

# Design optimization of mixed-flow pump impellers with various shaft diameters at the same specific speed<sup>†</sup>

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#### Abstract

In this study, the hydraulic performance of a mixed-flow pump depending on the impeller hub ratio was analyzed using Computational-fluid-dynamics (CFD). The impeller inlet shape varies according to the hub ratio even at the same specific speed. It is important to ensure an optimum impeller design according to the hub ratio in order for the impeller shape to provide the desired performance at constant specific speed. The design variables of inlet part for meridional plane and vane plane development were defined for optimum impeller design. The objective functions were defined as the total head and total efficiency of the mixed-flow pump impellers. The optimum impeller design was carried out by controlling the design variables of impeller inlet parts by using the Response-surface-method (RSM). The tendency of impeller design variables depending on the hub ratio was identified by analyzing the optimum impeller design. Further, the impeller shape was designed on the basis of the tendency of the design variables depending on the hub ratio. Finally, the performance of an impeller with the designed shape was verified by numerical analysis.

Keywords: Mixed-flow pump; Impeller; Hub ratio; Design optimization; Computational fluid dynamics (CFD); Response surface method (RSM)

#### 1. Introduction

The mixed-flow pump is fluid machinery that is widely used in many industries as well as domestically. Even though it is used in many different industries already, further researches aiming increase of energy efficiency are essential due to its enormous energy consumption. Therefore, pump designing for performance improvement is getting more important and it takes experience and theory to design a pump with improved performance.

The mixed-flow pump is used in various industries and has numerous applications. Therefore, its design specifications such as design flow rate, total head, and rotational speed need to be varied, even at the same specific speed. Thus, various impeller designs are required. In particular, as the pump size increases, the size of the shaft diameter also increases, even at the same specific speed. Thus, a larger hub ratio is required on the impeller meridional plane. As the impeller inlet shape varies according to the hub ratio, the impeller of a mixed-flow pump needs to be redesigned while considering the hub ratio in order to obtain the desired performance from a mixed-flow pump [1, 2].

Recently, studies about fluid machinery designing using optimization technique and CFD are actively carried out. Especially, researches on pump hydraulic design for improve performance are actively being carried out. Heo et al. have analyzed the flow-field of fluid machinery by using CFD and conducted a study of performance improvement in fluid machinery based on the analysis result [3]. Kim et al. have studied optimal designing of impeller and volute in centrifugal pump by using design of experiment as well as CFD [4].

Virendra et al. analyzed varying performance of turbine according to the changes of shaft diameter by using CFD. Virendra et al. analyzed changing efficiency of turbine according to the changes of shaft diameter and suggested few methods to improve efficiency [5].

Wen et al. analyzed the performance of mixed-flow pump according to the changes of diffuser hub radius by using CFD and tests. The changes of total head and efficiency according to diffuser hub radius were analyzed and the results of CFD were verified through performance experiments [6].

In this study, the impeller shape of the inlet part according to the hub ratio was analyzed and the impeller was optimally

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designed according to the hub ratio. For design optimization, the main design variables of impeller were designed using the Response surface method (RSM). The tendency of the impeller design variables according to the hub ratio was analyzed by using the optimally designed impeller shape. The impeller shape was designed from the tendency of the impeller design variables according to the hub ratio. Finally, the performance of the impeller shape was verified using numerical analysis.

#### 2. Mixed-flow pump impeller design method

Impeller is very important component of mixed-flow pump and it changes kinetic energy of fluids into pressure energy. In impeller designing, blade shape and sweep angles can be expressed as meridional plane and front plane. Meridional plane and front plane is expressed as cylindrical coordinate.

In Fig. 1, (a) expresses meridional plane of impeller while (b) and (d) show front plane and the-dimensional shape of the impeller respectively. Meridional plane represents the size and blade shape of the impeller as well as the information of r and z of cylindrical coordinate. Front plane depicts sweep angle depending on circular arc of the impeller as well as the information of r and  $\theta$  of cylindrical coordinate.

Although meridional and front plane represent blade shape and sweep angles, cylindrical coordinate provided from these planes also needs to be analyzed in order to understand blade angle distribution. Vane plane development can be expressed by using the information from the meridional and front plane of the impeller and one of its advantages is that blade angle distribution can easily be shown on it. In Fig. 1, (c) is twodimensional plane of vane plane development [7, 8].

