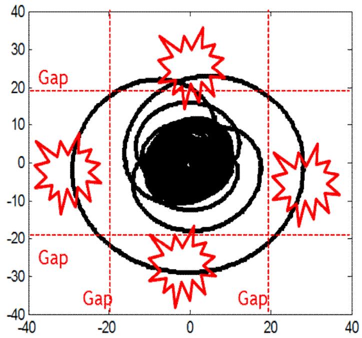


Fig. 4. Transient vibration response of the top loading washing machine with bigger capacity and a limited size cabinet (Abnormal vibrational conflict between driving part and cabinet).

In this paper, we introduce a quad horizontal damper for a compact top loading washing machine with bigger capacity and lower vibration. Each horizontal damper is connected between suspension bar and the outer tub of the top loading washing machine. The vibration energy of the outer tub is exhausted by the frictional damping force and viscous damping force. Four horizontal dampers are mounted on the top loading washing machine in the tangential direction of the outer tub, and each horizontal damper absorbs the vibration energy by a moving felt impregnated with grease during reciprocating motion. Using this technology, we increased the capacity of the top loading washing machine and dehydration performance was also improved by increasing the inner tub diameter and centrifugal force.

2. Dynamic simulation of top loading washing machine for transient vibration reduction

2.1 Dynamic modeling of top loading washing machine

The motion behavior of the top loading washing machine was confirmed by using the translational and rotational equations in the previous study [1]. In top loading washing machine, upper hydraulic balancer is simulated with two ball balancers with the degree of freedom in the plane rotation

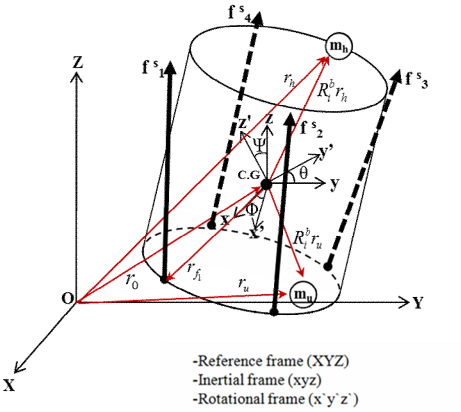


Fig. 5. Dynamic modeling of top loading washing machine and coordinate system.

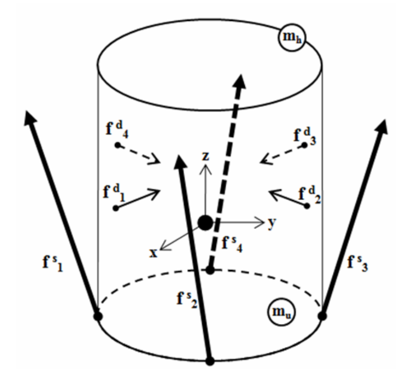


Fig. 6. Dynamic modeling of top loading washing machine with the quad horizontal damper system.

direction.

In the recent study, analysis on the dynamics of a top-loading washing machine and its equation of motion is developed [1]. m_u and m_h represent the mass of an unbalanced load and a hydraulic balancer, respectively. The tub and basket rotate in the three-dimensional coordinate system, as shown in Fig. 5, where the rotation angles, Φ, θ and ψ, represent the rotation about the x-axis, the rotation about the y-axis and the rotation about the z-axis of the rotating system. The origin for the rotation motion is set to the position of the center of mass of the tub and basket; the position of the center of mass of the tub and basket is denoted by r_o. As the same results with the recent study, the equations of motion of translation and rotation are obtained by Newton's second law of motion as follows, respectively.

