

# Adaptive control of a vehicle-seat-human coupled model using quasi-zero-stiffness vibration isolator as seat suspension†

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#### **Abstract**

We propose the quasi-zero-stiffness (QZS) vibration isolator as seat suspension to improve vehicle vibration isolation performance. The QZS vibration isolator is composed of vertical spring and two symmetric negative stiffness structures used as stiffness correctors. A vehicle-seat-human coupled model considering the QZS vibration isolator is established as a three degree-of-freedom (DOF) model; it is composed of a quarter car model and a simplified 1 DOF model combined vehicle seat and human body. This model considers the changing mass of the passengers and sets the total mass of the vehicle seat and human body as an uncertain parameter, which investigates the overload and unload conditions in practical engineering. To further improve the vehicle ride comfort, a constrained adaptive back stepping controller law based on the barrier Lyapunov function (BLF) is presented. The dynamic characteristic of the active vehicle-seat human coupled model under shock excitation was analyzed using numerical method. The results show that the designed controller law can isolate the shock excitation transmitted from the road to the passengers effectively, and both the vehicle and seat suspension strokes remain in the allowed stroke range.

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*Keywords*: Adaptive control; Quasi-zero-stiffness; Seat suspension; Vehicle-seat-human coupled model; Vibration isolator

# **1. Introduction**

In practical engineering, a low frequency vibration seriously affects human health and reduces working efficiency. Seat suspension can be used to attenuate high amplitude vibration transmitted from the road to the passengers in low frequency range and improve vehicle ride comfort [1]. As is well known, a linear seat suspension can provide an effective isolation when the excitation frequencies are larger than  $\sqrt{2}$  -times the natural frequency of the vibration system. Reducing spring stiffness can lead to a wider isolation frequency band, but results in a lower load bearing capacity and a larger static displacement between the seat and the vehicle floor. However, this dilemma can be eliminated by using quasi-zero-stiffness (QZS) vibration isolator [2, 3] as seat suspension. The QZS vibration isolator is composed of a load bearing elastic element providing positive stiffness and special mechanisms providing negative stiffness named as stiffness correctors. The load bearing elastic element is usually the vertical spring; after being loaded at the static equilibrium position, the positive stiffness of the vertical spring is exactly balanced by the negative stiffness provided by the stiffness correctors, then a small dynamic stiffness can be achieved to obtain a lower natural frequency and a wider isolation frequency band. So the QZS vibration isolator can have a high static stiffness with a small static displacement without sacrificing the load bearing capacity and a small dynamic stiffness to achieve a low natural frequency, which is superior to the linear vibration isolator. Many researchers have proposed various different types of stiffness correctors. Carrella et al. [4, 5], Kovacic et al. [6] and Wang et al. [7] considered a QZS vibration isolator by using inclined springs as stiffness correctors and investigated the static and dynamic characteristics theoretically. Robertson et al. [8], Zhou et al. [9] and Xu et al. [10] used electromagnetic or magnetic springs as stiffness correctors to build a QZS vibration isolator and studied the static and dynamic characteristics in detail. Liu et al. [11, 12] designed a QZS vibration isolator by using Euler buckled or sliding beams as stiffness correctors and analyzed the dynamic behavior theoretically. Shaw et al. [13] used a bistable composite plate as stiffness correctors to form a QZS vibration isolator and analyzed the dynamic response theoretically and experimentally. Ahn et al.

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[14] designed a QZS vibration isolator by using cam-roller mechanism as stiffness correctors and found the desired QZS characteristics can be achieved with properly designed cam geometry.

Le and Ahn [15] built a QZS vibration isolator composed of a positive stiffness mount and two symmetric negative stiffness structures for improving the vehicle seat isolation performance. They also designed a fuzzy sliding mode controller [16] or an adaptive intelligent back-stepping controller [17] to further improve the performance. In their studies, road excitation is applied to the vehicle floor, which is unsuitable for some practical engineering cases, because road excitation is filtered by vehicle suspension in both amplitude and frequency when transmitted to the vehicle floor [18]. Also, when loading a specific designed mass, the dynamic stiffness of the QZS vibration isolator is considered to be zero at the static equilibrium position, then the QZS characteristic can be obtained; the force-displacement and stiffness-displacement relationships about the static equilibrium position are symmetric. In fact, the static equilibrium position of the QZS vibration isolator is sensitive to the loaded mass; when the QZS vibration isolator loads a mass larger or smaller than the specific designed mass, the system would not be balanced at the original static equilibrium position, and the force-displacement and stiffness-displacement relationships about the new static equilibrium position would be asymmetric. So for the vehicle seat, when the mass of the passengers varies, the actual mass of the passengers can hardly match the specific designed mass, so the QZS vibration isolator would be in the overload or unload conditions. In addition, they neglected to investigate the vehicle and seat suspension strokes, which are two main important factors for the vehicle and seat suspensions design. So here we propose the QZS vibration isolator as seat suspension, con sider the overload and unload conditions, and apply road excitation directly to the vehicle wheel, which are more close to the actual conditions.

In the author's previous paper [19], a vehicle-seat-human coupled model considering the QZS vibration isolator is established as a 8 degree-of-freedom (DOF) model that is com posed of a quarter car model [20] (2 DOF), a seat suspension model (2 DOF) and a human body model [21] (4 DOF). Although the quarter car model is simple, it can provide qualitatively correct information for vehicle ride comfort studies in low frequency range. The seat suspension model includes two parts-seat frame and seat cushion. The human body model contains four important human body parts--thighs, lower torso, upper torso and head. In this paper, in order to consider the changing mass of the passengers, the vehicle-seat-human cou pled model is established as a 3 DOF model that is composed of a quarter car model (2 DOF) and a simplified 1 DOF model combined vehicle seat and human body. The total mass of the vehicle seat and human body is considered as an uncertain parameter, which can investigate the overload and unload conditions.

In Ref. [19], the dynamic characteristic of the vehicle-seat-



Fig. 1. Scheme of a QZS vibration isolator.

human coupled model under road shock and random excitations is analyzed using numerical method. The results show that when the QZS vibration isolator is used as seat suspen sion, the vehicle ride comfort improves effectively compared to the linear seat suspension, and the vehicle and seat suspen sion strokes remain in the allowed stroke range. When the vehicle-seat-human coupled model subjects to road shock excitation, as the road excitation displacement or the vehicle forward velocity increases, the peak acceleration value of human body increases for both kinds of seat suspensions, so the vehicle ride comfort becomes poorer. Since the vehicle seat-human coupled model proposed in this paper is an uncertain nonlinear system, and also to further improve the vehicle ride comfort, a constrained adaptive back-stepping controller law [22] based on the barrier Lyapunov function (BLF) [23, 24] is presented. This control law can achieve a good performance when nonlinearity and uncertain parameters exist in the dynamic model.

This paper is organized as follows. A QZS vibration isolator is presented and brief description of static analysis is shown in Sec. 2. In Sec. 3, the active vehicle-seat-human coupled model of 3 DOF using QZS vibration isolator as seat suspension is established. The constrained adaptive back-stepping controller law is presented in Sec. 4. In Sec. 5, the dynamic characteristic of this active vehicle-seat-human coupled model subject to shock excitation is obtained using numerical method, and the usefulness of the controller law is addressed. In Sec. 6, the comparison of the control effect for the active model with QZS and linear vibration isolators used as seat suspensions is discussed. Sec. 7 concludes the paper.

# **2. QZS vibration isolator model**

A QZS vibration isolator composed of vertical spring and two symmetric negative stiffness structures used as stiffness correctors is shown in Fig. 1. This model has been studied in detail in Ref. [25], so only brief description is presented here.

Fig. 1 shows when loading a mass *m*, the system is balanced at the static equilibrium position-equilibrium position 1. The stiffness of the vertical and horizontal springs is  $K_v$  and  $K_h$ ; the initial length of the horizontal springs is  $L_0$ ; the compression deformation of the horizontal springs when the system at the static equilibrium position is  $\lambda$ ; the length of the connecting element is *L*; *x* is the displacement of the mass from the



Fig. 2. Non-dimensional force-displacement and stiffness-displacement curves.

static equilibrium position and  $y$  is the displacement of the base excitation. The force-displacement and stiffness displacement relationships of the system are given as

Fig. 2. Non-dimensional force-displacement and stiffness-displacement  
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\begin{aligned}\n&K = (K_v - 2K_h) + \frac{2K_h(L - \lambda)L^2}{(L^2 - u^2)^{\frac{3}{2}}} &\text{(1) smaller than or equal to } \\&\text{was considered.} &\text{If the QZS vibration is\nform as} &\text{gent stiffness position at\ntion-equilibrium position 1\nunion of the QZS vibration is\ngent stiffness position at\ntion-equilibrium position 2\n
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\hat{F} = (1 - 2k)\hat{u} + 2k(1 - \hat{\lambda})\frac{\hat{u}}{\sqrt{1 - \hat{u}^2}}
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\begin{aligned}\n&2k(1 - \hat{\lambda}) &\text{(2)} &\text{to 3. The force-displacement in the image.}\n\end{aligned}
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Fig. 2. Non-dimensional force-displacement and stiffness-displacement  
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where  $\hat{u} = u/L$ ,  $\hat{\lambda} = \lambda/L$ ,  $k = K_n/K_v$ ,  $\hat{F} = F/(K_vL)$ ,  
 $\hat{K} = K/K_v$ .  
The active vehicle-**seat-human**  
if the stiffness of the QZS vibration isolator is zero at the  
triangle, then  $\hat{\lambda}_{\text{gen}} = 1/(2k)$ .  
if the differences of the QZS vibration isolator for various  
displacement curves of the QZS vibration isolator for various  
displacement curves of the QZS vibration isolator for various  
values of  $\hat{\lambda}$  when  $k = 1$  are shown in Fig. 2. It can be  
clearly seen that if  $\hat{\lambda} = \hat{\lambda}_{\text{gen}}$ , the positive stiffness provided by the  
respectively;  $c_1$  is the changing and stiffness of  
real spring is balanced by the negative stiffness provided by the  
respectively

 $\hat{F} = F/(K_v L)$ , 3. ACT  $\hat{K} = K / K_{v}$ .

If the stiffness of the QZS vibration isolator is zero at the static equilibrium position, the QZS characteristic can be obtained, then  $\hat{\lambda}_{qzs} = 1/(2k)$ .

values of  $\hat{\lambda}$  when  $k=1$  are shown in Fig. 2. It can be body, respectively, and  $m_3 = m_{31} + m_{32}$ ;  $c_1$ ,  $c_2$  and  $k_1$ ,  $k_2$ where  $u = x - y$ . Eq. (1) can be written in non-dimensional load to 2XS vibration isolations and the original term<br>
form as<br>  $\hat{K} = (1 - 2k)\hat{u} + 2k(1 - \hat{\lambda})\frac{\hat{u}}{\sqrt{1 - \hat{u}^2}}$ <br>  $\hat{K} = (1 - 2k) + \frac{2k(1 - \hat{\lambda})}{\sqrt{1 - \hat{u}^2}}$ <br> clearly seen that if  $\hat{\lambda} = \hat{\lambda}_{\text{max}}$ , the positive stiffness of the vertical spring is balanced by the negative stiffness provided by the stiffness correctors at the static equilibrium position, so the



Fig. 3. Original and new equilibrium positions of the QZS vibration isolator.

 $\left( L - \sqrt{L^2 - u^2} \right)$   $\sqrt{\frac{L^2 - u^2}{L^2 - u^2}}$  rium position. So to keep the stiffness positive,  $\hat{\lambda}$  should be stiffness-displacement<br>
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isolator maintain **Example 19 and 19** *L u*  $\lambda$ ) L<sup>2</sup> (1) smaller than or equal to  $\hat{\lambda}_{q_{\text{gs}}}$ . In this study, QZS characteristic can be achieved. If  $\hat{\lambda} > \hat{\lambda}_{\alpha s}$ , the stiffness is negative in the neighborhood of the static equilibrium position, then the system can be unstable, which is an undesirable condition. When  $\hat{\lambda} < \hat{\lambda}_{\text{gas}}$ , the stiffness of the QZS vibration isolator maintains a small positive value at the static equilibequilibrium position<br>  $\frac{0}{\hat{u}}$  0.2 0.4 0.6<br>
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ion, then the system can be unstable, which is an undesirable<br>
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inum position. So If the QZS vibration isolator is loaded with a mass smaller or larger than the specific designed mass, the mass will not load the QZS vibration isolator statically to its minimum tan gent stiffness position at the original static equilibrium position-equilibrium position 1, the QZS vibration isolator is in the unload or overload conditions, and the new static equilibrium tion 3. The force-displacement and stiffness-displacement relationships about this new static equilibrium position would be asymmetric, as can be clearly seen in Fig. 3.

