

Effect of continuum damage mechanics on springback prediction in metal forming processes[†]

Ali Nayebi* and Mehdi Shahabi

School of Mechanical Engineering, Shiraz University, Shiraz, Mollasadra, Iran

(Manuscript Received August 19, 2016; Revised January 5, 2017; Accepted January 15, 2017)

--

Abstract

The influence of considering the variations in material properties was investigated through continuum damage mechanics according to the Lemaitre isotropic unified damage law to predict the bending force and springback in V-bending sheet metal forming processes, with emphasis on Finite element (FE) simulation considerations. The material constants of the damage model were calibrated through a uniaxial tensile test with an appropriate and convenient repeating strategy. Holloman's isotropic and Ziegler's linear kinematic hardening laws were employed to describe the behavior of a hardening material. To specify the ideal FE conditions for simulating springback, the effect of the various numerical considerations during FE simulation was investigated and compared with the experimental outcome. Results indicate that considering continuum damage mechanics decreased the predicted bending force and improved the accuracy of springback prediction.

--

Keywords: Springback; Damage mechanics; Stiffness degradation; FE simulation; Metal forming

1. Introduction

Springback is defined as the change in a deformed part from a desirable configuration after unloading. It is one the com mon problems in sheet metal-forming processes. Disregarding this undesirable phenomenon causes problems, particularly in the assembly step, and increases tolerances and variability in the subsequent forming operations [1]. Therefore, prediction and compensation of springback are vital in nearly all metalforming processes. Springback is rapidly gaining considerable research interest. A review by Wagoner et al. [2] in 2005 revealed that the term "springback" does not appear in standard dictionaries, although this term had been used as early as the 1940s. According to the Thomson scientific database, only 334 technical papers discussing springback have been published since 1980. A recent investigation conducted in 2012 noted a dramatic change in previous reports [1]. Springback prediction was initially conducted by using only analytical methods. This approach is still employed by researchers [3, 4]. Analytical methods utilize simplified assumptions (e.g., perfect plastic behavior, constant Young's modulus). Therefore, the accuracy of these methods is low. The introduction of the Finite element method (FEM) caused a significant change in numerical work. Currently, FEM is regarded as an effective method by researchers. Yui et al. [5] investigated the effects of planar anisotropy coefficients and the yield function on springback characteristics through a numerical simulation. Da Sisva Botelho et al. [6] presented a simple and ordinary com parison of FE simulation, analytical, and experimental results on a draw-bending process. Farsi and Arezoo [7] experimentally investigated the value of springback and bending forces for a low-carbon steel material in a V-bending process. The capability of various hardening laws and their combination to predict the springback, wall-thinning, and cross-section deformation of titanium tubes in a rotary-draw tube-bending process was investigated by Shahabi and Nayebi [8]. Initially, researchers did not consider the importance of Bauschinger's effect but recognized its importance in subsequent investigations. Gau and Kinzel [9] reported a difference between the results of isotropic and kinematic hardening models in predicting the springback in sheet metal-forming processes. Their investigations indicated that the strain path changes the effect of the difference on springback prediction using the two models. Srinivasan et al. [10] presented predictive models for the bending force and final bend angle in air bending of an electrogalvanized steel sheet by employing the response surface methodology. Leu and Zhuang [11] developed a simplified approach by considering the thickness ratio, normal anisotropy, and strain-hardening exponent to estimate the springback of V-bending in reference to elementary bending theory. The researchers also experimentally and numerically investi-

