

## Robust target cascading for improving firing accuracy of combat vehicle<sup>†</sup>

Shinyu Kim, Woochul Lim, Hansu Kim, Namhee Ryu, Kihan Kwon,  
Sunghoon Lim, Seungjae Min and Tae Hee Lee\*

*Department of Automotive Engineering, Hanyang University, Seoul 04763, Korea*

(Manuscript Received December 16, 2015; Revised June 4, 2016; Accepted August 22, 2016)

### Abstract

Complex systems like combat vehicles contain numerous subsystems and components. Therefore, simultaneous consideration of hierarchy of a system, subsystems, and components is necessary for optimization. Multidisciplinary design optimization techniques have been researched to design the complex system. However most of these techniques premise integration process of total system which requires great time and cost. To reduce time and cost for the integration process, we introduce a target cascading technique that optimizes complex hierarchy system with several subproblems of each subsystem and component. Another challenge is to improve the firing accuracy of combat vehicle under various uncertainties. Robust design is therefore necessary to improve the firing accuracy of combat vehicles. To utilize these two concepts in optimization process, statistical information of firing angle is used as linking variables for problem formulation of robust target cascading. Furthermore, analysis of variance, surrogate modeling and statistical approach evaluating firing accuracy are employed to enhance efficiency of optimization. Finally, optimum design of a combat vehicle is achieved by using robust target cascading while improving firing accuracy.

*Keywords:* Analysis of variance; Combat vehicle; Firing accuracy evaluation; Robust target cascading; Surrogate model

### 1. Introduction

Combat vehicles are complex systems that consist of numerous related subsystems and components that share design variables and exchange responses. Therefore, to achieve optimal design, simultaneous consideration of the complex total system is necessary. Multidisciplinary design optimization (MDO) techniques have been developed to accomplish the simultaneous consideration and most of these techniques are needed on integration process of total system. Through the integration process, initial design with feasibility can be obtained. However, because design teams and experts work independently and have separate skills, the process for system integration requires much effort, time and cost. One type of MDO techniques is the Target cascading (TC) method, which has been developed to optimize complex system without system integration process [1]. TC optimizes total system with several subproblems of each subsystem and components. Therefore, the time and cost for the system integration process can be reduced. TC has been employed in the design of various products that have systems with complexity and hierarchical structure. Especially, it was widely employed in design of various types of vehicle like commercial bus [2], sport-utility

vehicle [3], and heavy truck [4]. In other fields, design of aircraft [5] and building [6] were performed using TC. According to research, TC provides optimum design that satisfies feasibility and adequacy without system integration process. In this research, TC is employed for the robust design of combat vehicles by considering the complexity and hierarchical structure of combat vehicles.

Combat vehicles have various performance such as fire-power, mobility, and survivability. Among these performances, firing accuracy is the most important performance that determines the striking power of a combat vehicle. However, there are various uncertainties that affect the firing accuracy of combat vehicle like environmental and operational uncertainties. According to previous researches, deflection of barrel, ballistic error, and weather error exist and those uncertainties make firing accuracy poor [7]. Due to these uncertainties, if firing is carried out under wrong aiming angle, it is certain that high firing accuracy cannot be guaranteed [8]. Therefore, to consider uncertainties and improve firing accuracy, research quantifying uncertainties and assessing firing accuracy was performed [9]. In this research, uncertainty of firing angle is considered and design optimization improving firing accuracy is performed through achieving robust landing point of shot.

In this research, the TC is employed to optimize a complex combat vehicle while saving cost and time for system integration process. Additionally, to achieve high firing accuracy,

\*Corresponding author. Tel.: +82 2 2220 0449, Fax.: +82 2 2220 2299

E-mail address: thlee@hanyang.ac.kr

<sup>†</sup>Recommended by Associate Editor Ki-Hoon Shin

© KSME & Springer 2016

Table 1. Specifications of APD.

