

Rolling element bearings absolute life prediction using modal analysis[†]

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Abstract

In this paper, the authors introduce an experimental procedure for predicting the fatigue life of each individual rolling element bearing separately using vibration modal analysis. The experimental procedure was developed based on a statistical analysis. A statistical analysis was performed to find an empirical model that correlates the dynamic load capacity of rolling bearings to their dynamic characteristics (Natural frequencies and damping). These dynamic characteristics are obtained from the frequency response function of each individual bearing that results from vibration modal analysis. A modified formula to the already known Lundberg-Palmgren life formula is proposed for rolling element bearings. Given the modified formula, one can predict the fatigue life of each individual rolling element bearing based on its dynamic characteristics. The paper compares the results from the modified formula with those from Lundberg-Palmgren formula. The modified formula provides an accurate prediction for the fatigue life of each individual bearing based on its dynamic characteristics. The experimental validation of the modified formula is considered for future work. Therefore, it can be used in various applications of rolling element bearings in machinery systems.

Keywords: Bearing life; Dynamic characteristics; Dynamic load rating; Modal analysis

1. Introduction

Rolling element bearings are fundamental elements in mechanical systems, with crucial applications in aerospace, machine tools and automotive industry. Their purpose is to allow rotational motion of shafts and last until the expected operating life under radial and thrust loading conditions [1, 2]. Bearings support static and dynamic loads in a machine for hundreds of millions of revolutions; therefore, their tendency to deteriorate or fail presents a crucial impact on the performance of machinery [3, 4]. Common failure modes of bearing elements include wear [5], pitting [6] and fatigue failure [7, 8].

The increasing demand for a reliable method to determine the bearing's life has inspired many researchers to develop different analytical and statistical methods. At the beginning of the 20th century, bearing manufacturers began to investigate the longevity of bearings. The first effort was by Stribeck, who conducted fatigue tests on full-scale bearings in 1896. This method was used as a basis for most bearing life determination theories. Palmgren and Lundberg applied the Weibull statistical analysis to assess the reliability of rolling element bearings [9-11]. They deduced the following formula where L_{10} is the life in (Millions of revolutions), P_{eq} is the

equivalent load in (N), C_D is the dynamic load capacity in (N), and p is an exponent (3 for balls - 10/3 for rollers).

$$L_{10} = \left(\frac{C_D}{P_{eq}} \right)^p \quad (1)$$

The equivalent load P_{eq} can be determined by using the following equation where F_r is the radial component of the applied load, F_a is the axial component of the applied load, X is the rotation factor, and Y is the thrust factor of the bearing.

$$P_{eq} = X \times F_r + Y \times F_a \quad (2)$$

The formulas used for calculating the dynamic load capacity C_D , as developed by Lundberg and Palmgren, are semi-empirical and are incorporated into the ANSI/ABMA and ISO standards [12, 13]. These formulas are as follows:

For radial ball bearings with $d \leq 25$ mm:

$$C_D = f_{cm} (i \cos \beta)^{0.7} Z^{2/3} d^{1.8} \quad (3)$$

For radial ball bearings with $d > 25$ mm:

$$C_D = f_{cm} (i \cos \beta)^{0.7} Z^{2/3} d^{1.4} \quad (4)$$

For radial roller bearings:

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$$C_D = f_{cm} (i L_t \cos \beta)^{7/9} Z^{3/4} d^{29/27}. \quad (5)$$

For thrust ball bearings with $\beta \neq 90^\circ$:

$$C_D = f_{cm} (i \cos \beta)^{0.7} (\tan \beta) Z^{2/3} d^{1.8}. \quad (6)$$

For thrust roller bearings with $\beta \neq 90^\circ$:

$$C_D = f_{cm} (i L_t \cos \beta)^{7/9} (\tan \beta) Z^{3/4} d^{29/27}. \quad (7)$$

For thrust ball bearings with $\beta = 90^\circ$:

$$C_D = f_{cm} i^{0.7} Z^{2/3} d^{1.8}. \quad (8)$$

For thrust roller bearings with $\beta = 90^\circ$:

$$C_D = f_{cm} (i L_t)^{7/9} Z^{3/4} d^{29/27}. \quad (9)$$

The material and geometry coefficient f_{cm} depends on the bearing type, material, processing method and rolling elements-races conformity. Representative values of f_{cm} are given from the ANSI/ABMA standards. The formulas for the dynamic load capacity C_D are dependent on:

1. Size of rolling elements ball or roller diameter, (d) and roller length, (L_t).
2. Number of rolling elements per row, (Z).
3. Number of rows of rolling elements, (i).
4. Contact angle, (β).
5. Material and geometry coefficient, (f_{cm}).