Designing of meridional plane and blade angle distribution is required in order to design impeller shape. The base meridional plane of impeller was designed by analyzing previous studies and data base while blade angle distribution was designed by using vane plane development. Design variables of meridional plane and vane plane development that controls impeller shape were identified in order to optimal designing of the impeller. Pump performance depending on the changes of design variables was analyzed by utilizing CFD while RSM was used in order to carry out optimal designing with improved pump performance. The performance of optimally designed impeller was verified using CFD and the result was compared with that of reference model. Table 1 shows the design specifications. Specific speed (rpm, m<sup>3</sup>/min, m) of impeller from mixed-flow pump is 550.

#### 2.1 Design variables in meridional plane

In order to design meridional plane which shows the information of impeller shape and size, design variables of meridional plane were defined as shown in Fig. 2.

Ordinary meridional plane is designed as arc type. However, making it arc type might not satisfy pump performance although the designing the curve of meridional plane can be Table 1. Design specifications.

Q [CMH]	Ht [m]	RPM
420	17.5	1780

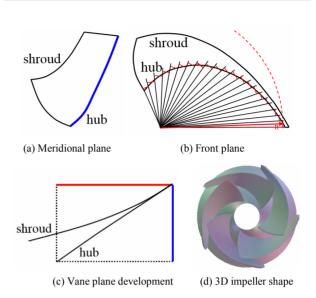


Fig. 1. Traditional design method of the impeller [7, 8].

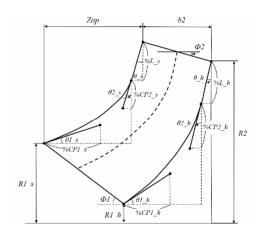


Fig. 2. Design variables for meridional plane [9].

simpler. Therefore, accurate definition of design variables for meridional plane is required to design meridional plane that satisfies performance.

In this study, Bezier curve was used for accurate designing of meridional plane. In Fig. 1, *R1* indicates the radius of inlet part and *R2* indicates the radius of exit part. *b2* is width of outlet part while *Ztip* indicates the length of axial direction between inlet and outlet of impeller shroud. *Phi1* is gradient of leading edge for impeller while *Phi2* is that of trailing edge for impeller. The curve on meridional plane was connected smoothly and bezier curve was used to smoothly connect inlet part and exit part of meridional plane.  $\theta I$  is a gradient of inlet part and  $\theta 2$  is that of exit part on the curve of meridional plane. %CP1 and %CP2 are control points of inlet part and outlet part that control Bezier curve. %L indicates section of straight line from outlet part of impeller while  $\theta$  indicates the gradient of straight lined section from the outlet part of impeller. Among design variables of meridional plane, *h* means hub while *s* indicates shroud [9].

#### 2.2 Design variables in vane plane development

Design variables of vane plane development were defined as shown in Fig. 3 in order to design vane plane development that represents information regarding distribution of impeller blade angles.

Vane plane development provides information of impeller inlet and outlet angles as well as distribution of impeller blade angles and it makes it easier to control inlet and outlet angles along with distribution of blade angles. Therefore, if design variables of vane plane development are defined accurately, the distribution of blade angles as well as impeller inlet and outlet angles can be designed easily. Vane plane development was designed in a way where impeller inlet angle is connected to impeller outlet angle smoothly. Therefore, beta distribution is expressed as linear distribution.

In Fig. 3, inlet angle of impeller was defined to be  $\beta I$  while

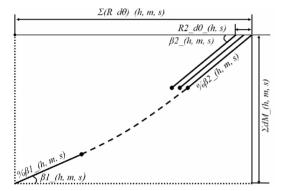


Fig. 3. Design variables for vane plane development [9].

Fig. 4. Comparison of meridional plane for hub ratios.

exit angle was defined to be  $\beta 2$ .  $\%\beta 1$  and  $\%\beta 2$  were defined to represent the identical sections from inlet and outlet angles and  $\%\beta 1$  and  $\%\beta 2$  were expressed as percentage of the length of meridional plane.  $\sum R_{-}d\theta$ , which is Y axis of vane plane development, is a sum of arc length while X axis  $\sum dM$  is a sum of meridional plane length. hub, mid-span, and shroud were defined as *h*, *m* and *s*, respectively in vane plane development [9].