^{*}Corresponding author. Tel.: +98 7136133029, Fax.: +98 7136287508

E-mail address: nayebi@shirazu.ac.ir

[†]Recommended by Associate Editor Jun-Sik Kim

[©] KSME & Springer 2017

gated the influence of geometrical and numerical process parameters. Lee and Park [12] optimized the structural and process parameters in a sheet metal-forming process. The relation between equivalent plastic strain and springback was investigated by Kadkhodayan [13] in consideration of a benchmark of NUMISHEET'93 2D draw bending and by using the com mercial FEM code. The theory of classical plasticity generally assumes that materials present a linear unloading behavior with a slope equal to the elasticity modulus, whereas experimental results show that the elasticity behavior in unloading is nonlinear and exhibits a slight curvature, which depends on plastic strain [14, 15]. Thus, researchers considered the changes in elasticity modulus in their investigations, and this consideration improved springback prediction [16-18]. In the current study, the damage mechanics concept was utilized to overcome the problem and improve the accuracy of modeling results. Continuum damage mechanics (CDM) is a useful approach to describe the material microscopic behavior accurately. Unlike most studies, the current study does not assume that Young's modulus is constant during plastic deformation. Instead, CDM based on the Lemaitre isotropic unified damage law was employed to describe the variations in Young's modulus during the forming process. CDM effectively describes plasticity behavior, such as kinematic hardening. Calibrating the material constants of the Lemaitre damage model requires cyclic (loading-unloading) experimental tests. Therefore, this process may be partly difficult and undesirable in many cases. The damage model was calibrated through simple uniaxial tests instead of complicated cyclic tests. To investigate the changes in material behavior and properties during plastic deformation, the concept of CDM based on the Lemaitre isotropic unified damage law was employed to provide effective springback prediction in metal-forming processes. Moreover, the material constants of the damage model were calibrated with a simple numerical approach based on FE simulation and the data of the uniaxial test.

2. CDM

Damage pertains to the gradual decrease in or sudden deca dence of the mechanical strength of materials through loading, thermal, and chemical effects. At the microscale, damage is the accumulation of micro stresses in a neighborhood of defects or interfaces and the breaking of bounds, both of which damage the material [19]. From a physical point of view, damage is related to plastic or irreversible strains and generally to strain dissipation either at the mesoscale, the scale of the representative volume element, or microscale. The concept of effective stress was introduced by Rabotnov in 1968. If S_D is the area of microcracks and microvoids in a cross section of a material and microforces do not act on the microsurface of microcracks and microvoids, effective stress ($\bar{\sigma}$), which is related to effective surface, can be conveniently derived as **CDM**
 SOM
 SECONS
 SUBM
 SUBM

$$
\overline{\sigma} = \frac{F}{S - S_D} \,. \tag{1}
$$

Fig. 1. 1D element (left: No damage, right: Damaged).

According to Fig. 1, by introducing the damage variable, (D $= S/S_D$) effective stress can be written in the following form [19].

$$
\overline{\sigma} = \frac{\sigma}{1 - D} \tag{2}
$$

When damage occurs, the yield function (*f*) is changed to [19]

$$
\sigma = (\sigma_y + R + X)(1 - D) \,. \tag{3}
$$

The elasticity modulus for the damaged material is defined [19] as

$$
\tilde{E} = E(1 - D) \tag{4}
$$

E. I. 1D element (left: No damage, right: Damaged).

According to Fig. 1, by introducing the damage variable, (D
 $\sqrt{S_{\text{FD}}}$) effective stress can be written in the following form

¹].
 $\sqrt{\sigma} = \frac{\sigma}{1 - D}$. (2)

Whe Eqs. (3) and (4) and the experiments show that the damage equally decreases yield stress, isotropic strain hardening stress, backstress, and elasticity modulus. The thermodynamics approach indicates that damage rate \overrightarrow{D} is associated with variable Y, which is the energy density release rate. Many obser vations and experiments have shown that damage is governed by plastic strain, which is introduced through the plastic multiplier λ as [20] $R + X$)(1-*D*). (3)

city modulus for the damaged material is defined
 D). (4)
 D). (4)
 and (4) and the experiments show that the damage

eases yield stress, isotropic strain hardening stress,

and elasticity modulu (4)

3) and (4) and the experiments show that the damage

decreases yield stress, isotropic strain hardening stress,

ss, and elasticity modulus. The thermodynamics ap-

microdes that damage rate \dot{D} is associated wit $\tilde{E} = E(1-D)$. (4)