#### **3. Active vehicle-seat-human coupled model**

The active vehicle-seat-human coupled model of 3 DOF using QZS vibration isolator as seat suspension is shown in Fig. 4; it is composed of a quarter car model (2 DOF) and a simplified 1 DOF model combined vehicle seat and human body:  $m_1$  is the mass of wheel,  $m_2$  is the mass of vehicle body,  $m_{31}$  and  $m_{32}$  are the masses of the vehicle seat and human for larger than the specific designed mass, the mass will not<br>lood the QZS vibration isolator statically to its minimum tan-<br>equalibrium position at the original static equilibrium posi-<br>tion-equilibrium position 1, the Q are the damping and stiffness of the wheel and vehicle body, respectively;  $c_3$  is the damping of the vehicle seat, and  $F_{s3}$ is the spring force of the vehicle seat;  $z_{1-3}$  are the displace-



Fig. 4. Active vehicle-seat-human coupled model.

ments of the corresponding masses,  $z_r$  is the road excitation displacement and is applied directly to the vehicle wheel; *U<sup>s</sup>* is the active input of the seat suspension system.

Denote  $m_{s3}$  as the specific designed mass of the vehicle seat and human body, when  $m_3 = m_{s3}$ , the dynamic equations of the active vehicle-seat-human coupled model in the corresponding static equilibrium positions of the masses are given by or the corresponding masses,  $z_r$  is the load exchation<br>cement and is applied directly to the vehicle wheel;  $U_s$ <br>active input of the seat suspension system.<br>note  $m_{s3}$  as the specific designed mass of the vehicle<br>and h

$$
\begin{array}{ll}\n\text{where } u_x = 0 \text{ (where } u_x = 0 \text{), } u_x = 0 \text{ (where } u_x = 0 \text{), and } u_y = 0 \text{ (where } u_y = 0 \text{), and } u_y = 0 \text{ (where } u_y = 0 \text{), and } u_y = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and } u_z = 0 \text{ (where } u_y = 0 \text{), and
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When the QZS and linear vibration isolators are used as seat suspension respectively,  $F_{s3}$  can be obtained as

$$
\begin{cases}\n m_2 \ddot{z}_2 = c_3 (z_3 - \dot{z}_2) + F_{s3} - c_2 (z_2 - \dot{z}_1) & (3) \text{ where } \mathcal{G} = 1 \\
 -k_2 (z_2 - z_1) - U_s(t) & \text{ and human term} \\
 m_3 \ddot{z}_3 = -c_3 (z_3 - \dot{z}_2) - F_{s3} + U_s(t).\n\end{cases}
$$
\nWhen the QZS and linear vibration isolators are used as seat when the QZS and linear vibration isolators are used as seat when the following expression respectively,  $F_{s3}$  can be obtained as where  $\mathcal{G}_{\text{min}}$  uncertainty in the linear term  $\mathcal{G} = \Omega_{\mathcal{G}} = \left\{ \begin{aligned}\n F_{s3} &= K_v (1 - 2k)(z_3 - z_2) & \text{where } \mathcal{G}_{\text{min}} \\
 &+ 2kK_v L (1 - \hat{\lambda}) \frac{(z_3 - z_2)}{\sqrt{L^2 - (z_3 - z_2)^2}} & \text{where } \mathcal{G}_{\text{min}} \\
 F_{s3} &= K_v (z_3 - z_2)\n \end{aligned}\n\right\}$ 

where  $K_v$  is the stiffness of linear seat suspension. In this case, the mass  $m<sub>3</sub>$  loads the QZS vibration isolator statically to its minimum tangent stiffness position at the static equilibrium position; the force-displacement and stiffness displacement relationships about the static equilibrium position are symmetric, which can be clearly seen in Figs. 1 and 3. When  $m_3$  is larger or smaller than the specific designed mass  $m_{s3}$ , the seat suspension system will not be balanced at the c original static equilibrium position,  $F_{s3}$  can be given in a general form, Eq. (4) can be written as

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\n
$$
\begin{cases}\nF_{s3} = K_v (1 - 2k)(z_3 - z_2 + u_n) - (m_3 - m_{s3})g \\
+2kK_v L(1 - \hat{\lambda}) \frac{(z_3 - z_2 + u_n)}{\sqrt{L^2 - (z_3 - z_2 + u_n)^2}}\n\end{cases}
$$
\n(5)  
\n
$$
F_{s3} = K_v (z_3 - z_2 + u_n) - (m_3 - m_{s3})g
$$
\nwhere  $u_n$  is the new static equilibrium position. When  $m_3 = m_{s3}$ ,  $u_n = 0$ ; when  $m_3 \neq m_{s3}$ ,  $u_n$  can be determined by solving Eq. (1) using the numerical method.  
\nDefine the following state variables  
\n
$$
\begin{cases}\nz_1 = x_1 & z_1 = x_2 & z_2 = x_3 \\
z_2 = x_4 & z_3 = x_5 & z_3 = x_6\n\end{cases}
$$
\neel.  
\nEq. (3) can be written in the state-space form  
\nis the road excitation  
\ne vehicle wheel;  $U_s$   
\n
$$
\begin{cases}\n\dot{x}_1 = x_2 \\
\dot{x}_2 = x_4 & z_3 = x_5 \\
1\end{cases}
$$
\n
$$
\begin{cases}\n\dot{x}_1 = x_2 \\
1\end{cases}
$$
\n(6)

 $m_3 = m_{s3}$ ,  $u_n = 0$ ; when  $m_3 \neq m_{s3}$ ,  $u_n$  can be determined by solving Eq. (1) using the numerical method.  $\sqrt{L^2 - (z_3 - z_2 + u_n)^2}$ <br>  $F_{s3} = K_v (z_3 - z_2 + u_n) - (m_3 - m_{s3}) g$ <br>
here  $u_n$  is the new static equilibrium position. When<br>  $v_3 = m_{s3}$ ,  $u_n = 0$ ; when  $m_3 \neq m_{s3}$ ,  $u_n$  can be determined by<br>
lving Eq. (1) using the numerical m where  $u_n$  is the new static equilibrium position. When

Define the following state variables

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z_1 = x_1 \t z_1 = x_2 \t z_2 = x_3 \n\dot{z}_2 = x_4 \t z_3 = x_5 \t \dot{z}_3 = x_6.
$$
\n(6)

Eq. (3) can be written in the state-space form

$$
\lim_{m_1} \frac{\text{Herm on the 1}}{\text{Vouley Bous Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 2}}{\text{Vouley Bous}} \int_{r_2}^{r_3} \frac{\text{Herm on the 3}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 4}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_2}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r_1}^{r_2} \frac{\text{Herm on the 1}}{\text{Vouley Bous}} \int_{r
$$

where  $\theta = 1/m$ , denotes the total mass of the vehicle seat and human body is an uncertain parameter. Assume that the extent of the uncertain parameter  $\theta$  is known and satisfies the following relationship:

$$
\mathcal{G} \in \Omega_{g} = \left\{ \mathcal{G} : \mathcal{G}_{\min} \le \mathcal{G} \le \mathcal{G}_{\max} \right\},\tag{8}
$$

 $(z_3 - z_2)$  seat in Eq. (7) using the state variables can be written in the **POSITIONS** of the masses are given<br>  $\begin{cases}\n\dot{x}_4 = \frac{1}{m_2} \left[ \frac{k_3 (x_6 - x_4) + r_{33} - c_2 (x_4 - x_3)}{k_2 - k_1} \right]\n\dot{x}_5 = x_6\n\end{cases}$ <br>  $z_2 - z_1$ <br>  $z_3 - z_2$ <br>  $(z_4 - z_1)$ <br>  $(z_5 - z_2)$ <br>
(*c*) and human body is an uncertain parameter. A where  $\mathcal{G}_{\text{min}}$  and  $\mathcal{G}_{\text{max}}$  are the lower and upper bounds of the uncertain parameter  $\theta$  and are assumed to be known parameters. The nonlinear and linear spring forces of the vehicle general form as  $m_3$  is an uncertain parameter:  $\Omega_g = \{ \mathcal{G} : \mathcal{G}_{min} \leq \mathcal{G} \leq \mathcal{G}_{max} \},$  (8)<br>  $\mathcal{G}_{min}$  and  $\mathcal{G}_{max}$  are the lower and upper bounds of the<br>
ain parameter  $\mathcal{G}$  and are assumed to be known pa-<br>
rs. The nonlinear and linear spring forces of the  $s \Omega_g = \{ \mathcal{G} : \mathcal{G}_{min} \leq \mathcal{G} \leq \mathcal{G}_{max} \},$  (8)<br> **s**  $\mathcal{G}_{min}$  and  $\mathcal{G}_{max}$  are the lower and upper bounds of the<br>
tain parameter  $\mathcal{G}$  and are assumed to be known pa-<br>
ers. The nonlinear and linear spring forces

$$
\begin{vmatrix}\n\dot{x}_s = x_6 \\
\dot{x}_6 = g(-c_3(x_6 - x_4) - F_{s3} + U_s(t))\n\end{vmatrix}
$$
\nHere  $g = 1/m_3$  denotes the total mass of the vehicle seat  
\nd human body is an uncertain parameter. Assume that the  
\ntent of the uncertain parameter  $g$  is known and satisfies  
\n $\varepsilon$  following relationship:  
\n $g \in \Omega_g = \{g : \mathcal{G}_{min} \leq g \leq \mathcal{G}_{max}\},$  (8)  
\nhere  $\mathcal{G}_{min}$  and  $\mathcal{G}_{max}$  are the lower and upper bounds of the  
\ncertain parameter  $g$  and are assumed to be known pa-  
\nmeters. The nonlinear and linear spring forces of the vehicle  
\nat in Eq. (7) using the state variables can be written in the  
\nneral form as  $m_3$  is an uncertain parameter:  
\n
$$
\begin{cases}\nF_{s3} = K_v \left(1 - 2k\right) \left(x_s - x_3 + u_n\right) - \left(m_3 - m_{s3}\right)g \\
+ 2kK_v L(1 - \hat{\lambda}) \frac{\left(x_s - x_3 + u_n\right)}{\sqrt{L^2 - \left(x_s - x_3 + u_n\right)^2}}\n\end{cases}
$$
 (9)  
\nFor the active seat suspension system, there are two main  
\nfromance requirements that should be considered in the  
\nntroller design: (1) The vehicle ride comfort should be im-  
\noved effectively, which means the designed controller

For the active seat suspension system, there are two main performance requirements that should be considered in the controller design: (1) The vehicle ride comfort should be im proved effectively, which means the designed controller should isolate vibrations transmitted from the road to the pasand passengers, although the uncertain parameter exists in the system. (2) When designing the controller, the vehicle and seat suspension strokes should be kept in a certain range to prevent the suspensions from hitting the limit block and de-*Y. Wang et al. / Journal of Mechanical Science*<br>sengers and stabilize the vertical motion of the vehicle body<br>and passengers, although the uncertain parameter exists in the<br>system. (2) When designing the controller, the rs and stabilize the vertical motion of the vehicle body<br>assengers, although the uncertain parameter exists in the<br>n. (2) When designing the controller, the vehicle and<br>uspension strokes should be kept in a certain range *<sup>s</sup> z z x x z z. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
agers and stabilize the vertical motion of the vehicle body where  $\delta_s$  is a positive constant at<br>
the following Lyapunov function<br> *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-298*<br>
ggers and stabilize the vertical motion of the vehicle body where  $\delta_i$  is a positive constant<br>
passengers, although the uncertain pa

$$
\begin{cases}\n|z_2 - z_1| = |x_3 - x_1| \le z_{s1\text{max}} \\
|z_3 - z_2| = |x_s - x_3| \le z_{s1\text{max}}\n\end{cases}
$$
\n(10)

where  $z_{s1\text{max}}$ ,  $z_{s2\text{max}}$  are the maximum vehicle and seat suspension strokes, respectively.

