

Rolling bearing fault diagnosis based on mean multigranulation decision-theoretic rough set and non-naive Bayesian classifier†

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Abstract

To analyze data from multi-level view, reduce computational burden, and improve fault diagnosis accuracy, a novel fault diagnosis method of rolling bearings based on mean multigranulation decision-theoretic rough set (MMG-DTRS) and non-naive Bayesian classifier (NNBC) is proposed in this paper. First, fault diagnosis features of rolling bearings in training samples are extracted to construct MMG-DTRS. Then, the significance degree of condition attribute in MMG-DTRS is defined to quantitatively measure the influence of condition attributes with respect to the decision ability of an information system. An attribute reduction algorithm based on MMG-DTRS is applied to acquire a lower dimensional condition attribute set, which reduces computational complexity and avoids the interference of irrelevant or redundant condition attributes. Finally, NNBC is constructed to classify rolling bearing conditions in test samples. The classification procedures by using NNBC are given. The performance of the proposed method is validated and the advantages are investigated by using a fault diagnosis experiment of rolling bearings. Experimental investigations demonstrate the proposed method is effective and reliable in identifying fault categories and fault severities of rolling bearings.

Keywords: Rolling bearing; Fault diagnosis; Mean multigranulation decision-theoretic rough set; Non-naive Bayesian classifier; Attribute reduction

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1. Introduction

Rolling bearings are one of the key components in rotating machinery, such as wind turbines, motors, and planetary gearboxes, etc. [1-3]. Due to harsh operating conditions caused by heavy load, high speed, or contamination, rolling bearings are prone to various faults such as crack, pitting and spalling, etc. [4, 5]. Once they suffer from the faults, they may lead to entire system shutdown, huge production losses or even catastrophic results. Therefore, fault diagnosis is a crucial task to guarantee healthy operation state of rolling bearings.

Recently, various data-driven fault diagnosis methods of rolling bearings have been put forward. Many intelligent techniques, such as deep neural network (DNN), support vector machine (SVM), and fuzzy logic (FL), have been investigated and developed as novel tools for rolling bearing fault diagnosis [6]. Deep learning as a novel machine learning tool possesses the capability to overcome the inherent disadvantages of traditional intelligent techniques [7, 8]. The most obvious advantage of deep learning models lies in that they can automatically discovery valuable information from raw data. The

most popular deep learning model is DNN, which has been extensively applied in rolling bearing fault diagnosis owing to its strong representation ability and simple structure [9-13]. Gan et al. [9] put forward a hierarchical diagnosis network (HDN) using deep learning. A two stage diagnosis strategy is used for hierarchical classification through a two-layer HDN. The first layer is utilized to classify fault types, and the second one is employed to realize fault severity reorganization. Guo et al. [10] proposed an adaptive hierarchical deep convolution neural network for fault type recognition and fault size evaluation of rolling bearings. Shao et al. [11] used ensemble deep auto-encoders (EDAEs) to propose an intelligent fault diagnosis method of rolling bearings. Different activation functions are regarded as the hidden functions of a series of autoencoders. EDAEs are constructed by these auto-encoders through unsupervised feature learning. Then, these learned deep features are considered as the input of classifiers for fault identification. Shao et al. [12] presented a fault diagnosis method of electric locomotive bearings using convolutional deep belief network (CDBN). Collected vibration data is compressed through an auto-encoder. A CDBN constructed through Gaussian visible units is used to learn deep features. The constructed CDBN is improved by an exponential moving average. Zhang et al. [13] constructed a deep convolu-

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tional neural network model for bearing fault diagnosis under variable working load and noisy environment. This model can achieve very high accuracy without any domain adaptation algorithm or any target domain information. However, determination of DNNs' structure and parameters requires a great number of training samples. High computational cost and low adaptation ability remains unsolved.

SVM, as a supervised machine learning technique, has been widely applied in fault diagnosis owing to its high prediction accuracy and outstanding generalization ability [14-19]. Besides, it does not need a large number of training samples to realize mode identification. Li et al. [15] combined hierarchical fuzzy entropy (HFE) with improved SVM to propose a bearing fault diagnosis method. HFE is utilized to extract fault features. Then, refined fault features are regarded as the input of the improved SVM to automatically identify fault patterns. Liu et al. [16] constructed a dictionary using roller bearing mechanism to put forward a diagnose method combining short-time matching and SVM. Practical bearing experimental results indicate the proposed method can classify a weak fault at the early stage from complex and non-stationary signals. Li et al. [17] presented a bearing fault diagnosis approach combining multiscale permutation entropy with improved SVM. Experimental results demonstrate this approach has high classification accuracy. Ziani et al. [18] proposed a bearing fault diagnosis strategy combining SVM with binary particle swarm optimization (BPSO). Regularized Fisher's criterion is considered as the fitness function of BPSO algorithm. Zheng et al. [19] put forward a fault detection and diagnosis method of rolling bearings combining composite multiscale fuzzy entropy (CMFE) with ensemble SVM. CMFE is used to measure the complexity of rolling bearing signals and extract fault features. The ensemble SVM is used for fault identification. Nevertheless, the parameter selection of SVM is a timeconsuming process and many parameter selection algorithms are not satisfactory.

FL technique transforms qualitative, vague or ambiguous data into numerical terms. Ambiguous judgments can be realized according to uncertain or imprecise information. Consequently, FL technique has been successfully used in rolling bearing fault diagnosis [20-25]. Straczkiewicz et al. [20] presented a fault diagnosis approach of rolling element bearings using FL technique for integration of vibration-based diagnostic features. Ziani et al. [21] combined multi-scale permutation entropy (MPE) and adaptive neuro fuzzy classifier (ANFC) to propose a bearing fault diagnosis method. MPE is used to extract fault features and refine these features. The refined features are considered as the input of ANFC to classify fault modes. Xu et al. [22] put forward an intelligent fault diagnosis approach using selective ensemble of multiple fuzzy classifiers. Optimal features are regarded as the input of the selective ensemble of multiple fuzzy classifiers to classify fault patterns. Sun et al. [23] presented a fault diagnosis strategy of rolling bearings integrating fuzzy evidence discovery with Dempster– Shafer evidence theory. Li et al. [24] introduced a probabilistic

fuzzy system to classify bearing faults. Each rule in the rulebased system can determine a fault and assign a probability to this fault. Experimental analyses demonstrate that this fuzzy system can match the effectiveness of other intelligent diagnosis methods. Although FL technique can achieve satisfactory diagnostic results by fuzzy decision rules, the procedures of acquiring fuzzy decision rules are very time-consuming. Moreover, construction of fuzzy system relies on human expertise and many parameter estimation algorithms are quite complicated.