However, the Lundberg-Palmgren model has the following limitations:

1. The model assumes that bearings of the same size and code are identically manufactured. Any individualities due to micro cracks, residual stresses, or lubrication variances are ignored in the model.
2. The L_{10} criterion itself presents uncertainties in the bearings' life calculation and forces the user to accept failure percentage of 10 % from the bearings' stock.
3. During the bearing service, environmental effects such corrosion and wear may have detrimental effect on the bearing performance and life [12].

Bearing manufacturers rely on this model to calculate the fatigue life of rolling element bearings. Due to these limitations the manufacturers, in conjunction with ANSI/ABMA, introduced life adjustment factors. However, the developed factors have a degree of uncertainty and result in inaccurate results due to the uniqueness of each bearing. A critical review is presented in the literature to summarize determination methods of the fatigue life of rolling element bearings [14]. The review classifies the methods into statistical, analytical and experimental. Experimental methods, such as accelerated life testing, are destructive. In some applications, destructive testing is inapplicable due to economic or technical considerations. Furthermore, the results obtained from such methods belong only to the destructed bearing. This means that these

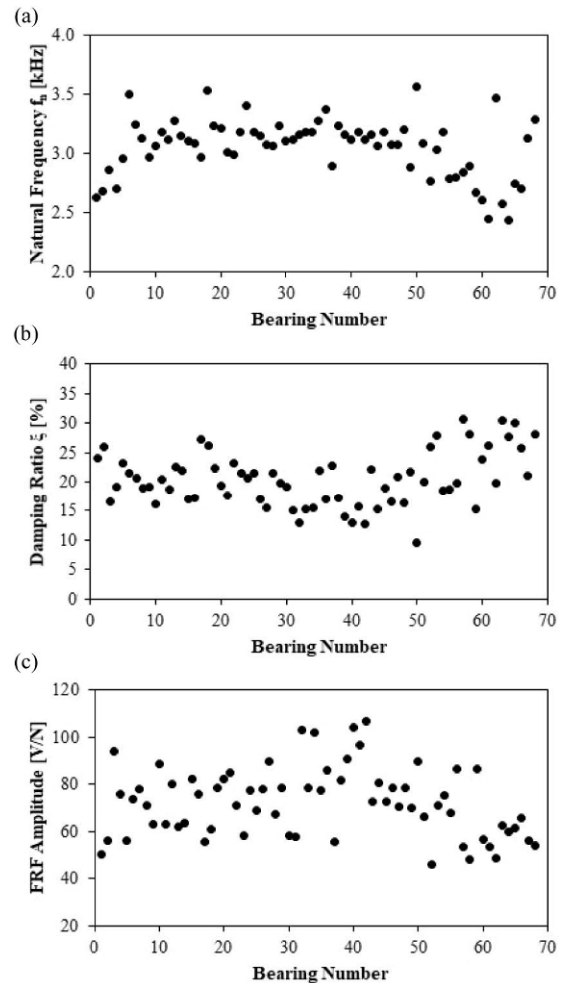


Fig. 1. Dynamic characteristics of rolling element bearings with the same code (6206ZZ): (a) Natural frequency f_n [Hz]; (b) damping ratio ξ [%]; (c) frequency response function FRF amplitude [V/N].

results cannot be applied to other bearings of the same type and size. The statistical approach presents some degree of confidence for decision making. In critical applications in the aerospace, aeronautics and automotive industry, the amount of risk from statistics makes it crucial for engineers to find other convenient methods for life prediction.

Fatigue life of individual bearings varies considerably, even if they are of the same size, material, heat treatment and operating conditions. According to Lundberg and Palmgren, bearings with the same code have the same dynamic load rating C_D and consequently the same fatigue life L_{10} . Modal analysis has been used for determining the dynamic characteristics of mechanical components [15]. Modal parameters are very sensitive to any changes in the mechanical properties and microstructure [16], as well as the presence of cracks and damages [17, 18]. Thus, modal analysis is a competitive, non-destructive method in various industrial applications. One of the recent advances in modal analysis application is the prediction of fatigue life for metallic materials [19-21]. Promising results obtained from studies in this area have motivated the

authors of this paper to consider the possibility of modal analysis to evaluate the fatigue life of assembled components rather than materials [22]. Fig. 1 shows the modal natural frequency f_n , damping ratio ξ , and frequency response function FRF amplitude at the first mode (Fundamental mode of vibration) for rolling element bearings from the same manufacturer and with the same code (6205ZZ).