# **4. Controller design**

To improve the vehicle ride comfort and maintain the vehicle and seat suspension strokes in a certain range, the controller *U<sup>s</sup>* is then designed. The constrained adaptive back stepping controller law is presented. It is convenient to use this (16), the following equation can be obtained: controller law when the nonlinearity and uncertain parameter exist in the system; this controller law is based on the BLF and can achieve a less conservatism than the classic quadratic Lyapunov function (QLF) in the controller design [22]. The constrained adaptive back-stepping controller based on BLF is composed of four main steps, which are shown as follows. if  $\frac{1}{2}$  of  $\frac{1}{2}$  where  $z_{\text{d,max}}$ ,  $z_{\text{d,max}}$  are the maximum vehicle and seat sus-<br>pension strokes, respectively.<br> **4. Controller design**<br> **4 Controller design**<br>
Fo improve the vehicle ride confort and maintain the vehi-<br>
and seat suspension strokes in a certain range, the control-<br> *U<sub>s</sub>* is then designed. The constrained adaptive back-<br> *U<sub>s</sub>* is then design

function and the virtual control function, which can be given The second derivative function of function  $V_5(e_5(t))$  is by

$$
e_{6}(t) = x_{6}(t) - \alpha(t) \tag{11}
$$

these two functions, which is given as

$$
e_{s}(t) = x_{s}(t) - x_{s}(t) \tag{12}
$$

Combining Eqs. (7), (11) and (12), the derivative function equality error function  $e_6(t)$  can be obtained as

$$
\dot{e}_s(t) = \dot{x}_s(t) - \dot{x}_{s_r}(t) = e_s(t) + \alpha(t) - \dot{x}_{s_r}(t) \,. \tag{13}
$$

The main objective of this step is to design the virtual control function  $\alpha(t)$  to ensure that the tracking trajectory error  $e_s(t) = x_s(t) - \alpha(t)$ . (11) also bounded,  $\vec{r}_s(e_t(t))$ <br>
Denote the reference trajectory function  $x_s(t)$  to the Lyapunov-like 1<br>
center trajectory of the vertical displacement function  $x_s(t)$  to the Lyapunov-like 1<br>
converge to stroke is maintained in the allowed range; that is, the vertical parameter g and let the error function  $e_g(t)$  as the estima-Denote the reference trajectory function  $x_s(t)$  as the reference trajectory of the vertical displacement function  $x_s(t)$  to the net tracking trajectory error function and let the error function  $e_s(t)$  as the difference be the error function  $ε_s(t)$  as the difference between<br>
the error function  $ε_s(t)$  as the difference between<br>
se two functions, which is given as<br>
to converge to zer<br>
set wo functions, which is given as<br>  $s_f(t) = x_s(t) - x_{s_f}(t)$ 

$$
\left|x_{\mathfrak{s}}(t)\right| < \delta_{\mathfrak{s}}\,,\tag{14}
$$

sengers and stabilize the vertical motion of the vehicle body where  $\delta_{s}$  is a positive constant and satisfies  $\delta_{s} > \varepsilon_{s}$ . Define the following Lyapunov function

and Technology 32 (7) (2018) 2973~2985  
\nwhere 
$$
\delta_s
$$
 is a positive constant and satisfies  $\delta_s > \varepsilon_s$ . Define  
\nthe following Lyapunov function  
\n
$$
V_s(e_s(t)) = \frac{1}{2} \ln \frac{\Delta_s^2}{\Delta_s^2 - e_s^2(t)}
$$
\nwhere  $\Delta_s = \delta_s - \varepsilon_s$ . Then the derivative function of function  
\n $V_s$  is given as  
\n
$$
\therefore (f_s(t)) = \frac{e_s(t) \dot{e}_s(t)}{e_s(t)} = \frac{e_s(t) (e_s(t) + \alpha(t) - \dot{x}_{s}(t))}{e_s(t)}
$$
\n(16)

 $V<sub>5</sub>$  is given as

and Technology 32 (7) (2018) 2973-2985  
\n2977  
\nwhere 
$$
δs
$$
 is a positive constant and satisfies  $δs > εs$ . Define  
\nthe following Lyapunov function  
\n
$$
Vs(es(t)) = \frac{1}{2} \ln \frac{Δs2}{Δs2 - es2(t)},
$$
\n(15)  
\nwhere  $Δs = δs - εs$ . Then the derivative function of function  
\n
$$
Vs is given as
$$
\n
$$
\dot{V}s(es(t)) = \frac{es(t)\dot{e}s(t)}{Δs2 - es2(t)} = \frac{es(t)(es(t) + α(t) - \dot{x}s(t))}{Δs2 - es2(t)}.
$$
\n(16)  
\nIf the virtual control function  $α(t)$  is chosen as  
\n
$$
α(t) = \dot{x}s(t) - ri(Δs2 - es2(t))es(t),
$$
\n(17)  
\nwhere  $ri$  is a positive value. Substituting Eq. (17) into Eq.  
\n(16), the following equation can be obtained:

If the virtual control function  $\alpha(t)$  is chosen as

$$
\alpha(t) = \dot{x}_{s_r}(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t), \qquad (17)
$$

where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.

If the virtual control function 
$$
\alpha(t)
$$
 is chosen as  
\n
$$
\alpha(t) = \dot{x}_{s_r}(t) - r_i(\Delta_s^2 - e_s^2(t))e_s(t),
$$
\n(17)  
\nhere  $r_i$  is a positive value. Substituting Eq. (17) into Eq.  
\n6), the following equation can be obtained:  
\n
$$
\dot{V}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_i e_s^2(t).
$$
\n(18)  
\nIf the error function  $e_s(t) = 0$ ,  $\dot{V}_s(e_s(t)) = -r_i e_s^2(t) \le 0$ ,

**Step 1:** Design the virtual control function  $\alpha(t)$  as the which implies that  $V_s(e_s(t)) \leq V_s(e_s(0))$  and  $V_s(e_s(t))$  is and seat suspension strokes in a certain range, the control-<br>  $U$ , is the referred advantive back-<br>  $\vec{v}$ , is then the system and calabitating dard the reformation dard<br>  $\vec{v}$  is the ref-strained advantive back-<br>
acti Ler  $U_s$  is then designed. The constrained adaptive back-<br>where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.<br>extraption controller law is presented. It is convenient to use this controller law when the nonline stepping controller law is presented. It is convenient to use this (16), the following equation can be obtained controller law when the nonlinearity and uncertain parameter  $\vec{r}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r e_s^2(t)$ .<br>
Ly **Example 1** achieve a less conservatism than the classic quadratic  $V_s(e_i(t)) = \frac{s_i}{\Delta_s^2 - e_s^2(t)} - r e_s^2(t)$ <br>
appunov function (QLF) in the controller design [22]. The<br>
starsing daphive back-stepping controller based on BLF is<br> composed of four main steps, which are shown as follows.<br> **Solution**  $\alpha(t)$  as the difference bureal control function  $\alpha(t)$  and denote<br>
the error function  $\alpha(t)$  as the difference between the actual<br>
the error function **Example 1.** Design the virtual control function  $\alpha_l(t)$  as the which implies that  $V_i\{e_i(t)\} \geq V_i\{e_i(0)\}$  and  $V_i\{e_i(t)\}$  is also be the vertical speed function  $\alpha_l(t)$  and denote be vertical speed function  $\alpha_l(t)$  and  $\frac{1}{2} \ln \frac{\Delta_s^2}{\Delta_s^2 - e_s^2(t)}$ , (15)<br>  $-\varepsilon_s$ . Then the derivative function of function<br>  $\frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_{s_r}(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
control function  $\alpha(t)$  is chosen as<br>  $\bigg) - r_i(\Delta_s^2 - e_s^$ *V<sub>s</sub>*( $e_s(t)$ ) =  $\frac{1}{2} \ln \frac{x_s}{\Delta_s^2 - e_s^2(t)}$ , (15)<br>
here  $\Delta_s = \delta_s - \varepsilon_s$ . Then the derivative function of function<br>
is given as<br>  $\dot{V}_s(e_s(t)) = \frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_{s_r}(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
ff t  $\vec{e}_2 = \frac{1}{2} \ln \frac{1}{\Delta_s^2 - e_s^2(t)}$ , (15)<br>  $\vec{o}_s - \vec{e}_s$ . Then the derivative function of function<br>
as<br>  $= \frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_{s}(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
all control function  $\alpha(t)$  is chosen as iere  $\Delta_s = \partial_s - \varepsilon_s$ . Ihen the derivative function of function<br>
is given as<br>  $\dot{V}_s(e_s(t)) = \frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_{s}(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
If the virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t)$  $\dot{Z}(\rho(t)) = -re^2(t) < 0$ ive function of function<br>  $\frac{+\alpha(t) - \dot{x}_{s_r}(t)}{-e_s^2(t)}$ . (16)<br>
is chosen as<br>
(17)<br>
tuting Eq. (17) into Eq.<br>
(18)<br>  $\dot{V}_s(e_s(t)) = -r_i e_s^2(t) \le 0$ ,<br>
(0)) and  $V_s(e_s(t))$  is<br>  $\dot{e}_s(t)$  is also bounded.<br>
function  $V_s(e_s(t))$  is<br>
(t))  $-\$  $\hat{V}_s(e_s(t)) = \frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_{s}(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
If the virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t) = \dot{x}_{s}(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_i$  is a positive value. Substit  $\vec{v}_s(e_s(t)) = \frac{e_s(t)\dot{e}_s(t)}{\Delta_s^2 - e_s^2(t)} = \frac{e_s(t)(e_s(t) + \alpha(t) - \dot{x}_s(t))}{\Delta_s^2 - e_s^2(t)}$ . (16)<br>
If the virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t) = \dot{x}_{s_r}(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_i$  is a positive value. Substit bounded, so  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded. The virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t) = \dot{x}_s(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_1$  is a positive value. Substituting Eq. (17) into Eq.<br>
(16), the following equation can be obtained:<br>  $\dot{V}_s(e_s(t)) = \frac{e_s$ If the virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t) = \dot{x}_{s_r}(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.<br>
(16), the following equation can be obtained:<br>  $\dot{v}_s(e_s(t)) = \$ If the virtual control function  $\alpha(t)$  is chosen as<br>  $\alpha(t) = \dot{x}_{s} (t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.<br>
16), the following equation can be obtained:<br>  $\dot{V}_s(e_s(t)) = \frac$  $\ddot{V}_s(e_s(t)) = -2r_1e_s(t)\dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_1e_s(t)\dot{e}_s(t)$  is  $\alpha(t) = \dot{x}_s(t) - r_1(\Delta_s^2 - e_s^2(t))e_s(t)$ , (17)<br>
where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.<br>
(16), the following equation can be obtained:<br>  $\dot{V}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_i e_s^2(t)$ . (18)<br>
If the error fun  $\dot{V}_s(e_s(t))$  is uniformly continuous. According  $\alpha(t) = x_s$ ,  $(t) - r_1(\Delta_s - e_s(t))e_s(t)$ , (17) into Eq.<br>
(16), the following equation can be obtained:<br>  $\dot{V}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_1e_s^2(t)$ . (18)<br>
If the error function  $e_s(t) = 0$ ,  $\dot{V}_s(e_s(t)) = -r_1e_s^2(t) \le 0$ ,<br>
which impl  $\dot{\mathcal{L}}_s(e_s(t)) \to 0$ , where  $r_i$  is a positive value. Substituting Eq. (17) into Eq.<br>
(16), the following equation can be obtained:<br>  $\vec{r}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_i e_s^2(t)$ . (18)<br>
If the error function  $e_s(t) = 0$ ,  $\vec{r}_s(e_s(t)) = -r_i e_s^2(t) \le 0$ to converge to zero asymptotically.  $\dot{V}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_i e_s^2(t)$ . (18)<br>
If the error function  $e_s(t) = 0$ ,  $\dot{V}_s(e_s(t)) = -r_i e_s^2(t) \le 0$ ,<br>
which implies that  $V_s(e_s(t)) \le V_s(e_s(0))$  and  $V_s(e_s(t))$  is<br>
bounded, so  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also  $\vec{v}_s(e_s(t)) = \frac{e_s(t)e_s(t)}{\Delta_s^2 - e_s^2(t)} - r_i e_s^2(t)$ . (18)<br>
If the error function  $e_s(t) = 0$ ,  $\vec{v}_s(e_s(t)) = -r_i e_s^2(t) \le 0$ ,<br>
which implies that  $V_s(e_s(t)) \le V_s(e_s(0))$  and  $V_s(e_s(t))$  is<br>
bounded, so  $e_s(t)$  is bounded, and  $\vec{e}_s(t)$  is also If the error function  $e_6(t) = 0$ ,  $\dot{V}_3(e_5(t)) = -r_1e_5(t) \le 0$ ,<br>which implies that  $V_3(e_5(t)) \le V_3(e_5(0))$  and  $V_3(e_5(t))$  is<br>bounded, so  $e_5(t)$  is bounded, and  $\dot{e}_5(t)$  is also bounded.<br>The second derivative function of f ch implies that  $V_s(e_s(t)) \le V_s(e_s(0))$  and  $V_s(e_s(t))$  is<br>
aded, so  $e_s(t)$  is bounded, and  $\hat{e}_s(t)$  is also bounded.<br>
second derivative function of function  $V_s(e_s(t))$  is<br>  $e_s(t) = -2r_i e_s(t)\hat{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t)\hat{e}_s(t)$  *so*  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded.<br>
d derivative function of function  $V_s(e_s(t))$  is<br>  $\frac{-2r_i e_s(t) \dot{e}_s(t)}{s_s(t)}$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$  is<br>
ed,  $\dot{V}_s(e_s(t))$  is uniformly continuous. According<br> second derivative function of function  $V_s(e_s(t))$  is<br>  $e_s(t) = -2r_i e_s(t) \dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$  is<br>
bounded,  $\ddot{V}_s(e_s(t))$  is uniformly continuous. According<br>
he Lyapunov-like lemma, when  $t \rightarrow 0$ ,  $\ddot{V}_s(e$ lies that  $V_s(e_s(t)) \leq V_s(e_s(0))$  and  $V_s(e_s(t))$  is<br>
o  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded.<br>
d derivative function of function  $V_s(e_s(t))$  is<br>  $-2r_i e_s(t) \dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$  is<br>
ed,  $\dot{V}_s(e_s(t))$ so  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded.<br>
and derivative function of function  $V_s(e_s(t))$  is<br>  $=-2r_i e_s(t)\dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t)\dot{e}_s(t)$  is<br>
ded,  $\ddot{V}_s(e_s(t))$  is uniformly continuous. According<br>
yapuno ich implies that  $V_s(e_s(t)) \le V_s(e_s(0))$  and  $V_s(e_s(t))$  is<br>
unded, so  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded.<br>
e second derivative function of function  $V_s(e_s(t))$  is<br>  $(e_s(t)) = -2r_i e_s(t)\dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t)\dot{e}_s$ unded, so  $e_s(t)$  is bounded, and  $\dot{e}_s(t)$  is also bounded.<br>
e second derivative function of function  $V_s(e_s(t))$  is<br>  $(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$  is<br>
bounded,  $\ddot{V}_s(e_s(t))$  is uniformly contin e second derivative function of function  $V_s(e_s(t))$  is<br>  $(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$ , so  $\ddot{V}_s(e_s(t)) = -2r_i e_s(t) \dot{e}_s(t)$  is<br>
bounded,  $\dot{V}_s(e_s(t))$  is uniformly continuous. According<br>
the Lyapunov-like lemma, when  $t \to 0$ ,  $\ddot{V}_$ 