In recent years, multigranulation rough sets (MGRSs) have attracted considerable attention for their multi-level view [26, 27]. They describe lower and upper approximations via multiple granulation relations to realize approximate approximation. Different from traditional rough sets, MGRSs are constructed according to a series of equivalence relations rather than a single one. Therefore, they have been successfully applied in attribute reduction and fault diagnosis fields. Qian et al. [28] introduced an importance measure method of condition attributes to realize attribute reduction based on pessimistic MGRS model. Tan et al. [29] developed belief reduction algorithms to find pessimistic lower (or upper) approximate reduct by evidence theory-based characteristics. Several measurements by using the belief and plausibility functions are introduced, and attribute reduction algorithms based on these measurements are given. Zhang et al. [30] proposed a fault diagnosis approach of steam turbine based on single-valued neutrosophic MGRSs over two universes. Single-valued neutrosophic MGRSs over two universes are defined. Then, general decision rules are constructed using rough sets over two universes for fault diagnosis. Zhang et al. [31] combined interval-valued hesitant fuzzy sets with MGRS to present a novel fault diagnosis method. Experimental results demonstrate that this method can increase fault diagnosis accuracy and greatly reduce uncertainty. During the attribute reduction using MGRSs, seeking common ground while removing differences is the most frequently used reduction strategy. It can be considered as a conservative reduction strategy, which means that it retains common rules while removing inconsistent rules. Hence, most of the attribute reduction algorithms by using MGRSs are based on pessimistic MGRS model. However, compared to optimistic MGRS model, constraint conditions of pessimistic MGRS model are excessively harsh. Sensitive and essential condition attributes are probably removed and fault diagnosis accuracy is reduced in practical application.

To overcome this problem, based on mean multigranulation decision-theoretic rough set (MMG-DTRS) and non-naive Bayesian classifier (NNBC), a novel fault diagnosis method of rolling bearings is put forward in this paper. First, fault diagnosis features of rolling bearings in training samples are extracted to construct MMG-DTRS. Then, an attribute reduction algorithm based on MMG-DTRS is applied to acquire a lower dimensional condition attribute set. Finally, NNBC is constructed to classify rolling bearing conditions in test samples. Experimental results demonstrate that this method can effectively and accurately identify fault categories and fault severities of rolling bearings.

briefly introduces the preliminaries of MGRSs, MMG-DTRS and NNBC. A fault diagnosis method of rolling bearings based on MMG-DTRS and NNBC is proposed in Sec. 3. The experimental results are analyzed and discussed in Sec. 4. Finally, conclusions are drawn in Sec. 5.

2. Preliminaries

2.1 Multigranulation rough sets

MGRSs were introduced by Qian et al. [26, 27]. Different from traditional rough sets, MGRSs are constructed according to a series of equivalence relations rather than a single one. Some basic concepts about MGRSs are summarized as follows.

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\left\langle \sum_{i=1}^{m} B_i(X), \overline{\sum_{i=1}^{m} B_i}(X) \right\rangle
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2.2 Mean multigranulation decision-theoretic rough set

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Fig. MGRSs were introduced by Oixn et al. [26, 27]. Different $\sum_{i=1}^{n} R_i (X) = \sum_{i=1}^{n} R_i(-X)$ (5)

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 $\mathbf{S} = \mathbf{S} \mathbf{S} + B_i$, $\mathbf{S} = \mathbf{S} \mathbf{S} + B_i$, **Proposition** 1. Let $U, A \subseteq U \setminus V$, be pure the paramosic paramosic paramosic paramosic metallity means that in $\sum_{i=1}^{n} B_i(X) = \{x \in U : [x]_n \subseteq X \setminus \dots \setminus [x]_n \subseteq X\}$.
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H₃..., $B_{\pm} = C$, then $\forall X \subseteq U$, the optimistic lower through legranular structures, it ne $\sum_{i=1}^{m} B_i(X) = \{x \in U : [x]_{B_i} \subseteq X \setminus [x]_{B_n} \subseteq X \}$ (1) 2.2 Mean multigranu
 $\sum_{i=1}^{m} B_i(X) = -\sum_{i=1}^{n} B_i(-X)$ (2) Decision-theoretic
 $\sum_{i=1}^{m} B_i(X) = -\sum_{i=1}^{n} B_i(-X)$ (2) Decision-theoretic

DTRSS introduce Bases of the e $\sum_{n=0}^{\infty} B_n(X) = \{x \in U : |x|_n \in X \setminus \{x\}_n \in X\}$... $\sqrt{|x|_n} \in X$ and $\sum_{n=0}^{\infty} B_n(X) = \sum_{n=0}^{\infty} B_n(X) = \sum_{n=0}^{\infty} B_n(X) = \sum_{n=0}^{\infty} B_n(X)$... (3) **Decision-theoretic rough sets** (OTRSs) [32], as a form of probabilisti Decision-theoretic rough sets (DTRSs) [32], as a form of probabilistic tools, possess very strong theoretical basis. DTRSs introduce Bayesian decision procedure to minimize decision costs. They provide a systematic strategy to set the threshold parameters according to loss functions. In the Bayesian decision procedure, a finite set of states can be writ-In the optimistic MGRS model, the word "optimistic"
means that in multiple granular structures, it needs only at
least a granular structure to satisfy with the inclusion condition
between knowledge granule and target conc in the following transface increases model in multiple granular structures, it needs only at least a granular structure to satisfy with the inclusion condition between knowledge granule and target concept. In the pessimis $\sum_{i=1}^{n} B_i (X) =$
 $\langle x \in U : [x]_{n_i} \cap X \neq \emptyset \setminus [x]_{n_i} \cap X \neq \emptyset \rangle ... \setminus [x]_{n_i} \cap X \neq \emptyset \rangle$. (6)

