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Effects of weight on bit on torsional stick-slip vibration of oilwell drill string[†]

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Abstract

The stick-slip phenomenon is a type of dysfunction detrimental to the drilling operation. Field application shows that stick-slip phenomenon is inclined to appear when using a large Weight on bit (WOB). In this paper, effects of the WOB on the stick-slip vibration are investigated. Based on a lumped torsional pendulum model of the drilling system, equation of motion of the drill bit is obtained. By using parameters commonly used in field applications, the bit dynamics are analyzed and the stick-slip vibrations are discussed. During the stick-slip motions, the negative damping effect occurs in the transition from the stick phase to the slip phase. With the increasing WOB, the bit behavior may change from the stable motion to the stick-slip vibration once the WOB reaches the critical value. In case of stick-slip vibration, the phase trajectory ultimately converges to a limit cycle which represents periodical bit motion. With increases in the WOB, the limit cycle enlarges. For cases without stick-slip vibrations, the drill bit vibrates damply and finally converges to a state of uniform motion. The results presented in this paper can be applied to interpret some of the field phenomena related to WOB.

Keywords: Weight on bit; Stick-slip vibration; Drill string; Frictional torque; Limit cycle

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1. Introduction

Stick-slip vibration induced by friction occurs in many industrial applications [1], and one of the systems that encounter stick-slip phenomenon is the rotary drilling system [2]. Due to a large length-to-diameter ratio, the oilwell drill string often experience stick-slip vibrations which are characterized by: (i) The stick phase in which the bit remains still, and (ii) the slip phase in which the maximum velocity of the bit becomes several times the velocity of the rotary table [3, 4]. Fig. 1 shows the measurements of the drilling system during the drilling process, wherein the stick-slip phenomenon is clearly shown. For the drilling engineering, the stick-slip vibration is undesirable because it causes premature tool failures, reduces the Rate of penetration (ROP), adversely affects the borehole quality, and increases the drilling cost [2, 5, 6].

In general, both the rock strength and rock hardness increase with an increase in well depth [7], so a large Weight on bit (WOB) is used to guarantee the ROP. In addition, the stiffness of the drill string decreases with the increase in the length-to-diameter ratio. These two factors will result in or aggravate the stick-slip vibration [8]. Along with an increasing number of deep wells, the significance of stick-slip vibration on the drilling efficiency becomes increasingly conspicuous [9]. To minimize the stick-slip vibration during the drilling process, two common measures are usually taken: Reduce the WOB and/or increase the rotary table velocity. However, reducing the WOB will lead to decrease in ROP and increase in the rotary table velocity will aggravate other modes of vibration [10].

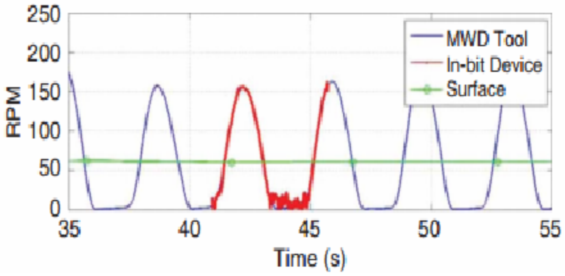

Fig. 1. Measurements of the drill string during stick-slip [4].

The investigation of stick-slip vibration in drill string is traced back to the research by Belokobyl'skii and Prokopov in 1982 [11], followed by subsequent efforts on this subject in Refs. [12-17]. However, most of these publications focus on reporting the stick-slip vibration in drilling system or presenting approaches to suppress such phenomena [2, 18-22]. Most of the measures which are taken to suppress vibration are

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based on drilling experiences, and the mechanisms for interpreting the stick-slip phenomenon or suppression approaches are scarce.

For a certain vibration mode, it not only couples with other vibration modes, but also contains sudden jumps in the vibration amplitude, leading to challenges in analyzing the problem. Drill string dynamics have been studied in the industry using both linear and nonlinear models, which present quantitative insight of the drilling process [23, 24]. For example, many vibration modes can be obtained through analyzing a large finite element model (many degrees of freedom) [25]. However, these models are so complex that it is difficult to understand why certain vibration modes occur. Low dimensional model, which can be researched with analytical solutions, can provide (to some extent) qualitative information in certain vibration phenomenon (for example the stick-slip vibration studied in this work) in the drilling application. Based on this, Cunha Lima [26] presented a low dimensional model to study the stability of stick-slip vibration.