The results show variations in the dynamic characteristics of the bearings, although they have the same code and dynamic load rating (14000 N). More than 30 %, 60 % and 50 % variations were observed in natural frequency, damping ratio, and FRF amplitude consequently. The results impose questions regarding their dynamic performance, which in turn is expected to affect life expectancy of the bearings.

Due to the nature and complexity of a bearing assembly, no one bearing of a certain type is guaranteed to have the same life as other bearings produced by the same manufacturers on the same machines at the same time on the same batch. This leads to the need for an experimental procedure on each bearing to predict its remaining fatigue life. One attempt to overcome this problem showed an experimental procedure to predict the remaining life of bearing nondestructively [23]. The method gives a relative life index by comparing a known life bearing dynamic parameters with those of the tested bearing. This methodology is a helpful guide for the users during acceptance tests of new bearings. However, no information is given regarding the absolute life or number of revolutions that a bearing will survive during its service life.

Recent studies focus on predicting the remaining service life of bearings based on system parameter measurement [24, 25]. A study correlated the remaining life of bearings to two main parameters: Surface temperature and vibrations [26]. The study used a method to contaminate bearings at various levels. Temperature, vibration levels, and the time of failure were measured and statistically analyzed. However, the study did not present a clear quantitative relationship between the correlated parameters and life. Furthermore, this approach is valid only during the service life, i.e., for condition monitoring and supervision and not as an acceptance test of new elements.

The main objective of the proposed methodology is to modify the Lundberg-Palmgren model for an accurate prediction of the fatigue life of rolling element bearings using their dynamic characteristics. The paper presents the determination of the dynamic characteristics of rolling element bearings using experimental modal analysis. Then, it explains the statistical procedure that was used for correlating the dynamic load rating to the dynamic characteristics. A modified formula to predict the fatigue life of rolling element bearings using their dynamic characteristics is presented.

2. Experimental modal analysis

Four different sets of rolling element bearings (A, B, C and D) were chosen in this study. Each set consists of five replicates (Bearings with the same code). A represents a set of

Table 1. Load ratings of the tested bearings.

Set	Bearing number	Basic load ratings (N)	
		Dynamic (C_D)	Static (C_0)
A	6006ZZ	13200	8300
B	7206B	20500	13500
C	NU205EW	29300	27700
D	22212EAE4	142000	174000
	22212EAE4H	178000	174000

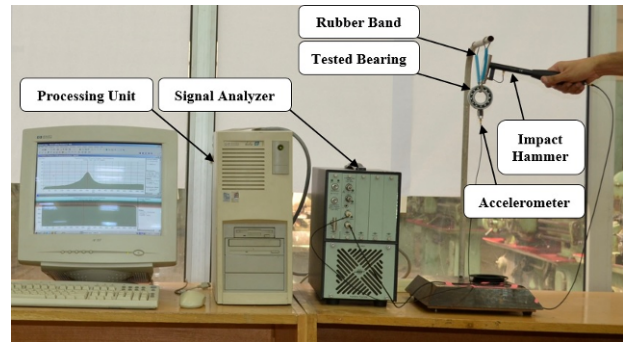


Fig. 2. Modal testing setup for rolling element bearing.

deep groove ball bearings 6006ZZ, B represents a set of angular contact ball bearings 7206B, C represents a set of cylindrical roller bearings NU205EW, and D represents a set of spherical roller bearings 22212EAE4. The nominal load ratings of each set were suggested by the supplier as shown in Table 1. Each bearing was labeled with a letter representing the set followed by a number that represents the replicate (e.g., Bearing A1).

2.1 Test setup

The dynamic characteristics of each bearing were determined using the modal analysis setup shown in Fig. 2. The bearing under test was freely hung and excited by a moderate impulse from an instrumented impact hammer. The impact hammer generates pulses up to 5 kN for 0.2 milliseconds long. The impulse signal was measured using a force transducer built into the impact hammer. The corresponding response was measured using a vibration sensor (Piezoelectric uniaxial accelerometer) attached by wax to the outer ring of the bearing [27]. The accelerometer measures from 0.01 to 10000 m/s^2 .