In the optimistic MGRS model, the word "optimistic"

means that in multiple granular structures, it reeds only at

least a gra $\sum_{i=1}^{D_i} D_i \quad (A) =$
 $\{x \in U : [x]_n \cap X \neq \emptyset \lor [x]_n \cap X \neq \emptyset \lor ... \lor [x]_{n_i} \cap X \neq \emptyset\}$. (6)

In the optimistic MGRS model, the word "optimistic"

necess that in multiple granular structures, it reeds only at

least a granu taking actions of a_p , a_N and a_B , respectively, when a sample belongs to *X*. Similarly, λ_{12} , λ_{22} and λ_{32} represent the loss for taking the correspondence actions when the sample belongs to \sim *X*. For a sample *x*, the expected loss associated with ween knowledge granule and target concept. In the pessi-
tive MGRS model, "pessimistic" means that in multiple
nular structures, it needs all granular structures to satisfy
the inclusion condition between knowledge granul

$$
R(a_1|[x]) = \lambda_1 P(X|[x]) + \lambda_2 P(\sim X|[x])
$$

$$
R(a_2 | [x]) = \lambda_{21} P(X | [x]) + \lambda_{22} P(\sim X | [x]),
$$

\n
$$
R(a_3 | [x]) = \lambda_{31} P(X | [x]) + \lambda_{32} P(\sim X | [x]).
$$

risk decision rules:

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\n
$$
R(a_2 | [x]) = \lambda_{31}P(X | [x]) + \lambda_{22}P(-X | [x]),
$$
\n
$$
R(a_3 | [x]) = \lambda_{31}P(X | [x]) + \lambda_{32}P(-X | [x]).
$$
\n
$$
= \lambda_{31}P(X | [x]) + \lambda_{33}P(-X | [x]).
$$
\nThe Bayesian decision procedure suggests the minimum-
\n
$$
R(a_3 | [x]) = \lambda_{31}P(X | [x]) + \lambda_{32}P(-X | [x]).
$$
\nThe Bayesian decision procedure suggests the minimum-
\n
$$
R(b) If P(X | [x]) ≥ \gamma
$$
 and $P(X | [x]) ≥ \alpha$, then decision rules:
\n $x ∈ POS(X);$
\n $x ∈ POS(X);$
\n $x ∈ POS(X);$
\n $x ∈ NEG(X);$
\n $x ∈ NEG(X);$
\n $x ∈ NEG(X);$
\n $x ∈ \frac{\lambda_{32} - \lambda_{33}}{\lambda_{31} - \lambda_{32} - (\lambda_{11} - \lambda_{32})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}, \gamma = \frac{\lambda_{12} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{11} - \lambda_{22})}.$ \n
$$
P = \frac{\lambda_{22} - \lambda_{22}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{21} - \
$$

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(a₂ [[x]) = $\lambda_2 P(X | [x]) + \lambda_2 P(-X | [x])$,
 $BN_{\sum a}^{M}(X) = \sum_{i=1}^{n} B_i^{M}(X) = \sum_{i=1}^{n} B_i^{M}(X)$ (10)

Faxis and decision procedure suggests t In many real applications, such as data analysis, attribute reduction, and mode identification of multi-source data with high dimensions, DTRS is not suitable for these cases. To overcome this issue, Qian et al. [33] combined MGRS and DTRS to propose MMG-DTRS. With respect to MGRSs, when the loss function is assigned, the conditional probability of a sample within a target concept in granular structures can be calculated as follows: $\beta = \frac{\lambda_{22} - \lambda_{23}}{(\lambda_{21} - \lambda_{22}) - (\lambda_{31} - \lambda_{32})}$ (with respect to *A*, α_{21} and paplication, and more determinisments, such as data analysis, attribute to the clocide $x \in BND(X)$.

In many real application, and mode id

$$
E(P(X \mid x)) =
$$

{P(X | [x]_{B_1}) + P(X | [x]_{B_2}) + ... + P(X | [x]_{B_m})} / m (7)

by B_i and $P(X|[x]_{B_i})$ is the conditional probability of the can be computed by the mean value of *m* conditional probabilities. According to this idea, MMG-DTRS can be defined as follows.

decision-theoretic lower approximation, upper approximation, $, \alpha$

follows. The huge
\nwhere
$$
[x]_{B_i}
$$
 ($1 \le i \le m$) is the equivalence class of x induced
\nby B_i and $P(X | [x]_{B_i})$ is the conditional probability of the
\nequivalent class $[x]_{B_i}$ with respect to X. The joint probability
\nmean be computed by the mean value of m conditional prob-
\nabelities. According to this idea, MMG-DTRS can be defined
\nas follows.
\n**Definition 3** [33]. Let $(U, A = C \cup D, V, f)$ be an IS in which
\nthe decision-theoretic lower approximation, upper approximation,
\nand boundary region of X are denoted by $\sum_{i=1}^{m} B_i(X)$,
\n $\sum_{i=1}^{m} B_i(X)$ and $BN_{\sum_{i=1}^{m} B_i}(X)$, respectively,
\n $\sum_{i=1}^{m} B_i(X)$ and $BN_{\sum_{i=1}^{m} B_i}(X)$, respectively, with respectively
\n $\sum_{i=1}^{m} B_i(X)$ is obviously
\n $\sum_{i=1}^{m} B_i(X)$ and $BN_{\sum_{i=1}^{m} B_i}(X)$, respectively, with respectively, $\sum_{i=1}^{m} B_i(X)$ is obviously
\n $\sum_{i=1}^{m} B_i(X)$ is not suitable to their
\n $\sum_{i=1}^{m} B_i(X)$ is not suitable
\n $\sum_{i=1}^{m} B_i(X)$ is not suitable.
\n $\sum_{i=1}^{m} B_i(X)$ is not sufficient to be a solution.
\n $\sum_{i=1}^{m} B_i(X)$ is not suitable.
\n $\sum_{i=1}^{m} B_i(X)$ is not sufficient to be a solution.
\n $\sum_{i=1}^{m} B_i(X)$ is

$$
\sum_{i=1}^{m} B_{i}^{M,\beta} (X) = U -
$$
\n
$$
\{x \in U : \{P(X | [x]_{B_{1}}) + P(X | [x]_{B_{2}}) + ... + P(X | [x]_{B_{m}})\} / m \leq \beta\}
$$
\n(9)