For the drilling engineers, how to explain the stick-slip vibration related to WOB and take measures to control this phenomenon are some of the most important questions. In this paper, effects of WOB on the stick-slip phenomenon of the drill string are to be investigated. Firstly, a mechanical model similar to that of Cunha Lima [26] is presented. Secondly, dynamics of a drilling system with the drill string length of 3000 m is analyzed. Finally, the bit dynamics are discussed and some conclusions are obtained. This paper aims to explain the reason why reducing WOB will suppress stick-slip phenomenon. In addition, the method of determining a critical WOB so as to mitigate stick-slip vibration is also presented.

2. Theoretical model

During the drilling process, the drill string (includes drill pipes and drill collars) is driven by the rotary table to rotate forward and the drill bit scrapes the formation. For the stick-slip behavior in a vertical drill string, the interaction between the drill bit and the rock formation is the main source. In this research, the drilling system is regarded as a lumped pendulum and the contact between the drill bit and the rock formation will be regarded as Coulomb friction. Fig. 2 shows the mechanical model of the drilling system studied in this paper, which is obtained by referring to the model of Cunha Lima [26]. For the analysis in this paper, K_D and J correspond to the equivalent drill string stiffness and equivalent moment of inertia, respectively; L_P and L_C are the length of the drill pipes and length of the drill collars, respectively; D_P and d_P are the outside diameter and inside diameter of the drill pipes, respectively; and D_C and d_C are the external diameter and internal diameter of the drill collars, respectively. The expressions of J and K_D can be given as

$$J = \frac{J_r}{3} = \frac{\pi\rho\left[L_P\left(D_P^4 - d_P^4\right) + L_C\left(D_C^4 - d_C^4\right)\right]}{96} \tag{1}$$

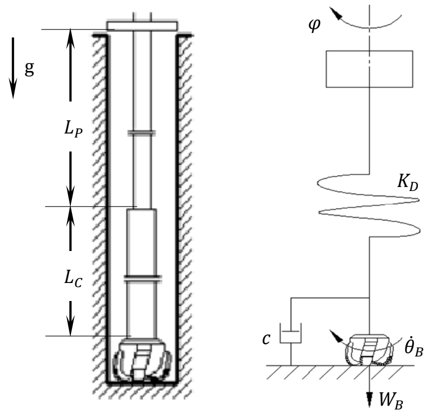

Fig. 2. Mechanical model used to represent the drill string [26].

$$K_D = \frac{K_P K_C}{K_P + K_C} \tag{2}$$

with

$$K_P = \frac{\pi G\left(D_P^4 - d_P^4\right)}{32 L_P} \tag{3}$$

$$K_C = \frac{\pi G\left(D_C^4 - d_C^4\right)}{32 L_C} \tag{4}$$

where J_r, ρ, and G denote, respectively, the real polar moment of inertia, density, and shear modulus of the drill string.

In this study, the bit is supposed to be in a critical phase of transiting from the stick phase into the slip phase at the time $t = 0$, which also means that the driving torque from the drill string overcomes the frictional torque at $t = 0$. As the table always rotates clockwise, the bit accelerates to rotate clockwise once the stick phase ends. In case of stick-slip phenomenon, energy conversion frequently occurs in the drilling system.

To understand the bit dynamics, the D'Alembert's principle is used. Dynamical responses of the drill bit in the slip phase can be adequately described by the following equation

$$J\ddot{\theta}_B + c\dot{\theta}_B + K_D\left(\theta_B - \varphi t\right) - \mu_K W_B \bar{R}_B = 0 \tag{5}$$

where the left four terms of the equation are the inertia force, damping force, restoring force, and friction force, respectively, θ_B denotes the angular rotation of the drill bit, $(\dot{\ })$ means the derivative with respect to time, c denotes the viscous damping coefficient, φ denotes the rotary table velocity, μ_K denotes the kinetic friction coefficient, W_B denotes the WOB, and $\bar{R}_B = 2R_B/3$ corresponds to the equivalent bit radius and R_B denotes the real bit radius [27]. What should be stressed is that $\bar{R}_B$ is obtained through the integral method, which makes $\bar{R}_B$ different from the expressions in Refs. [10, 28-30].