Eqs. (3) and (4) and the experiments show that the damage

ally decreases yield stress, isotropic strain hardening stress,

Ackristess, and elasticity modulus. The thermodynamics ap-

ackristes, and e *s* (*b*) and (4) and the experiments show that the damage
ecreases yield stress, isotropic strain hardening stress,
s, and elasticity modulus. The thermodynamics ap-
dicates that damage rate *D* is associated with vari-
 27.

and (4) and the experiments show that the damage

creases yield stress, isotropic strain hardening stress,

and elasticity modulus. The thermodynamics ap-

icides that damage rate *D* is associated with vari-

eich i s and experiments have shown that damage is governed
since and experiments have shown that damage is governed
astic strain, which is introduced through the plastic mul-
 \vec{i} as [20]
 $=\frac{\partial F_{\rho}}{\partial Y}$ if $P > P_{\rho}$. (5)
my

$$
\dot{D} = \frac{\partial F_D}{\partial Y} \qquad \text{if} \qquad P > P_D. \tag{5}
$$

Many experimental results have also shown that F_D must be a nonlinear function of *Y*. Therefore, a simple and good choice can be presented as follows [20]: c strain, which is introduced through the plastic mul-
as [20]
 $\frac{F_{D}}{DY}$ if $P > P_{D}$. (5)
experimental results have also shown that F_{D} must be
exact function of *Y*. Therefore, a simple and good choice
resented as ions and experiments have shown that damage is governed
plastic strain, which is introduced through the plastic mul-
ier λ as [20]
 $\dot{D} = \frac{\partial F_D}{\partial Y}$ if $P > P_D$. (5)
Many experimental results have also shown that F_D ² $\frac{\partial F_p}{\partial Y}$ if $P > P_p$. (5)

experimental results have also shown that F_D must be

ear function of Y. Therefore, a simple and good choice

resented as follows [20]:
 $\frac{S}{(s+1)(1-D)} \left(\frac{Y}{S}\right)^{s+1}$. (6)

titre pres if $P > P_D$. (5)

rimental results have also shown that F_D must be

notion of Y. Therefore, a simple and good choice

ted as follows [20]:
 $\frac{S}{\sqrt{(1-D)}}(\frac{Y}{S})^{s+1}$. (6)

resented a damage criterion by using the thermo-
 x as [20]
 $= \frac{\partial F_p}{\partial Y}$ if $P > P_p$. (5)

my experimental results have also shown that F_p must be

linear function of Y. Therefore, a simple and good choice

presented as follows [20]:
 $= \frac{S}{(s+1)(1-D)} (\frac{Y}{S})^{s+1}$. (6)

$$
F_D = \frac{S}{(s+1)(1-D)} \left(\frac{Y}{S}\right)^{s+1}.\tag{6}
$$

Lemaitre presented a damage criterion by using the thermodynamics of irreversible processes [20].

$$
\dot{D} = \left(\frac{\sigma_{eq}^2 R_{\nu}}{2ES(1 - D)^2}\right)^s \dot{p}
$$
\n(7)

$$
R_{\nu} = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2.
$$
 (8)

Fig. 2. Process of calibrating the damage model constants.

Material constants "s" and "S" must be specified. The material constants of the Lemaitre isotropic unified damage law (*s* and *S*) are conventionally specified through fatigue tests. The uniaxial tensile test was used to extract the desirable data and calibrate the material constants instead of utilizing other types of mechanical tests (e.g., cyclic loading-unloading test). The material constants of the Lemaitre damage model were obtained in the current study according to an iterative process as follows: See CAS destribute the damage model constants.

All constants "s" and "S" must be specified. The mate-

tank of the Lemaitre isotropic unified damage law
 λ are conventionally specified through fatigue test.