() () () ² ²¹ ²² *R a x P X x P X x* |[] |[] ⁼ ^l ⁺ ^l : |[] , () () () ³ ³¹ ³² *R a x P X x P X x* |[] |[] ⁼ ^l ⁺ ^l : |[] . The Bayesian decision procedure suggests the minimum-() () () 1 , , ¹ ¹ *^m i ⁱ ^M ^M m m M ⁱ ⁱ ^B ⁱ ⁱ BN X B X B X* b ^a ⁼ ⁼ ⁼ ⁼ - ^å å å (10) When the thresholds^a ^b > , the MMG-DTRS possesses the then decision *x POS X* ^Î () ;

where α , β are two thresholds.

following decision rules:

$$
(MP1) If \{P(X | [x]_{B_1}) + P(X | [x]_{B_2}) + ... + P(X | [x]_{B_m})\} / m \ge \alpha,
$$

$$
(MN1) \text{ If } {P(X | [x]_{B_1}) + P(X | [x]_{B_2}) + ... + P(X | [x]_{B_m})} \land m \leq \beta,
$$

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 $R(a_1 | [x]) = \lambda_n P(X | [x]) + \lambda_n P(-X | [x])$,
 $R(a_2 | [x]) = \lambda_n P(X | [x]) + \lambda_n P(-X | [x])$,

The Bayesian decision procedure suggests the minimum-

where $\$ *et al. / Journal of Mechanical Science and Technology 32 (11) (2018) 5201-5211*
 $P(-X \mid [x])$,
 $B N_{\sum_{n=0}^{N} A_n}(X) = \sum_{i=1}^{\infty} B_i(X)$ (10)

ure suggests the minimum-

where α , β are two thresholds.

When the threshol purnal of Mechanical Science and Technology 32 (11) (2018) 5201–5211

(x)],

(x)].
 $BN_{\sum_{i=0}^{m}R_i}^{M} (X) = \sum_{i=1}^{m} B_i$ (X) $\sum_{i=1}^{m} B_i$ (X) (10)

gests the minimum-

where *α*, *β* are two thresholds.

When the thre or al. *Journal of Mechanical Science and Technology 32 (11) (2018) 5201-5211*
 $(-X | [x])$,
 $BN^u_{\sum_{i=0}^u} A_i (X) = \sum_{i=1}^{m} B_i (X)$ (10)
 $BN^u_{\sum_{i=0}^u} A_i (X) = \sum_{i=1}^{m} B_i (X)$ (10)
 $\sum_{i=1}^{m} B_i (X)$ (10)
 $\sum_{i=1}^{m} B_i (X)$ *mal of Mechanical Science and Technology 32 (11) (2018) 5201-5211*

(1),
 $BN_{\sum_{i=1}^{n}R_i}^{M}(X) = \sum_{i=1}^{\infty} B_i^{M}(X) - \sum_{i=1}^{\infty} B_i(X)$ (10)

ests the minimum-

where α , β are two thresholds.

When the thresholds α Fechnology 32 (11) (2018) 5201-5211
 $BN_{\sum_{i=1}^{n}B_{i}}^{M} (X) = \sum_{i=1}^{m} B_{i}^{M} (X) - \sum_{i=1}^{m} B_{i} (X)$ (10)

nere α , β are two thresholds.

When the thresholds $\alpha > \beta$, the MMG-DTRS possesses the

llowing decision rul Fechnology 32 (11) (2018) 5201-5211
 $BN_{\sum_{i=1}^{n}R_i}^{M,B}(X) = \sum_{i=1}^{m} B_i(X)$ (10)
 $BN_{\sum_{i=1}^{n}R_i}^{M,B}(X) = \sum_{i=1}^{m} B_i(X)$ (10)

nere α , β are two thresholds.

When the thresholds $\alpha > \beta$, the MMG-DTRS possesses the
 d Technology 32 (11) (2018) 5201-5211
 $BN_{\sum_{i=1}^{n} R_i}^{M} (X) = \sum_{i=1}^{m} B_i^{M} (X) - \sum_{i=1}^{m} B_i(X)$ (10)

where α , β are two thresholds.

When the thresholds $\alpha > \beta$, the MMG-DTRS possesses the

following decision rule Fechnology 32 (11) (2018) 5201-5211
 $BN_{\sum_{i=0}^{n}R_i}^{M,\beta}(X) = \sum_{i=1}^{\infty} B_i(X)$ (10)

nere α , β are two thresholds.

When the thresholds $\alpha > \beta$, the MMG-DTRS possesses the

When the thresholds $\alpha > \beta$, the MMG-DTRS *d Technology 32 (11) (2018) 5201–5211*
 $BN_{\sum_{n}^{M}A}^{M}(X) = \sum_{i=1}^{m} B_{i}^{M}(X) - \sum_{i=1}^{m} B_{i}(X)$ (10)

where α , β are two thresholds.

When the thresholds $\alpha > \beta$, the MMG-DTRS possesses the

following decision rul but also realizes data analysis from multi-level view. Furthermore, it overcomes the shortcoming that constraint conditions of pessimistic MGRS model are excessively harsh. Thus, MMG-DTRS extends the wider applications such as data analysis and attribution reduction of multi-source information.

2.3 Non-naive Bayesian classifier

E P X x () () [|] ⁼ () () () ¹ ² { |[] |[] ... |[] } / *^B ^B ^B^m P X x P X x P X x m* ⁺ + + (7) The Hayesian tectoon procedure suggests the minimum-

ink decision rates:
 \sqrt{B} are two thresholds. $\alpha > \beta$ are two thresholds $\alpha > \beta$, the MMG-DINS prosesses the
 $\alpha > \beta$ $\alpha > \beta$ and $P(X \mid |x|) \geq \alpha$, then decision α and $|x| = kP(X | x|) + kP(-X | x|)$.
 $\int_{X_1^2} f(X) \left(\sum_{i=1}^{n} \frac{X_i}{\sqrt{n}} \right) dx$ (10)
 $\int_{X_2^2} f(X) \left(\sum_{i=1}^{n} \frac{X_i}{\sqrt{n}} \right) dx$ (10)
 $\int_{X_1^2} f(X) \left(\sum_{i=1}^{n} \frac{X_i}{\sqrt{n}} \right) dx$ (10)
 $\int_{X_2^2} f(X) \left(\sum_{i=1}^{n} \frac{X_i}{\sqrt{n}} \right) dx$ (10)
 EXECTE THE CONDIGE CONT AN THE CONDIGNATION TO THE SET AND THE CONDUCION and THE CONDUCION (CONDUCION THE SECTION THE SECTION OF CONDUCION THE SECTION OF THE SECTION (*P***) and** *D***(***P***) and** *D***(***P***) and** *D***(***P***) and** *D***(** 1 $\pi_1 \to -\pi_2$