$$\begin{aligned}(m+m_u+m_h)\ddot{\boldsymbol{r}}_o &= \sum_{k=1}^{4}\boldsymbol{f}_k + (m+m_u+m_h)\boldsymbol{g}\\ &-m_u\left[(\ddot{\boldsymbol{r}}_u)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i\right]\\ &-m_h\left[(\ddot{\boldsymbol{r}}_h)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i\right]\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{d\boldsymbol{L}_b}{dt} + \boldsymbol{\omega}_b\times\boldsymbol{L}_b &= \boldsymbol{T}_a + \boldsymbol{R}_i^b(\psi,\theta,\phi)\sum_{i=1}^{4}\boldsymbol{r}_{fi}\times\boldsymbol{f}_i\\ &+m_u\boldsymbol{r}_u\times\boldsymbol{R}_i^b(\psi,\theta,\phi)\left[\boldsymbol{g}-\left\{\ddot{\boldsymbol{r}}_o + (\ddot{\boldsymbol{r}}_u)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i\right\}\right]\\ &+m_h\boldsymbol{r}_h\times\boldsymbol{R}_i^b(\psi,\theta,\phi)\left[\boldsymbol{g}-\left\{\ddot{\boldsymbol{r}}_o + (\ddot{\boldsymbol{r}}_h)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i\right\}\right].\end{aligned} \tag{2}$$

In the rotating coordinate system, the equation of rotational motion for the hydraulic balancer is

$$\begin{aligned}\frac{dL_h}{dt} + \dot{\beta}\times L_h &= m_h\boldsymbol{R}_i^b(\psi,\theta,\phi)\boldsymbol{e}_z\boldsymbol{r}_h\times\boldsymbol{R}_i^b(\psi,\theta,\phi)\times\\ &\left[\boldsymbol{g}-\left\{\ddot{\boldsymbol{r}}_o + (\ddot{\boldsymbol{r}}_h)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i\right\}\right]\\ &-c_h\dot{\beta}\,.\end{aligned} \tag{3}$$

In case of applying the quad horizontal damper to top loading washer, Dynamic modeling is modified as shown in Fig. 6.

Four horizontal dampers are applied to the upper part of the outer tub for reducing transient vibration of top loading machine. In that case, the equations of motion of translation and rotation are modified as follows

$$\begin{aligned}(m+m_u+m_h)\ddot{\boldsymbol{r}}_o &= \sum_{k=1}^{4}\boldsymbol{f}_k^s + \sum_{l=1}^{4}\boldsymbol{f}_l^d + (m+m_u+m_h)\boldsymbol{g}\\ &-m_u\left[(\ddot{\boldsymbol{r}}_u)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i\right]\\ &-m_h\left[(\ddot{\boldsymbol{r}}_h)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{d\boldsymbol{L}_b}{dt} + \boldsymbol{\omega}_b\times\boldsymbol{L}_b &= \boldsymbol{T}_a + \boldsymbol{R}_i^b(\psi,\theta,\phi)\sum_{i=1}^{4}\boldsymbol{r}_{fi}\times\boldsymbol{f}_i^s + \boldsymbol{R}_j^b(\psi,\theta,\phi)\sum_{j=1}^{4}\boldsymbol{r}_{fj}\times\boldsymbol{f}_j^d\\ &+m_u\boldsymbol{r}_u\times\boldsymbol{R}_i^b(\psi,\theta,\phi)\left[\boldsymbol{g}-\left\{\ddot{\boldsymbol{r}}_o + (\ddot{\boldsymbol{r}}_u)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_u)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_u)_i\right\}\right]\\ &+m_h\boldsymbol{r}_h\times\boldsymbol{R}_i^b(\psi,\theta,\phi)\left[\boldsymbol{g}-\left\{\ddot{\boldsymbol{r}}_o + (\ddot{\boldsymbol{r}}_h)_i + 2(\boldsymbol{\omega}_b)_i\times(\dot{\boldsymbol{r}}_h)_i + (\boldsymbol{\omega}_b)_i\times(\boldsymbol{\omega}_b)_i\times(\boldsymbol{r}_h)_i\right\}\right].\end{aligned} \tag{5}$$

In Eqs. (4) and (5), $\sum_{l=1}^{4}\boldsymbol{f}_l^d$ shows the damping force in the translation motion of the each horizontal damper and $\boldsymbol{R}_j^b(\psi,\theta,\phi)\sum_{j=1}^{4}\boldsymbol{r}_{fj}\times\boldsymbol{f}_j^d$ shows the damping force in the rotation motion of the each horizontal damper.