All the s **Example 12**

2. Process of calibrating the damage model constants.

Alaterial constants "s" and "S" must be specified. The matternal constants "s" and "S" must be specified. The matternal constants of the Lemanite istorb **EXERCED**
 EXERCES Select as desirable value
 EXERCES
 EXERCES Select as desirable value
 EXERCES
 EXE

Considering the uniaxial case leads to $R_v = 1$ and $p = \varepsilon_p$. the best ones; otherwise, this process will be repeated.
Therefore, Eq. (7) is simplified as If desirable agreement is not achieved for s = 1, setting S =

$$
\dot{D} = \left(\frac{\sigma_{eq}^2}{2ES(1-D)^2}\right)^s \dot{\varepsilon}_p.
$$
\n(9)

By integrating Eq. (9), damage parameter D can be obtained in terms of other effective parameters. The experimental results indicate that $s = 1$ for most materials [19]. The FE simulation in the uniaxial tensile test based on ASTM (E8m) is conducted considering $s = 1$ and various values of "S". Notably, no specific and exclusive approach is available to identify the initial guess of "S" or "s". However, as an approximate general rule, for a material with yield stress less than 300 MPa, $s < 0.5$ is recommended as an initial guess and $s > 0.5$ is considered for yield stress more than 300 MPa. This approach is not a general and accurate law, and the choice of the initial guess is empirical. To investigate the certainty of the selected material constants ("s" and "S"), the stress-strain curve of the simulation outputs is compared with that of the experimental

Fig. 4. Sketch of the V-bending test [7].

results. If the difference between the stress-strain curves obtained experimentally and numerically is less than a certain value (e.g., 10 %), the values of "s" and "S" can be considered the best ones; otherwise, this process will be repeated.

1 is recommended, and the process above is repeated to obtain a reasonable "s". Fig. 2 depicts this process. According to this process, the values of $s = 1$ and $S = 0.4$ MPa result in a maximum error of 5 % between the numerical and experimental results (Fig. 3). Good agreement between the simulation and experimental results can provide reasonable assurance on the selected material constants.

3. V-bending sheet metal forming process

The investigated V-bending problem in the study is shown in Fig. 4. All the geometrical conditions and mechanical properties are presented according to Farsi and Arezoo [7]. The sheet is a low-carbon steel that is frequently used in various industries. Its dimensions are $L = 80$ mm, $W = 50$ mm and $t =$ 0.95 mm with mechanical properties of E = 193 GPa, σ_v = 155 MPa and $v = 0.3$. The geometrical characteristics of the vbending test are as follows: die angle = 90 (deg.), punch angle $= 84$ (deg.), and punch radius $= 1$ mm.

Table 1. Compression of various considerations to model the unloading step.

Automatic stabilization method of unloading step modeling Springback	
Specify dissipated energy fraction	0.03
Specify damping factor	2.51
Use damping factor from previous general step	2.56

4. FE simulation

To simulate the V-bending test, the forming and springback steps were simulated with an implicit solver. The die and punch were regarded as analytical rigid surfaces, and the sheet was considered a deformable part. According to a previous study [7], $\mu = 0.1$ is considered the friction coefficient among all involved surfaces. The die was fixed during the simulation of the sheet metal-forming process, but the punch was moved downward to make a 90° V-shape part. After the loading step, the unloading step was simulated to achieve springback modeling. Similar to the loading step, FE modeling of the unloading step was conducted with an implicit solver. Thus, a gen eral static step was generated in consideration of nonlinearity effects (i.e., "on" case of Nlgeom). "Specify damping factor" was regarded as automatic stabilization. However, obtaining a reasonable estimate for the damping factor would be difficult unless a value is known from the output of previous runs. The damping factor depends on the amount of damping, mesh size, and material behavior. The adaptive automatic stabilization scheme, in which the damping factor can vary spatially and with time, is an effective alternative approach. In this case, the damping factor is controlled by the convergence history and the ratio of the energy dissipated by viscous damping to the total strain energy. The damping factor is determined in such a manner that the dissipated energy for a given increment with characteristics similar to that in the first increment is a small fraction of the extrapolated strain energy. The fraction is called the dissipated energy fraction and has a default value of $2 \times 10-4$. Several choices are available for modeling the unloading (springback) step. To select the best approach, FE simulation of the springback step was conducted with each method, and the results were compared and are shown in Ta ble 1. Table 1 indicates that considering the "specify damping factor" case produces the best results. Feifei et al. [21] performed a partly comprehensive FE investigation to specify the best suggestions for modeling the unloading step. Their investigation indicated that considering the "specify damping factor" approach results in improved springback prediction. Their investigation also indicated that the initial increment size does not significantly influence the simulation results.