La many real applications, such as data analysis, attitude bot also realizes data analysis from the section and model identification of multi-selections, and not be denoted by the DIRS. The many redivisor $\sum_{i=1}^{M,a} B_i(X)$, tools due to their str $\sum_{n=1}^{\infty} p(x)$ tools due to their structure simplicity, high computational **Example 10**
 Example 10 α and α ensions, DTRS is not aniable for these cases. To and pressimistic MGRS model are excessively busts. Thus,

to the same of positively denoted and the same of positively denoted and the same propose MMG-DTRS. With respect t $E(P(X | x)) =$
 ${P(X | [x]_n) + P(X | [x]_n) + ... + P(X | [x]_{n_n})} / m$ (7) the existing mode of

where $[x]_n (1 \le i \le m)$ is the equivalence class of x induced

where $[x]_n (1 \le i \le m)$ is the conditional probability of the existing mode of

where sign dimensions, DTRS_3 is not anitobe for these cases. To of passinistic MGRS model are excessively laash. Thus,

negroes the is such that if 3π cases in the simulation reduction reduction and the simulation reduc **Example the condition, such as ANN, SVM, and classification, such as ANN, SVM, the causal feature)** $\int_R f(x) \left[x \right]_{B_n} + ... + P\left(X | \left[x \right]_{B_n} \right) / m$ **(7) the existing mode classification method of the existing mode classificat** Les function is assigned, the conditional probability

les electrics is any methods have been developed

les follows:

less functions, such as NOV, SVM, and Democratic Research

less functions, such as NNN, SVM, and Democ $\hat{\mu}$ dimensions. DTRS is ato said to lead one cases. To at persionics MRS much also are exceeded at the sign of the same of the sign μ is the proper what is the proper whole is the proper word in the sign particles $s1_n$) + $P(X | [s1_n]$ +... + $P(X | [s1_n])$ /m

(7) the existing mode classification methods are summar

($1 \le i \le m$) is the cquivalence class of x induced these methods in practice. Let N be the size of furion

($1 \le i \le m$) is th itities. According to this idea, MMG-DTRS can be defined in pactical applications. Bay

bolows.

bolom the basis of statistics
 i tools on the basis of statistics
 $B_1, ..., B_m \subseteq C$, then $\forall X \subseteq U$, the mean multigranulation
 calculated as follows:
 $E(F(X \mid x)) + F(X \mid (x), y) = 0$
 $E(S(X \mid x)) + F(X \mid (x), y) = 0$
 $(F(X \mid x)) + F(X \mid (x), y) = 0$
 $F(X \mid x) =$ Recently, many methods have been developed for mode classification, such as ANN, SVM, and Dempster–Shafer theory (DST), etc. Nevertheless, two main disadvantages of the existing mode classification methods are summarized as follows. The huge computational burden prohibits the use of these methods in practice. Let *N* be the size of training samples, the training complexities of ANN, SVM and DST are (*B* are two thresholds

When the thresholds $x > \beta$, the MMG-DTRS possesses the

following decision rules:

(MPI) If $\{P(X | [x]_n] + P(X | [x]_n] + ... + P(X | [x]_n] \}$ / $m \ge \alpha$,

then decision $x \in POS(Y)$;

(MNI) If $\{P(X | [x]_n) + P(X |$ ignore the dependence among features in the acquired samples in practical applications. Bayesian classifiers are identification tools on the basis of statistical theory. Among Bayesian classifiers, naive Bayesian classifiers (NBCs) are a simple and efficient probabilistic tool, which have been successfully applied to identify fault modes [34]. The classification performance of NBCs is obviously superior to traditional mode identification efficiency, and less storage requirement. A key step in the classification strategy of NBCs is to estimate probability density function (PDF) from training data set. However, the common methods of PDF estimation are on the basis of the assumption that all attributes are independent and this hypothesis is not suitable for many real-world applications [35]. To address this issue, NNBC using the optimal bandwidth selection was proposed to ignore the independence assumption among attributes and replace marginal PDF estimation by joint PDF estimation [36]. The class label of a sample is determined as the following: nethods in practice. Let *N* be the size of training sam-

he training complexities of ANN, SVM and DST

he training complexities of ANN, SVM and DST

he dependence among features in the acquired samples

the dependence a In the probabilistic tool, which have been successfully applied
dentify fault modes [34]. The classification performance of
Cs is obvived y superior to traditional mode identification
ls due to their structure simplicity, naive Bayesian cassiners (NBCs) are a simple and em-
probabilistic tool, which have been successfully applied
probabilistic tool, which have been successfully applied
entify fault modes [34]. The classification performanc Solution, which have been successingly applied to the total studient from the theorem supplied that modes [34]. The classification performance of the subspace of the strate probability, high computational less storage req

$$
w = \arg \max \left\{ \frac{n_k}{N} P(x \mid w_k) \right\} =
$$

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\n
$$
\arg \max \left\{ \frac{1}{Nh_k^{(d)}} \sum_{i=1}^{n_k} \left[k \left(\frac{x_1 - x_{i1}^{(k)}}{h_k}, \frac{x_2 - x_{i2}^{(k)}}{h_k}, \dots, \frac{x_d - x_{id}^{(k)}}{h_k}, \right) \right] \right\}
$$
\nTraining samples
\nthere *d* is the attribute vector dimension and $N = \sum_{k=1}^{c} n_k$
\nhere *c* is the class number. Besides, h_k is the optimal band-
\nAn IS. attribute reduction