**Step 2:** Design a constrained adaptive back-stepping controller law for  $U<sub>s</sub>(t)$ ; when the uncertain parameter  $\theta$  exthe virtual control function  $\alpha(t)$ . The derivative function of the tracking trajectory error function  $e_s(t)$  is guaranteed<br>nverge to zero asymptotically.<br>**ep 2:** Design a constrained adaptive back-stepping con-<br>r law for  $U_s(t)$ ; when the uncertain parameter  $\theta$  ex-<br>n the system, the sign a constrained adaptive back-stepping con-<br>  $U_s(t)$ ; when the uncertain parameter  $\mathcal{F}$  ex-<br>
tem, the vertical speed function  $x_6(t)$  can track<br>
ntrol function  $\alpha(t)$ . The derivative function of<br>  $e_6(t)$  can be obta nction  $e_s(t)$  is guaranteed<br>laptive back-stepping con-<br>necertain parameter  $\theta$  ex-<br>if function  $x_s(t)$  can track<br>The derivative function of<br>ed as<br> $x_{s}$ ,  $(t) + U_s(t) - \alpha(t)$ <br>(19)<br> $t) + U_s(t)$ . aso bounded,  $r_s(e_s(t))$  is antioning continuous. According<br>to the Lyapunov-like lemma, when  $t \to 0$ ,  $\dot{V}_s(e_s(t)) \to 0$ ,<br>then the tracking trajectory error function  $e_s(t)$  is guaranteed<br>to converge to zero asymptotically.<br>**St** converge to zero asymptotically.<br> **Seep 2:** Design a constrained adaptive back-stepping con-<br>
ller law for  $U_s(t)$ ; when the uncertain parameter  $\theta$  ex-<br>
in the system, the vertical speed function  $x_s(t)$  can track<br>
virtua

$$
\begin{cases}\n\dot{e}_6(t) = \mathcal{G}\Big(-c_3\big(x_6(t) - x_4(t)\big) - F_{s3}(t) + U_s(t)\big) - \dot{\alpha}(t) \\
= \mathcal{G}G(t) - \dot{\alpha}(t) \\
G(t) = -c_3\big(x_6(t) - x_4(t)\big) - F_{s3}(t) + U_s(t).\n\end{cases}
$$
\n(19)

Define the function  $\tilde{\mathcal{G}}(t)$  as the estimate of the uncertain tion error, which is given as

$$
e_{\mathfrak{s}}(t) = \tilde{\mathfrak{S}}(t) - \mathfrak{S} \,. \tag{20}
$$

$$
V\left(e_s(t), e_s(t), e_s(t), t\right) = V_s\left(e_s(t)\right) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t).
$$
 Lyapunov-like lemma, v  
\n
$$
\rightarrow 0, \text{ then the error fu}
$$
\n(21)

*e t t* <sup>J</sup> ( ), ) using Eq. (17) is given as ( ( ) ( ) ( ) ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 5 6 <sup>2</sup> 5 6 1 5 2 2 5 5 1 6 6 5 <sup>6</sup> 2 2 5 5 2 1 1 5 , , , . *V e t e t e t t re t* <sup>J</sup> <sup>J</sup> <sup>J</sup> <sup>J</sup> <sup>J</sup> - & & % & & & % (22) If the control law *U t* 3 6 4 3 *U t c x t x t F t s s e t t r e t* = - +

If the control law  $U<sub>s</sub>(t)$  is chosen as

$$
U_s(t) = c_3(x_6(t) - x_4(t)) + F_{s3}(t)
$$
  
\n
$$
= r_1e_s^2(t) + r_s^{-1}e_s(t)\dot{\tilde{S}}(t).
$$
  
\nSubstituting the system can  
\n
$$
U_s(t) = c_3(x_6(t) - x_4(t)) + F_{s3}(t)
$$
  
\n
$$
+ \frac{1}{\tilde{S}}\left(\dot{\alpha}(t) - r_2e_6(t) - \frac{e_s(t)}{\Delta_s^2 - e_s^2(t)}\right)
$$
  
\n
$$
= r_2
$$
 is a positive value. Substituting Eq. (23) into Eq.  
\n
$$
V(e_s(t), e_s(t), t) = -r_1e_s^2(t) - r_2e_6^2(t)
$$
  
\n
$$
+ e_s(t)(r_s^{-1}\dot{\tilde{S}}(t) - e_6(t)G(t)).
$$
  
\n(24)  
\n
$$
\left.\begin{array}{c}\n0 \\
-\frac{k_1 + r_2}{m} \\
0 \\
\frac{k_2}{m_2}\n\end{array}\right\}
$$

where  $r_2$  is a positive value. Substituting Eq. (23) into Eq. (22), the following equation can be obtained

$$
\dot{V}\big(e_s(t), e_s(t), e_s(t), t\big) = -r_i e_s^2(t) - r_2 e_s^2(t) \n+ e_s(t) \bigg(r_s^{-1} \dot{\tilde{\mathcal{B}}}(t) - e_s(t) G(t)\bigg).
$$
\n(24)

If the adaptive law is chosen as the projection type [26]

$$
\dot{V}(e_{s}(t), e_{s}(t), e_{s}(t), t) = -r_{i}e_{s}^{2}(t) - r_{2}e_{s}^{2}(t)
$$
\n
$$
+e_{s}(t)\left(r_{s}^{-1}\dot{\tilde{g}}(t) - e_{s}(t)G(t)\right).
$$
\n(24)\n\nIf the adaptive law is chosen as the projection type [26]\n
$$
\dot{\tilde{g}}(t) = proj_{\tilde{g}}\left(r_{s}e_{s}(t)G(t)\right)
$$
\n
$$
= \begin{cases}\n0 & \tilde{\tilde{g}}(t) = \mathcal{G}_{\text{max}} \text{ and } r_{s}e_{s}(t)G(t) > 0 \\
0 & \tilde{\tilde{g}}(t) = \mathcal{G}_{\text{max}} \text{ and } r_{s}e_{s}(t)G(t) < 0\n\end{cases}
$$
\n(25)\n
$$
= \begin{cases}\n0 & \tilde{\tilde{g}}(t) = \mathcal{G}_{\text{max}} \text{ and } r_{s}e_{s}(t)G(t) > 0 \\
0 & \tilde{\tilde{g}}(t) = \mathcal{G}_{\text{min}} \text{ and } r_{s}e_{s}(t)G(t) < 0\n\end{cases}
$$
\nDefine the matrix function of the matrix function of the matrix function  $\tilde{f}$  and  $\tilde{f}$  are bounded, which implies that  $\tilde{f}(t)$  and  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  are  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  are  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  are  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  and  $\tilde{f}$  and  $\$ 

where  $r_{\rm g}$  is a tunable positive value.

Substituting Eq. (25) into Eq. (24), then  $V(e<sub>s</sub>(t), e<sub>6</sub>(t))$ ,  $e_g(t)$ ,  $t$ ) =  $-r_1e_5^2(t) - r_2e_6^2(t) \le 0$ ; this indicates that  $V(e_5(t))$ , Fire adaptive law is chosen as the projection type [26]<br>
If the adaptive law is chosen as the projection type [26]<br>  $\hat{\beta}(t) = proj_{\delta}(r,e_{\delta}(t))G(t)$ <br>  $= \begin{cases} 0 & \hat{\beta}(t) = g_{\text{m}} \text{ and } r_{\beta}e_{\delta}(t)G(t) > 0 \\ 0 & \hat{\delta}(t) = g_{\text{m}} \text{ and } r_{\beta}$  $e_s(t)$  are bounded, which implies that  $\dot{e}_s(t)$  and  $\dot{e}_s(t)$  are also bounded. The second derivative function of function If the adaptive law is chosen as the projection type [26]<br>  $\dot{\tilde{\theta}}(t) = proj_s(r_se_s(t)G(t))$ <br>  $= \begin{cases} 0 & \tilde{\theta}(t) = \theta_{\text{max}} \text{ and } r_se_s(t)G(t) > 0 \\ 0 & \tilde{\theta}(t) = \theta_{\text{max}} \text{ and } r_se_s(t)G(t) < 0 \end{cases}$ <br>  $= \begin{cases} 0 & \tilde{\theta}(t) = \theta_{\text{max}} \text{ and } r_se_s(t)G(t) < 0 \\ 0 & \$  $\hat{\beta}(t) = \proj_g \{r_s e_s(t)G(t)\}$ <br>  $= \begin{cases} 0 & \hat{\beta}(t) = \beta_{\text{max}} \text{ and } r_s e_s(t)G(t) > 0 \\ 0 & \hat{\beta}(t) = \beta_{\text{max}} \text{ and } r_s e_s(t)G(t) < 0 \\ r_s e_s(t)G(t) & \text{otherwise} \end{cases}$ <br>  $= \begin{cases} 0 & \hat{\beta}(t) = \beta_{\text{max}} \text{ and } r_s e_s(t)G(t) < 0 \\ r_s e_s(t)G(t) & \text{otherwise} \end{cases}$ <br>  $= \begin{cases} 0 & \text{otherwise} \end{cases}$ <br> f the adaptive law is chosen as the projection type [26]<br>  $\hat{\beta}(t) = p\alpha_{j_0} \{r_{\rho} \epsilon_n(t)G(t)\}$ <br>  $= \begin{cases} 0 & \tilde{\beta}(t) = \theta_{mn} \text{ and } r_{\rho} \epsilon_n(t)G(t) > 0 \\ 0 & \tilde{\beta}(t) = \theta_{mn} \text{ and } r_{\rho} \epsilon_n(t)G(t) < 0 \end{cases}$ <br>  $= \begin{cases} 0 & \tilde{\beta}(t) = \theta_{mn} \text{ and } r_{\rho} \$ 

$$
\ddot{V}\big(e_{s}(t), e_{s}(t), e_{s}(t), t\big) = -2r_{1}e_{s}(t)\dot{e}_{s}(t) - 2r_{2}e_{s}(t)\dot{e}_{s}(t). \tag{26}
$$

