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# Unsupervised identification of arbitrarily-damped structures using time-scale independent component analysis: Part I †

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**Abstract**

In this study, a new method is proposed to identify the dynamic parameters of structures with higher accuracy compared to current methods. First, the wavelet-transformed representation of system responses is extracted from measured responses, and then the independent component analysis is used to achieve the modal characteristics. The simulation results of a multi-degree-of-freedom system illustrate that this method is capable of accurately identifying the modal information of lightly- and highly-damped structures. It is represented that continuous wavelet transform, due to its adaptive time-frequency resolution, is more efficient to be incorporated into independent component analysis compared to Short time Fourier transform (STFT). The latter is unable to accurately determine the modal response, especially at higher frequencies, while the proposed method can identify the system with marked accuracy. The efficiency of proposed method is also investigated under additive noise. Results shown that for highly- and lightly- damped system, the proposed method is able to capture the modal parameters especially in higher frequencies of vibration, along with the modal assurance criterion values with satisfactory accuracy, which indicates the robustness of the procedure compared to other available methodologies.

*Keywords*: Independent component analysis; Seismic responses; Wavelet transform; Unsupervised identification

## 1. Introduction

Difficulties associated with measuring input excitation have caused additional costs associated with input-output approaches for identification of dynamic characteristics of structures. In this regard, signal-based output-only algorithms are used to identify modal parameters of dynamical systems with implementation of available structural responses. System identification is the mathematical representation of dynamical systems [1-5] using experimental data, which are mainly categorized into modal parameter identification, structural-model parameter identification and control-model identification [6-9] in which the latter has been mostly utilized for flexible structures [10]. Observer Kalman filter identification and system Markov parameters are among widely used methods as modal identification approaches. He et al. [5] utilized wind-induced vibration for modal parameter extraction based on stochastic subspace identification. They also studied the effect of noise on modal identification process and eventually concluded that their approach could have satisfactory verification of Vincent Thomas bridge modal parameters located in Los Angeles. However, in most of the real-world applications it is difficult to measure the input excitation. Output-only identification approaches are emerged as robust methods for operational modal identification of structures. In this regard, a few studies based on output-only approaches are conducted to extract natural frequencies, damping ratios and mode shapes. Moaveni et al. [6] integrated Natural excitation technique (NExT) and ERA (Eigensystem realization algorithm) in order to perform output-only identification of a full-scale 7-story building resulting in appropriate modal parameters for a given damage level. Reynders et al. [7] performed modal identification based on kernel principal component analysis in order to omit environmental and operational influences. Additionally, they proposed a damage-sensitive feature algorithm for civil engineering structures.

Identification of dynamic characteristics of structures based on signal-based approaches has attracted a significant number of studies. In this respect, wavelet analysis as one of the time-scale representations of signals has been implemented to identify time-frequency content of signals with sufficient precision due to its appropriate resolution variation. Slavic et al. [8]

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performed Gabor wavelet in order to identify modal damping ratios, they consequently decreased the edge effect associated with wavelet transform. Lardies and Gouttebroze [11] estimated damping ratios, natural frequencies and mode shapes using modified Morlet mother wavelet with higher frequency resolution rather than conventional Morlet wavelet to provide more satisfactory modal separation. Kougioumtzoglou and Spanos [12] utilized harmonic wavelets in order to generate time-dependent wavelet-based frequency response functions for identification of linear and nonlinear multi-degree-of-freedom systems concluding that this procedure could be appropriately performed in the presence of noise-corrupted signals.

Blind source separation (BSS) techniques are proposed to extract source signals [13] from observed ones which are conventionally known as Cocktail-party problems. This technique has been transmitted to structural dynamics in the last decade; McNeill [14] has compared Second order blind identification (SOBI) and gradient descent minimization in the context of BSS problem in mechanical engineering. He further estimated modal parameters using SOBI by competitive accuracy compared with state-of-the-art methods. Hazara and Narasimhan [15] utilized the property of invariance in stationary wavelet transform in order to be used in the method of SOBI to appropriately separate the modal characteristics of a system. They observed that their method can satisfactorily estimate natural frequencies of the UCLA Factor building. However, their method is computationally-inefficient. Sadhu et al. [16] integrated stationary wavelet packet transform and principal component analysis for modal identification with a set of limited sensor networks. They further declared that their decentralized method could be used for mobile sensor networks as well. While their approach is unique but the wavelet packet transform is not capable of accurately identify all the frequencies and some data is missed. Sadhu et al. [17] implemented an updated method based on SOBI in which modal characteristics of vibration are identified using a new set of weighted covariance matrices. Their method is accurate, however, extensive prior knowledge about covariance matrices in advance is required, and it cannot be done as an unsupervised approach. Sadhu et al. [18] have also proposed the usage of stationary wavelet packet transformed data in parallel factor decomposition for modal identification. They stated that their method overcomes the deficiencies of parallel factor decomposition approach. This method also utilizes wavelet packet transform in which some data are eliminated and subsequently, some frequency data could be missed. Abazarsa et al. [19] have proposed an improved technique based on SOBI for extraction of modal parameters using limited number of sensors. They further contend that estimated mode shape matrix would not be square due to incomplete observation of structural responses. But, the improved SOBI is complicated and hard-to-implement in real-world applications.