, where $\boldsymbol{R}_j^b(\psi,\theta,\phi)$ is the 3×3 rotation matrix corresponding to (ψ,θ,ϕ).

From Eqs. (3)-(5), displacement of outer tub with and without a quad horizontal damper system in case of unbalance weight 1.2 kg in the lower position of the basket can be derived using Runge-Kutta method as shown Fig. 7.

As a simulated results, displacement of outer tub with a quad horizontal damper system is much smaller than that of without quad horizontal dampers by 36~37 %. The amount of displacement of outer tub is deeply dependent on the magnitude of horizontal damping force. In simulation, the damping force of each horizontal damper is given by 15 N. Even if the damping force of horizontal dampers have higher, vibrational transfer from outer tub to the outer cabinet of washing machine abnormally increase at higher spin speed. Therefore, optimization of damping force of horizontal damper is very important for reduction of transient vibration of outer tub at low spin speed and steady state vibration at high spin speed at the same time.

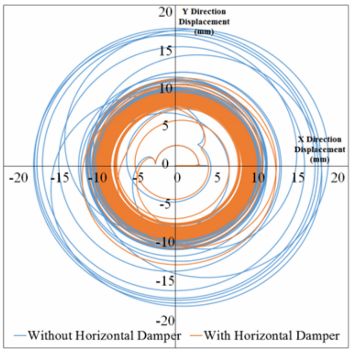


Fig. 7. X-Y directional horizontal displacement of outer tub with or without a quad horizontal damper in case of 1.2 kg unbalance weight.

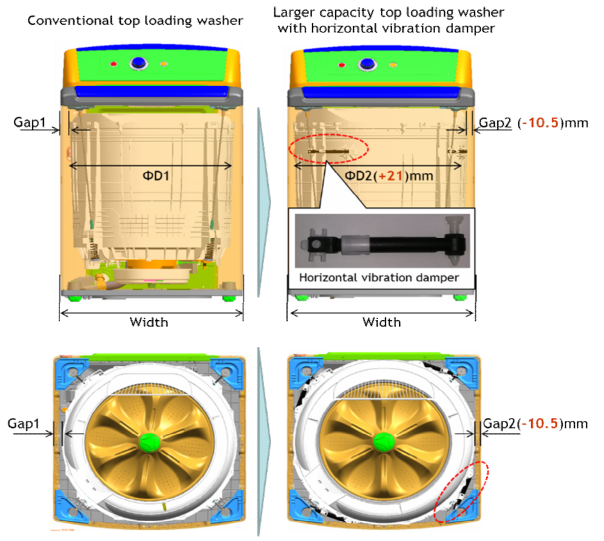


Fig. 8. Comparison of conventional top loading washing machine and a top loading washing machine with a horizontal damper.

2.2 Design parameter of a quad horizontal damper system

A typical top loading washing machine is designed to ensure a sufficient gap (Gap1) between the driving part and the cabinet of the washing machine, so that they do not collide even if vibration occurs, as shown in Fig. 8.

However, in order to design a top loading washing machine with a higher capacity within a limited outer size, the displacement generated by the driving part must be smaller than the gap (Gap2). Therefore, a damper for reducing vibration in the horizontal direction must be applied. As shown in Fig. 8, four horizontal dampers are installed to achieve a 40 % reduction in vibration of the driving part in the horizontal direction. In this case, the capacity of the washing machine may increase by approximately +0.4 to +0.5 cubic feet.

As shown in Fig. 9, the horizontal damper has both frictional damping force and viscous damping force. The Piston rod moves back and forth inside the main body. The polyurethane felt impregnated with grease is wrapped over the piston rod with a sufficient thickness so that vibration is reduced. The thickness (t_2) of the felt is sufficiently larger than the depth (t_1)

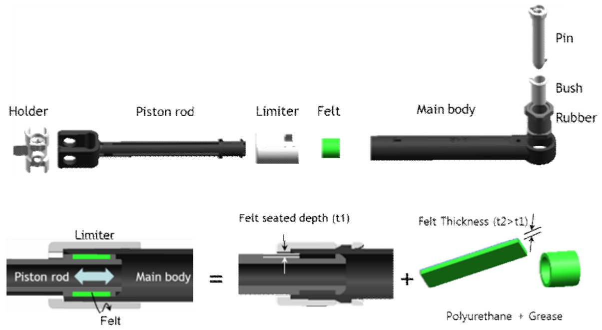


Fig. 9. Structure of horizontal damper with friction and viscous damping forces.