Eq. (5) is a second-order inhomogeneous ordinary differential equation and its solution contains a general solution of its homogeneous equation as well as a particular solution of Eq. (5). Consequently, solution of the equation of motion can be expressed as

$$\theta_B = Ae^{-\xi\omega_n t}\sin\left(\sqrt{1-\xi^2}\,\omega_n t+\psi\right)+\varphi t-\frac{\mu_K W_B \bar{R}_B}{K_D} \tag{6}$$

where $\omega_n=\sqrt{K_D/J}$ denotes the natural frequency, and $\xi=c/\left(2J\sqrt{K_D/J}\right)$ refers to the viscous damping ratio, and A and ψ represent the amplitude and phase angle, respectively. These two parameters are dependent on the initial state of the bit and can be written as

$$A=\sqrt{\left(\theta_{B0}+\frac{T}{K_D}\right)^2+\left[\frac{\varphi-\dot{\theta}_{B0}+\xi\omega_n\left(\theta_{B0}+\frac{T}{K_D}\right)}{\omega_n\sqrt{1-\xi^2}}\right]^2} \tag{7}$$

$$\psi=\arctan\frac{\omega_n\sqrt{1-\xi^2}\left(\theta_{B0}+\frac{T}{K_D}\right)}{\varphi-\dot{\theta}_{B0}+\xi\omega_n\left(\theta_{B0}+\frac{T}{K_D}\right)} \tag{8}$$

where θ_{B0} denotes the initial angular displacement, $\dot{\theta}_{B0}$ denotes the initial angular velocity, and T denotes the driving torque. Since the bit starts from the stick state (moves to the slip phase at the end of the stick phase at $t=0$), the initial angular displacement of the bit θ_{B0} is a function of the static friction coefficient μ_S

$$\theta_{B0}=\frac{\mu_S W_B \bar{R}_B}{K_D}\,. \tag{9}$$

3. Dynamic responses of bit for different WOB

In this study, the data corresponding to a drill string with 3000 m in length are applied. Parameters for the drilling system are listed in Table 1 and these parameters are determined by referring to the drilling field, which are commonly used in the drilling application. The friction coefficients and damping ratio are used by referring to the Refs. [28, 30].

Since the purpose of this paper is to reveal the effects of WOB on the stick-slip vibration of oilwell drill string. For the drilling system analyzed in this section, a common WOB is 160 kN. Without loss of comparability, different WOB values which are determined on the basis of the common value are used: 180 kN, 160 kN, 140 kN, 120 kN and 100 kN. Because the rotary table rotates at a constant velocity, the drill bit keeps rotating clockwise (as can be seen from the bit dynamics). During the stick state, the bit remains stationary and the rotary table keeps rotating. During the slip phase, the drill bit abruptly accelerates due to the distortion of the drill string in the stick phase and then rotates forward. For different WOB values, the corresponding relative angular displacements (referred to as $\theta_{Br}=\theta_B-\varphi t$) between the rotary table and the drill bit can be obtained.

Table 1. Parameters of the drilling system.

Parameter	Value	Parameter	Value
L_D (m)	3000	R_B (mm)	108
L_P (m)	2800	φ (rpm)	100
L_C (m)	200	ρ (kg/m^3)	7850
D_P (mm)	127	G (Pa)	8.0×10^{10}
d_P (mm)	108.6	μ_S	0.8
D_C (mm)	165.1	μ_K	0.5
d_c (mm)	57.2	ξ	0.1

Case 1: =180 kN

$$\theta_{Br}=-13.85e^{-0.164t}\sin\left(1.63t+0.99\right)-19.31\,. \tag{10}$$

Case 2: =160 kN

$$\theta_{Br}=-12.73e^{-0.164t}\sin\left(1.63t+0.94\right)-17.16\,. \tag{11}$$

Case 3: =140 kN

$$\theta_{Br}=-11.62e^{-0.164t}\sin\left(1.63t+0.89\right)-15.02\,. \tag{12}$$

Case 4: =120 kN

$$\theta_{Br}=-10.57e^{-0.164t}\sin\left(1.63t+0.82\right)-12.87\,. \tag{13}$$

Case 5: =100 kN

$$\theta_{Br}=-9.56e^{-0.164t}\sin\left(1.63t+0.74\right)-10.73\,. \tag{14}$$

Eqs. (10)-(14) show the dynamics of the drill bit in the slip phase for different drilling conditions (also referred to as different WOB values). In order to obtain the bit dynamics in the whole drilling process, one of the five cases is analyzed in detail. Without loss of generality, case 2 wherein W_B = 160 kN is selected.