Parameters	Specification	Unit
Upper body mass	12965	kg
Lower body mass	4200	kg
Wheelbase	4.2	m
Track	2.54	m
Height	1.9	m

robust optimization technique is employed. Unlike previous TC researches, the statistical information of firing angle is evaluated and used as linking variables to employ TC and robust optimization technique together. This paper is organized as follows: In Sec. 2, modeling of the combat vehicle is described. The entities of each level of a system as well as brief information of theory and implementation are described. The design variables and responses of each entity are defined in this section. In Sec. 3, concepts of TC and Robust target cascading (RTC) are introduced. In addition, Analysis of variance (ANOVA) and surrogate modeling technique is employed to increase efficiency of optimization process. RTC of combat vehicle problem is formulated and optimization of combat vehicle is performed. In the end, conclusion is presented in Sec. 4.

## 2. Modeling of combat vehicle

### 2.1 Computational modeling of combat vehicle

In this research, combat vehicle is modeled by benchmarking 6-wheel Autonomous platform demonstrator (APD) which developed at National Robotics Engineering Center (NREC). Brief specifications of APD is listed in Table 1. In this research, five main performances of APD are evaluated: firing accuracy, longitudinal firing stability, lateral firing stability, acceleration performance and comfortability. Combat vehicle is modeled into 4 parts: ballistic model, firing stability model, mobility model and suspension spring model. Specific descriptions of modeling, input and output are written in the following section.

#### 2.1.1 Ballistic model

Firing accuracy is evaluated by using a ballistic model. Under assumption of existence of uncertainty of pitch and yaw angles of firing, which makes landing point of shot different, firing accuracy is evaluated based on variances of pitch and yaw angles. To evaluate firing accuracy, exterior ballistic model and firing accuracy estimation model based on bivariate normal distribution is employed.

Landing point of shot is calculated by exterior ballistic model. Point-mass trajectory model is employed as exterior ballistic model [10, 11]. Point-mass trajectory model simplifies simulation by assuming projectile as a point-mass. Gravitational force and drag force affected by static atmosphere are considered in simulation. The equation of motion for point-

mass is expressed as following equation:

$$\frac{d\mathbf{V}}{dt} = -C_D^* V_p \mathbf{V} + \mathbf{g} \quad (1)$$

where  $\mathbf{V}$  denotes velocity vector of trajectory and  $\mathbf{g}$  is vector of gravitational acceleration. Initial condition of differential equation is given through pitch and yaw angles of firing. The equation can be separated into equation of each coordinate. The trajectory of point-mass can be calculated by solving following equations:

$$\begin{aligned} \dot{V}_x &= -C_D^* V_p V_x \\ \dot{V}_y &= -C_D^* V_p V_y - g \\ \dot{V}_z &= -C_D^* V_p V_z \end{aligned} \quad (2)$$

where  $x$  and  $z$  denotes coordinate of longitudinal and lateral firing direction respectively.  $y$  is coordinate of vertical direction.  $C_D^*$  and  $V_p$  are calculated as following equations:

$$\begin{aligned} C_D^* &= \frac{\rho A C_D}{2mp} \\ V_p &= \sqrt{V_x^2 + V_y^2 + V_z^2} \end{aligned} \quad (3)$$

where  $\rho$ ,  $A$ ,  $C_D$  and  $mp$  denote density of air, cross section area of projectile, drag coefficient of projectile, and mass of projectile, respectively.

The differential equations are solved using ode45 solver in MATLAB software and modularized in function form. The function evaluates coordinate of landing point of shot from given pitch and yaw angles of firing.

Based on landing points of shots, firing accuracy can be calculated. In evaluation of firing accuracy, Monte Carlo simulation (MCS) is commonly used [12]. However, MCS needs a lot of samples and this increases cost and time spent in evaluation. MCS based method evaluates firing accuracy according to following equation:

$$\alpha_N = \frac{N_s}{N} \quad (4)$$

where  $\alpha_N$  denotes firing accuracy evaluated through MCS,  $N$  is total number of shots and  $N_s$  is number of shots which strike target. Due to characteristics of MCS, large cost and time needed in evaluation, MCS based method is hard to employ in optimization that performs repetitive evaluation.