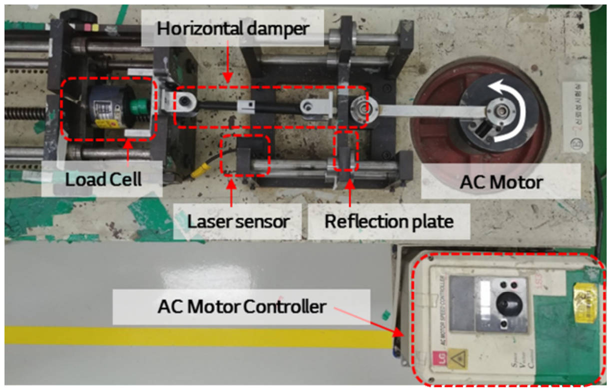


Fig. 10. Experimental test jig for measuring the damping force of horizontal damper.

of the space through which the felt is wound, so that the frictional and viscous forces of the felt material can be generated to reduce vibration.

2.2.1 Measurement of damping force of a horizontal damper

The experimental equipment was set up as shown in Fig. 10 to measure the damping force of the horizontal damper.

The piston rod of the horizontal damper is fixed to the load cell, and the main body of the horizontal damper is connected to the rotating shaft, which is driven by the AC motor. Further, the laser sensor mounted on the test desk can measure the displacement of the main body, so that the hysteresis loop can be measured as shown in Fig. 11. The total damping energy of the horizontal damper can be calculated by integrating the area of the hysteresis curve. Fig. 11 shows the hysteresis curve of the damping energy generated when the piston rod of the horizontal damper moves back and forth. The unbalanced load causes the driving part of the top loading washing machine to vibrate in the horizontal direction. At this time, the piston rod connected with the suspension bar, which is fixed with upper corner of the cabinet, moves forward toward the driving part and absorbs vibration with the damping energy of the area, as shown in the upper part of Fig. 11. Because the driving part continuously whirls in the opposite direction to the horizontal damper, the piston rod of the horizontal damper absorbs vibration with the damping energy of the area, as shown in the low-

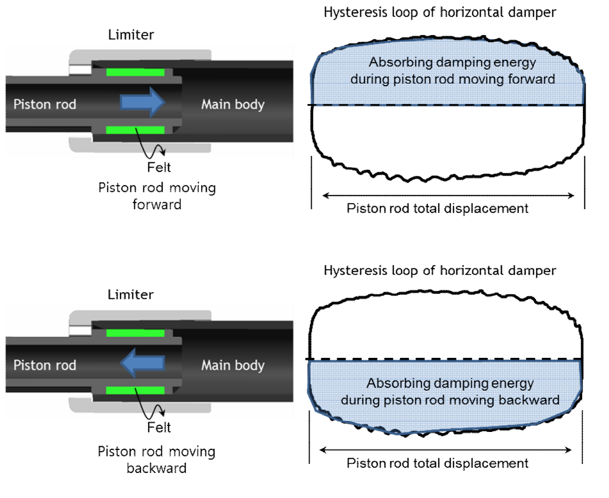


Fig. 11. Damping energy of the horizontal damper as the piston rod moves back and forth.

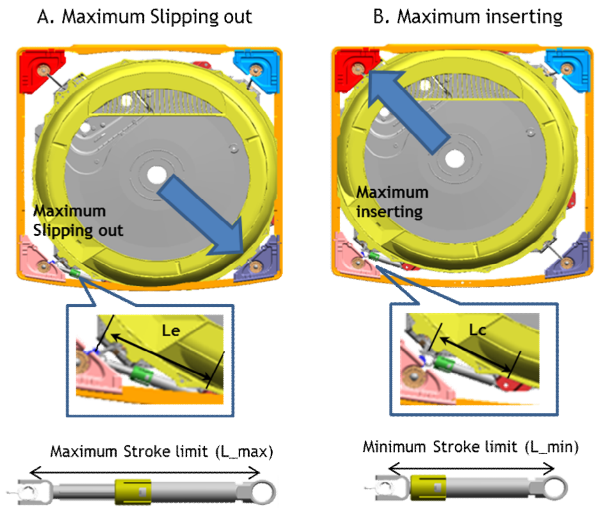
er part of Fig. 11.