For case 2, the drill bit dynamics in the slip phase is written as

$$\theta_B=-12.73e^{-0.164t}\sin\left(1.63t+0.94\right)+10.47t-17.16\,. \tag{15}$$

In the previous section, the bit is supposed to be in a critical phase of transiting from the stick phase into the slip phase. As a result, the drill bit begins to rotate in the clockwise direction at t =0. By taking the derivative of Eq. (15) with respect to time, the absolute bit velocity can be obtained and is presented in Fig. 3. In order to vividly reveal the time of drill bit step-

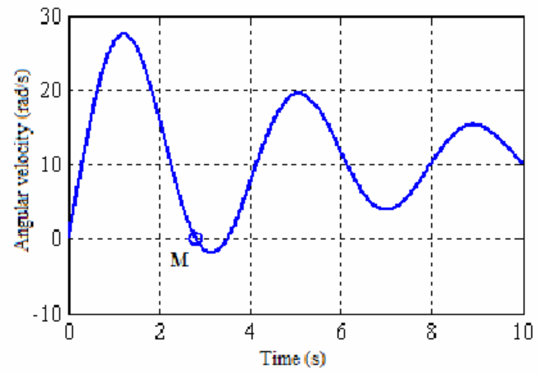

Fig. 3. Absolute angular velocity of the drill bit obtained by taking the derivative of the Eq. (15) with respect to time.

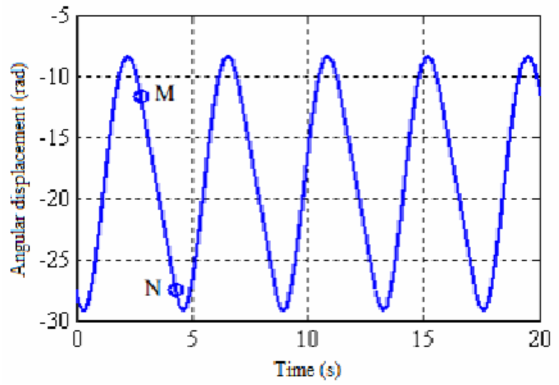

Fig. 4. Relative angular displacement obtained by Eq. (11).

ping into the stick phase (Point M), the absolute angular velocity of the drill bit is used.

As can be seen from the figure, the drill bit accelerates and then decelerates. Because the drill bit begins with a stationary state (stick phase) and has a velocity less than rotary table velocity in the first small period of time, so the $|\theta_{Br}|$ (absolute value of the relative angular displacement) increases (Fig. 4). After reaching the rotary table velocity (about 10.47 rad/s), the drill bit continues to accelerate because the driving torque (depends on the relative angular displacement) is larger than the resisting force (friction force and damping force are included). During this period, since the drill bit rotates faster than the rotary table, the relative angular displacement decreases, leading to a decrease in the driving torque. On the contrary, the damping force increases due to the increase in the angular velocity. Once the driving torque decreases to a value less than the resisting torque, the drill bit begins to decelerate. However, during the decelerating process, $|\theta_{Br}|$ continues to increase because the drill bit rotate faster than the rotary table (the drill bit lags behind the rotary table). Finally, the drill bit velocity becomes zero (Point M shown in Figs. 3 and 4) at 2.81 s. At t =2.81 s, the corresponding θ_{Br} is -11.7 rad (a negative value means the drill bit lags behind the rotary table).

Eq. (9) indicates that the stick phase cannot be broken if the $|\theta_{Br}|$ is less than 27.5 rad. Since the bit velocity reaches zero (Point M) and the corresponding $|\theta_{Br}|$=11.7 rad (less than 27.5 rad), the drill bit remains stationary, neither moving clockwise nor anti-clockwise beyond 2.81 s. As a result, only the response in the time period [0, 2.81s] of Fig. 3 represents the bit behavior in the slip phase. That is to say, the curve before the point M is available to illustrate the bit motion in the slip phase. Once the slip phase ends, the drill bit steps into the stick phase in which the bit remains still while the rotary table rotates at a constant velocity. During this phase, the driving torque increases, preparing for the next slip phase. The time period of the stick phase is 1.51 s, corresponding to an increase of the $|\theta_{Br}|$ from 11.7 rad to 27.5 rad.

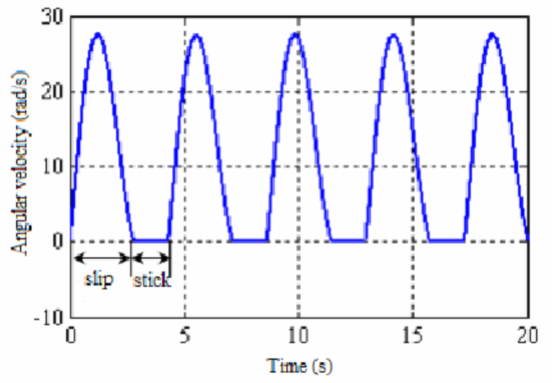

Fig. 5. Absolute angular velocity of the drill bit.