# 3. Mixed-flow pump impeller design according to hub ratio

Shaft diameter is calculated using shaft power and rotational speed. Shaft diameter increases when flow rate increases at the same specific speed. Pump designing is difficult for a fixed hub ratio when flow rate increases at the same specific speed. The optimally designed Ns550 mixed-flow pump in a previous study was used as the reference model. Each hub ratio in the reference model was increased by 5 % from the impeller diameter. The hub ratio indicates the ratio of the impeller inlet hub diameter and outlet shroud diameter. Fig. 4 shows the impeller meridional plane depending on the hub ratio. Fig. 4 indicates that as the hub diameter increased, the inlet shape of the meridional plane varied according to the following conditions:

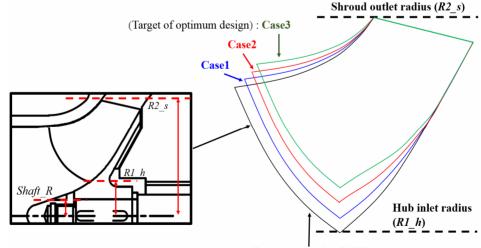
(1) The inlet area of the meridional plane was designed to be identical to that of the reference model.

(2) When the hub diameter of the inlet part increased, the axial positions of the inlet hub remained unchanged.

(3) The gradients of the impeller inlet part were identical.

(4) When the hub diameter of the inlet part increased, the shroud diameter of the inlet part and the axial position of the inlet shroud that satisfied (1)~(3) were selected.

The optimum impeller design was carried out on the meridional plane, the hub ratio of which is 0.15.  $\theta I h$ , which is the



Reference model (Ns550)

Measurement device	Range	Uncertainty
Torque meter	0~200 Nm	$\pm 0.2$ %
Flow meter	0~900 m <sup>3</sup> /hr	$\pm 0.2$ %
Rotational sensor	0~20000 r/min	$\pm0.02~\%$
Absolute pressure transducer	0~200 kPa	$\pm0.25\%$
Differential pressure transducer	0~500 kPa	$\pm 0.2$ %

Table 2. Specifications of measurement devices.

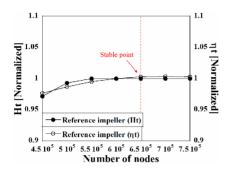


Fig. 5. Result of grid dependency test.

incline angle of the hub meridional curve in the inlet part, and  $i\beta 1_h$  and  $i\beta 1_s$ , which are the incidence angles of the hub and shroud, respectively, were chosen as the impeller design variables.

#### 4. Numerical analysis method

Commercial program ANSYS CFX-16.0Ver. was utilized for flow-field analysis of internal impeller as well as verification of performance for impeller.

Structured grid was generated by using turbo-grid for internal flow passage of the impeller. The impeller has 5 blades and they have identical blade shape. Therefore, numerical analysis was carried out using periodic condition on only one blade domain. After the mesh test, 650000 grids were used for the impeller analysis. Fig. 5 shows the result of grid test on the impeller.

Atmospheric pressure was applied to impeller inlet part while mass flow-rate was given to outlet part. The shape of numerical analysis for the impeller as well as boundary condition are shown in Fig. 6. Working fluid of impeller is water [10-12]. RANS (Reynolds average Navier-Stokes) equation was used for turbulent flow analysis for incompressible fluid as water is incompressible flow-field. SST (Shear stress transport) k-w model was used as turbulent model which makes it possible to predict flow separation of flow-field for the impeller [13-15].

#### 5. Validation of reference model

Reference model is an optimally designed shape that has been developed through previous studies. The reference model was optimally designed by applying CFD and optimization method on impeller design variables (Meridional plane and

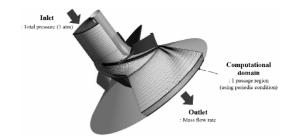


Fig. 6. Boundary conditions & grid system for impeller calculation.