In this research, to save cost and time, statistical approach using bivariate normal distribution is employed. Probability density function (PDF) of bivariate normal distribution is expressed as follows:

$$f = \frac{1}{\sqrt{(2\pi)^2 |\Sigma|}} \exp\left(-\frac{1}{2}(\mathbf{p} - \boldsymbol{\mu})^T \Sigma_p^{-1} (\mathbf{p} - \boldsymbol{\mu})\right) \quad (5)$$

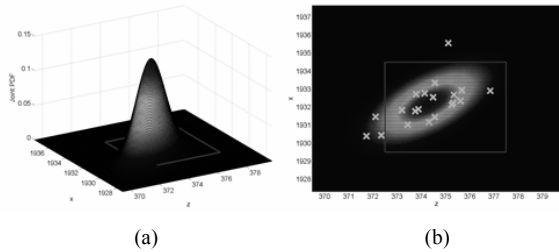


Fig. 1. (a) 3-D plot; (b) 2-D contour plot describing PDF of landing point and target area.

where  $\mathbf{p} = [x, z]^T$

$$\boldsymbol{\mu} = [\mu_x, \mu_z]^T$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_z \\ \rho\sigma_x\sigma_z & \sigma_z^2 \end{bmatrix}$$

Note that  $\boldsymbol{\mu}$  denotes expected value of each marginal distribution and  $\boldsymbol{\Sigma}$  denotes matrix of covariance and these are parameters of bivariate normal distribution.

Probability of landing in domain is expressed as bivariate normal distribution and its parameters are determined by maximum likelihood estimation. Based on estimated PDF of bivariate normal distribution, firing accuracy can be calculated using follows:

$$\alpha = \iint_{\text{target}} f \, dx dz \tag{6}$$

where  $\alpha$  represents firing accuracy evaluated by statistical approach and target is target domain represented by size and position of target.

By integrating the PDF of probability of landing over specified target domain, firing accuracy can be evaluated. Fig. 1 shows the estimated bivariate normal distribution and the integrating area given by the target domain.

Based on statistical approach using bivariate normal distribution, 20 times of firing was carried out to evaluate firing accuracy. Because of variances of pitch and yaw angles, 20 landing points of shots are scattered on ground. Position of center of target is assumed same as landing point of shot fired with mean pitch angle and mean yaw angle.

To improve firing accuracy, robustness of points of impact should be guaranteed. Therefore, in this research, robust optimization concept is employed because high firing accuracy can be obtained by minimizing variance of firing angle.

In conclusion, firing accuracy of combat vehicle is evaluated in ballistic model. Based on variance of pitch and yaw angles, landing points of shots are calculated through point-mass trajectory model function. Then firing accuracy of according landing points is evaluated through statistical approach based on bivariate normal distribution.

### 2.1.2 Firing stability model

Firing stability model evaluates longitudinal firing stability, lateral firing stability, and variance of firing angle based on pitch, yaw and roll angles generated according to firing. To evaluate these responses, interior ballistic model and dynamic model are modeled.

Interior ballistic model is modeled to calculate recoil force generated from firing. Le Duc empirical model is employed as interior ballistic model [13]. Through Le Duc empirical model, pressure occurs in caliber tube is calculated with respect to travel distance of projectile. Le Duc empirical model expresses velocity of projectile as following equation:

$$v = \frac{ad_x}{b + d_x} \tag{7}$$

where  $v$  and  $d_x$  denote velocity and travel distance of projectile.  $a$  and  $b$  are constant coefficients. By applying Newton’s laws of motion and law of conservation of energy, pressure in caliber is calculated as following equation:

$$p = (mp + 0.5mc) \frac{v^2}{2AL} \tag{8}$$

where  $mc$  and  $L$  denote mass of propellant and length of caliber respectively. Based on the pressure, recoil force can be calculated by multiplying cross section area of projectile,  $A$ .

In summary, recoil force is evaluated based on gun parameters which consist of mass of projectile and propellant and caliber. The internal ballistic model based on the Le Duc empirical model is implemented as a function form using MATLAB software.

To evaluate firing stability based on recoil force, a dynamic model is composed. According to specification of benchmarked APD, 6-wheel full vehicle model is composed by employing mass, spring, damper and tire components in commercial software MapleSim. Additionally, the turret and gun are attached on top of the combat vehicle to perform firing. Turret and gun consist of dampers and springs and these are designed to absorb recoil force from firing. Based on recoil force evaluated using Le Duc empirical model, dynamic model analyzes change of pitch, roll and yaw angles after firing. Dimensions of chassis of combat vehicle and parameters of turret and gun are considered as design variables in this dynamic model.