*The Because the velocity of the error function*  $e_g(t)$  does not<br>
Because the velocity of the error function  $e_g(t)$  does not<br>
ve to be restricted, define the following Lyapunov function<br>  $V(e_s(t), e_s(t), e_s(t), t) = V_s(e_s(t)) + \frac{1}{2}e_s^$ have to be restricted, define the following Lyapunov function  $e_6(t), e_9(t), t$  is uniformly continuous. According to the Journal of Mechanical Science and Technology 32 (7) (2018) 2973<br>
tion  $e_s(t)$  does not So  $\ddot{V}(e_s(t), e_s(t), e_s(t), t)$ <br>
Lyapunov function  $e_s(t), e_s(t), t)$  is uniformly<br>  $e_s(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ . Lyapunov-like lemma, whe<br>  $\rightarrow 0$ , th *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
ause the velocity of the error function  $e_s(t)$  does not<br>
be restricted, define the following Lyapunov function<br>  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t$ Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985<br>
the velocity of the error function  $e_s(t)$  does not<br>  $\oint_e(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\hat{V}(e_s(t), e_s(t), t)$ <br>
restricted, define the fol *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973~2985*<br>
Because the velocity of the error function  $e_s(t)$  does not<br>  $V(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\dot{V}(e_s(t),$ <br>
we to be restricted, def *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
f the error function  $e_s(t)$  does not<br>  $= V_s(e_s(t)) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $= V_s(e_s(t)) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $\rightarrow$ Then the derivative function of function  $V(e_s(t), e_s(t))$  and  $V(e_s(t), e_s(t), t)$  is given as<br>  $V(e_s(t), e_s(t), t) = V_s(e_s(t)) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^2e_s^2(t)$ .<br>
Then the derivative function  $V(e_s(t), e_s(t), e_s(t))$  and  $V(e_s(t), e_s(t), t)$  is uniformly contin *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
the error function  $e_s(t)$  does not<br>  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $t$ ) is also bound<br>  $e_s(t) = e_s(t) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1$ *e<sub>s</sub>*(*t*) *e*<sub>s</sub>(*t*) *e*<sub>s</sub> *F. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
the error function  $e_s(t)$  does not<br>  $e_s(t)$  and  $\frac{1}{2}e_s^*(t) + \frac{1}{2}e_s^*(t) + \frac{1}{2}e_s^*(t)$ .<br>  $\frac{1}{2}(e_s(t)) + \frac{1}{2}e_s^*(t) + \frac{1}{2}e_s^*(t$ *estal. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br> *cor* function  $e_s(t)$  does not<br>  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $t$  is also bound<br>  $e_s(t) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $(21)$  Lyapunov *F. Wang et al. / Journal of Mechanical Science and Technology 32 (?) (2018) 2973-2985*<br>
of the error function  $e_s(t)$  does not<br>  $e_s(t)e_s(t)$ ,  $e_s(t)e_s(t)$ ,  $e_s(t)$ , *et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
function  $e_s(t)$  does not<br>  $\leftarrow \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $\leftarrow \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $\leftarrow \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2$ ocity of the error function  $e_s(t)$  does not<br> *reform* the following Lyapunov function<br> *reform* the following type (*r*),  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$ ,  $e_s(t)$  is uniformly continuous. According to<br> *cs*(*r*),*t*) =  $V$ *urnal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
on  $e_s(t)$  does not<br>
So  $\vec{V}(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\vec{V}(e_s(t),$ <br> *v*<sub>2</sub>(*v*), does not<br>  $\vec{V}(e_s(t), e_s(t), t)$  is uniformly continuous. Accordin *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985*<br>
f the error function  $e_s(t)$  does not<br>  $= V_s(e_s(t)) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>  $= V_s(e_s(t)) + \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t)$ .<br>
Lyap Y. Wang et al. /Journal of Mechanical Science and Technology 32 (7) (2018) 2973-2985<br>
the error function  $e_s(t)$  does not<br>
the following Lyapunov function<br>  $\begin{aligned}\ne_6(t), e_s(t), t) &\text{ is uniformly continuous.} \end{aligned}$  According Lyapunov function<br>  $\begin{$ For  $u(x, t) = \frac{1}{2} e_{\hat{g}}^2(t) + \frac{1}{2} r_{\hat{g}}^2(e_{\hat{g}}(t))$ <br>
So  $\vec{v}(\epsilon_1(t), e_{\hat{g}}(t), t)$  is also bounded,  $\vec{v}(\epsilon_2(t), t)$ <br>
e crior function  $e_{\hat{g}}(t)$  does not<br>  $\vec{v}(\epsilon_1(t), t) = \frac{1}{2} e_{\hat{g}}^2(t) + \frac{1}{2} r_{\hat{g}}^2 e_{\hat{g}}^2$ *Y. Wang et al. / Journal of Mechanical Science and Technology 32 (?) (2018) 2973-2985*<br>
of the error function  $\varepsilon_s(t)$  does not<br>  $\ell = V_s(e_s(t)) + \frac{1}{2}e_s^*(t) + \frac{1}{2}r_s^*e_s^*(t)$ .<br>  $\ell = 1$  and  $\ell = 1$  and  $\ell = 1$  and  $\ell = 1$  a The error function  $e_s(t)$  does not<br>  $\begin{aligned}\n\mathbf{F}_s^T(t) &= \frac{1}{2}e_s^2(t) + \frac{1}{2}r_s^{-1}e_s^2(t) +$ *d Technology 32 (7) (2018) 2973~2985*<br>
So  $\ddot{V}(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\dot{V}(e_s(t), t)$ <br>  $(t), e_s(t), t)$  is uniformly continuous. According to the<br>
rapunov-like lemma, when  $t \rightarrow 0$ ,  $\dot{V}(e_s(t), e_s(t), e_s(t), t)$ <br>  $\ddot{V}(t)$ , and Technology 32 (7) (2018) 2973-2985<br>
So  $\ddot{V}(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\dot{V}(e_s(t), e_s(t), e_s(t), t)$  is uniformly continuous. According to the<br>
Lyapunov-like lemma, when  $t \rightarrow 0$ ,  $\dot{V}(e_s(t), e_s(t), e_s(t), t)$ <br>  $\rightarrow 0$ , then th and Technology 32 (7) (2018) 2973-2985<br>
So  $\ddot{V}(e_s(t), e_s(t), t)$  is also bounded,  $\dot{V}(e_s(t), e_s(t), t)$  is uniformly continuous. According to the<br>
Lyapunov-like lemma, when  $t \rightarrow 0$ ,  $\dot{V}(e_s(t), e_s(t), e_s(t), t)$ <br>  $\rightarrow 0$ , then the error  $\dot{Z}(e(t)) e(t) e(t) t$ and Technology 32 (7) (2018) 2973-2985<br>
So  $\vec{V}(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\vec{V}(e_s(t), e_s(t), t)$ <br>  $e_s(t), e_s(t), t)$  is uniformly continuous. According to the<br>
Lyapunov-like lemma, when  $t \rightarrow 0$ ,  $\vec{V}(e_s(t), e_s(t), e_s(t), t)$ <br>  $\rightarrow 0$ and Technology 32 (7) (2018) 2973-2985<br>
So  $\vec{V}(e_s(t), e_s(t), t)$  is also bounded,  $\vec{V}(e_s(t), e_s(t), t)$ <br>  $e_s(t), e_s(t), t)$  is uniformly continuous. According to the<br>
Lyapunov-like lemma, when  $t \to 0$ ,  $\vec{V}(e_s(t), e_s(t), e_s(t), t)$ <br>  $\to 0$ , th and Technology 32 (7) (2018) 2973-2985<br> **So**  $\vec{V}(e_s(t), e_s(t), t)$  is also bounded,  $\vec{V}(e_s(t), e_s(t), t)$ <br>  $\vec{U}(e_s(t), e_s(t), t)$  is uniformly continuous. According to the<br> **Lyapunov-like lemma, when**  $t \rightarrow 0$ ,  $\vec{V}(e_s(t), e_s(t), e_s(t), t)$ <br> *U U U U I Colorary* 32 *(i)* (2018) 2973-2985<br> **So**  $\ddot{V}(e_s(t), e_s(t), e_s(t), t)$  is also bounded,  $\dot{V}(e_s(t), t)$ <br>  $(t), e_s(t), t)$  is uniformly continuous. According to the<br>
apunov-like lemma, when  $t \rightarrow 0$ ,  $\dot{V}(e_s(t), e_s(t),$ *sehnology* 32 (7) (2018) 2973-2985<br>  $\vec{v}$  ( $e_s(t)$ ,  $e_e(t)$ ,  $e_s(t)$ ,  $t$ ) is also bounded,  $\vec{v}'(e_s(t)$ ,<br>  $\vec{v}_e(t)$ ,  $\vec{v}_s(t)$ ,  $\vec{v}_s(t)$  is uniformly continuous. According to the<br>
bunov-like lemma, when  $t \rightarrow 0$ ,  $\vec{$ 

(*t*) four states. Let  $e_s(t) = 0$  and  $\dot{e}_s(t) = 0$ , the control law **Step 3:** Ensure the zero dynamics of the system is stable. The constrained adaptive back-stepping controller law is a second-order error dynamic controller law; the original system is a sixth-order system, so the zero dynamics of the system has  $U<sub>s</sub>(t)$  is given as

$$
U_{s}(t) = c_{s}(x_{6}(t) - x_{4}(t)) + F_{s3}(t) + m_{3}\ddot{x}_{s}(t).
$$
 (27)

 $(t) + r_g^{-1}e_g(t)\theta(t)$ . the system can be obtained as Substituting Eq. (27) into Eq. (7), then the zero dynamics of

is a complete that the complete state of the complete state of the complete state of the complete state of the

$$
V(\xi_1(t),\xi_1(t),\xi_2(t),\tau) = r_1(\xi_1(t)) + \frac{r_2(\xi_2(t)) + \frac{r_2(\xi_2(t
$$

Define the positive function  $V(x(t)) = x^{T}(t) Hx(t)$ , where the matrix  $H > 0$  is a positive matrix. Then the derivative

$$
\dot{V}(x(t)) = \dot{x}^{T}(t)Hx(t) + x^{T}(t)H\dot{x}(t) \n= x^{T}(t)(A^{T}H + HA)x(t) + 2x^{T}(t)Hw(t).
$$
\n(29)

$$
w(t) = \begin{vmatrix} m_1 \ b_1 \end{vmatrix} \left[ \frac{m_1}{m_1} + \frac{m_1}{m_1} \right].
$$
  