Independent component analysis (ICA), originally developed in neuroscience, is implemented as an unsupervised algorithm for separation of modal responses implementing measured story responses. Poncelet et al. [20] performed modal identification based on SOBI and ICA approaches using free and random responses of mechanical systems. They identified modal parameters for very weakly damped structures. Zang et al. [21] conducted ICA in order to decompose time domain structural responses and then have estimated mode shape matrix in the presence of higher damping. Kerschen et al. [22] estimated mode shape matrix for linear structures based on ICA. They remarked that this method does not need stabilization charts which are required for conventional modal identification. However, this method could not identify structures with high damping ratios. Yang and Nagarajaiah [23] proposed an algorithm based on performing Short time Fourier transform (STFT) on structural responses and evaluating modal parameters by FastICA procedure. This method could extract the modal damping ratios for just highly damped structures, however, limitation of frequency resolution of STFT leads to inaccurate modal identification estimation, specifically for closely-spaced modes. Furthermore, comparison of the results of previous works (Yang and Nagarajaiah [23]) with this study, it is observed that higher frequencies which are extremely important for structural health monitoring would be missing in structures with numerous degrees of freedom. Yang and Nagarajaiah [23] also proposed a method using discrete wavelet transformed data as the input of independent component analysis for damage detection of time-varying systems, concluding that their approach can localize spikes due to a degradation in stiffness of the system.

In this study, the aggregation of wavelet transform with ICA is implemented to recover the modal time-history responses under free and forced vibration. The highly-damped structure is further analyzed. Furthermore, by implementation of this newly-developed method, the damping ratios, natural frequencies, mode shapes as well as modal time-history responses are accurately obtained which makes it a novel approach in modal identification of structures.

## 2. Wavelet transform

In the recent decades, Wavelet transform (WT) has emerged as a time-scale representation of signals instead of conventional time-frequency transforms such as STFT and Wigner-Ville distribution [24, 25]. WT is a convolution-based transform, which is defined as indicated in Eq. (1) [25].

$$W(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty}x(t)\psi^{*}\left(\frac{t-b}{a}\right)dt \tag{1}$$

in which a and b are dilation and shift parameters, respectively. Mother wavelet $\psi(t)$ is a square-integrable function and * denotes complex conjugate. Satisfying equal energy at all scales, the term $1/\sqrt{a}$ is used as a normalization factor. The dilated and shifted version of mother wavelet is called basis wavelet which is similar to sinusoidal basis functions in Fourier transform (FT). Eq. (2) represents that mother wavelet must have zero mean in order to be used as the basis function [25].

$$\int_{-\infty}^{+\infty} \psi(t)\mathrm{d}t = 0 . \tag{2}$$

One important property of the mother wavelets is their vanishing moments. The greater number of vanishing moments would result in higher smoothness of the wavelet function. Vanishing moment of a wavelet transform is defined in Eq. (3) [23, 25].

$$M_p = \int_{-\infty}^{+\infty} t^p \psi(t)\mathrm{d}t = 0,\ p = 0,1,2,\ldots,n . \tag{3}$$

These functions are required to satisfy admissibility condition to be assumed as mother wavelet. This condition is indicated in Eq. (4) showing zero value in frequency domain at frequency of zero.

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < +\infty \tag{4}$$

$$\left.|\Psi(\omega)|^2\right|_{\omega=0} = 0 . \tag{5}$$

Like any other convolution-based transform, WT also must meet the requirement of Heisenberg uncertainty principle indicated in Eq. (6) [23].

$$\Delta_{\phi(t)} * \Delta_{\Phi(f)} \geq \frac{1}{4\pi} . \tag{6}$$

It can be inferred that in frequency domain, this transform is a set of filters in different scales. In which the lower scales, the larger frequency bands and the smaller amplitude are.

STFT works with constant resolution, which is used for determining the sinusoidal frequency, phase content of the response data by dividing a long time signal into shorter parts, and compute the Fourier transform separately on each part to capture the Fourier spectrum. In general, STFT is not highly useful for analyzing time-variant, non-stationary signals.

In contrast to what is expected from STFT, wavelet transform is capable of having multiple basis functions such as Morlet, Mexican hat and Complex frequency B-spline, suitable for various signal-processing applications.

## 3. Theoretical background of independent component analysis

### 3.1 Basic concept

ICA is used as an efficient method to provide the linear combination of coefficients. By implementation of the recorded response signals $x_1(t)$ and $x_2(t)$ as the weighted sum of the unknown sources $s_1(t)$ and $s_2(t)$, this method could estimate the attributed weighting parameters with high accuracy.

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) \tag{7}$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t) \tag{8}$$

where $a_{11}$, $a_{12}$, $a_{21}$ and $a_{22}$ are the weighting parameters depending on source-to-recorder distances. The Eqs. (7) and (8) could be indicated in generalized matrix format which is represented in Eq. (9):

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{9}$$

where $\mathbf{x}(t)$ and $\mathbf{s}(t)$ are the vectors of recorded and source signals, respectively. And, **A** is the mixing matrix of coefficients.