As a result, changes of pitch, roll and yaw angles are calculated. Fig. 2 shows changes of pitch and yaw angle after longitudinal firing. As described in Fig. 2, maximum absolute value from pitch angle of firing is used to evaluate longitudinal firing stability. Likewise, maximum roll angle after lateral firing is used to evaluate lateral firing stability. To evaluate variance of firing angle, pitch and yaw angles, candidate period for next firing is assumed as shown in Fig. 2. Therefore, variance of firing angle is evaluated by calculating variance of pitch and

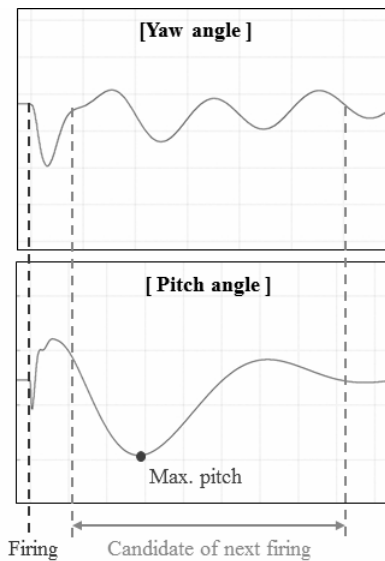


Fig. 2. Response evaluation of firing stability model.

yaw angles of candidate period of next firing.

Consequently, specifications of chassis, gun and turret are considered design variables in firing stability model. Based on these design variables, Le Duc interior ballistic function evaluates reaction force. Then dynamic model, composed using MapleSim, evaluates behavior of combat vehicle, pitch, roll and yaw angles. Longitudinal firing stability, lateral firing stability and variance of firing angle are evaluated.

**2.1.3 Mobility model**

In mobility model, acceleration performance and comfortability of combat vehicle are evaluated. To evaluate these performances, chassis model, powertrain model, suspension model, tire model and road profile are modeled using MapleSim.

Same chassis model used in firing stability model was also used. Because combat vehicle used in this research is hybrid combat vehicle, powertrain consists of internal combustion engine, battery, electric in-wheel-motor, controller and etc. These components are also modeled using MapleSim library. For suspension model, swing arm type suspension which used in APD is employed. Tire is modeled using Fiala tire model introduced by Fiala and extended by the developers of MSC Adams.

Acceleration performance is simulated by running on a flat road profile. It is evaluated based on required time for acquiring specific speed. Comfortability is evaluated by passing a specific bump on the flat road profile. Maximum vertical acceleration after passing bump is measured to evaluate comfortability.

In summary, specifications of chassis, which are also considered in firing stability model, and specifications of powertrain, suspension and tire are considered as design variables in mobility model. Based on these design variables, the velocity and vertical acceleration are evaluated through a dynamic

Table 2. Input and output of each models.

Level	Model	Output	Input	# of d.v.
System	Ballistic	Firing accuracy	$\sigma^2_{fire}$	2
Subsystem	Firing stability	$\sigma^2_{fire}$ , firing stability	Chassis	6
			Gun	3
			Turret	6
	Mobility	Comfortability acceleration perf.	Chassis	6
			Powertrain	4
			Suspension	5
			Tire	3
Component	Suspension spring	Stiffness coef. mass, index	Spring	3

model function, and these values represent the acceleration performance and comfortability of combat vehicle.

**2.1.4 Suspension spring model**

At the component level, the suspension spring design needed for suspension of mobility model is considered. Because spring design is related to suspension model, this affects design of mobility model. Simple spring design method based on mathematical equation is employed. The stiffness coefficient of spring is evaluated through the follows:

$$k = \frac{d^4 G}{8D^3 N_a} \tag{9}$$

where  $d$ ,  $D$  and  $N_a$  represent wire diameter, coil diameter and effective number of turns respectively and these are considered as design variables. The shear modulus of spring material is  $G$ . Evaluated stiffness coefficient of spring is used in the mobility model. Additionally, spring mass and spring index are evaluated and used as performance of suspension spring.