2.2 Measurement procedures

The following procedures were performed to determine the dynamic characteristics of each rolling bearing separately:

1. The tested bearing was hung in a rigid support using an elastic rubber band as shown in Fig. 3.
2. Four repeated light impacts were implied to the outer ring of the bearing using an impact hammer.
3. The vibration response to each hammer blow was meas-

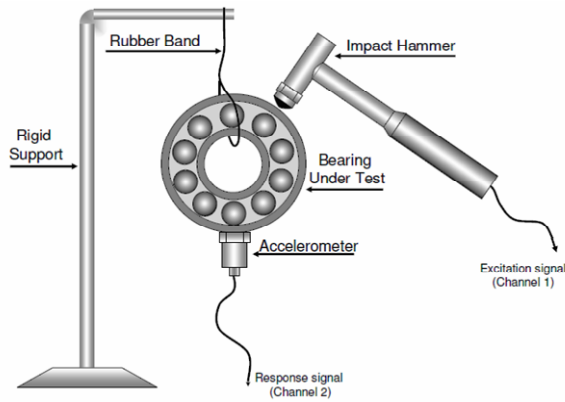


Fig. 3. Schematic diagram of the experimental modal analysis setup for rolling element bearing [23].

ured using an attached accelerometer.

4. Both impulse and response signals were recorded and processed using a signal analyzer equipped with acquisition front-end. The analyzer arithmetically averaged the results of the four impacts to obtain the Frequency response function (FRF) plots.
5. The FRF plots were used to determine the dynamic characteristics of each bearing using the analysis explained in the forthcoming sections.

2.3 Assumptions

The following assumptions were considered during the experiment for accurate determination of FRF:

1. The weight of the accelerometer was less than 10 % of the bearing weight.
2. Anti-aliasing filters were applied to the force and vibration signals to avoid leakage of non-periodic signals.
3. Transient and exponential windowing functions were selected for excitation and response signals, respectively, beside windows overlapping to avoid data loss from individual windowing.
4. The transfer function was monitored by a corresponding correlation function called the coherence function. Only resonant modes with almost unity coherence values were considered for the analysis.
5. The location of the measurement, excitation, and fixation points was the same for all tested bearings.
6. The vibration modes of the rigid support were ignored, in which only the first vibration mode of the bearing was considered for analysis.

2.4 Analysis procedures

The excitation signal was recorded in both time waveform (Time domain) and auto spectrum (Frequency domain) as shown in Fig. 4. Similarly, the response signal was recorded in both time waveform and auto spectrum as shown in Fig. 5. The FRF was obtained from the spectra of both excitation and response as explained in Eq. (10).

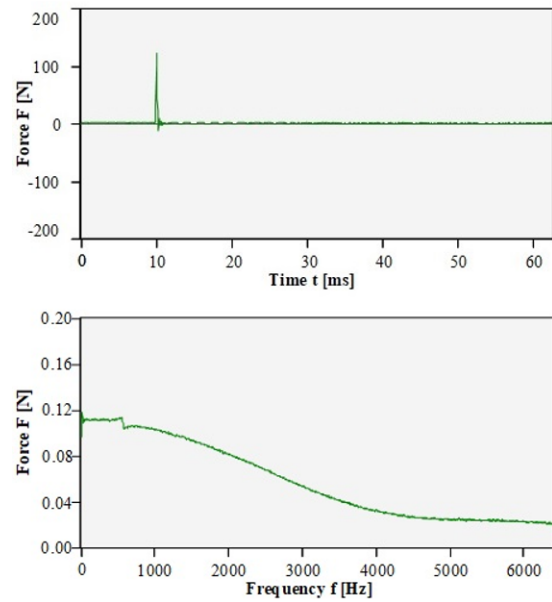


Fig. 4. Time waveform and auto spectrum of the impulse force (Excitation) applied to bearing A1.