2.2.2 Horizontal damper length

The maximum displacement that can occur in the driving part in the top loading washing machine is substantially the same as the gap between the driving part and cabinet. The piston stroke in the horizontal damper is maximum when the driving part is tightly fitted to the cabinet, as shown in Fig. 12(a), while it is minimum in the opposite direction when the driving part is closely attached to the cabinet, as shown in Fig. 12(b). If the maximum and minimum strokes of the horizontal damper are L_e and L_c, respectively, the condition that the horizontal damper can effectively reduce the vibration within the gap between the cabinet and the driving part is that L_e should be smaller than $L_{\_max}$ when the horizontal damper piston completely slips out from the main body, and L_c should be larger than $L_{\_min}$ when the horizontal damper piston is completely inserted into the main body of the horizontal damper. If this limit is not satisfied, the piston in the horizontal damper may completely detach from the main body or be damaged owing to excessive compression when the driving part vibrates extremely.

2.2.3 Optimization of damping force of horizontal damper

As mentioned previously, as a general top loading washing machine does not have any device to reduce the horizontal vibration of the driving part, the noise during high-speed dehydration is mainly generated by the unbalanced force due to the unbalanced weight. However, when four horizontal dampers are connected to the driving part and the cabinet, as suggested in this paper, vibration transfers from the driving part to the cabinet. Therefore, if the damping force of the horizontal damper is excessively large, then vibration transmission during high-speed dehydration increases, and the noise generated by the washing machine is louder compared with that of conventional top loading washing machines without horizontal



Length(mm)		
Maximum Stroke	Minimum stroke	Parameter condition
Le	Lc	Le < L_max Lc > L_min

Fig. 12. Stroke change of the horizontal damper piston according to driving part position. And the stroke limit of the horizontal damper piston.

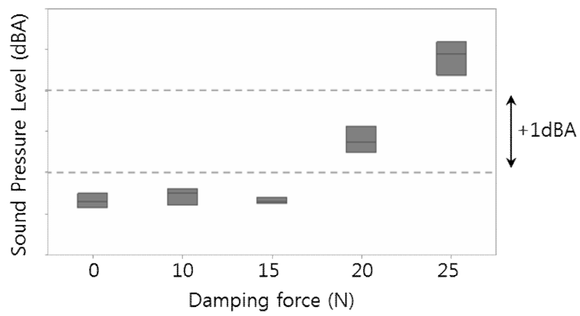


Fig. 13. Sound pressure level at an RPM of 700 (dehydration process) according to the damping force of horizontal damper.

dampers. On the contrary, if the damping force of the horizontal damper is too small, it may be ineffective because the performance of reducing the horizontal vibration of the driving part is not sufficient to reduce the horizontal vibration of the driving part. Therefore, the optimal damping force of the horizontal damper should be able to sufficiently reduce the vibration of the driving part at a low RPM and should have a low damping force at a high RPM.

Fig. 13 shows the noise level of the top loading washing machine during the high-speed dehydration process according to the damping force of the horizontal damper. As shown in the experimental results, when the damping force of the horizontal damper is approximately 10–15 N, the noise level is similar to that of a washing machine without a horizontal damper. However, when the horizontal damping force is more than 20 N, the noise level increases by more than +1–+2 dBA.