**2.2 Hierarchical structure of combat vehicle**

According to modeling of combat vehicle described in Sec. 2.1, input and output of each model are summarized in Table 2 where  $\sigma^2_{fire}$  represents the vector of  $\sigma^2_{pitch}$  and  $\sigma^2_{yaw}$  which are variance of pitch angle and variance of yaw angle, respectively. Firing stability includes longitudinal firing stability and lateral firing stability. In addition, design variables of each parameter groups are described in Table 3. As shown in Table 3, ballistic model and firing stability model share variance of pitch and yaw angle. Likewise, firing stability model and mobility model share chassis parameter and spring model shares stiffness coefficient with mobility model. Considering input and output relationship of each shared variables between these parts, hierarchical structure of combat vehicle is composed as shown in Fig. 3.

Finally, as shown in Fig. 3, hierarchical modeling of combat vehicle is composed of three levels. System level consists of

Table 3. Description of design variables of each parameter groups.

Parameter group	Symbol	Description	Current value	Unit
Chassis	$M_{body}$	Body mass	7000	kg
	$L_F$	Longitudinal distance between C.G. and front suspension	1.2	m
	$L_M$	Longitudinal distance between C.G. and middle suspension	0.6	m
	$L_R$	Longitudinal distance between C.G. and rear suspension	1.1	m
	$H$	Vertical distance between C.G. and suspension	0.186	m
	$W$	Lateral distance between C.G. and suspension	1.1	m
Gun	$mp$	Mass of projectile	19.5	g
	$mc$	Mass of propellant	1.28	g
	$D_{gun}$	Diameter of gun	0.105	m
Turret	$K_{turret}$	Spring stiffness of turret	8.50E+06	N/m
	$C_{turret}$	Damping coefficient of linear motion	50000	Ns/m
	$K_{gun}$	Spring stiffness of trunnion in vertical direction	4.00E+06	N/m
	$C_{gun}$	Damping coefficients of linear motion	44,300	Ns/m
	$K_{turret,h}$	Spring stiffness in the horizontal direction of turret	5.00E+07	N/m
	$K_{gun,h}$	Spring stiffness of trunnion in horizontal direction	5.00E+07	N/m
Powertrain	$Tq_{max}$	Maximum torque	950	Nm
	$GR$	Motor gear ratio	10	-
	$C_{bat}$	Rated battery capacity	62.3	Ah
	$RPM_{gen}$	Engine reference RPM for generation	2000	rpm
Suspension	$K_f$	Stiffness coefficient of front suspension	300000	N/m
	$K_m$	Stiffness coefficient of middle suspension	300000	N/m
	$K_r$	Stiffness coefficient of rear suspension	300000	N/m
	$C_{sus}$	Damping coefficient of suspension	30000	Ns/m
	$M_{sus,body}$	Mass of suspension (actuator)	170	kg
Tire	$R_{tire}$	Radius of tire	0.5	m
	$K_{tire}$	Stiffness coefficient of tire	800000	N/m
	$C_{tire}$	Damping coefficient of tire	1600	Ns/m
Spring	$d$	Wire diameter	0.03	m
	$D$	Mean diameter	0.16	m
	$N_a$	Number of turns	6.592	-

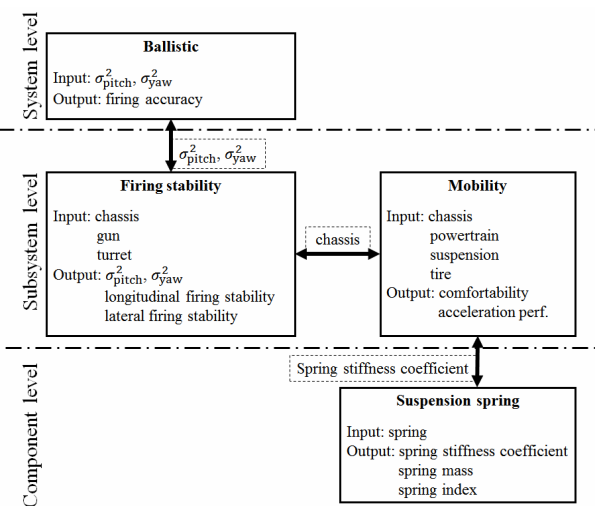


Fig. 3. Hierarchical structure of combat vehicle.

ballistic model and subsystem level consists of firing stability model and mobility model. The very bottom level, component level consists of suspension spring model. Linking variables between models are described in Fig. 3. Variance of pitch and yaw angle, chassis variable and suspension spring stiffness coefficient are linking variables between models. Based on the hierarchical structure of combat vehicle, RTC of combat vehicle is performed.