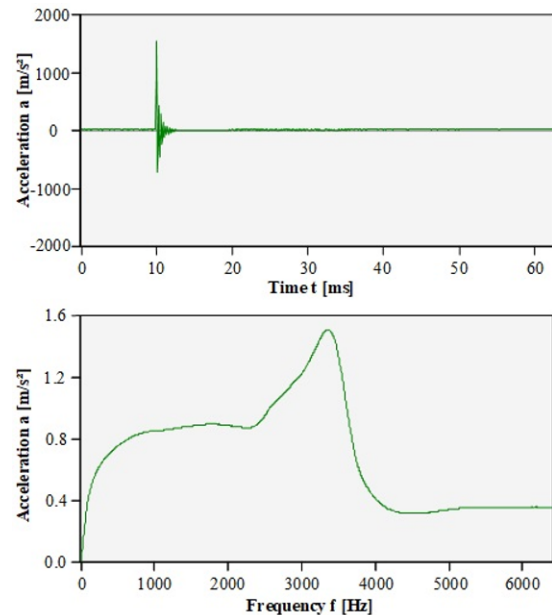


Fig. 5. Time waveform and auto spectrum of the vibration amplitude (Response) resulting from bearing A1.

$$FRF = \frac{\text{Response}}{\text{Excitation}} = \frac{\text{Acceleration (a)}}{\text{Force (F)}} \tag{10}$$

The frequency range of interest for the tested bearing was found to be within 0 – 6400 Hz. Fig. 6 shows the FRF obtained from impact excitation of bearing (A1). Fig. 6(a) represents the FRF showing the first mode of vibration for bearing (A1), and Fig. 6(b) shows the corresponding coherence function to the first dominant mode. The coherence value was around 99.4 % (Approaching unity), corresponding to the first

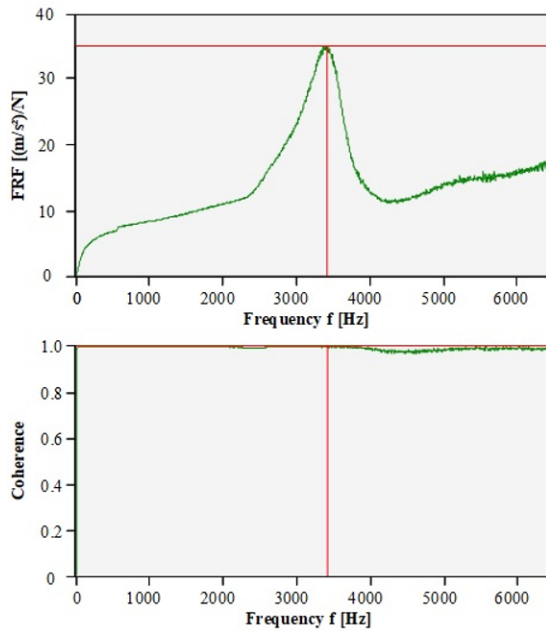


Fig. 6. The FRF and coherence resulting from bearing A1.

mode. Modal parameters (Natural frequency, damping ratio, and FRF magnitude) were extracted from the first mode of vibration (Mode of concern) for each bearing.

3. Proposed methodology

The proposed methodology consists of two main parts: (1) Developing an empirical model that can predict the dynamic load rating C_D of a bearing using its dynamic characteristics, and (2) modifying the equation of the equivalent applied load P_{eq} to include the dynamic characteristics of the bearing. Then, the Lundberg-Palmgren equation was modified to include the dynamic characteristics of rolling element bearings.

3.1 Prediction of the dynamic load rating

A statistical analysis procedure was constructed as shown in Fig. 7. Various combinations of modal parameters were developed: f_n , ξ , FRF, $\xi/f_n \cdot FRF$, ξ/f_n , ξ/FRF and $1/f_n \cdot FRF$. Pearson correlation test [28–31] was conducted between each of these parameters and the resultant dynamic load rating C_D using XLSTAT software. The tests show the linear and monotonic dependency of the dynamic load rating on the dynamic characteristics. The analysis showed that the parameter $\xi/f_n \cdot FRF$ has a strong influence on the dynamic load rating (High correlation coefficient of 0.995, high determination coefficient R^2 of 0.990, p-value less than 0.05).