where *d* is the attribute vector dimension and $N = \sum_{k=1}^{c} n_k$ where *c* is the class number. Besides, h_k is the optimal band-
An IS, attribute reduction width for *k*th class. It can be computed by the following equation: s the attribute vector dimension and $N = \sum_{k=1}^{c} n_k$

the class number. Besides, h_k is the optimal band-

kth class. It can be computed by the following equa-
 $\frac{4d}{\sqrt{k\left|\sum_{k}\right|^{\frac{1}{2}}\left\{2tr\left(\sum_{k}^{-1}\sum_{k}^{-1}\right)+tr^2\left(\sum$ *I* is the attribute vector dimension and $N = \sum_{k=1}^{c} n_k$

is the class number. Besides, h_k is the optimal band-

or *k*th class. It can be computed by the following equa-
 $\left(\frac{4d}{n_k|\Sigma_k|^{\frac{1}{2}}\left\{2tr(\Sigma_k^{-1}\Sigma_k^{-1})+tr^2(\Sigma$

$$
h_{k} = \left(\frac{4d}{n_{k}|\Sigma_{k}|^{-\frac{1}{2}}\left\{2tr\left(\Sigma_{k}^{-1}\Sigma_{k}^{-1}\right)+tr^{2}\left(\Sigma_{k}^{-1}\right)\right\}}\right)^{\frac{1}{d+4}}
$$
(12)

2 Γ ($\sqrt{2}$ Γ ($\sqrt{2}$) $\left\{ \begin{array}{c} \left(k\right) \ \left[\begin{array}{c} 0 \ 1 \end{array} \right] \end{array} \right\} \ \left[\begin{array}{c} \left(k\right) \ \left[\begin{array}{c} 0 \ 0 \end{array} \right] \end{array} \right], \ \ \ldots, \ \left\{ \begin{array}{c} \sigma_d^{(k)} \ \left[\begin{array}{c} 0 \ 1 \end{array} \right] \end{array} \right\} \ .$

$$
k(x) = \frac{1}{(\sqrt{2}\pi)^{d}} \exp\left(-\frac{xx^{r}}{2}\right).
$$
 (13)

From Eq. (11), the class label of the sample *x* represents the probability that the sample *x* belongs to this class. The larger the class label is, the higher the probability that the sample *x* belongs to this class is. Accordingly, the class of the sample *x* can be determined through the maximum class label. Recent research demonstrates that NNBC can obtain the optimal classification performance among the existing NBCs. Moreover, it can achieve satisfactory diagnosis effectiveness without high time consumption and memory requirement.

3. Fault diagnosis method of rolling bearings based on MMG-DTRS and NNBC

3.1 Fault diagnosis framework

MMG-DTRS retains fault-tolerant ability of DTRS, and can realize data analysis from multi-level view. It also overcomes the shortcoming that constraint conditions of pessimistic MGRS model are excessively harsh. Moreover, NNBC based on the optimal bandwidth selection ignores the independence assumption among attributes and replaces marginal PDF estimation by joint PDF estimation. It can obtain satisfactory diagnosis accuracy without high time consumption and memory requirement. Therefore, we integrate MMG-DTRS with NNBC to propose a novel fault diagnosis method of rolling bearings, and the fault diagnosis framework is shown in Fig. 1. $B_1, B_2, ..., B_m \subseteq C$ and condition attribute set $C = \{c_1, c_2, ..., c_n\}$. First, fault diagnosis features of rolling bearings in training then $\forall c_i \in C(1 \le i \le n)$, the significance degree of condition samples are extracted to construct MMG-DTRS. Then, an attribute reduction algorithm based on MMG-DTRS is applied to acquire a lower dimensional condition attribute set. Finally, NNBC is constructed to classify rolling bearing conditions in test samples.

Fig. 1. Fault diagnosis framework of rolling bearings based on MMG-DTRS and NNBC.

3.2 Attribute reduction based on MMG-DTRS

Attribute reduction of condition attribute set is a vital topic in data-driven fault diagnosis methods. It removes irrelevant or redundant condition attributes and retains the decision ability of an IS. That is, the positive region determined by condition attributes after attribute reduction is consistent with the one before attribute reduction. Consequently, to quantitatively measure the influence of condition attributes with respect to the decision ability of an IS, the significance degree of condition attribute in MMG-DTRS can be defined as follows: **Example 1 Definition Definition Definition Example 1**. Fault diagnosis framework of rolling bearing solard on MMG-DTRS
Attribute reduction based on MMG-DTRS
Attribute reduction based on MMG-DTRS
Attribute r Constractive Contribute (Constrained main $\frac{1}{2}$ Rolling bearing conditions

Fault diagnosis framework of rolling bearings based on MMG-

and NNBC.

Thit and the reduction of condition attribute set is a vital topic

d Classification by using NNBC

Maria by Unitary Colling bearing conditions

ework of rolling bearings based on MMG-

based on MMG-DTRS

condition attribute set is a vital topic

gnosis methods. It removes irrelevant

turbi **based on MMG-DTRS**

of condition attribute set is a vital topic

equenosis methods. It removes irrelevant

attributes and retains the decision abil-

e positive region determined by condi-

ribute reduction is consistent *^C X U D B X Marlome reuncion of condition attribute set of <i>MARO 2116*

Attribute reduction of condition attribute set is a vital topic

data-diver fault diagnosis methods. It removes irrelevant

ordernal data and retains the decisi the optimal hundwidth
 Examples Simular Manufold Compute the class labels of
 the interperent function
 I Classification by using NNBC
 I Classification by using NNBC
 I Classification by using conditions
 Definition 5. Let $(U, A = C \cup D, V, f)$ and D is the positive region determined by condition attribute safter attribute reduction is consistent with the pare before attribute reduction. Consequently, to quantitatively measu 3.2 Attribute reduction based on MMG-DTRS