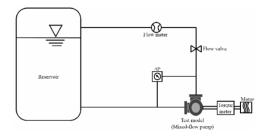


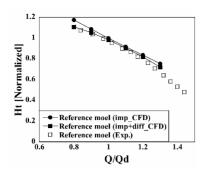
Fig. 7. Schematic diagram of experimental apparatus for a mixed-flow pump [16].

vane plane development). The performance of optimally designed reference model has been verified with CFD and performance test. The experiment and CFD results were normalized by the CFD results of design-flow rate for the reference model.

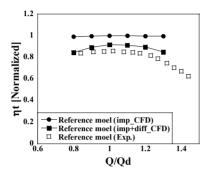
Fig. 7 shows an experiment apparatus for pump performance test. The pump performance test consists of a reservoir, a pressure gauge, an electromagnetic flowmeter, a flow control valve and a torque meter. The electrical motor that was connected to test pump was controlled by using an inverter. The range and uncertainty of main measurement device from the experiment apparatus are shown in Table 2 [16].

In order to verify the performance of reference model, the numerical analysis results and experimental results were compared and analyzed. The performance curve of the reference model was verified by using CFD and experiment. Fig. 8 is the comparison of CFD and experimental result of the reference model.

Fig. 8(a) is the comparison of total head curve from numerical analysis and experimental result while Fig. 8(b) is the comparison of total efficiency curve. Diffuser loss is confirmed when the results of impeller CFD and the result of impeller and diffuser CFD are analyzed. The result of performance experiment for the reference model includes impeller and diffuser. In Fig. 8, CFD result of the impeller and diffuser is similar to the performance curve of performance test result. However, the test results were smaller than the numerical results. That is because factors such as bell mouth effect, tip clearance effect, disk friction loss, and leakage loss are not included in the numerical analysis results. The tendencies of performance curves are similar between the numerical analysis results and experimental results from the reference model.



(a) Performance evaluation of the total head



(b) Performance evaluation of the total efficiency

Fig. 8. Comparative analysis of the performance evaluation for reference model (Experiment vs. numerical analysis).

#### 6. Optimally designed impeller

The optimum impeller design was carried out using Response surface method (RSM). The purpose of RSM is to find the optimal conditions for a design variable that could optimize the response variable [17, 18]. In RSM, the objective functions are required in order to analyze the performance depending on the changes of design variables. Objective functions were defined on total head curve and total efficiency curve which are performance curves of a pump. Fig. 9 shows total head curve and total efficiency curve that defined objective functions. A pump has its total head fixed at design specification and designed to have high efficiency. Therefore, the objective functions were defined to be total head and total efficiency at design flow rate. Total head and total efficiency are calculated by Eqs. (1) and (2).

$$Ht = \frac{Pt\_out - Pt\_in}{\rho g} \tag{1}$$

$$\eta t = \frac{(Pt\_out - Pt\_in) \times Q}{\tau \times \omega} .$$
<sup>(2)</sup>

In Eq. (1),  $Pt\_out$  indicates the outlet total pressure and  $Pt\_in$  indicates the Inlet total pressure.  $\rho$  is the density while g indicates the acceleration of gravity. In Eq. (2), Q is the flow rate and  $\tau$  is the torque.  $\omega$  is the Angular velocity.

The changes of total head and total efficiency depending on the changes of design variables were analyzed. Optimum design was carried out based on the analysis result. Proper ex-

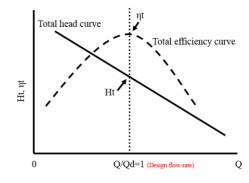
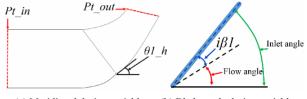


Fig. 9. Objective functions (Total head, total efficiency).



(a) Meridional design variable (b) Blade angle design variable

Fig. 10. Impeller design variables for RSM.

perimental conditions need to be selected as design variables for optimum design. Experimental conditions of design variables were selected by utilizing central composite [19-21].

As for the optimum design of impeller,  $i\beta I_h$ ,  $i\beta I_s$  and  $\theta I_h$  were selected to be main design variables using RSM.  $i\beta I_h$  and  $i\beta I_s$  are incidence angles of hub and shroud respectively. Incidence angle means the difference of inlet angle and flow angle.  $\theta I_h$  means gradient of inlet part of hub curve on meridional plane. Fig. 10 shows main design variables for optimum design.