Define the positive function  $V(x(t)) = x^T(t)Hx(t)$ , where  
the matrix  $H > 0$  is a positive matrix. Then the derivative  
function of  $V(x(t))$  using Eq. (28) is given as  

$$
\dot{V}(x(t)) = \dot{x}^T(t)Hx(t) + x^T(t)H\dot{x}(t)
$$

$$
= x^T(t)(A^T H + H A)x(t) + 2x^T(t)Hw(t).
$$
  
The characteristic equation of matrix A is given as  

$$
|A - sI| = s^4 + \frac{c_1m_2 + c_2m_1 + c_2m_2}{m_1m_2} s^3 + \frac{k_1m_2 + k_2m_1 + k_2m_2 + c_1c_2}{m_1m_2} s^2 + \frac{c_1k_2 + c_2k_1}{m_1m_2} s + \frac{m_1m_2}{m_1m_2}.
$$
 (30)

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\nIt can be verified that the four eigenvalues of the matrix A  
\nwe negative real parts. Let A<sup>T</sup>H + HA = -N, where the  
\nmix N > 0 is a positive matrix.  
\nis a tunable positive value. Then the following inequality  
\nis a tunable positive value. Then the following inequality  
\n
$$
\hat{V}(x(t)) \leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} -x^T(t)Nx(t) + \frac{1}{T}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} 311 & \frac{m_{\text{max}}}{m_{\text{max}}} & 80 \text{ kg} & k_1 \\ -80 \text{ kg} & k_1 & 31000 \text{ N/m} \\ +\frac{1}{2}w^T(t)w(t) \end{pmatrix}
$$
\n
$$
\leq \begin{pmatrix} 311 & \frac{m_{\text{max}}}{m_{\text{max}}} & 80 \text{ kg} & k_1 \\ -80 \text{ kg} & k_1 & 31000 \text{ N/m} \\ +\frac{1}{2}w^T(t)w(t) & 1 & 0 \\ +\
$$

$$
\lambda_{\min}\left(H^{-1}N\right) - \frac{1}{\eta} \lambda_{\max}\left(H\right) \ge h_1 \,,\tag{32}
$$

$$
\dot{V}\big(x(t)\big) \le -h_1 V\big(x(t)\big) + h_2 \tag{33}
$$

$$
\mathbb{E}\left(\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)
$$
\n
$$
\mathbb{E}\left(\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)
$$
\n
$$
\mathbb{E}\left(\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)
$$
\n
$$
\mathbb{E}\left(\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)\right)
$$
\n
$$
\mathbb{E}\left(\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)\right)
$$
\n
$$
\mathbb{E}\left(\frac{\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)
$$
\n
$$
\mathbb{E}\left(\frac{\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)\right)
$$
\n
$$
\mathbb{E}\left(\frac{\left(-\lambda_{\min}(H^{-1}N)+\frac{1}{\eta}\lambda_{\max}(H)\right)V(x(t))\right)
$$
\n<math display="block</math>

Then the following inequality can be obtained:

$$
\left|x_i(t)\right| \le \sqrt{\frac{h}{\lambda_{\min}(H)}} \quad i = 1 \cdots 4 \,. \tag{35}
$$

**Step 4:** Select the appropriate tunable parameters to maintain the vehicle and seat suspension strokes in the allowed range. Using Eq. (35), these two constraints can be expressed as

$$
|x_{i}(t)| \leq \sqrt{\frac{h}{\lambda_{\min}(H)}} \quad i = 1 \cdots 4. \qquad (35)
$$
\nStep 4: Select the appropriate tunable parameters to main-  
\nthe vehicle and seat suspension strokes in the allowed  
\nage. Using Eq. (35), these two constraints can be expressed  
\n
$$
\begin{cases}\n|x_{2} - z_{1}| = |x_{3} - x_{1}| \leq |x_{3}| + |x_{1}| \leq 2 \sqrt{\frac{h}{\lambda_{\min}(H)}} \\
|x_{3} - z_{2}| = |x_{5} - x_{3}| \leq |x_{5}| + |x_{3}| \leq \delta_{5} + \sqrt{\frac{h}{\lambda_{\min}(H)}}.\n\end{cases}
$$
\nStep 4: Select the appropriate tunable parameters to main-  
\nthe value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can find that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that the value of the table, we can use that

Table 1. Parameter values of the active vehicle-seat-human coupled model.

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It can be verified that the four eigenvalues of the matrix $A$ have negative real parts. Let $ATH + HA = -N$ , where the	Table 1. Parameter values of the active vehicle-seat-human coupled model.					
matrix $N > 0$ is a positive matrix.	Parameter		Value	Parameter		Value
Note $2x^{r}(t)Hw(t) \le x^{r}(t)HHx(t)/\eta + \eta w^{r}(t)w(t)$ , where $\eta$ is a tunable positive value. Then the following inequality	$m_{\rm l}$		$60 \text{ kg}$	$c_{1}$		$7$ Ns/m
can be obtained:	m <sub>2</sub>		375 kg	$c_{2}$		1425 Ns/m
	$m_{s3}$		$70 \text{ kg}$	$c_{3}$		830 Ns/m
$\overrightarrow{V}(x(t)) \leq \begin{pmatrix} -x^T(t)Nx(t)+\frac{1}{\eta}x^T(t)HHx(t) \\ +\eta w^T(t)w(t) \end{pmatrix}$	$m_{\text{3min}}$		60 kg	$k_{1}$		200000 N/m
	$m_{3\text{max}}$		$80 \text{ kg}$	$k_{2}$		15000 N/m
(31) $\leq \left[\left(-\lambda_{\min}\left(H^{-1}N\right)+\frac{1}{\eta}\lambda_{\max}\left(H\right)\right)V\left(x(t)\right)\right]+ \eta w^{T}\left(t\right)w(t)$	$K_{v}$		31000 N/m			
	Table 2. Controlled parameter values of the active system.					
If the matrices $H$ , $N$ and the tunable positive value $\eta$	Parameter	$r_{g}$	$r_{1}$	r <sub>2</sub>	$\Delta_{\varsigma}$	$\delta_{\rm s}$
are chosen properly, the following inequality can be ensured	Value	0.01	100	100	0.05	0.08
$\lambda_{\min}\left(H^{-1}N\right)-\frac{1}{n}\lambda_{\max}\left(H\right)\geq h_1,$ (32)	ues and the positive matrix $H$ is chosen whose eigenvalues					If the parameters $\delta_{\zeta}$ and h are chosen to be smaller val-
where $h_1$ is a positive value. Assume $\eta w^T(t) w(t) \leq h_2$ , then Eq. (31) can be transformed into	are larger. The following inequalities can be ensured:					
$\dot{V}(x(t)) \le -h_1 V(x(t)) + h_2$ . (33)				$2\sqrt{\frac{h}{\lambda_{\min}(H)}} \leq z_{\text{slmax}} \quad \delta_{\varsigma} + \sqrt{\frac{h}{\lambda_{\min}(H)}} \leq z_{\text{slmax}}.$		(37)
Using Eq. (33), the inequality of the positive function $V(x(t))$ can be given as	satisfied.					Then Eq. (10) can be guaranteed and the two constraints are

Table 2. Controlled parameter values of the active system.



$$
2\sqrt{\frac{h}{\lambda_{\min}(H)}} \le z_{\text{slmax}} \quad \delta_{\text{s}} + \sqrt{\frac{h}{\lambda_{\min}(H)}} \le z_{\text{slmax}}. \tag{37}
$$