### 3. Robust target cascading of combat vehicle

#### 3.1 Target cascading formulation using quadratic penalty function

Most MDO techniques need total system integration process before performing optimization. However, in reality, because design of each subsystems and components are carried out independently, total system integration process requires

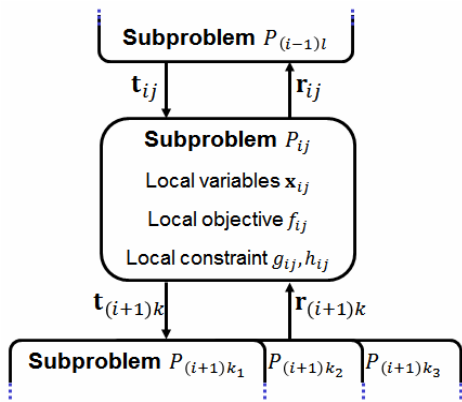


Fig. 4. Diagram of subproblem in target cascading.

great cost and time. TC is developed to save cost and time consumed in this process. TC optimizes total system with optimization of small subproblems.

Diagram of TC is shown in Fig. 4. For each entities of system level, subsystem level and component level, subproblems exist. Linking variables or targets exist between subproblems to consider coupling relation between subproblems. Thus same linking variable acts as variable in the one subproblem, while acting as response in the other subproblem. To eliminate differences from duality, penalty function is added in objective function of each subproblems. In this research, TC using quadratic penalty function is used and its general subproblem is given as the follows [14]:

$$\begin{aligned}
 \min_{\bar{\mathbf{x}}_{ij}} \quad & f_{ij}(\bar{\mathbf{x}}_{ij}) + \pi(\mathbf{c}_{ij}) + \sum_{k \in \mathcal{L}_{ij}} \pi(\mathbf{c}_{(i+1)k}) \\
 & = f_{ij}(\bar{\mathbf{x}}_{ij}) + \|\mathbf{w}_{ij}^T \mathbf{c}_{ij}\|_2^2 + \sum_{k \in \mathcal{L}_{ij}} \|\mathbf{w}_{(i+1)k}^T \mathbf{c}_{(i+1)k}\|_2^2 \\
 \text{s.t.} \quad & \mathbf{g}_{ij}(\bar{\mathbf{x}}_{ij}) \leq 0 \\
 & \mathbf{h}_{ij}(\bar{\mathbf{x}}_{ij}) = 0 \\
 & \bar{\mathbf{x}}_{ij} = [\mathbf{x}_{ij}, \mathbf{r}_{ij}, \mathbf{t}_{(i+1)k_1}, \dots, \mathbf{t}_{(i+1)k_n}] \\
 & \mathbf{c}_{ij} = (\mathbf{t}_{ij} - \mathbf{r}_{ij})
 \end{aligned} \tag{10}$$

where  $f_{ij}$ ,  $\mathbf{g}_{ij}$  and  $\mathbf{h}_{ij}$  are the original objective function and constraints of subproblem or component. Objective function is composed of original objective function and quadratic penalty function,  $\pi(\mathbf{c}_{ij})$ . Inconsistency between target linking variable,  $\mathbf{t}_{ij}$ , and response linking variable,  $\mathbf{r}_{ij}$ , is represented as  $\mathbf{c}_{ij}$ . In this formulation, these variables represent nominal values and stochastic properties of linking variables cannot be propagated between levels. Therefore, to evaluate stochastic property of system level performance, like robustness, modification of formulation is necessary. Penalty weight  $w$  for the inconsistency is multiplied to improve convergence and it is updated with every iteration. Penalty weight update scheme is calculated as follows:

$$w^{(\kappa+1)} = \beta w^{(\kappa)} \tag{11}$$

where  $\beta$  represents update parameter. The update parameter should be larger than 1 and generally use  $2 < \beta < 3$ . Convergence of TC is determined by difference of inconsistencies between previous iteration and current iteration. Mathematical expression of convergence criterion is as follows:

$$\|c^{(\kappa)} - c^{(\kappa+1)}\|_{\infty} \leq \tau \tag{12}$$

where  $\kappa$  represents iteration number. Tolerance of convergence is  $\tau$  and it is determined by designer.