The direct method and the full Newton solution technique were utilized to solve the FE equations. The other conditions governing the problem, such as loading and boundary conditions, were similar to those in the loading step, except that the die moved upward at the end of V-bending. The sheet used was meshed with 9954-node bilinear plane strain quadrilateral

Table 2. Effect of CDM on springback prediction (deg.).

Experimental = 2.00 [7]			
Hardening model	With damage	Without damage	
ĪΗ	$2.13(6.5\%)$	$2.56(28\%)$	
I KH	$1.74(13\%)$	$1.71(14.5\%)$	

Fig. 5. Mesh generation of the sheet.

elements, with five elements through the sheet thickness. Reduced integration and hourglass control were also considered. To economize the solving time, only the middle part of sheet was meshed with small elements. The implicit approach was employed to solve the FE equations, and convergence of the results did not occur. Fig. 5 illustrates the mesh generation of the sheet in the present study. iction (deg.).
 $\frac{D[7]}{D[7]}$ Without damage
 $\frac{2.56 (28 \%)}{1.71 (14.5 \%)}$
 $\frac{1.71 (14.5 \%)}{1.71 (14.5 \%)}$

the sheet thickness. Re-

col were also considered.

the middle part of sheet

is the middle part of sheet

is the ¹ 2.13 (0.3 %) 2.30 (2.6 %)

(*H* 1 2.14 (13 %) 1.71 (14.5 %)

(*H* 1 1.74 (13 %) 1.71 (14.5 %)

(*H* 1 1.74 (13 %) 1.71 (14.5 %)

(*H*) five elements through the sheet thickness. Re-

tegration and hourglass control we IN EXECUTE: $\frac{2.15 (0.376)}{2.171 (14.5\%)}$

LATE 1.74 (13 %) 1.71 (14.5 %)

5. Mesh generation of the sheet.

EXECUTE: $\frac{1}{171 (14.5\%)}$

5. Mesh generation of the sheet.

economize the solving time, only the middle part *p* $\frac{2.15 (0.376)}{2.17 (14.5\%)}$ $\frac{2.36 (26.76)}{1.71 (14.5\%)}$
 p x

The isotropic hardening model of $\sigma = 370.52(\varepsilon_n^{0.16})$ and the Ziegler linear kinematic hardening model were utilized to describe the hardening behavior of the material [22].

$$
\dot{x} = C \frac{1}{\sigma_0} (\sigma - x) \dot{\varepsilon}_p \ . \tag{10}
$$

The test data obtained from a half cycle of unidirectional tension or compression must be linearized because this simple model can predict only linear hardening. Linear kinematic hardening requires two data pairs to define the linear behavior: the stress at zero plastic strain (σ0; yield stress) and the stress at an arbitrary finite plastic strain value (σ). To employ Ziegler's linear kinematic hardening law, $\sigma_0 = 155$ MPa at $\varepsilon_p = 0$ and $\sigma = 296$ MPa at $\varepsilon_p = 0.26$ were specified. 0 and σ = 296 MPa at ε_p = 0.26 were specified.

5. Results and discussion

The FE simulation of the V-bending test based on the aforesaid notes was conducted, and the following results were obtained. Table 2 shows the effect of CDM on the springback prediction for various hardening models. The values in parentheses indicate the relative error between numerical and ex perimental results (e.g., $((2.56-2)/2)*100 = 28$ %). Utilizing the hardening models separately cannot describe material hardening models correctly. In fact, material behavior in actual cases necessitates that the yield surface be translated and expanded at the same time. Thus, considering the isotropic and kinematic hardening laws separately is insufficient. These laws should be combined until improved modeling is achieved. Considering the changes in material properties by using CDM improved the accuracy of springback prediction of the sheet to between 28 % and 6.5 % in the IH case with respect to the experimental results. The capability of the hardening models and CDM in bending force prediction was investigated as well. The results are presented in Table 3. Considering CDM during the simulation decreased the prediction of bending force in the

Table 3. Effect of CDM on bending force prediction (N.).