## **5. Numerical simulations**

human coupled model under shock excitation were studied.<br>  $(v(0))$   $V(x(0)) \ge \frac{h_2}{h}$  (34) The parameter values of this coupled model [20, 21] used in cices *H*, *N* and the tunable positive value  $\eta$ <br>
operly, the following inequality can be ensured<br>  $\left(-\frac{1}{\eta}\right) = \frac{1}{n} \lambda_{\text{max}}(H) \ge h_i$ ,<br>  $\left(-\frac{1}{n} \lambda_{\text{max}}(H) \ge h_i\right)$ <br>  $= \frac{1}{n} \lambda_{\text{max}}(H) \ge h_i$ ,<br>  $= \frac{1}{n} \lambda_{\text{max}}(H$  $\left(\frac{2h}{h}\right)^2 + \frac{1}{h}\lambda_{\text{max}}(H) \ge h,$ <br>  $\left(\frac{2h}{h}\right)^2 + \frac{1}{h}\lambda_{\text{max}}(H) \ge h,$ <br>  $\left(\frac{2h}{h}\right)^2 + \frac{1}{h}\lambda_{\text{max}}(H) \ge h,$ <br>  $\left(\frac{2h}{h}\right)^2 + \frac{1}{h}\lambda_{\text{max}}(H) \ge \frac{1}{h}\lambda_{\text{max}}(H)$ <br>  $\left(\frac{2h}{h}\right)^2 + \frac{h}{h}\lambda_{\text{max}}(H) \ge \frac{1}{h}\lambda_{\text{max}}(H$ and the tunable positive value *n*<br>
bilowing inequality can be ensured<br>  $f(z(0)) \ge \frac{h_1}{h_1}$ . (32)<br>  $\frac{1}{2} \int \frac{h}{\sqrt{h_{\text{min}}(H)}} \le z_{\text{d,max}}$ <br>  $\frac{1}{2} \int \frac{h}{\sqrt{h_{\text{min}}(H)}} \le z_{\text{d,max}}$ <br>  $\frac{1}{2} \int \frac{h}{\sqrt{h_{\text{min}}(H)}} \le z_{\text{d,max}}$  $h_1$ <br>
the simulations are listed in Table 1. The maximum vehicle *h*  $h = \frac{1}{\pi} \lambda_{\text{max}}(H) \ge h_1$ ,<br>
is a positive value. Assume  $\eta w^T(t)w(t) \le h_2$ , then<br>
is a positive value. Assume  $\eta w^T(t)w(t) \le h_2$ , then<br>  $\lambda = -\hbar V(x(t)) + h_2$ .<br>
Eq. (33), the inequality of the positive function<br>  $\lambda = \frac{1}{$ <sup>-1</sup>*N*)  $-\frac{1}{\eta} \lambda_{\text{max}}(H) \ge h_i$ ,<br>
is a positive value. Assume  $\eta w^r(t)w(t) \le h_2$ , then<br>
is a positive value. Assume  $\eta w^r(t)w(t) \le h_2$ , then<br>
and the positive matrix *H* is chosen who<br>
is a positive value. Assume  $\eta w^r(t$  $(H^{-1}N) - \frac{1}{\eta} \lambda_{\text{max}}(H) \ge h_1$ ,<br>  $h_1$  is a positive value. Assume  $\eta w^T(t)w(t) \le h_2$ , then<br>  $\text{max of } \frac{1}{\lambda} \sum_{i=1}^n (h_1 - h_2) \le -h_1 V(x(t)) + h_2$ .<br>
(1)  $\le -h_1 V(x(t)) + h_3$ .<br>
(3) and the transformed into<br>  $\left(\frac{1}{\lambda} \sum_{i=1}^n (H$ From Eq. (34), it can be verified that *V*  $(x(t))$  is bounded.<br>  $\mathcal{F}(x(t)) \leq -hV(x(t)) + h_z$ .<br>  $\mathcal{F}(x(t)) \leq -hV(x(t)) + h_z$ .<br>
(33)<br>  $\mathcal{F}(x(t)) \leq -hV(x(t)) + h_z$ .<br>
(33)<br>  $\mathcal{F}(x(t)) \leq \frac{h}{\lambda_{\min}}(t) \leq \frac{h}{\lambda_{\min}}(t) \leq \frac{h}{\lambda_{\min}}(t)$ <br>  $\mathcal{$  $(x(t)) \le -h_f V(x(t)) + h_2$ .<br>
(33)  $\sqrt{\lambda_{mn}(H)}$ <br>
(33)  $\sqrt{\lambda_{mn}(H)}$ <br>
(a) can be guaranteed and the positive function<br>
(x(t))  $\le [V(x(0)) - \frac{h_1}{h_1}]e^{-\lambda t} + \frac{h_2}{h_1} \le h$ <br>
5. **Numerical simulations**<br>
(x(t))  $\le [V(x(0)) - \frac{h_1}{h_1}]e^{-\lambda t} +$  $\chi(x(t)) \le -h_1 V(x(t)) + h_2$ .<br>  $\chi(x(t)) \le \left[ V(x(0)) - \frac{h_1}{h_1} \right] e^{-\lambda t} + \frac{h_2}{h_1} \le h$ <br>  $\chi(x(t)) \le \left[ V(x(0)) - \frac{h_1}{h_1} \right] e^{-\lambda t} + \frac{h_2}{h_1} \le h$ <br>  $\chi(x(t)) \le \left[ V(x(0)) - \frac{h_1}{h_1} \right] e^{-\lambda t} + \frac{h_2}{h_1} \le h$ <br>  $\chi(x(t)) \ge \left[ V(x(0)) - \frac{h_1}{h_1} \right] e$  $\overline{\lambda_{\min}(H)}$   $l=1\cdots 4$ . (35)  $\overline{V}(e_s(t), e_s(t), e_s(t), t)$  converges to zero faster.  $\delta_s$  and  $\varepsilon_s$ (a)  $\left(\frac{h}{\lambda_{\min}}(H)\right) \le -h_i V(x(t)) + h_i$ .<br>
Eq. (33), the inequality of the positive function<br>  $\left(\frac{V(x(0))}{h_i}\right) \le \left(\frac{h_i}{h_i} + \frac{h_i}{h_i} \le h\right)$ <br>  $\left(\frac{h_i}{h_i} - V(x(0))\right) \ge \frac{h_i}{h_i}$ <br>  $\left(\frac{34}{h_i} - V(x(0))\right) \ge \frac{h_i}{h_i}$ .<br>  $\left(\frac{34}{h_i$ *x*  $\left| \frac{F(x(0))}{\lambda_{\frac{n}{n}}} - F(x(0)) \right| \le \frac{\lambda_{\frac{n}{n}}}{\lambda_{\frac{n}{n}}}$ . (34) The parameter values of this coupled n the simulations are listed in Table 1. The simulations are listed in Table 1. The simulations are listed in Table 1.  $|2\frac{r_2}{h_t} - V(x(0)) - V(x(0))| < \frac{r_2}{h_t}$ .<br>
suspension stroke  $(z_{\text{atim}})$  and seat suspension stroke  $(z_{\text{atim}})$  and seat suspension stroke  $(z_{\text{atim}})$  and 0.1 m, respectively reached that  $V(x(t))$  is bounded.<br>  $r_s$ ,  $r_t$  an  $\left| \frac{V(x(0))}{2\frac{h}{h_1}} - V(x(0)) \right| \ge \frac{h_2}{h_1}$  (34) The parameter values of this coupled mode the simulations are listed in Table 1. The constraints are listed in Table 1. The sumpresion stroke  $(z_{n_{\text{max}}})$  and seat suspe  $\left|\frac{2\frac{3x}{h}}{h} - V(x(0)) - V(x(0)) \right| < \frac{1}{h}$ , suspension stroke  $(z_{\text{r,max}})$  and seat suspension<br>
"From Eq. (34), it can be verified that  $V(x(t))$  is bounded.<br>  $\left|\frac{1}{h} - \frac{1}{h} - V(x(t))\right| \leq \sqrt{\frac{h}{\lambda_{\text{max}}(H)}}$  is bounded.<br>  $\left$ The dynamic characteristics of the active vehicle-seat suspension stroke ( $z_{\text{sum}}$ ) and seat suspension stroke ( $z_{\text{sum}}$ ) were chosen as 0.15 m and 0.1 m, respectively. The controlled parameter values of the active system are shown in Table 2.  $r<sub>g</sub>$ ,  $r<sub>1</sub>$  and  $r<sub>2</sub>$  are positive values as can be seen in Sec. 4;  $r_1$  and  $r_2$  are chosen as larger values and  $r_3$  is chosen as  $2\sqrt{\frac{h}{\lambda_{min}(H)}} \leq z_{\text{d,max}}$   $\delta_s + \sqrt{\frac{h}{\lambda_{min}(H)}} \leq z_{\text{d,max}}$ . (37)<br>
Then Eq. (10) can be guaranteed and the two constraints are<br>
satisfied.<br>
5. **Numerical simulations**<br>
The dynamic characteristics of the active vehicle-se  $\dot{V}_s(e_s(t))$  and **Frame (Value 3) V Consumpted** and the two constraints are satisfied.<br> **5. Numerical simulations**<br> **7.** The dynamic characteristics of the active vehicle-seat-<br>
human coupled model under shock excitation were studied. **5. Numerical simulations**<br> **5. Numerical simulations**<br>
The dynamic characteristics of the active vehicle-seat-<br>
human coupled model under shock excitation were studied.<br>
The parameter values of this coupled model [20, 21 ons are listed in Table 1. The maximum vehicle<br>troke  $(z_{\text{sum}})$  and seat suspension stroke  $(z_{\text{sum}})$ <br>as 0.15 m and 0.1 m, respectively. The controlled<br>dues of the active system are shown in Table 2.<br> $r_z$  are positive val parameter values of this coupled model [20, 21] used in<br>simulations are listed in Table 1. The maximum vehicle<br>ension stroke  $(z_{\text{rlam}})$  and seat suspension stroke  $(z_{\text{rlam}})$ <br>e chosen as 0.15 m and 0.1 m, respectively. T *th* and seat surface and d.1 m, respectively. The controlled active system are shown in Table 2. we values as can be seen in Sec. 4; la red in Table 1. The maximum vehicle<br>the in Table 1. The maximum vehicle<br>and 0.1 m, respectively. The controlled<br>e active system are shown in Table 2.<br>sitive values as can be seen in Sec. 4;<br>as larger values and  $r_g$  is ch rameter values of this coupled model [20, 21] used in<br>ulations are listed in Table 1. The maximum vehicle<br>ion stroke ( $z_{\text{d,max}}$ ) and seat supersion stroke ( $z_{\text{d,max}}$ )<br>onesn as 0.15 m and 0.1 m, respectively. The control mulations are listed in Table 1. The maximum vehicle<br>mulations are listed in Table 1. The maximum vehicle<br>nsion stroke ( $z_{i,\text{max}}$ ) and seat suspension stroke ( $z_{i,\text{max}}$ )<br>chosen as 0.15 m and 0.1 m, respectively. The con

The shock excitation is caused by a discrete irregularity on the road, such as a bump or a pothole. When the road excitation is assumed to be shock excitation, the road excitation displacement  $z<sub>r</sub>$  can be given as

$$
z_r = \begin{cases} \frac{A_r}{2} \left( 1 - \cos\left(\frac{2\pi V}{l}t\right) \right) & 0 \le t \le \frac{l}{V} \\ 0 & t > \frac{l}{V} \end{cases} \tag{38}
$$

 $m_3 = m_{s3}$ .

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Table 3. Peak acceleration value of human body, maximum vehicle suspension stroke and seat suspension stroke for passive system when $m_3 = m_{s3}$ .				Table 4. Effect of the mass $m3$ on the peak acceleration resp human body and maximum seat suspension stroke for passive with linear seat suspension.					
Parameter	$\ddot{z}_{3p}$ /(m/s <sup>2</sup> )	$\left z_{2}-z_{1}\right _{\text{max}}/m$	$\left z_{_3}-z_{_2}\right _{\rm max}$ /m	$m3$ /kg	60	65	70	75	
Linear	8.4826	0.1129	0.0145	$\ddot{z}_{3p}$ /(m/s <sup>2</sup> )	9.2133	8.8337	8.4826	8.1572	
QZS $\hat{\lambda} = 0.4$	5.4666	0.1128	0.0185	$ z_3 - z_2 _{\text{max}}/m$	0.0134	0.014	0.0145	0.015	
QZS $\hat{\lambda} = 0.45$	5.0632	0.1128	0.0192						
QZS $\hat{\lambda} = 0.5$	4.6629	0.1128	0.0199	10 <sub>1</sub>	10 <sub>1</sub> $\overline{\mathbf{A}}$				



Fig. 5. Acceleration response of human body, vehicle suspension stroke and seat suspension stroke for passive system.

where  $A_r$  is the height of the bump, *l* is the length of the t bump and *V* is the vehicle forward velocity. In this simulation, these parameters are chosen as  $A_r = 0.1$  m,  $l = 2$  m and  $V = 60$  km/h.

The structural parameters of the QZS vibration isolator are chosen as  $k=1$  and  $L=0.2$  m; when the parameter  $\lambda$ takes different values, the time histories of acceleration response of human body, vehicle suspension stroke and seat suspension stroke for passive system are shown in Fig. 5. In **Example 1.**  $-(283 \text{ h} - 0.01)$ <br>  $-(283 \text{ h} - 0.01)$ <br> **Example 1.**  $-(283 \text{ h} - 0.01)$ <br> **Example 1.**  $-(283 \text{ h} - 0.01)$ <br> **Example 1.**  $-(283 \text{ h} - 0.01)$ <br> **Example** this case,  $m_3 = m_{s3} = 70 \text{ kg}$ , the corresponding peak accelera-

Table 4. Effect of the mass  $m_3$  on the peak acceleration response of human body and maximum seat suspension stroke for passive system with linear seat suspension.

and Technology 32 (7) (2018) 2973~2985					
Table 4. Effect of the mass $m1$ on the peak acceleration response of human body and maximum seat suspension stroke for passive system with linear seat suspension.					
$m3$ /kg	60	65	70	75	80
$\ddot{z}_{3p}$ /(m/s <sup>2</sup> )	9.2133	8.8337	8.4826	8.1572	7.855
$\left z_{3}-z_{2}\right _{\rm max}/\rm{m}$	0.0134	0.014	0.0145	0.015	0.0154
10 5	10 5 $\ddot{\mathbf{0}}$				



Fig. 6. Effect of the mass  $m_3$  on the acceleration response of human body and seat suspension stroke for passive system with linear seat suspension.

tion value of human body, maximum vehicle suspension stroke and seat suspension stroke for passive system are shown in Table 3.

It can be seen that when the QZS vibration isolator is used as seat suspension, the peak acceleration value of human body is smaller than the linear seat suspension, the vibration attenuation time becomes shorter and the vehicle ride comfort improves effectively. When  $\lambda$  increases, which indicates the dynamic stiffness of the QZS vibration isolator is smaller in the whole displacement range, the peak acceleration value of human body decreases.

The time histories of vehicle suspension stroke are almost the same for these two kinds of seat suspensions. When using QZS vibration isolator as seat suspension, the seat suspension stroke is larger, because the dynamic stiffness of the QZS vibration isolator is smaller than the stiffness of the linear seat suspension in the displacement range. When  $\lambda$  increases, the maximum seat suspension stroke increases, but remains in the allowed stroke range. In the following analysis,  $\lambda = 0.5$  was considered. It can be seen that when the QZs vibratom isolator is used<br>as east suspension, the peak acceleration value of human body<br>is smaller than the linear seat suspension, the vibration at-<br>tenuation time becomes shorter and the

The unload and overload conditions were then considered, . The effects of the mass  $m_3$  on the

Table 5. Effect of the mass  $m_3$  on the peak acceleration response of human body and maximum seat suspension stroke for passive system with QZS vibration isolator as seat suspension.