### 3.2 Analysis of variance and surrogate modeling

Before performing RTC of combat vehicle, ANOVA and surrogate modeling were performed to improve efficiency of optimization process.

#### 3.2.1 Analysis of variance

Current combat vehicle model has a number of design variables. In particular, the firing stability model has 15 design variables, and mobility model has 18 design variables. Employing large number of design variables in optimization process makes convergence of optimum design worse and increases cost and time for optimization. In this research, ANOVA is employed with respect to firing stability model and mobility model to select significant design variables among the initial design variables [15].

In firing stability model, ANOVA was performed with respect to 4 responses: variances of pitch and yaw angles, longitudinal firing stability and lateral firing stability. In mobility model, 2 responses are considered: Comfortability and acceleration performance. Design of experiment for ANOVA is performed using 2-level orthogonal array. The result of ANOVA is listed in Tables 4 and 5. Values in tables represent the p-value of each design variables for each response. In this research, design variables with p-value lower than 0.01 are screened as significant design variables. For the chassis design variables, which are used in both firing stability model and mobility model, the intersection of the ANOVA results of the two models were selected to minimize number of linking variables. As a result, design variables of each models are screened as table through ANOVA. Significant variables which used in optimization are described in bold face in Tables 4 and 5. In firing stability model, 3 significant design variables are selected among 15 initial design variables. Likewise, 18 design variables are screened into 5 design variables in mobility model.

#### 3.2.2 Surrogate modeling

Exterior ballistic model in ballistic model and dynamic model in firing stability model and mobility model consist of differential equations. Therefore, these models take dozens of seconds for single analysis and employing these high fidelity models in optimization process increases cost for optimization.

Table 4. ANOVA result of firing stability model.

Group	Design variable	p-value			
		$\sigma^2_{pitch}$	$\sigma^2_{yaw}$	Longi.	Lat.
Chassis	$M_{body}$	<b>0.000</b>	<b>0.181</b>	<b>0.000</b>	<b>0.285</b>
	$L_F$	0.000	0.171	0.000	0.060
	$L_M$	0.000	0.181	0.000	0.125
	$L_R$	0.000	0.171	0.000	0.084
	$H$	0.032	0.181	0.669	0.440
	$W$	0.741	0.171	0.963	0.001
Gun	$mp$	<b>0.000</b>	<b>0.946</b>	<b>0.860</b>	<b>0.377</b>
	$mc$	<b>0.000</b>	<b>0.944</b>	<b>0.829</b>	<b>0.281</b>
	$D_{gun}$	0.901	0.946	0.992	0.979
Turret	$K_{turret}$	0.849	0.944	0.984	0.980
	$C_{turret}$	0.835	0.946	0.986	0.969
	$K_{gun}$	0.814	0.944	0.985	0.966
	$C_{gun}$	0.758	0.946	0.989	0.972
	$K_{turret,h}$	0.817	0.944	0.992	0.969
	$K_{gun,h}$	0.802	0.171	0.890	0.226

Table 5. ANOVA result of mobility model.

Group	Design variable	p-value	
		Comfortability	Acc. Perf.
Chassis	$M_{body}$	<b>0.000</b>	<b>0.000</b>
	$L_F$	0.132	0.448
	$L_M$	0.473	0.707
	$L_R$	0.432	0.878
	$H$	0.494	0.483
	$W$	0.338	0.810
Powertrain	$Tq_{max}$	<b>0.105</b>	<b>0.000</b>
	$GR$	<b>0.447</b>	<b>0.000</b>
	$C_{bat}$	0.383	0.950
	$RPM_{gen}$	0.966	0.990
Suspension	$K_f$	<b>0.003</b>	<b>0.587</b>
	$K_m$	0.142	0.480
	$K_r$	0.283	0.874
	$C_{sus}$	0.257	0.876
	$M_{sus,body}$	0.020	0.030
Tire	$R_{tire}$	<b>0.086</b>	<b>0.000</b>
	$K_{tire}$	0.260	0.569
	$C_{tire}$	0.897	0.966

In this research, surrogate modeling is employed to enhance efficiency of optimization process. Surrogate model is a mathematical model which predicts response model based on sample points. Efficiency of optimization can be enhanced by replacing high fidelity model by surrogate model.