Experimental = 950 [7]				
Hardening model	With damage	Without damage	Decrease $(\%)$	
		692		
I KH		705		

Fig. 6. Stress distribution within the sheet (up: Bending, down: Springback).

V-bending sheet metal forming process.

This decrease can be justified. The bending force of a V- w bending process can be calculated analytically with Eq. (11) X [23].

$$
F = \frac{\sigma_y W t^2}{D_f} \left(\frac{2}{1 + \mu \sin \theta - \cos 2\theta} \right).
$$
 (11) σ_f

 D_f equals 1.3 for rectangular sections. Eq. (11) shows that v the bending force is directly related to the yield stress. Eq. (3) μ indicates that CDM decreases the yield stress. Furthermore, θ Mkaddem et al. [24] conducted FE prediction of material damage distribution within a workpiece during wiping die bending processes. They found that damage mechanics decreases the predicted loading force. Considering the damage mechanics in predicting the bending force resulted in a mean difference of approximately 5 % compared with the case without damage. In conclusion, although considering the damage mechanics improves the accuracy of springback prediction significantly, its effect on predicting the bending force is not evident. Fig. 6 depicts the stress distribution within the V-bent sheet before and after springback. A portion of elasticity strains was released after the occurrence of springback. Thus, the stress level in the sheet metal decreased.

6. Conclusions

Considering the Young's modulus variation caused by plastic deformation is an effective strategy to simulate metal forming processes. CDM can describe the material's inelastic behavior. Combining CDM and plasticity models improved the precision of springback prediction through FE simulations. Utilizing CDM to describe the variation in the elasticity modulus and plastic follow behavior enhanced the accuracy of springback prediction by approximately 20 %. An iterative approach based on FE simulation was used as a convenient method to calibrate the constants of the damage model. Con sidering the induced damage by the CDM model in the metalforming process decreased the predicted bending force.

Nomenclature-

- C : Kinematic hardening modulus
- D : Damage parameter
- D_f : Die opening factor
- E : Young's modulus

E : Elasticity modulus
	- Ẽ : Elasticity modulus of damaged material
- F : Force
- F_D : Dissipative damage potential function
- L : Sheet length
- p : Accumulated plastic strain
- P_D : Damage threshold accumulated plastic strain
- R : Stress due to isotropic hardening
- R_v : Triaxiality function
- s, S : Material constant
- t : Sheet thickness
	- : Sheet width
	- : Back-stress
- σ_0 : Initial size of yield surface
- σ_{eq} : Von-Mises equivalent stress
- σ_{H} : Hydrostatic stress
- σ_v : Yield stress
- $\varepsilon_{\rm p}$: Plastic strain
- υ : Poisson ratio
- µ : Friction coefficient
- : Angle of bent sheet

References

- [1] R. H. Wagoner, H. Lim and M. G. Lee, Advanced issues in springback, *International Journal of Plasticity*, 45 (2012) 3- 20.
- [2] R. H. Wagoner, J. F. Wang and M. Li, Springback, ASM Handbook, *ASM: Materials Park* (2006) 733-755.
- [3] K. C. Chan, Theoretical analysis of springback in bending of integrated circuit lead frames, *Journal of Material Processing Technology*, 91 (1991) 111-115.
- [4] H. Li, H. Yang, F. F. Song, M. Zhan and G. J. Li, Springback characterization and behaviors of high-strength Ti-3Al- 2.5V tube in cold rotary draw bending, *Journal of Material Processing Technology*, 212 (1987) 1973- 1987.
- [5] L. Yuqi, H. Ping and W. Jinchen, Springback simulation and analysis of strong anisotropic sheet metals in U-channel bending process, *Acta Mechanica Sinica*, 18 (2002) 264-273.
- [6] T. Da Sisva Botelho, E. Bayraktar and G. Inglebert, Com-

parison of experimental and simulation results of 2D-draw bend springback, *Journal of Achievement in Materials and Manufacturing Engineering*, 18 (2006) 275-278.