The stick-slip vibration is characterized by the repeated alternation of stick phases and slip phases. Figs. 4 and 5 show the angular displacement and angular velocity of the drill bit. The stick-slip vibration of the drill bit is piecewise, and the time histories shown in Figs. 4 and 5 are comprised of many curves and straight lines. For example, the MN part shown in Fig. 4 is a straight line, corresponding to the stick phase of the drill bit. As reflected in Fig. 5, the amplitude of the absolute angular velocity (about 28 rad/s) is much higher than the rotary table velocity (about 10.47 rad/s), which is in agreement with the analyses by Khulief et al. [24] and Saldivar et al. [29].

Based on the bit dynamics analyzed above, we may deduce that the time periods of the drill bit in both slip and stick phases are dependent on the WOB. For a certain WOB, if the absolute angular velocity of the drill bit does not reach zero after t = 0, the stick-slip vibration will not occur and the drill bit will vibrate damply toward a value equal to the rotary table velocity. By applying the analytical method used in case 2, the dynamics of the drill bit can be obtained. The periodical characteristics for different WOB are shown in Table 2, where P_S denotes the period of the stick phase, P_K denotes the period of the slip phase and P denotes the period of the stick-slip vibration.

As shown in Table 2, stick-slip phenomena appear for only three of the five WOB. Although the damping and frequency are consistent for different WOB, different WOB have different periodical characteristics. Since the time periods of both

Table 2. Characteristics of the bit behavior.

W_B (kN)	180	160	140	120	100
P_S (s)	2.71	2.81	2.96	NA	NA
P_K (s)	1.75	1.51	1.22	NA	NA
P (s)	4.46	4.32	4.18	NA	NA

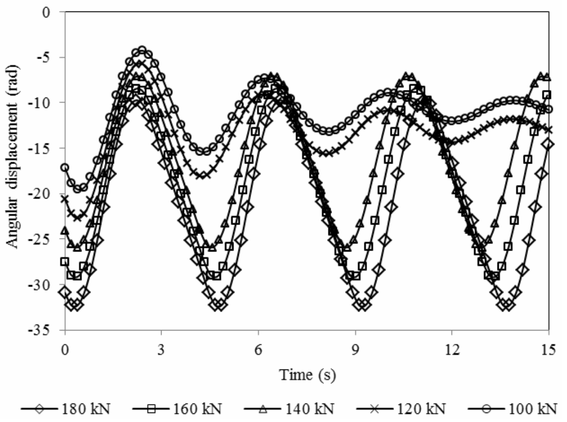

Fig. 6. Angular displacement between the bit and the table.

stick and slip phases depend on the WOB and the bit dynamics are not smooth, the period of stick-slip motions are different when stick-slip phenomenon occurs. For cases without the stick-slip vibration (Cases 4 and 5), the vibration period does not vary. In case of the stick-slip vibration, P_S increases and P_K decreases with a decrease in the WOB, resulting in a decreasing stick-slip vibration period. For the cases W_B = 120 kN and 100 kN, periodic motion or rather stick-slip vibration does not occur. The following discussions illustrate the different dynamic responses using the laws of bit dynamics.

Figs. 6 and 7 show the time histories of the relative angular displacement and relative angular velocity, respectively. In Fig. 6, a negative value of angular displacement indicates the drill bit lags behind the rotary table. As shown in the figures, the stick-slip phenomenon appears only for the cases W_B = 180 kN, 160 kN and 140 kN. For other cases without the stick-slip vibration, the drill bit vibrates damply and tends to converge into a uniform motion.

At the initial stage, the rotary table keeps rotating at a velocity of φ, but the drill bit accelerates from a velocity of 0, leading to a decrease in the angular displacement amplitude (absolute value increases). Then the bit velocity increases under the action of the driving torque. The accelerating process continues until the driving torque is less than the resisting torque. During this process, the damping force increases and the driving torque decreases. Then, the drill bit starts to decelerate (Figs. 6 and 7). Once the bit velocity decreases to 0, it steps into the stick phase. Since the rotary table rotates at a constant velocity and the drill bit remains still, a relative angular velocity equal to -10.47 rad/s can be obtained. In the case of stick-slip vibration, amplitudes of both the angular displacement and velocity increase with the increasing WOB.

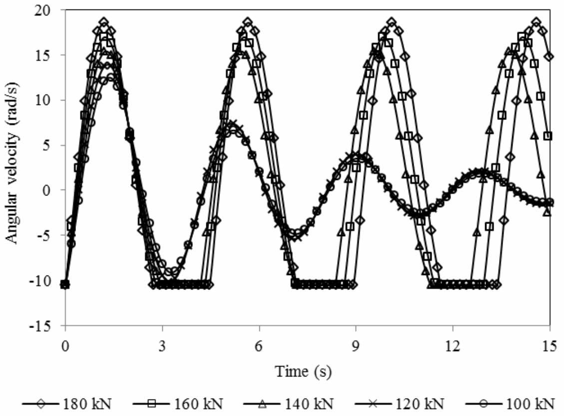

Fig. 7. Angular velocity of the drill bit versus that of the rotary table.