Attribute reduction of condition attribute set is a vital topic

in data-driven fault diagnosis methods. It removes irrelevant

or redundant condition attribute sad retains the d andant condition attributes and retains the decision abil-
an IS. That is, the positive region determined by conditing
thicutes after attribute reduction is consistent with the
fore attribute reduction. Consequently, to q

attribute set*C* with respect to decision attribute *D* in MMG-DTRS is defined as

$$
S_{C}(D) = \frac{\sum\left\{X \in U / \{D\} : \sum_{i=1}^{m} B_{i}(X) \right\}}{|U|}.
$$
 (14)

attribute c_i with respect to decision attribute D in MMG-DTRS is defined as **Definition 4.** Let $(U, A = C \cup D, V, f)$ be an IS in which $B_2, ..., B_m \subseteq C$, then the classification quality of condition
ibute set *C* with respect to decision attribute *D* in MMG-
RS is defined as
 $S_c(D) = \frac{\sum_{i=1}^{M} X \in U / \{D\} : \$

$$
Sig(c_i, C, D) = S_c(D) - S_{c-\{c_i\}}(D). \tag{15}
$$

Definition 6. Attribute c_i in condition attribute set C with re-
 $\begin{vmatrix} \text{An } I S = (U, A = C \cup D, V, f) \end{vmatrix}$, an attribute spect to decision attribute *D* in MMG-DTRS is not the lower subset $B_1, B_2, ..., B_m \subseteq C$, and loss function

overcomes the shortcoming of harsh constraint conditions of pessimistic MGRS model. Furthermore, the significance degree of condition attribute in MMG-DTRS can be used to quantitatively measure the influence of condition attribute with respect to the decision ability of an IS. Therefore, an attribute reduction algorithm based on MMG-DTRS is proposed to acquire a lower dimensional condition attribute set. The flowchart of the proposed algorithm is shown in Fig. 2, and the general procedures are summarized as follows.

Step 1: Calculate the threshold α according to the loss func-

If the significance degree $Sig(c_i, C, D) = 0$,

tion attribute set *C* with respect to decision attribute *D* in MMG-DTRS;

condition attribute c_1 with respect to decision attribute *D* in MMG-DTRS;

condition attribute c_1 is redundant, or else the condition attribute c_i is indispensable;

Step 5: Repeat steps 3 and 4 to the other condition attributes, till the last one;

Step 6: Remove all the redundant condition attributes and generate a lower dimensional condition attribute set.

3.3 Classification by using NNBC

From the definitions of NNBC, it can be observed that NNBC based on the optimal bandwidth selection ignores the independence assumption among attributes and replaces marginal PDF estimation by joint PDF estimation. From Eq. (5), it can be noted that the larger the class label is, the higher the probability that the sample *x* belongs to this class is. The class of a sample *x* can be determined through the maximum class label. Therefore, NNBC is used to classify rolling bearing conditions in test samples in this paper. The classification procedures of a test sample *x* by using NNBC are summarized as follows.

Input: A test sample x and training samples after attribute reduction.

Output: The class of the test sample *x* .
Step 1: The optimal bandwidth h_k for *k*th class and the Gaussian kernel function $k(x)$ are representatively calculated according to training samples by Eqs. (12) and (13);

Step 2: Compute the class labels of the test sample *x* by Eq. (11);

Step 3: Determine the class of the test sample *x* according to the maximum class label.

Fig. 2. Flowchart of the attribute reduction algorithm based on MMG-DTRS.

In the process of classification by NNBC, each class of the test sample *x* represents a rolling bearing condition. The class label of a sample x represents the probability that the sample *x* belongs to corresponding rolling bearing condition. The larger the class label is, the higher the probability that the sample *x* belongs to corresponding rolling bearing condition is. Consequently, the rolling bearing condition of the test sample *x* is determined through the maximum class label.

4. Experimental investigations

4.1 Experimental setup

Solution and the test since the system and the system of the system A rolling bearing test-rig was used to evaluate the proposed method. The rolling bearing test-rig is illustrated in Fig. 3. A variable speed motor is employed to drive a rotating shaft supported by two rolling bearings. The rotating shaft is connected with the drive motor by a coupling. The strategy reduces the transmission and misalignment influences. Three weights are used to apply radial load. One SR150M acoustic emission (AE) sensor is mounted on the bearing housing to acquire AE signals. In the experiment, there are seven rolling bearing conditions: normal condition (NC), an inner race fault with a fault sizes of 0.4 mm (IRF-0.4 mm), an inner race fault with a fault sizes of 0.8 mm (IRF-0.8 mm), an outer race fault with a fault sizes of 0.4 mm (ORF-0.4 mm), an outer race fault with a fault sizes of 0.8 mm (ORF-0.8 mm), and a rolling

Fig. 3. Rolling bearing test-rig.

Fig. 4. Seven rolling bearing conditions: (a) NC; (b) IRF-0.4 mm; (c) IRF-0.8 mm; (d) ORF-0.4 mm; (e) ORF-0.8 mm; (f) REF; (g) CF.

element fault (REF), and combined fault (CF: IRF-0.8 mm and ORF-0.8 mm), respectively. Seven rolling bearing conditions are illustrated in Fig. 4.

Radial load was used to simulate three operation states of rolling bearings. The rotation speeds of the drive motor were adjusted to 400 rpm, 800 rpm and 1200 rpm, respectively. AE signals of rolling bearings were acquired at a sampling frequency of 96 kHz. There were 10 samples in each rolling bearing operation state. Thus, 90 samples were collected for each bearing condition and there were a total of 630 samples. The AE signals of seven rolling bearing conditions are shown in Fig. 5.

4.2 Experimental results

By adding white noise, ensemble empirical mode decomposition (EEMD) can automatically project the composed components of a signal onto a uniform reference frame to solve mode mixing problem [37]. Consequently, it has been applied in extracting fault features of rolling bearings [38, 39]. In this experiment, EEMD is used to decompose the AE signals into several intrinsic mode functions (IMFs). Both time domain and frequency domain features are extracted to characterize rolling bearing conditions. These features contain standard deviation, kurtosis, shape factor and impulse factor in the time domain, and mean frequency, root mean square frequency, standard deviation frequency and spectrum peak ratio in the

Fig. 5. AE signals of seven rolling bearing conditions: (a) NC; (b) IRF-0.4 mm; (c) IRF-0.8 mm; (d) ORF-0.4 mm; (e) ORF-0.8 mm; (f) REF; (g) CF.

frequency domain. The first three maximum IMF values of each feature are regarded as fault features of rolling bearings. Accordingly, a total of 24 fault features are acquired to form condition attributes.