### 3.2 Solution by independent component analysis

The independency of each source is presumed in ICA method. This method searches for de-mixing matrix **W** according to components to be independent as possible [23], while **W** is approximate inverse of matrix **A**. In addition, ICA is developed based on the central limit theorem meaning that the aggregate of independent random variable has higher tendency towards the Gaussian behavior than that of the non-Gaussian original data. It is significant to note that the non-Gaussian sources data is obtained by maximizing the contrast functions such as following negentropy function, indicated in Eq. (10), in a probabilistic sense [13, 23].

$$J(y_i) \propto \left\{E\left[G(y_i)\right] - E\left[G(v)\right]\right\}^2 \tag{10}$$

where J(yi) is the contrast function, E is the expectation operator $v$ is standardized Gaussian variable and G is non-quadratic function.

FastICA [27] seeks for each row of matrix **W** such that the Eq. (11) would be obtained reasonably according to the maximization of contrast function. Eq. (12) is the relation of the matrix of coefficient with de-mixing matrix and Eq. (13) represents the recovered components associated with ith row of **W**.

$$\mathbf{s}(t) = \mathbf{W}\mathbf{x}(t) \tag{11}$$

$$\mathbf{W} = \mathbf{A}^{-1} = \mathbf{\Phi}^{-1} \tag{12}$$

$$y_i(t) = w_i x(t) \tag{13}$$

where x(t) is the system response of the structure. $\mathbf{\Phi}$ is the modal matrix establishing by considering each mode shape as a column of $\mathbf{\Phi}$. **W** is the inverse matrix of $\mathbf{\Phi}$ and $\mathbf{w_i}$ is the components of the demixing matrix.

### 3.3 Blind source separation incorporating wavelet transform function

The governing equation of motion for a Multi-degree-of-freedom (MDOF) is:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(\mathrm{t})+\mathbf{K}\mathbf{x}(\mathrm{t})=\mathbf{f}(\mathrm{t}) \quad (14)$$

where **M**, **C** and **K** are the mass, damping and stiffness matrices of the system, respectively. **f**(t) is the external force vector applied to the system. The modal expansion of the response could be obtained as represented in Eqs. (15) and (16):

$$\mathbf{x}(t)=\mathbf{\Phi}\mathbf{q}(t)=\sum_{i=1}^{N}\varphi_i q_i(t) \quad (15)$$

$$\mathbf{q}(t)=\mathbf{\Phi}^{-1}\mathbf{x}(t) \quad (16)$$

where q(t) is the modal response of the structure. Wavelet transform is elaborated to estimate the characteristics of lightly and highly damped structures. By applying the Continuous wavelet transform (CWT) to both sides of Eq. (15), Eq. (17) is yielded:

$$\mathbf{X}(a,b)=\mathbf{\Phi}\mathbf{Q}(a,b)=\sum_{i=1}^{N}\varphi_i Q_i(a,b) \quad (17)$$

where $\mathbf{X}(a,b)$ is wavelet transform of $\mathbf{x}(t)$ and $\mathbf{Q}(a,b)$ could be obtained with implementation of the Eq. (17). a and b are dilation and shift parameters; subsequently, concatenating and combining the results would lead to Eq. (18):

$$\mathbf{X}_{ab}=\mathbf{\Phi}\mathbf{Q}_{ab}\,. \quad (18)$$

Concatenation is conducted through mapping time-frequency for each nodal response, with a matrix form. The matrix is further transformed to a one-dimensional space. In this regard, consider a signal with N time instances. The first N values of the concatenated vector are the wavelet coefficients in frequency $f_i$ followed by N values of wavelet transformed data for the next analyzed frequency and so on.

Considering modal independency of the time-domain modal responses, the aimed sources could be evaluated by implementation of ICA method regarding the post wavelet-transformed data of $\mathbf{X}_{ab}$. Hence, the independent sources in time-scale BSS model after implementing ICA yields in Eq. (19):

$$\mathbf{Q}_{ab}=\mathbf{W}\mathbf{X}_{ab} \quad (19)$$

where $\mathbf{Q}_{ab}$ are supposed to be sparse as possible. The matrix A could be obtained with the aid of Eq. (19). Afterwards, the normal modes' responses could be evaluated using the matrix W regarding the Eq. (20) as follow:

$$\mathbf{Q}_{ab}=\mathbf{W}\mathbf{X}_{ab}\,. \quad (20)$$

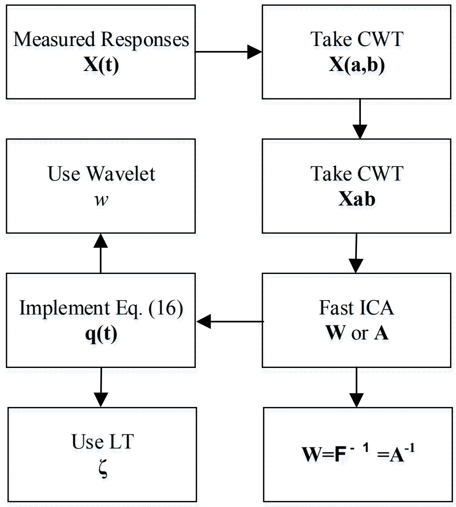


Fig. 1. CWT-ICA flowchart for modal investigation.