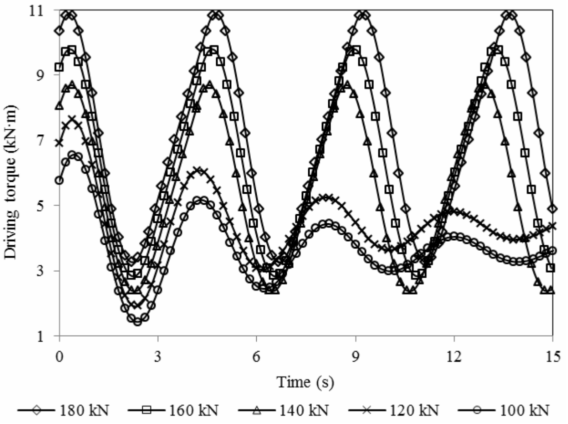

Fig. 8. Driving torque applied onto the drill bit.

Fig. 8 shows the responses of the driving torque transferred from the drill string to the drill bit for different WOB. The driving torque is calculated by multiplying the relative angular displacement with the torsional stiffness of drill string. As the torsional stiffness is a constant for a given structure, plots of the driving torque present a contrary shape relative to that of the relative angular displacement.

As can be seen from Fig. 8, the amplitude of the driving torque increases with the increasing WOB. In the case of stick-slip vibration, the drill bit cannot overcome the maximum static frictional torque and thus remains stationary in the stick phase. Consequently, in the stick phase, the driving torque on the drill bit equals the frictional torque (with an opposite direction). In the case without stick-slip vibration, the driving torque vibrates damply and finally converges to a constant value equal to the kinetic frictional torque.

Fig. 9 shows the responses of the frictional torque applied onto the bit. The period characteristics (shown in Table 2) for both the stick phase and the slip phase can be explicitly found in this figure. As shown in the figure, for the cases W_B = 180 kN, 160 kN and 140 kN that stick-slip vibrations occur, plots of the frictional torque on the bit are segmented. During the

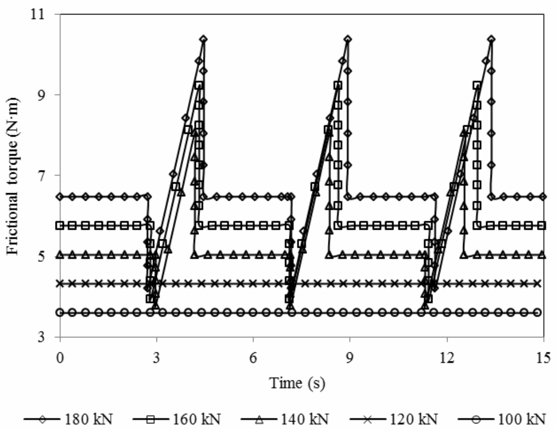

Fig. 9. Frictional torque applied onto the bit.

slip phase, the frictional torque equals the constant kinetic friction torque. Once the slip phase ends, the frictional torque drops to a small value which is a static frictional torque determined by the relative angular displacement and the drill string stiffness. In the stick phase, the frictional torque increases uniformly to its maximum value (referred to as the maximum static frictional torque) and the slope of the inclined line is determined by the rotary table velocity. The frictional torque drops once again from the maximum static frictional torque to the kinetic frictional torque at the time the driving torque overcomes the maximum static frictional torque. During the slip phase, the frictional torque is constant, corresponding to the segmented horizontal lines. In case of stick-slip vibration, amplitudes of the frictional torque increases with increasing the WOB. In cases without stick-slip vibration, the frictional torque is the kinetic frictional torque, which means the frictional torque remains constant (corresponds to the continuous horizontal lines). Similarly, values of the frictional torque decreases with the increasing WOB in cases without stick-slip vibration.

Figs. 10 and 11 present the phase plane of the bit motion relative to the table motion for different WOB. As can be seen from the two figures, there are two types of phase trajectories for different WOB. For the cases W_B = 180 kN, 160 kN and 140 kN that stick-slip vibrations occur, the phase trajectories of the bit motion are closed loops (limit cycle) and each loop comprises of a curve and a straight line. With the increasing WOB, the limit cycle enlarges outwardly and the centre point moves left. For the cases W_B = 120 kN and 100 kN, the phase trajectories shrink around focal points of their own and the plots are spirals, which means the bit converges to a stable state (uniform motion). The focal points moves left with the increasing WOB, corresponding to an increase in the relative angular displacement between the drill bit and the rotary table.