To validate the performance of the proposed method, 504 samples are regarded as training samples and 126 samples are used for test in this study. The ratio of training to test samples is 4:1. The attribute reduction algorithm based on MMG-DTRS is applied to acquire a lower dimensional condition attribute set from the training samples. The threshold α is set to 0.7. By using principle component analysis (PCA), the scatter plots of the training samples with the entire condition attributes and the ones with the lower dimensional condition attribute set are shown in Fig. 6. It can be observed that the lower dimensional condition attribute set provides better separation with a distinct clustering distribution among seven rolling bearing conditions. It is because the condition attributes in the acquired lower dimensional condition attribute set are

Condition	Ratio of training to test samples	Training accuracy (%)	Test accuracy (%)	Average α ccuracy $(\%)$
NC.	4:1	100	77.78	95.56
$IRF-0.4$ mm	4:1	100	83.33	96.67
$IRF-0.8$ mm	4:1	100	88.89	97.78
$ORF-0.4$ mm	4:1	100	88.89	97.78
$ORF-0.8$ mm	4:1	100	94.44	98.89
REF	4:1	100	83.33	96.67
CF	4:1	100	88.89	97.78

Table 1. Diagnosis accuracy of seven rolling bearing conditions.

Fig. 6. Scatter plots of the training samples: (a) With the entire condition attributes; (b) with the lower dimensional condition attribute set.

more sensitive during clustering performance, which avoids the interference of irrelevant or redundant condition attributes.

NNBC is constructed to classify rolling bearing conditions in the test samples. Diagnosis accuracy of seven rolling bearing conditions is listed in Table 1, where average accuracy is calculated according to all the samples of each rolling bearing condition. As listed in Table 1, the average accuracy of each condition is more than 95 %. Thus, the proposed method can accurately identify fault categories and fault severities of rolling bearings.

4.3 Comparison analyses

Toanalyze the influence of the threshold α on average accuracy**,** the attribute reduction algorithm based on MMG-DTRS and the other one based on DTRS are, respectively, applied to acquire lower dimensional condition attribute sets. Then, rolling bearing conditions in test samples are classified by using NNBC. Relation curves between average accuracy and the

Fig. 7. Relation curves between average accuracy and the threshold α .

Fig. 8. Diagnosis results of four methods.

threshold α are depicted in Fig. 7. It can be observed that average accuracy increases first and then decreases along with the increase of the threshold α . It is because smaller threshold values will lead to constraint conditions being loose, while irrelevant or redundant condition attributes are probably retained. Larger threshold values will lead constraint conditions to be harsh, while sensitive and essential condition attributes are probably removed. Accordingly, the best diagnostic performance can be achieved by reasonable threshold value selection in practical applications. Although average accuracy is influenced by the threshold α , the proposed method still possesses higher average accuracy than the other one based on DTRS. The reason is that MMG-DTRS can realize data analysis from multi-level view and describe a target concept through fusing multiple granular structures rather than a single one. Therefore, the proposed method is exactly suitable to identify various types of rolling bearing faults.

To further validate the effectiveness, the proposed method is compared to the traditional data-driven fault diagnosis methods (FL, SVM and DNN) using the same samples. In this experiment, fuzzy toolbox available in MATLAB 2016a is used to construct the fuzzy inference engine. Fault diagnosis based on SVM is performed by MATLAB SVM toolbox, whose penalty parameter C and Gaussian kernel parameter γ are set to 120 and 10, respectively. A three-layer DNN is applied and the number of hidden layer nodes is set as 100. The ratios of training to test samples are 2:1, 3:1 and 4:1, respectively. Fig. 8 illustrates diagnosis results of four methods. It can be noted that the average accuracy of four methods increases along with the ratio enhancement. The higher the ratio of training to test samples is, the better the diagnosis effectiveness is. Compared with the other methods, the proposed

method always achieves the highest average accuracy with three ratios. This is because more training samples possess more diagnosis information, which can realize more accurate determination of the model structure and parameters. Besides, the lower dimensional condition attribute set provides better separation with a distinct clustering distribution among rolling bearing conditions. In addition, NNBC ignores the independence assumption among attributes and replaces marginal PDF estimation by joint PDF estimation. We suggest using the proposed method under the following condition. A great number of vibration signals can be acquired from historical records. Moreover, these historical records contain various conditions of rolling bearings.

4.4 Discussions

(1) According to the above experimental results, the diagnosis accuracy of the proposed method obviously outperforms the traditional data-driven fault diagnosis methods. The main reasons are summarized as follows. The attribute reduction algorithm based on MMG-DTRS is applied to acquire sensitive condition attributes, which can maximize classification separation and avoid the interference of irrelevant or redundant condition attributes. Also, NNBC based on the optimal bandwidth selection ignores the independence assumption among attributes and replaces marginal PDF estimation by joint PDF estimation.

(2) In the comparison analyses, the proposed method obtains higher average accuracy than the other method based on DTRS, despite the influence of the threshold α . The reasons are that MMG-DTRS retains fault-tolerant ability of DTRS, realizes data analysis from multi-level view, and overcomes the shortcoming that constraint conditions of pessimistic MGRS model are excessively harsh. In addition, the attribute reduction algorithm based on MMG-DTRS can remove redundant condition attributes and decrease the number of random variables in NNBC. Thus, the proposed method is capable of improving diagnosis accuracy and reducing computational burden.

(3) In the experimental investigations, seven rolling bearing conditions covering typical fault categories and different fault severities are used to validate the performance of the proposed method. Accordingly, it is a representative case of rolling bearing fault diagnosis. According to the comparison analyses, the comparison results demonstrate the effectiveness, reliability and generalization ability of the proposed method. Thus, this method is exactly suitable to identify various types of rolling bearing faults.

5. Conclusions

A novel fault diagnosis method of rolling bearings combining MMG-DTRS with NNBC was put forward. MGRSs, MMG-DTRS, and NNBC were first reviewed. Then, the fault diagnosis framework was illustrated. A fault diagnosis experiment of rolling bearings was utilized to validate the performance of this method. The comparison analyses demonstrate that the diagnosis accuracy of this method is much superior to that of the traditional methods. Besides, the proposed method is capable of improving diagnosis accuracy and reducing computational burden. Therefore, it is effective and reliable in identifying fault categories and fault severities of rolling bearings.

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