Y. Wang et al. / Journal of Mechanical Science and Techno Table 5. Effect of the mass $m1$ on the peak acceleration response of human body and maximum seat suspension stroke for passive system with QZS vibration isolator as seat suspension. $m_{\lambda}$ /kg 60 65 70 75 80 $\ddot{z}_{3p}$ /(m/s <sup>2</sup> ) 6.0708 5.4499 4.6629 4.7096 4.6862 $\left z_{_3}-z_{_2}\right _{\rm max}$ /m 0.0174 0.0185 0.0199 0.0198 0.0198				
				Table 6. P suspension QZS vibra
				Param
				Line
				QZ.
				$QZS+1$



Fig. 7. Effect of the mass  $m_3$  on the acceleration response of human body and seat suspension stroke for passive system with QZS vibration isolator as seat suspension.

acceleration response of human body and seat suspension stroke for passive system with these two kinds of seat suspen sions are shown in Figs. 6 and 7, respectively. The corresponding peak acceleration value of human body and maximum seat suspension stroke for passive system with these two kinds of seat suspensions are shown in Tables 4 and 5, respectively. The vehicle suspension strokes are almost the same with different mass  $m_3$ , so it is not shown here.  $\begin{bmatrix}\n\frac{1}{5} & 0 & 0 \\
0 & 0 & 0\n\end{bmatrix}\n\begin{bmatrix}\n\frac{1}{5} & 0 & 0 \\
0 & \frac{1}{5} & \frac{1}{5} & 0\n\end{bmatrix}\n\begin{bmatrix}\n\frac{1}{5} & -m_3 - 86 \\
\frac{1}{5} & -m_3 - 69 \\
\frac{1}{5} & \frac{1}{5} & 0\n\end{bmatrix}\n\begin{bmatrix}\n\frac{1}{5} & 0.05 \\
\frac{1}{5} & \frac{1}{5} & 0\n\end{bmatrix}$ <br>
4.0.15<br>
4.7. Effect

When  $m_3 < m_{s3}$ , these two kinds of seat suspensions are in<br>a upload condition: as the mass  $m_1$  degrees the neek by bration isolator as seat suspension when  $x_{s_1}(t) = 0$  ( $m_3 = 60$ ). the unload condition; as the mass  $m_3$  decreases, the peak acceleration value of human body increases while the maximum seat suspension stroke decreases. If the QZS vibration isolator is used as seat suspension, the peak acceleration value of human body increases more than the counterpart of the For the mission with the acceleration response of numan body and seat suspension.<br>
Holyand seat suspension. Solution in Figs. 6 and 7, respectively, The correct of the same suspension.<br>
Solution as seat suspension.<br>
Solut linear seat suspension. When  $m_3 > m_3$ , these two kinds of ment( $x_{s}$ (t)) is designed as  $x_{s}$ (t) = 0, the acceleration reseat suspension are in the overload conditions; the increase of the mass  $m_3$  has less effect on the peak acceleration value of s human body and the maximum seat suspension stroke for the QZS vibration isolator. For linear seat suspension, as the mass  $m<sub>3</sub>$  increases, the peak acceleration value of human body seat decreases while the maximum seat suspension stroke in creases. In the following simulations,  $m_3 = 60 \text{ kg} < m_{s3}$ , the body, maximum vehicle susp

Table 6. Peak acceleration value of human body, maximum vehicle suspension stroke and seat suspension stroke for the active system with

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Table 6. Peak acceleration value of human body, maximum vehicle suspension stroke and seat suspension stroke for the active system with QZS vibration isolator as seat suspension when $x_{s_r}(t) = 0$ ( $m_3 = 60$ ).			
Parameter	$\ddot{z}_{3p}$ /(m/s <sup>2</sup> )	$\left z_2-z_1\right _{\text{max}}/m$	$\left z_{3}-z_{2}\right _{\text{max}}/m$
Linear	9.2133	0.1129	0.0134
			0.0174
QZS	6.0708	0.1128	



Fig. 8. Acceleration response of human body, vehicle suspension stroke and seat suspension stroke for the active system with QZS vi-

two kinds of seat suspension in the unload conditions were considered.

When the reference trajectory of the human body displacesponse of human body, vehicle suspension stroke and seat suspension stroke for the active system with QZS vibration isolator as seat suspension are shown in Fig. 8. The corresponding curves of the passive system for these two kinds of seat suspensions are also plotted in the same figure for com parison. The corresponding peak acceleration value of human body, maximum vehicle suspension stroke and seat suspension stroke are shown in Table 6.

For the active system, when  $x_s$ ,  $(t) = 0$ , the acceleration re-<br>
For the active system, when  $x_s$ ,  $(t) = 0$ , the acceleration re-<br>
momes of human body is zero in the whole time domain; the<br>
hicle ride comfort improves more sponse of human body is zero in the whole time domain; the vehicle ride comfort improves more effectively than the passive system, and the vehicle suspension stroke is almost the same with the passive system, while the seat suspension stroke is larger, so both the vehicle and seat suspension strokes remain in the allowed stroke range. Overall, the designed controller law can isolate the shock excitation transmitted from the road to the passengers effectively and both the vehicle and seat suspension stroke constraints are satisfied. 9982 <br> **Example 12** *r**x***<sub>m</sub> <b>c** *x*<sub>m</sub> *x*<sub>m</sub> (*t*) = 0, the acceleration re-<br>
For the active system, when  $x_{s_r}(t) = 0$ , the acceleration re-<br>
popuse of luman body is zero in the whole time domain; the<br>
popuse of luma

As can be seen in Fig. 8, when the reference trajectory remains in the allowed stroke range. This can be improved by setting a specific designed reference trajectory in a determined time instead of the zero reference trajectory, and the deter mined time  $t_{\lambda}$  can be changed to adjust the peak acceleration value of human body and seat suspension stroke to high or low values. The reference trajectory can be designed as a specific polynomial function, which can be given as Example 1. This can be improved by<br>
ension stroke becomes larger although<br>
troke range. This can be improved by<br>
and reference trajectory in a determined<br>
be reference trajectory, and the deter-<br>
hanged to adjust the peak e seat suspension stroke becomes larger although<br>
e allowed stroke range. This can be improved by<br>
ific designed reference trajectory in a determined<br>
of the zero reference trajectory, and the deter-<br>  $\int_a$  can be changed e system, and the vehicle suspension stroke is almost the<br>
new with the passive system, while the seat suspension stroke<br>
new with the vehicle and seat suspension stroke<br>
in an the allowed stroke range. Overall, the desig

$$
x_{5r}(t) = \begin{cases} a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 & 0 \le t \le t_d \\ 0 & t > t_d \end{cases}
$$
 (39)

following equations:

ic polynomial function, which can be given as  
\n
$$
x_{s_r}(t) = \begin{cases} a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 & 0 \le t \le t_d \\ 0 & t > t_d \end{cases}
$$
\n(39)  
\nthere be the coefficients  $a_i$ ,  $i = 0 \cdots 4$  are determined by the  
\nlowing equations:  
\n
$$
\begin{cases} x_{s_r}(0) = a_0 = x_s(0) & x_{s_r}(0) = a_1 = x_6(0) \\ x_{s_r}(t_d) = a_0 + a_1t_d + a_2t_d^2 + a_3t_d^3 + a_4t_d^4 = 0 \\ x_{s_r}(t_d) = a_1 + 2a_2t_d + 3a_3t_d^2 + 4a_4t_d^3 = 0 \\ x_{s_r}(t_d) = 2a_2 + 6a_3t_d^2 + 4a_4t_d^3 = 0. \end{cases}
$$
\n(40)  
\nEq. (40) can ensure that  $e_s(0) = \dot{e}_s(0) = 0$  and the refer-

tion, that is,  $x_{s_r}(t) \in C^2$  and in the determined tim

cific polynomial function, the acceleration response of human body, vehicle suspension stroke, seat suspension stroke and control input force for the active system with QZS vibration isolator as seat suspension are shown in Fig. 9. In this case Sollowing equations:<br>  $x_5 = 0.02$ <br>  $x_6(t_0) = a_0 = x_5(0)$   $\dot{x}_5(t_0) = a_1 = x_6(0)$ <br>  $x_6(t_6) = a_0 + a_4t_4 + a_4t_3^2 + a_4t_4^3 = 0$ <br>  $\ddot{x}_5(t_6) = a_1 + 2a_5t_4 + 3a_4t_4^3 + 4a_4t_4^3 = 0$ <br>  $\ddot{x}_5(t_6) = a_1 + 2a_5t_4 + 3a_4t_4^3 + 4a_4t_4^$ zero reference trajectory are also plotted in the same figure for  $x_{s_r}(t) \neq 0$  ( $m_3 = 60$ ). comparison. The corresponding peak acceleration value of human body, maximum vehicle suspension stroke, seat suspension stroke and control input force are shown in Table 7.  $\begin{vmatrix} x_{1x}(t_0) = a_1 + 2a_2t_d + 3a_tt_d^2 + 4a_tt_d^3 = 0 \\ x_{1x}(t_0) = 2a_3 + 6a_tt_d^2 + 4a_tt_d^3 = 0. \end{vmatrix}$ <br>
Eq. (40) can ensure that  $e_x(0) = \dot{e_x}(0) = 0$  and the refer-<br>
ce trajectory  $x_{1x}(t)$  is a second order differentiable func-<br>

specific polynomial function and with the increase of the determined time  $t_d$ , the peak acceleration value of human body decreases although the vibration attenuation time becomes longer. The vehicle suspension stroke is almost the same for these two different reference trajectories and changes a little when the determined time  $t_{\textit{d}}$  takes different values. The



Fig. 9. Acceleration response of human body, vehicle suspension stroke, seat suspension stroke and control input force for the active system with QZS vibration isolator as seat suspension when

maximum seat suspension stroke and control input force are smaller than the counterparts of the zero reference trajectory and decrease as the determined time  $t_d$  increases.

## **6. Discussion**

When the linear vibration isolator and QZS vibration isolator are used as seat suspension. respectively, it is of interest to compare the control effect of the active system with these two

Table 7. Peak acceleration value of human body, maximum vehicle suspension stroke, seat suspension stroke and control input force for the active system with QZS vibration isolator as seat suspension when

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Table 7. Peak acceleration value of human body, maximum vehicle suspension stroke, seat suspension stroke and control input force for				
the active system with QZS vibration isolator as seat suspension when $x_{5r}(t) \neq 0$ ( $m_3 = 60$ ).				
Parameter	$x_{5r} = 0$	$t_0 = 0.5s$	$t_a = 1$ s	$t_a = 2s$
$\ddot{z}_{3p}$ /(m/s <sup>2</sup> )	$\theta$	0.1601	0.04	0.0101
$\left z_2-z_1\right _{\text{max}}/m$	0.1123	0.1123	0.1122	0.1123
$ z_3 - z_2 _{\text{max}}$ /m	0.033	0.0277	0.024	0.0233

Table 8. Peak acceleration value of human body, maximum vehicle suspension stroke, seat suspension stroke and control input force for the active system with two kinds of seat suspensions ( $m_3 = 60$ ).<br>
The active system with two kinds of seat suspensions ( $m_3 = 60$ ).



kinds of seat suspensions. The comparison is clearly shown in Fig. 10. The corresponding peak acceleration value of human body, maximum vehicle suspension stroke, seat suspension stroke and control input force are shown in Table 8.

The acceleration response of the human body, vehicle suspension stroke and seat suspension stroke are almost the same for these two kinds of seat suspension when the reference the control input force is different; when using QZS vibration isolator as seat suspension, the control input force becomes much smaller.

#### **7. Conclusion**

(1) When the QZS vibration isolator is used as seat suspen sion for the passive system, the peak acceleration value of human body is smaller than the linear seat suspension, the vibration attenuation time becomes shorter and the vehicle ride comfort improves effectively. The changing trends of vehicle suspension stroke are almost the same as the linear seat suspension. The seat suspension stroke is larger but remains in the allowed stroke range.

(2) When these two kinds of seat suspensions are in the unload condition, as the mass  $m_3$  decreases, the peak acceleration value of human body increases, while the maximum seat suspension stroke decreases. While in the overload condition, the increase of the mass  $m_3$  has less effect on the peak  $\alpha$ acceleration value of the human body and the maximum seat suspension stroke for the QZS vibration isolator; for the linear



Fig. 10. Acceleration response of human body, vehicle suspension stroke, seat suspension stroke and control input force for the active system with two kinds of seat suspension ( $m<sub>3</sub> = 60$ ).

seat suspension, as the mass  $m_3$  increases, the peak acceleration value of human body decreases while the peak seat suspension stroke increases. The vehicle suspension strokes are almost the same with different mass  $m_3$ .

(3) When the constrained adaptive back-stepping controller law based on BLF is used for the active system and the refer ence trajectory is designed as zero reference trajectory, the acceleration response of human body is zero in the whole time domain; the vehicle ride comfort improves more effectively than the passive system, the vehicle suspension stroke is almost the same with the passive system, and the seat suspen sion stroke is larger but remains in the allowed stroke range. The seat suspension stroke can be improved by setting the reference trajectory as the specific polynomial function, and the determined time  $t_a$  can be changed to adjust the peak acceleration value of human body and seat suspension stroke to high or low values. Then the maximum seat suspension stroke and control input force are smaller than the counterparts of the zero reference trajectory and decrease when the deter mined time  $t_{d}$  increases.

(4) When the controller law is used for these two kinds of seat suspensions, the acceleration response of human body, vehicle suspension stroke and seat suspension stroke are almost the same for the two different reference trajectories. But the control input force is different; when using QZS vibration isolator as seat suspension, the control input force becomes much smaller.

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