Nonlinear regression analysis was conducted between the dynamic load rating C_D as the dependent variable (Y) and the parameter $\xi/f_n \cdot FRF$ as the independent variable (X). Linear, quadratic, β polynomial, exponential, and natural logarithmic models were studied as potential regression models. Fig. 8

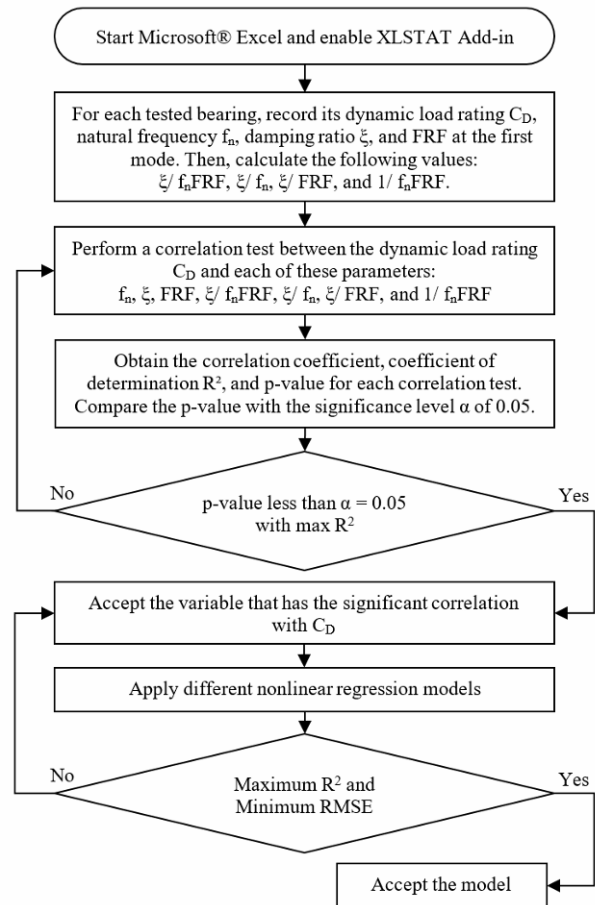


Fig. 7. Statistical analysis flow chart.

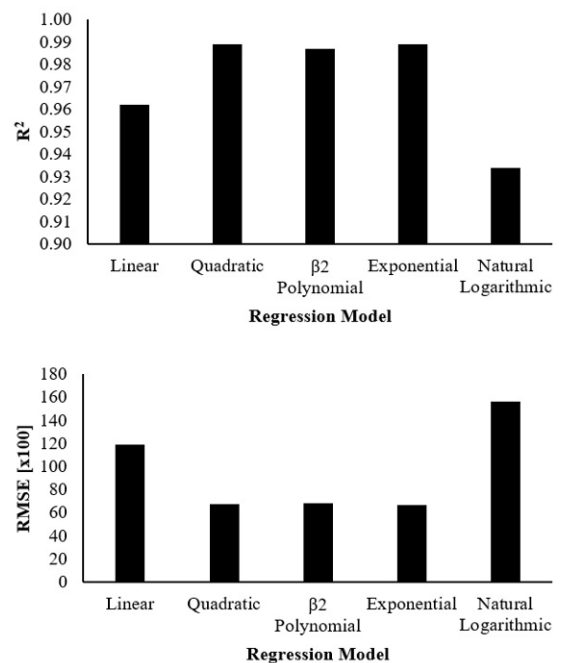


Fig. 8. The coefficient of determination R^2 and the root mean squares of the errors RMSE for regression models between C_D and $\xi/f_n \cdot FRF$.

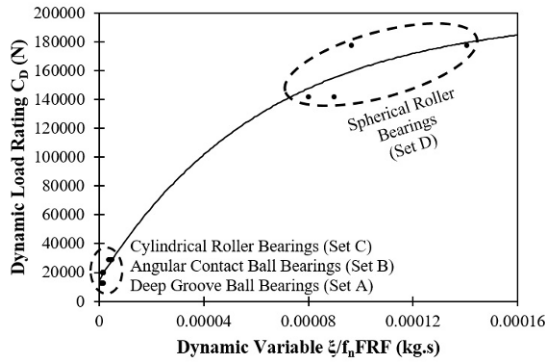


Fig. 9. The developed exponential model for the dynamic load rating.

shows the determination coefficient R^2 and the Root mean squares of the errors (RMSE) for each nonlinear regression model. The results showed that the exponential model is the best fitting nonlinear regression model (High determination coefficient R^2 of 0.989 and low RMSE). The empirical model shown in Fig. 9 was developed using the four different sets of bearings where each set consisted of five replicates.

The predicted model shown in Fig. 9 correlates the dynamic characteristics of a rolling element bearing to its dynamic load rating. The spherical roller bearings have higher C_D and ξ/f_n FRF than ball and cylindrical bearings. By substituting the dynamic characteristics of bearings (ξ/f_n FRF) in the exponential model, the actual dynamic load rating values of these bearings were predicted. The model determines the dynamic load rating of each individual bearing separately given the dynamic characteristics of this bearing.