By implementation of Continuous wavelet transformed Independent component analysis (CWT-ICA), the natural frequencies and damping ratios are estimated implementing Logarithm-decrement technique (LDT) on modal responses. The order of matrix and the corresponding mode shape analyzed by CWT-ICA may not be precise; however, by evaluating the power spectrum attributed to each modal response the consecutive mode shapes and attributed natural frequencies could be derived easily.

## 4. Combination of wavelet transform and independent component

The CWT-ICA method can extract modal characteristics by performing unsupervised blind source separation from the measured structural responses to address the mentioned accuracy issues associated with previous works. Despite the previously common methods (e.g. STFT-ICA), implementation of CWT-ICA would lead to agreeable assessment of modal characteristics of the structures, especially with closely-spaced modes and higher damping. This method could be summarized in four major steps:

First, the measured system responses would be gathered, and the wavelet transformation of the responses would be estimated.

Second, the wavelet transformed data would be concatenated to produce one-dimensional time-scale models.

Third, the wavelet transformed responses would be analyzed and subsequently decomposed by ICA to provide the demixing matrix W. The normal modes further could be extracted from demixing matrix.

Fourth, the demixing matrix would be used to recover the time history responses attributed to each mode. These responses are to be used in continuous wavelet transform to evaluate the natural frequencies as well. Additionally, the modal damping ratios are estimated based on LDT incorporating the corresponding time-frequency analytical signal with

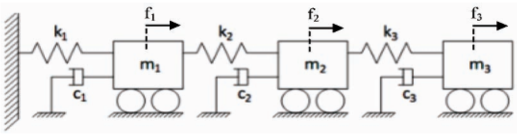


Fig. 2. A sample multi degree of freedom system for numerical simulation.

each of natural frequencies.

The unsupervised WT-ICA method with capability of modal identification by extracting the modal responses with the measured structural data is summarized in a flowchart shown in Fig. 1.

## 5. Numerical modeling

To validate the proposed CWT-ICA method, a numerical study has been conducted. The linear mass-spring-damper model is considered as indicated in Fig. 1. The equations of motion are:

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}. \quad (21)$$

The parameters are assumed as $m_1 = m_2 = m_3 = 1$ (N), and the stiffness is set as $k_1 = k_2 = k_3 = 2$ (N/m); the damping is considered $c_1 = c_2 = c_3 = \alpha m$ (N/s) where $\alpha = 0.04$ and $\alpha = 0.1$ are considered to investigate the highly- and lightly- damped structure in this study. It is noted that f1, f2 and f3 are functions of time.

In this regard to evaluate the effectiveness of the CWT-ICA for modal identification, the system was excited by inducing initial conditions of $x(0)=\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T (m)$ and $\dot{x}(0)=\begin{bmatrix}0 & 1 & 0\end{bmatrix}^T (m/s)$. Free vibration responses were produced based on the methods mentioned by Chopra [28] with sampling frequency of 20 Hz, and then analyzed by CWT. Subsequently, the measured responses concatenated and implemented to be solved by FastICA procedure. Mixing matrix was obtained and the normal modes were extracted which subsequently utilized to produce the modal responses.

## 6. Validation of CWT-ICA under free vibration excitation

### *6.1 Modal validation*

The exact natural frequencies evaluated by Eigen value analysis and mode shapes is once again identified by CWT-ICA frequencies of the three-DOF system which is indicated in Tables 1 and 2. The correlation between the identified and exact data is estimated by the Modal assurance criterion

Table 1. Exact and identified values of natural frequencies (Hz) by CWT-ICA in noise-free vibration.

| Mode (j) | Exact | CWT-ICA | | MAC (%) | |
|---|---|---|---|---|---|
| | | a = 0.04 | a = 0.1 | a = 0.04 | a = 0.1 |
| 1 | 0.1002 | 0.1006 | 0.0997 | 99.97 | 99.99 |
| 2 | 0.2807 | 0.2819 | 0.2795 | 99.71 | 99.95 |
| 3 | 0.4056 | 0.4078 | 0.4026 | 99.68 | 99.95 |

Table 2. Exact and identified values of mode shapes by CWT-ICA in noise-free vibration.

| Mode (j) | Exact | CWT-ICA | |
|---|---|---|---|
| | | a = 0.04 | a = 0.1 |
| 1 | $[1.00,1.80,2.25]^T$ | $[1.00,1.80,2.17]^T$ | $[1.00,1.80,2.17]^T$ |
| 2 | $[1.00,0.45,-0.80]^T$ | $[1.00,0.44,-0.72]^T$ | $[1.00,0.44,- 0.72]^T$ |
| 3 | $[1.00,-1.25,0.55]^T$ | $[1.00,-1.319,0.48]^T$ | $[1.00,-1.319,0.48]^T$ |

Table 3. Exact and identified values of modal damping ratios by CWT-ICA in noise-free vibration.

| Mode (j) | Exact | CWT-ICA | | MAC (%) | |
|---|---|---|---|---|---|
| | | a = 0.04 | a = 0.1 | a = 0.04 | a = 0.1 |
| 1 | 3.18 | 7.94 | 3.18 | 8.07 | 3.18 |
| 2 | 1.13 | 2.84 | 1.12 | 2.83 | 1.13 |
| 3 | 0.79 | 1.96 | 0.79 | 1.97 | 0.79 |