4. Discussion and conclusions

Stick-slip vibration is excited by the friction on the drill bit, and therefore, the existence of frictional torque is necessary for the occurrence of the stick-slip phenomenon. As the frictional torque is proportional to the WOB, it can be further deduced that the WOB is necessary for the stick-slip phenomenon. Assuming a limiting case with W_B = 0, the stick-slip phenomenon certainly will not occur. The cases studied in the previous section indicate that the stick-slip phenomenon occurs for three of the five drilling conditions. The following discussions explains the principles underlyng the stick-slip phenomenon and the conditions for the stick-slip vibration to occur in a given drilling system.

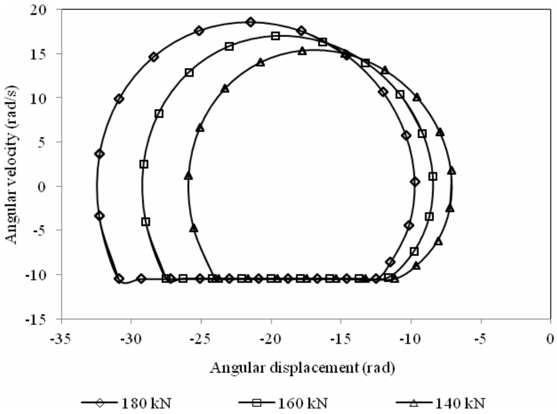

Fig. 10. Phase plane of the drill bit dynamics relative to the rotary table dynamics (Cases 1-3).

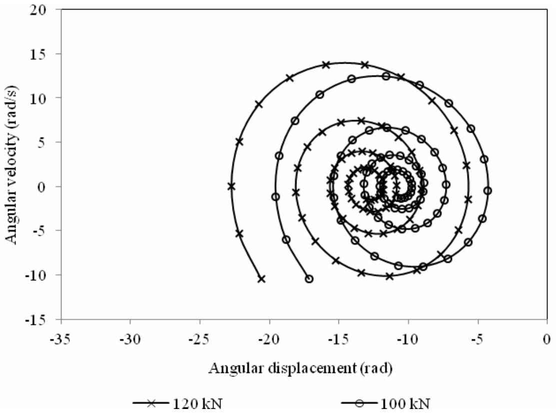

Fig. 11. Phase plane of the drill bit dynamics relative to the rotary table dynamics (Cases 4 and 5).

For the 3000 m drilling system with W_B = 180 kN, 160 kN and 140 kN, the stick-slip vibration occurs and the final state of the drill bit exhibits a stable periodical motion. For a given drilling system, periodic motions appear on the drill bit and the phase trajectory ultimately converges to a limit cycle once the stick-slip phenomenon appears. In the phase plane, the phase trajectory corresponds to a stick-slip vibration is a closed loop comprised of a curve and a straight line.

During the slip phase, the frictional torque on the drill bit equals the kinetic frictional torque which remains constant. At the critical state of the bit transiting from the stick phase to the slip phase, the frictional torque changes from the maximum

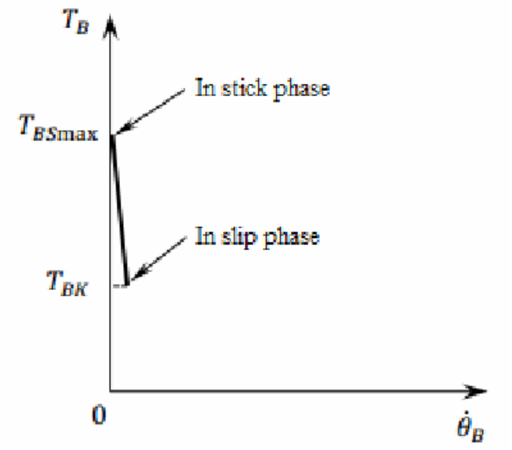

Fig. 12. Frictional torque varies with respect to bit velocity.

static frictional torque to the kinetic frictional torque. The frictional torque decreases with the increasing angular velocity (e.g. the bit velocity increase from 0 to a positive number), which means the derivatives of the frictional torque with respect to velocity is negative (the frictional torque decreases while the bit velocity increases), see Fig. 12. That is to say, there is a frictional torque drop at the moment transiting from the stick phase to the slip phase, and this phenomenon can be regarded as the negative damping effect.