- [7] M. A. Farsi and B. Arezoo, Bending force and springback in V-die bending of perforated sheet metal components, *Jour nal of Brazilian Society Mechanical Science Engineering*, 1 (2011) 45-51.
- [8] M. Shahabi and A. Nayebi, Springback FE modeling of titanium alloy tubes bending using various hardening models, *Structural Engineering and Mechanics*, 56 (2015) 369-383.
- [9] J. T. Gau and G. L. Kinzel, A new model for springback prediction in which the Bauschinger effect is considered, *International Journal of Mechanical Sciences*, 43 (2001) 1813- 1832.
- [10] R. Srinivasan, D. Vasudevan and P. Padmanabhan, Prediction of bend force and bend angle in air bending of electro galvanized steel using response surface methodology, *Journal of Mechanical Science and Technology*, 27 (2013) 2093-2105.
- [11] J. J. Lee and G. J. Park, Optimization of the structural and process parameters in the sheet metal forming process, *Journal of Mechanical Science and Technology*, 28 (2014) 605-619.
- [12] D. K. Leu and Z. W. Zhuang, Springback prediction of the vee bending process for high-strength steel sheets, *Journal of Mechanical Science and Technology*, 30 (2016) 1077-1084.
- [13] M. Kadkhodayan, An investigation into the influence of deformable dies on the springback of circular plate, *Scientia Iranica*, 13 (2006) 201-205.
- [14] F. Yoshida and T. Uemori, A model of large-strain cyclic plasticity describing the Bauschinger effect and work hardening stagnation, *International Journal of Plasticity*, 18 (2002) 661-686.
- [15] A. Ghaei, D. Green and A. Taherizadeh, Semi-implicit numerical integration of Yoshida-Uemori two-surface plasticity model, *International Journal of Mechanical Sciences*, 52 (2010) 531-540.
- [16] H. Kim, M. Kimchi, N. Kardes and T. Altan, Effects of variable elastic modulus on springback predictions in stamping advanced high-strength steels (AHSS), *10th International Conference on Technology of Plasticity* (2011) 628- 633.
- [17] S. Chatti and N. Hermi, The effect of non-linear recovery

on springback prediction, *Journal of Computers and Structures*, 89 (2011) 1367-1377.

- [18] M. Vrh, M. Halilovič and B. Starman, A new anisotropic elasto-plastic model with degradation of elastic modulus for accurate springback simulations, *International Journal of Material Forming*, 4 (2011) 217-225.
- [19] J. Lemaitre, *A course on damage mechanics*, Springer Verlag, Berlin (1992).
- [20] J. Lemaitre and R. Desmorat, *Engineering damage me chanics*, Springer Verlag, Berlin (2005).
- [21] S. Feifei, Y. He, L. Heng, Z. Mei and L. Guangjun, Springback prediction of thick-walled high-strength titanium tube bending, *Chinese Journal of Aeronautics,* 26 (2013) 1336- 1345.
- [22] J. Lemaitre and J. L. Chaboche, *Mechanics of solid materials*, Cambridge University Press (1990).
- [23] M. A. Eltantawie and E. Elsoaly, A static approach for determination of bending force and springback during proc ess, *International Journal of Mechanical Engineering and Robotic Researches*, 1 (2012) 1-13.
- [24] A. Mkaddem, R. Hambli and A. Potiron, Comparison between Gurson and Lemaitre damage models in wiping die bending processes, *International Journal of Advanced Manufacturing Technology*, 23 (2004) 451-461.

Ali Nayebi received his Ph.D. degree in mechanical engineering from University of Rennes, France, in 2002. He is currently a Professor of Mechanical Engineering in Shiraz University, Iran. His interests include plasticity, continuum damage mechanics, FGM, and creep and cyclic loading.

Mehdi Shahabi received his M.Sc. degree in Mechanical Engineering from Shiraz University in 2013. His interests include plasticity, FE simulation, metal formation, and continuum damage mechanics.