(MAC) indicated in Eq. (22). In addition, the CWT-ICA calculated modal damping ratios are indicated and compared with exact results in Table 3. The results show that the CWT-ICA could precisely estimate the natural frequency and modal damping ratios with marginal error.

$$MAC(\tilde{\varphi}_i, \varphi_i) = \frac{(\tilde{\varphi}_i^T \times \varphi_i)^2}{(\tilde{\varphi}_i^T \times \tilde{\varphi}_i)(\varphi_i^T \times \varphi_i)}. \quad (22)$$

### *6.2 Modal responses*

System displacement responses along with modal displacement responses produced by CWT-ICA for the cases of a = 0.1 and a = 0.04 are considered in this section. It is noted that the system responses are associated with multiple frequencies. The separated modal responses with single component frequencies produced by the proposed method from the structure's displacement responses (Figs. 3 and 5) could be observed in Figs. 4 and 6.

It is represented that WT-ICA method regardless of the damping ratio is capable of estimating the natural frequencies and mode shapes only by using the structural responses with satisfactory accuracy (Tables 1-3).

The proposed WT-ICA method is able to capture the modal responses of a structure regardless of highly- or lightly-damped conditions. Results show satisfactory prediction of modal responses, while, other methods such as ICA, STFT-ICA are not as robust as the mentioned method especially for

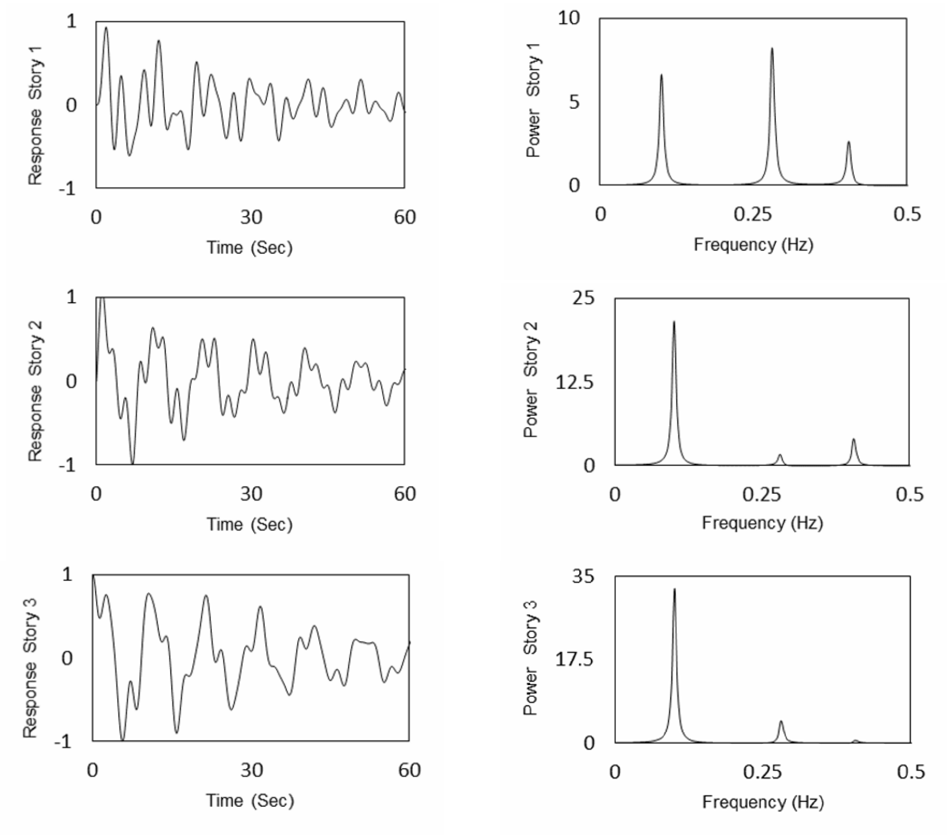


Fig. 3. Free vibration displacement time-histories (a = 0.1).