3.2 Determination of the actual equivalent dynamic load

As the inner and outer rings of rolling elements are constantly loaded when bearings rotate, bearings which are subjected to cyclic loading and unloading will fail due to fatigue after a certain running time. Hence, it is considered that a vibrational load is dynamically induced in the bearing due to fatigue cyclic loading and unloading during operation. Hence, the term P_{eq} in Eq. (1) should be modified by the dynamicity of the applied load. As the dynamic force increases the displacement of vibration increases, which means that the vibration dynamic force is proportional to the displacement of vibration as shown in Eq. (11).

$$\frac{X_{dy}}{X_{st}} = \frac{P_{dy}}{P_{eq}}, \tag{11}$$

where X_{dy} is the vibration displacement due to a dynamic vibration load P_{dy} , X_{st} is the static deflection, and P_{eq} is the equivalent static force applied to the bearing.

If the bearing operates under n rpm (ω rad/sec), the frequency of the forced vibration response f (The excitation frequency) is:

$$f = \frac{\omega}{2\pi} = \frac{n}{60}. \tag{12}$$

The amplitude of the forced vibration response X_{dy} is:

$$X_{dy} = \frac{X_{st}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}, \quad \text{where } r = \frac{f}{f_n} \tag{13}$$

where f_n is the natural frequency of the bearing, ξ is its damping ratio, and r is the frequency ratio.

By combining Eqs. (11) and (13), the dynamic applied load will be:

$$P_{dy} = \frac{P_{eq}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}, \quad \text{where } r = \frac{f}{f_n}. \tag{14}$$

3.3 Development of the modified fatigue life equation

The Lundberg-Palmgren equation Eq. (1) is used for calculating the fatigue life of the rolling element bearing based on the dynamic load rating C_D and the equivalent static load P_{eq} . The equation does not include the dynamic characteristics of the bearing. The dynamic load rating will be replaced by a predicted dynamic load rating that can be determined from the exponential model in Fig. 9. This predicted dynamic load rating depends on the dynamic characteristics of the bearing that was determined using experimental modal analysis. The equivalent applied load will also be replaced by a dynamic load P_{dy} that can be determined from Eq. (14). This dynamic load depends on both the dynamic characteristics of the bearing and its operating conditions. Hence, the Lundberg-Palmgren equation will be modified as follows:

$$\begin{aligned} L_{D10} &= \left(\frac{\text{Predicted } C_D}{P_{dy}} \right)^p \\ &= \left(\sqrt{(1-r^2)^2 + (2\xi r)^2} \right)^p \left(\frac{\text{Predicted } C_D}{P_{eq}} \right)^p \\ &= a_D \left(\frac{\text{Predicted } C_D}{P_{eq}} \right)^p, \end{aligned} \tag{15}$$

where a_D is the dynamic factor:

$$a_D = \left(\sqrt{(1-r^2)^2 + (2\xi r)^2} \right)^p. \tag{16}$$

At resonance, ($f = f_n$ and $r = 1$), the dynamic applied load will be minimal ($a_D = 2\xi^p$) and the predicted life will be short. At resonance with undamped systems ($\xi = 0$ and $r = 1$), the dynamic load will lead to zero remaining life (Vibration

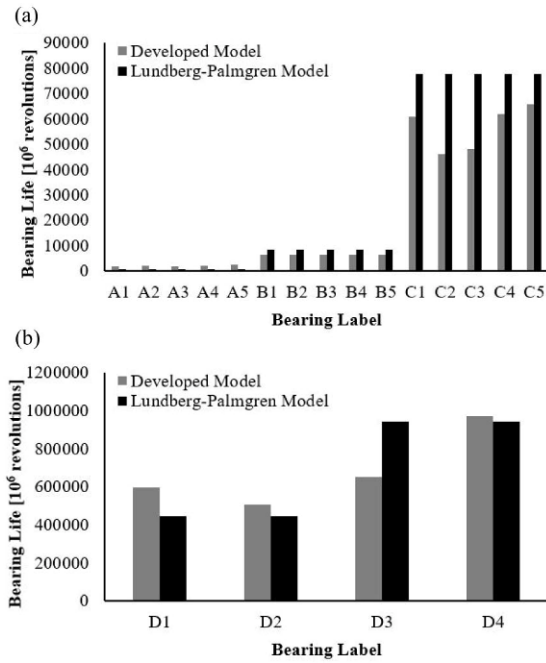


Fig. 10. The predicted and calculated life from the developed model and Lundberg-Palmgren model consequently: (a) Bearing sets A, B, C; (b) bearing set D.

resonance). The developed model can be used to predict the absolute fatigue life of rolling element bearings using their dynamic characteristics. It is a modification to the Lundberg-Palmgren model to include the dynamicity of rolling bearings.