When the WOB reaches to a certain level, the stick-slip phenomenon of the drill bit will be excited. In other words, a big enough WOB is necessary for the occurrence of stick-slip phenomenon in the drill string. On the condition that the other parameters are constant, the stick-slip phenomenon is more likely to occur at a larger WOB. For a drilling system with determined parameters, however, there is only one critical WOB. From the five cases analyzed, the critical WOB is between 120 kN and 140 kN for the 3000 m drilling string presented in Table 1, because stick-slip vibration appears when the WOB is lager than 140 kN and disappears when it is less than 120 kN. Then, a WOB equals to 130 kN is selected. For different WOB values, the dynamics of the drill bit are different. By obtaining the dynamics of the drill bit with W_B =130 kN, vibration characteristics (like Figs. 3-5) can be obtained. It is found that stick-slip vibration occurs when W_B =130 kN. Further, the critical WOB is determined to be between 120 kN and 130 kN. By selecting other WOB values, for example 125 kN (without stick-slip), 127.5 kN (with stick-slip), 126 kN (with stick-slip), the area wherein stick-slip vibration appears becomes shorter and shorter. In this way, the critical WOB of generating the stick-slip vibration can be determined through a trial and error iterative procedure.

For the drilling system and the system parameters listed in Table 1, the critical WOB for the stick-slip phenomenon is 125.9 kN. Specific to the drilling system analyzed in this paper, the stick-slip vibration will occur only if W_B > 125.9 kN. Of course, what should be stressed is that different critical WOB will be obtained for the same drilling structure with different operating parameters or for different drilling structures.

Based on the bit dynamics, the law for different WOB can be found. With the increasing WOB, the bit behavior may change from a constant motion to the stick-slip vibration. For the cases with W_B = 120 kN and 100 kN analyzed in this study, the bit finally stabilizes at a uniform motion and the angular displacements at which the bit lags behind the rotary table are determined by the frictional torque and the torsional stiffness of drill string. For the cases with W_B = 180 kN, 160 kN and 140 kN, the amplitudes of the dynamics of the stick-slip increases with the increasing WOB. In addition, centre point of the limit cycle corresponding to the bit motion moves left and the limit cycle enlarges with the increasing WOB.

The analysis presented above shows that the stick-slip vibration is prone to occur at a big WOB and thus a small WOB should be used. In addition, premature tool failure is more likely to occur at a big WOB. However, the bit scrapes the formation under the action of WOB and too small a WOB is insufficient to crush hard rocks and affects the drilling efficiency. As a result, overall considerations should be given during the drilling process to reduce the vibration and to enhance the drilling efficiency.

For a given drilling system, the critical WOB for the appearance of stick-slip vibration can be obtained by using the method provided in this paper. During the drilling process, a WOB less than the critical value should be used to avoid the stick-slip vibration. In this way, the stick-slip phenomenon is suppressed and the drilling efficiency may be improved. In fact, many other factors influence the stick-slip phenomenon, such as the drill bit structure, the rock formation and the fluid damping. When matching these parameters properly, the stick-slip vibration may be suppressed.

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Nomenclature

K_D : Equivalent drill string stiffness
J : Equivalent moment of inertia of the drill string
J_r : Real moment of inertia of the drill string
L_D : Length of drill string
L_P : Length of drill pipes
L_C : Length of drill collars
D_P : External diameter of the drill pipes
d_P : Internal diameter of the drill pipes

D_C : External diameter of the drill collars

d_C : Internal diameter of the drill collars

t : Time

Q : External force of the drilling system

θ_B : Angular displacement of the drill bit

$\left|\theta_B\right|$: Absolute value of the angular displacement of the drill bit

$(\dot{\;})$: Derivative with respect to time

c : Viscous damping coefficient

φ : Rotary table velocity

μ_K : Kinetic friction coefficient

μ_S : Static friction coefficient

W_B : Weight on bit

R_B : Real bit radius

$\bar{R}_B$: Equivalent bit radius

ω_n : Natural frequency

ξ : Damping ratio

A : Vibration amplitude related to initial conditions

Ψ : Phase position related to initial conditions

θ_{B0} : Initial angular displacement of the bit

$\dot{\theta}_{B0}$: Initial angular velocity of the bit

θ_{Br} : Relative angular displacement between the drill bit and rotary table.

T : Driving torque

ρ : Density of the drill string

G : Shear modulus of the drill string

P : Period of the stick-slip vibration

P_S : Time interval of the stick phase

P_K : Time interval of the slip phase

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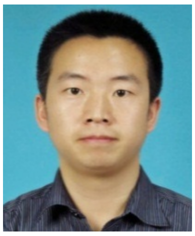

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