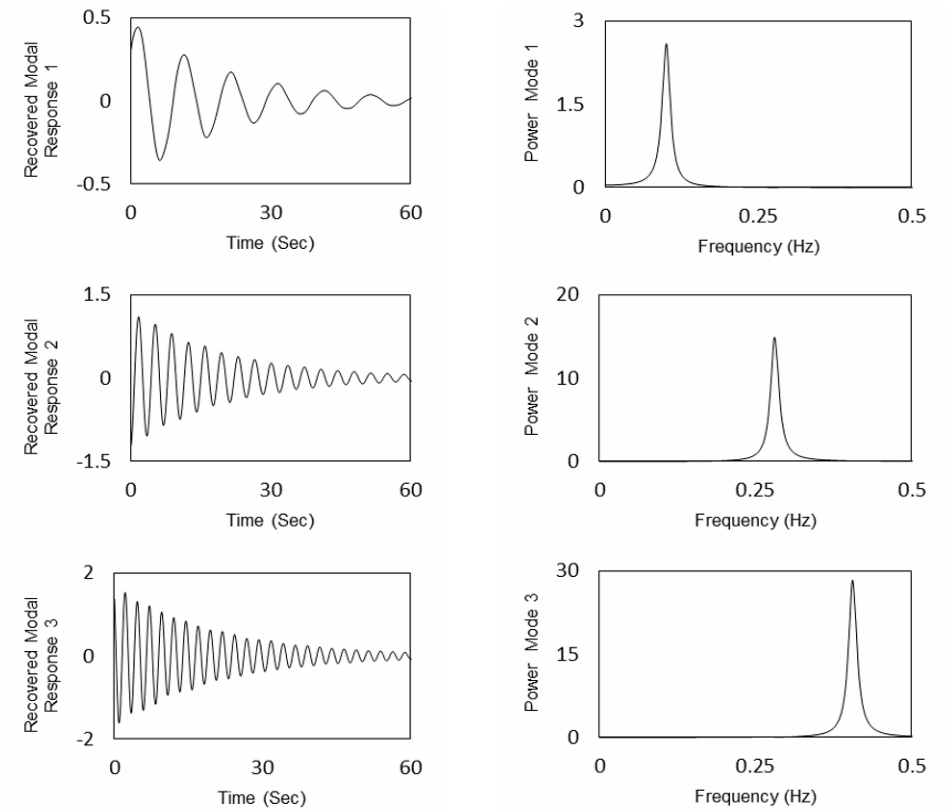


Fig. 4. Modal responses evaluated by WT-ICA (a = 0.1).

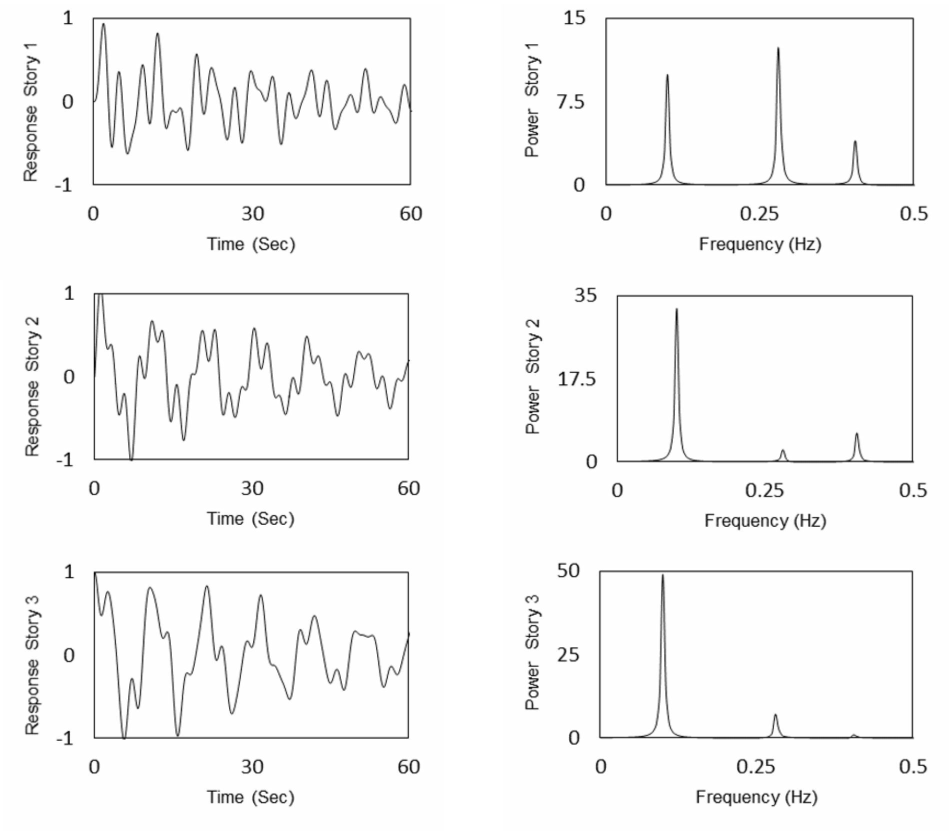


Fig. 5. Free vibration response to the initial conditions (a = 0.04).