As for optimum design of impeller, regression analysis was used for response optimization of design variables. Regression analysis is numerical anticipation of mathematical shape in order to analyze association between design variables and objective functions. Curvilinear regression analysis is a type of regression analysis where the relationship between design variables and objective functions is assumed to be curved. Regression analysis is presented in Eq. (3).

$$\hat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 \times X_1 + \widehat{\beta}_2 \times X_2^2 + \dots + \widehat{\beta}_n \times X_n^n.$$
(3)

*X* indicates the design variables and  $\hat{\beta}$  is regression coefficient.  $\hat{Y}$  is established by regression analysis. Here, the standard error of regression is less than 5 %. Hence, the accuracy of the curvilinear regression analysis is superior in this study.

Optimum design of the impeller was carried out on the meridional plane, as shown by Case 3 in Fig. 4. The hub ratio in Case 3 is 0.15. A main effects plot was used in order to analyze the tendencies of response variables according to the impeller design variables. Fig. 11 shows the optimization flowchart for optimum design of impeller.

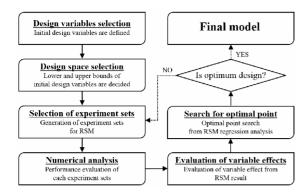


Fig. 11. Optimization flow chart.

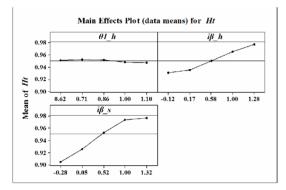


Fig. 12. Main effects plot for total head.

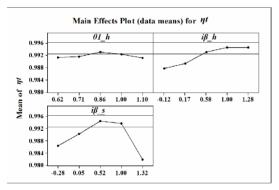


Fig. 13. Main effects plot for total efficiency.

Figs. 12 and 13 show the main effects plots. In Fig. 12, the total head increases as  $i\beta 1\_h$  and  $i\beta 1\_s$  increase. However,  $\theta 1\_h$  tends to be the highest total head at 0.71. In Fig. 13, the total efficiency is highest when  $\theta 1\_h$  is about 0.85, while it is highest at  $i\beta 1\_h$  and  $i\beta 1\_s$  values of 1.0 and 0.52, respectively. The optimum impeller design was carried out such that the efficiency could be improved since the total head according to changes in the design variables of impeller satisfies the design specifications.

When main effects flot is analyzed, the tendencies of total head and total efficiency depending on the changes of design variables can be predicted. In order to numerically express total head and total efficiency depending on the changes of design variables, regression analysis shown in Eq. (3) was

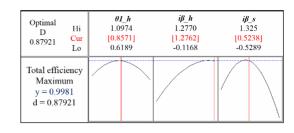


Fig. 14. Plot for response optimization.

used. Optimal condition of design variables can be found if total head and total efficiency are digitized according to the changes of design variables. Objective function for optimum design was set to be maximum efficiency. Total head is influenced significantly by the shape of impeller exit part. The shape of impeller exit part according to hub ratio is same as the reference model. Therefore, total head was excluded from the objective function.

The plot of response optimization shown in Fig. 14 was drawn using RSM. With regard to the result of response optimization, the total efficiency was predicted to be 0.9981 when  $\theta I_h = 0.86$ ,  $i\beta I_h = 1.28$  and  $i\beta I_s = 0.52$ . This impeller performance result was verified using numerical analysis [22]. The design and objective functions were normalized by the design flow rate of the reference model.

### 7. Tendency analysis of impeller design variables according to hub ratio

The tendency of the design variables according to changes in the inlet part hub diameter was analyzed using the reference model and the Case 3 shape constructed on the basis of previous studies. Fig. 15 shows the tendencies of  $\theta 1_h$ ,  $i\beta 1_h$  and  $i\beta 1_s$ . The hub ratio in each case improved by a value of 0.05 from the reference model hub ratio, which was 0. The hub ratio in Cases 1-3 were 0.05, 0.1 and 0.15, respectively. In Figs. 15(a) and (c),  $\theta 1_h$  and  $i\beta 1_s$  decrease as the hub ratio increases. However, in Fig. 15(b),  $i\beta 1_h$  increases as the hub ratio increases. By analyzing the tendencies of the design variables in Fig. 15, we can observe that the tendencies of  $\theta 1_h$ and  $i\beta 1_s$  are opposite to that of  $i\beta 1_h$ .