Figs. 14 and 15 show that the transient vibration at low

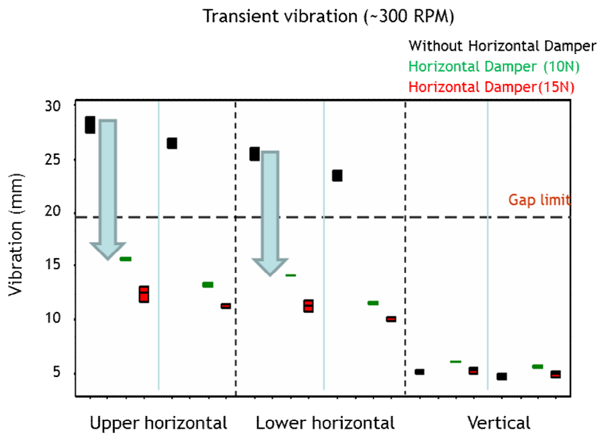


Fig. 14. Transient horizontal vibration according to the damping force of horizontal damper below 300 RPM.

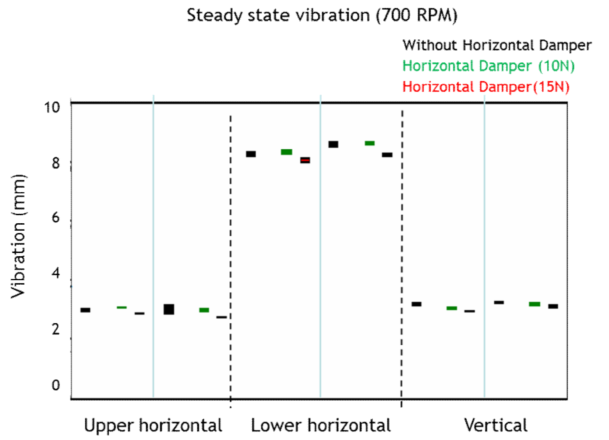


Fig. 15. Steady-state vibration according to the damping force of the horizontal damper at 700 RPM.

RPM and steady-state vibration at high RPM, respectively, increase owing to vibration transmission during high-speed dehydration when using a horizontal damper with a damping force of 10–15 N. The magnitude of the transient horizontal vibration should be smaller than the gap, which is the distance between the cabinet and outer tub of top loading washing machine. In case the damping force of the horizontal damper is 10–15 N, this condition is satisfied.

Further, it is desirable that the vibration of the driving part during high-speed dehydration should be substantially unchanged even when the horizontal damper is applied, compared with the case without a horizontal damper. As mentioned above, if the damping force of the horizontal damper is too large, the vibration transmission, as well as the noise, during high-speed dehydration will increase. Thus, the magnitude of steady-state vibration must be the same as that without a horizontal damper.

As a result, the minimum allowable damping force of the horizontal damper is 10 N and the maximum allowable damping force is 15 N. Under these conditions, the horizontal vibration of the driving part was improved by approximately 35~45 % at low RPM compared with the conventional top loading washing machine, and the horizontal vibration hardly changed at high RPM. At the same time, there was no increase in noise due to the horizontal damper during high-speed dehydration.

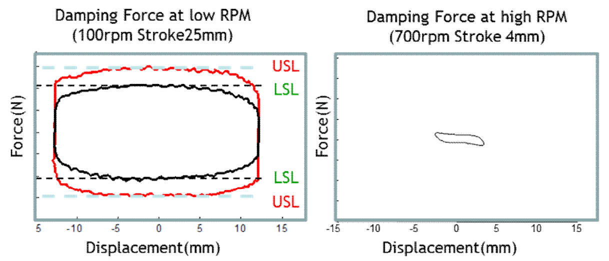


Fig. 16. Damping force of the horizontal damper measured using the damping force test jig (Fig. 11) according to RPM.

3. Conclusions

In this paper, a new horizontal damper technique is studied to reduce the transient horizontal vibration in a top loading washing machine. The horizontal damper is connected between the driving part of the top loading washing machine and the suspension bar attached to the cabinet to reduce the horizontal vibration of the driving part of the washing machine. In order to effectively reduce vibration, four horizontal dampers are connected in the same direction as the direction of motion of the driving part of the washing machine. The horizontal damper has friction force and viscous force owing to the reciprocating piston movement of the grease-impregnated felt. The minimum allowable damping force is 10 N, and the maximum allowable damping force is 15 N, in order to reduce the vibration of the driving part by 35~45 % at low RPM and to prevent additional vibration of the driving part or increase noise at high RPM. Fig. 16 shows the measured damping force according to RPM for a single horizontal damper. The left side of Fig. 16 shows the damping force of the horizontal damper at 100 RPM for a stroke of 25 mm. LSL and USL stand for the lower specification limit and upper specification limit of the damping force of the horizontal damper, which are 10 N and 15 N, respectively. The right side of Fig. 16 shows the damping force of the horizontal damper at 700 RPM for a stroke of 4 mm, which are sufficiently small to be ignored. As a result, no additional noise or abnormal vibration is generated at high speed.