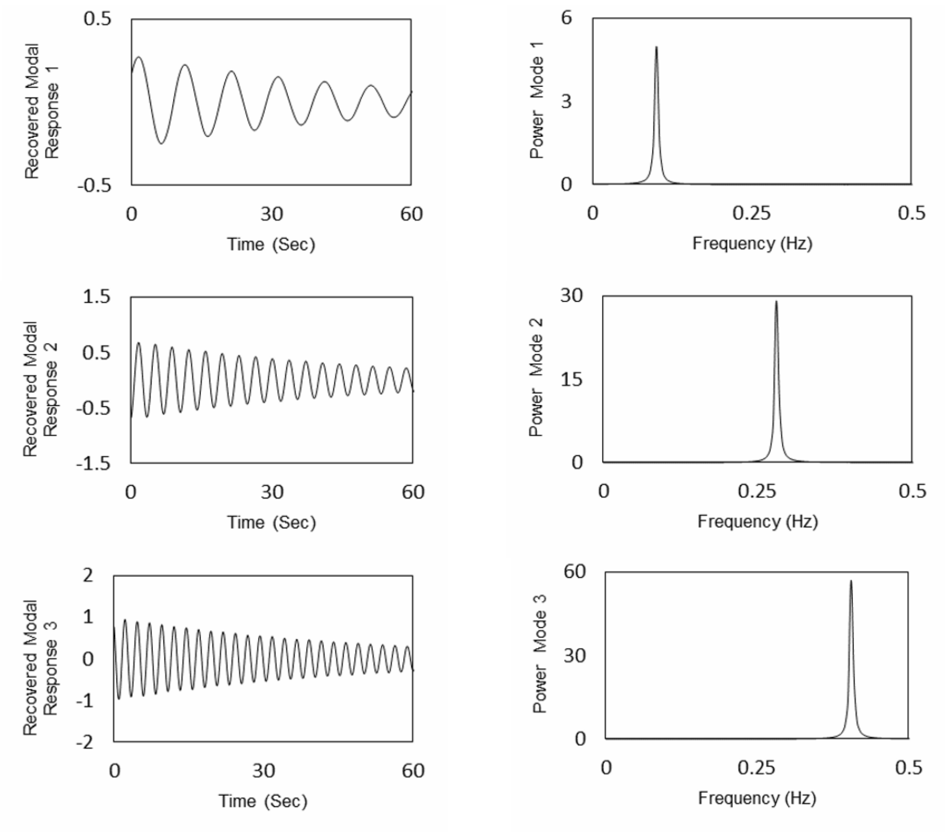


Fig. 6. Modal responses evaluated by WT-ICA (a = 0.04).

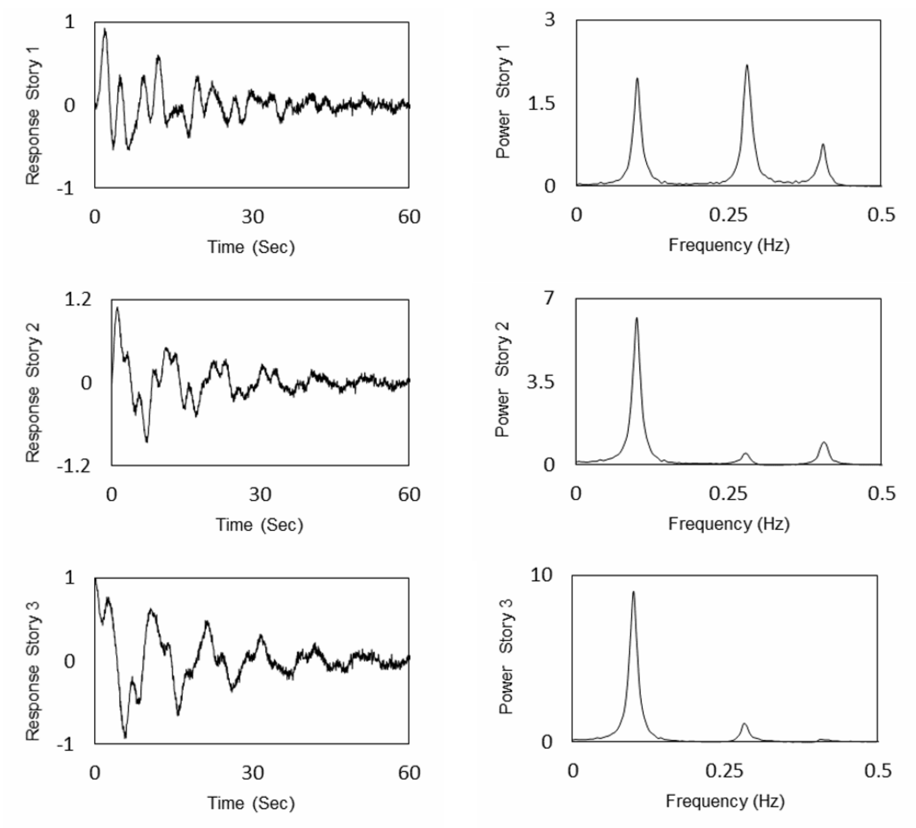


Fig. 7. Modal response of noise-corrupted system (SNR = 10) with a = 0.1 in time-domain.