The equations of tendency for  $\theta l_h$ ,  $i\beta l_h$  and  $i\beta l_s$  are as follows, respectively:

- $\theta l_h = 1 0.95 \times \text{hub ratio}$  (4)
- $i\beta 1$   $h = 1 + 1.85 \times \text{hub ratio}$  (5)

$$i\beta 1 \quad s = 1 - 3.17 \times \text{hub ratio}$$
 (6)

The impeller shape is designed using the design variable tendencies shown in Fig. 15.

The designed impeller shape is shown in Fig. 16. In Fig. 16, Cases 1 and 2 are designed using the design variable tendencies shown in Fig. 15. An optimally designed impeller shape from a previous study was chosen to be the reference model. Case 3 shows the optimally designed shape with a hub ratio of

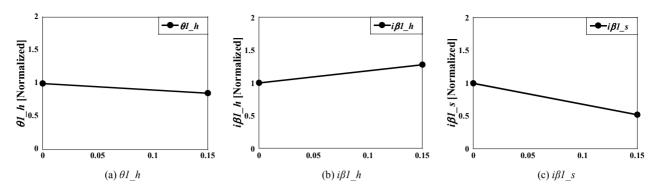


Fig. 15. Tendency of design variables.

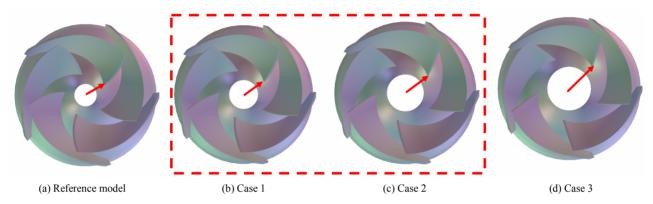
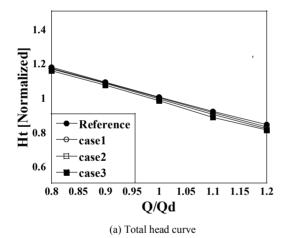


Fig. 16. Comparison of three-dimensional geometry impeller.



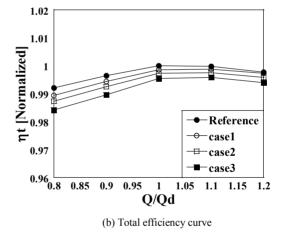


Fig. 17. Comparison of total head and total efficiency curve.

0.15. Fig. 16 shows a comparison of the impeller shapes according to changes in the hub ratio. Impeller inlet shapes are different depending on the hub ratio. The other design variables of impeller were fixed to be design variables of the reference model.

# 8. Analyses of numerical analysis results according to hub ratio

Fig. 17 shows a comparison between the total head and total efficiency curves for the impeller depending on the hub ratio.

The impeller designed on the basis of the design variable tendencies was subjected to performance verification conducted using numerical analysis [23]. In Fig. 17(a), the tendencies of the total head curves depending on the impeller shape are similar. Similarly, the total heads at the design flow rate are almost identical. In Fig. 17(b), the tendencies of the total efficiency curves according to the impeller shape are similar, and the maximum efficiency is expected at the design flow rate. The total efficiency in Cases 1 and 2, which are based on the design variable tendencies, is lower than that of the reference model at the design flow rate. However, the total efficiency in

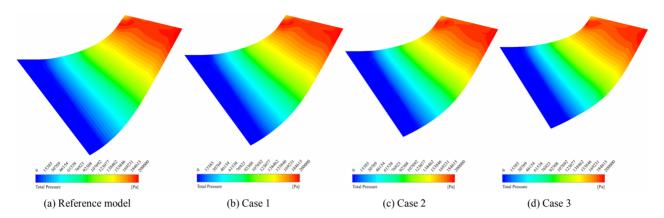
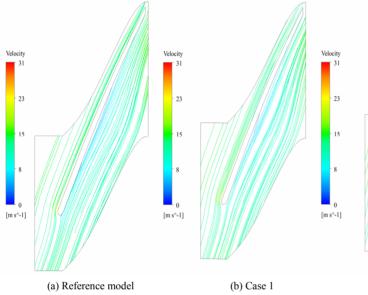


Fig. 18. Comparison of the total pressure contour on the meridional plane.