As shown in Fig. 17, when the horizontal damper is installed, the vibration of the driving part of the top loading washing machine was reduced by 35~45 %. The results indicated that the vibration of the driving part becomes smaller than the gap between the cabinet and the outer tub of the driving part of the top loading washing machine, and the dehydration process can be normally performed without striking the cabinet.

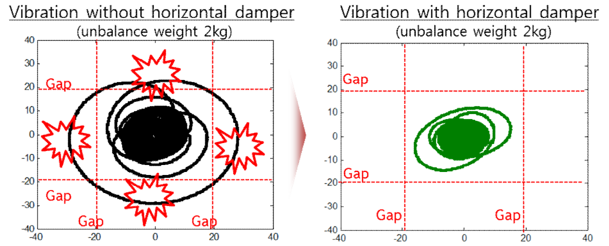


Fig. 17. Reduction in transient horizontal vibration of the top loading washing machine with quad horizontal dampers by 35~45 % compared with conventional top loading washing machine without quad horizontal dampers.

The horizontal dampers investigated studied in this study not only increases the capacity of the top loading washing machine without increasing the size of the cabinet, but also reduces the vibration caused by excessive unbalanced weight.

The quad horizontal damper technology was certified as New Excellent Technology by the Ministry of Trade Industry and Energy on April 23, 2014 under the title "New Horizontal Damper Technology for Large Capacity Design of Top Loading Washer". (Certification Number 0826). Further, the top loading washing machine developed using this technology received the IR52 award in June 2014 under the title "High Capacity, High Speed Top Loader Washer" in recognition of its superiority and innovation (The 33rd week's Award of 2014).

Nomenclature

M_t : Total mass of washing machine (driving part)
R : Outer radius of hydraulic balancer
R_u : Radius of rotation
X : Displacement of washing machine (driving part)
ω : Angular velocity
ω_n : Resonance angular velocity of driving part
ω_b : Angular velocity of the basket
ρ : Density of salt water
H : Height of hydraulic balancer
F_b : Balancing force
D : Diameter of outer tub
Gap : Distance between outer tub and cabinet
t_1 : Felt seated depth
t_2 : Felt thickness
L_e : Maximum stroke of horizontal damper
L_c : Minimum stroke of horizontal damper
$L_{\_max}$: Maximum stroke limit of horizontal damper
$L_{\_min}$: Minimum stroke limit of horizontal damper
m : Total mass except balls & unbalanced masses
m_h : Mass of a ball
m_u : Unbalanced mass
I_{xx} : Mass moment of inertia
I_{yy} : Mass moment of inertia
I_{zz} : Mass moment of inertia
r_t : Radius of a tub
r : Gyration radius of balls
r_c : Radius of a cabinet
a : Gyration radius of an unbalanced mass
l_o^1 : Height of c.g. from origin
h : Height of a ball from c.g.
h_c : Height of a cabinet from c.g.
b_o : Height of an unbalanced mass from origin
b : Height of an unbalanced mass from c.g.
h_s : Height of suspensions at tub from c.g.
K : Stiffness of suspensions
c : Damping coefficient of suspensions
c_h : Damping coefficient between hydraulic balancer and the basket
c_b : Damping coefficient between balls and the basket
L_b : Angular momentum of the basket
L_h : Angular momentum of the hydraulic balancer
f^s : Moment force of Suspension
f^d : Moment force of quad horizontal damper
e_z : Unit Vector of z axis
L : Angular momentum
I : Moment of Inertia
T_a : Actuation torque that an electric motor generates to rotate the basket

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