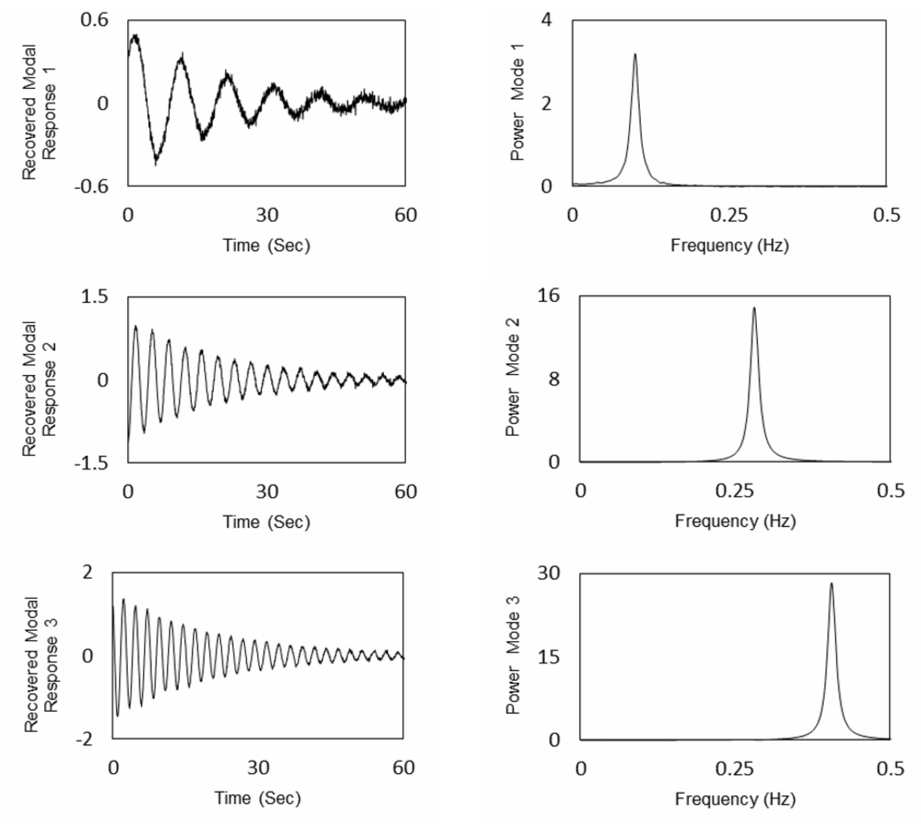


Fig. 8. Modal response of noise-corrupted system (SNR = 10) with a = 0.1 in frequency-domain evaluated by WT-ICA.

Table 4. Exact and identified values of natural frequencies (Hz) by CWT-ICA in noise-free vibration.

| Mode (j) | Exact | CWT-ICA (%) |
|---|---|---|
| 1 | 7.94 | 8.05 |
| 2 | 2.84 | 2.66 |
| 3 | 1.96 | 1.90 |

Table 5. Exact and identified values of natural frequencies (Hz) by CWT-ICA in noisy vibration.

| Mode (j) | Exact | CWT-ICA | MAC (%) |
|---|---|---|---|
| 1 | 0.1002 | 0.0997 | 99.98 |
| 2 | 0.2807 | 0.2801 | 99.60 |
| 3 | 0.4056 | 0.4051 | 99.06 |

higher frequencies. The damping values whether low or high would not have effect on the robustness of the WT-ICA method, while it could have some effects on other system identification methodologies (e.g. STFT-ICA).

Additionally, the higher damping case could be well separated to the single modal components with sufficient precision; however, the time-domain ICA method without implementation of wavelet transform could only evaluate the lightly damped structures [17].

## 7. Validation of CWT-ICA in free vibration with additive noise

To evaluate the validity of CWT-ICA for the additive noise conditions, a Signal-to-noise-ratio (SNR) of 10 dB is considered and added to original time-history data to contaminate the responses. Then, the responses are analyzed with the aid of the WT-ICASNR which is indicated in Eq. (23) [18]:

$$SNR = 20\log_{10}\left(\frac{RMS(Signal)}{RMS(Noise)}\right) \tag{23}$$

where RMS is the root mean square of the signal. The robustness of CWT-ICA could be observed in Tables 4 and 5, where the natural frequencies, damping ratios and MAC values are evaluated and compared. The SNR of 10 db (RMS 31.6 %) is used based on the suggestions by the studies done previously on noisy environmental effects [23]. It is concluded that CWT-ICA method could separate the modal responses and identify the modal characteristics based on the noisy responses. Figs. 7 and 8 show the system responses and recovered modal responses, respectively. It is noted that CWT-ICA method could evaluate the modal responses under heavy noise with sufficient accuracy due to its capability to sparse time-frequency domain even in noisy condition (Tables 4 and 5).

## 8. Conclusion

The time-scale representation of signals using wavelet transform provides the capability of time and frequency domain system identification regarding the linear and nonlinear data. In this study, wavelet transform was initially discussed and compared with other methodologies. The appropriate frequency resolution and the usage of different basis functions were indicated as two main advantages of this transform. In addition, the procedure of independent component analysis as a well-established method to extract modal characteristics of signals was elaborated. The separation process of the modal responses and extraction of the dynamic characteristics from concatenated data were delineated. To obtain the modal parameters of a three-DOF system, first the system is excited to generate system responses and then the wavelet transform was incorporated on time-domain responses, and finally FastICA procedure was performed on wavelet-transformed data to recover the structural modal responses. Two levels of damping are considered to analyze the effect of light and high values of damping on the robustness of the method. Moreover, the effect of noise on the proposed method is examined. As the results have shown, the CWT-ICA could be used as a strong and precise blind source separation output-only identification regarding lightly and highly damped structures.

In addition, CWT-ICA represented satisfactory accuracy in identification of higher frequencies of vibration, which makes it more robust compared to other methods such as short time Fourier transform. The damping ratios could be precisely estimated even with higher precision if the free vibration responses are considered. The natural frequencies and modal damping ratios extracted by CWT-ICA had sufficient compatibility with the results of exact procedures, and the correlation between the exact and identified mode shapes was indicated with high values of estimated MACs.

The structure with higher damping could be well separated to the single modal components with sufficient precision; however, the time-domain ICA and STFT-ICA methods could only evaluate the lightly damped structures with limited number of modes. It is indicated that implementation of the proposed method for the structure with proportion damping could lead to accurate results; however, for the non-proportion damping conditions, detailed investigations are needed. The implementation of this work for structural applications is under further studies.

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