4. Results and discussion

The fatigue life of the tested bearings was predicted using the developed model given its dynamic characteristics and operating conditions. The results from the developed model were compared with those obtained using the Lundberg-Palmgren model. The bearings were assumed to operate under an axial load of $F_a = 500$ N, a radial load of $F_r = 1000$ N, and at a rotational speed $n = 1000$ rpm ($f = 16.667$ Hz).

The dynamic load rating C_D , rotation factor X , and thrust factor Y of each bearing were extracted from the bearing catalogue. Then, the equivalent applied load P_{eq} in each bearing was calculated using Eq. (2). The fatigue life L_{10} was calculated according to Lundberg-Palmgren equation Eq. (1).

The dynamic characteristics of each bearing: f_n , ξ , FRF, and $(\xi/f_n, FRF)$ were obtained using the experimental modal analysis. Using the developed exponential model shown in Fig. 9, the dynamic load rating of each bearing was predicted from the dynamic variable $(\xi/f_n, FRF)$. The dynamic applied load P_{dy} of each bearing was also predicted using Eq. (14). Then, the predicted fatigue life L_{D10} was obtained using Eq. (15).

The results obtained from the developed model were compared with those calculated from the Lundberg-Palmgren model as shown in Fig. 10. Among the same bearing set, the

results showed that the predicted life using the developed model fluctuates based on the dynamic characteristics of the bearing. But the calculated life using Lundberg-Palmgren model showed the same values for the same bearing set.

The dynamic variable $(\xi/f_n, FRF)$ increases at high damping ratio, as well as low natural frequency and FRF value. It was found that increasing the dynamic variable increases the dynamic load rating and consequently increases the fatigue life. This means that a rolling bearing with high damping ratio and low natural frequency will have high dynamic load rating and consequently long fatigue life. That can be attributed to the fact that a high damping ratio will reduce the amount of vibration during the bearing operation and a low natural frequency will keep the bearing away from the resonance conditions. This dynamic variable showed a good influence on the dynamic load rating of rolling element bearing.

On the other hand, the dynamic characteristics affect the dynamic applied load according to Eq. (14) and consequently will affect the fatigue life. Increasing the damping ratio will also cause a decrease in the dynamic applied load P_{dy} and therefore will increase the fatigue life. The calculated applied load P_{eq} is constant for bearings with the same code and independent of the operating speed. However, the dynamic applied load P_{dy} depends on the dynamic characteristics of the bearing and the operating conditions.

5. Conclusions

The capability of experimental modal analysis to predict the reliability of rolling element bearings was investigated. The authors conducted a statistical analysis to empirically develop a model that predicted the dynamic load rating C_D of each bearing given its dynamic characteristics. The empirical model was developed using an experimental study on four different sets of rolling element bearings. Each set consisted of five bearings from the same manufacturer with the same code. According to Lundberg-Palmgren theory, the bearings in the same set will have the same calculated life since they have the same bearing code. However, the experimental modal analysis results showed significant differences in the dynamic characteristics of bearings with the same code (Bearings in the same set). This means the actual fatigue life of these bearings will be affected by variations in the dynamic characteristics. Detailed statistical analyses were performed on the experimental results and an empirical formula was developed that predicts the dynamic load capacity of each individual bearing from its dynamic characteristics.

The equivalent load P_{eq} was modified based on a theoretical derivation to include the vibration data and dynamic characteristics of the bearing. Based on the modified dynamic load rating and equivalent dynamic load P_{dy} , the Lundberg-Palmgren equation was modified to include the dynamicity of rolling element bearings. The results showed that the fatigue life of rolling element bearings is dependent on their dynamic characteristics (Experimental modal analysis results) and the

operating conditions (Applied load, vibration load, and operating speed). The predicted equivalent dynamic load proved to be dependent on the running speed unlike the calculated dynamic load (Lundberg-Palmgren formula). Further investigations should focus on testing the methodology in assessing the reliability of other mechanical assemblies.

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