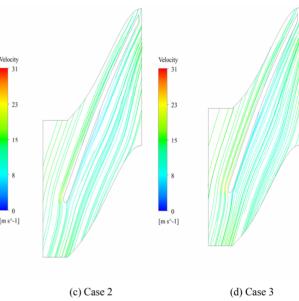


Fig. 19. Comparison of the streamline on the blade to blade at mid-span.

these cases is higher than that in the optimally designed Case 3 for which the hub ratio is 0.15.

Fig. 18 compares total pressure contour of each impeller shape. From total pressure distribution of each shape, it is obvious that the tendencies of total pressure distribution are very similar. That is because the loss of total pressure is minuscule with similar total head curves according to changing hub ratio as it is seen from the total head curves in Fig. 17(a).

Fig. 19 compares streamline of each impeller shape. As shown in Fig. 19, the streamline of fluid flow for each shape is very smoothly. That is also due to the fact that the loss of total efficiency is minimum according to the changes of hub ratio as it is seen from the total efficiency curves in Fig. 17(b).

It can be said that the performance of the impeller with a shape that was designed using the design variable tendencies depending on changes in the hub ratio is almost identical to that of impeller with the optimally designed shape. Moreover, this impeller could satisfy the design specifications at the same specific speed.

## 9. Conclusions

In this study, the impeller shape and performance according to the hub ratio at an unchanged specific speed were analyzed. The impeller was designed on the basis of design variable tendencies according to the hub ratio. The following conclusions have been drawn from this research:

(1) The result of performance test on the reference model with numerical analysis can be credited because the tendencies of performance curves are similar when numerical and experimental result of reference model are compared and analyzed.

(2) The impeller shape can be designed according to the hub ratio in the same impeller area, and the optimum design can be determined using design variables of the impeller inlet part.

(3) The tendency of design variables can be analyzed according to the hub ratio, and the impeller shape depending on the various hub ratios can be designed on the basis of the design variable tendencies.

(4) The total head curve for the impeller, which is con-



structed using the design variable tendencies, is expected to be almost identical to that for the impeller with the optimally designed shape.

(5) The total efficiency curve for the impeller, which was constructed using the design variable tendencies, is expected to be almost identical to that for the impeller with the optimally designed shape. Further, the total efficiency is highest at the design flow rate.

(6) The impeller shape that satisfies the design specifications could be designed on the basis of design variable tendencies according to changes in the hub ratio.

(7) The hub ratio that is related to shaft diameter is very important when designing impeller of mixed flow pump. Performance analysis depending on hub ratio is important as the performance of mixed-flow pump varies according to hub ratio. Impeller shape that satisfies pump performance from various shaft diameters can be designed if pump performance depending on hub ratio is analyzed.

### Acknowledgements

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#### Nomenclature-

CFD	: Computational fluid dynamics
RSM	: Response surface method
Ht	: Total head
M	: Meridional length
N	: Rotational speed
Ns	: Specific speed
Pt_in	: Inlet total pressure
Pt_out	: Outlet total pressure
Q	: Flow rate
Qd	: Design flow rate
<i>R1</i>	: Radius
R2	: Radius of diffuser inlet
Ztip	: Axial direction from inlet/outlet parts of the shroud
g	: Acceleration of gravity
h	: Hub
т	: Mid-span
S	: Shroud
$\Phi l$	: Inclined angle of the leading edge
$\Phi 2$	: Inclined angle of the trailing edge
β1	: Inlet angle of impeller
<i>iβ1</i>	: Incidence angle
$\theta l$	: Inlet angle from meridional curve
θ2	: Outlet angle from meridional curve
%L	: Straight line from outlet part
%CP1	: Control points of inlet part
%CP2	: Control points of outlet part
%β1	: Portion of same inlet blade angle

#### $\%\beta 2$ : Portion of same outlet blade angle

- $\rho$  : Density
- $\omega$  : Angular velocity
- $\tau$  : Torque

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