To increase prediction accuracy of surrogate model, Optimal Latin hypercube design (OLHD) is performed to deter-

Table 6. Number of samples and error of kriging model.

	# of d.v.	# of samples			Response	RMSE
		OLHD	MDD	Total		
Ballistic	2	6	12	18	Firing accuracy	0.077
Firing stability	3	10	20	30	$\sigma^2_{pitch}$	9.8e-5
					$\sigma^2_{yaw}$	0.033
					Longi.	5.1e-4
					Lat.	1.7e-4
Mobility	5	21	42	63	Comfort.	3.0e-3
					Acc. perf.	0.083

mine initial sample points and Maximin distance design (MDD) is performed sequentially [16, 17]. The kriging surrogate model, which is suitable for computational experiment, is employed as surrogate model [18, 19]. As a result, kriging surrogate model of 7 responses are constructed. The number of sample points and normalized RMSE are listed in Table 6.

### 3.3 Robust target cascading formulation of combat vehicle

Based on hierarchical structure of combat vehicle and ANOVA result, optimization formulation for RTC is formulated. Unlike other TC formulations, statistical information, the variance of firing angle, is employed as linking variable to evaluate robustness of performance in system level. By employing variance, statistical properties can be propagated between system and subsystem level. The objective of optimization is maximizing firing accuracy. In constraint, all other performances except firing accuracy are set to have better performance than current design.

Subproblem of ballistic model is formulated as follows:

$$\begin{aligned} \max_{\bar{x}_{11}} \quad & f_{accuracy}(\bar{x}_{11}) - \pi(\bar{x}_{11}) \\ \bar{x}_{11} = & [\sigma^2_{pitch}, \sigma^2_{yaw}] \end{aligned} \tag{13}$$

where objective function consists of firing accuracy and inconsistencies of linking variables, i.e., variance of firing angle.

Subproblem of firing stability model is formulated as follows:

$$\begin{aligned} \min_{\bar{x}_{22}} \quad & \pi(\bar{x}_{22}) \\ \text{s.t.} \quad & g_{longi.}(\bar{x}_{22}) \leq 0 \\ & g_{lat.}(\bar{x}_{22}) \leq 0 \\ \bar{x}_{22} = & [\sigma^2_{pitch}, \sigma^2_{yaw}, M^U_{body}, mp, mc] \end{aligned} \tag{14}$$

where longitudinal firing stability and lateral firing stability are configured in constraints. Local design variables of firing stability model are represented as  $mp$  and  $mc$ .

Subproblem of mobility model is as follows:

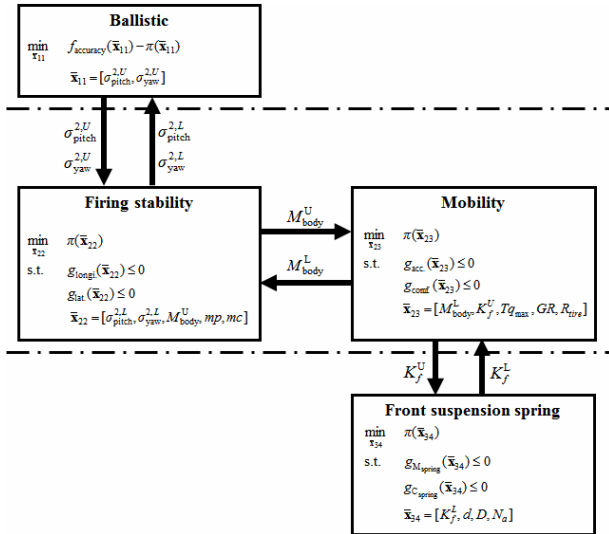


Fig. 5. Formulation for robust target cascading.

$$\begin{aligned}
 & \min_{\bar{x}_{23}} \pi(\bar{x}_{23}) \\
 & \text{s.t.} \quad g_{acc}(\bar{x}_{23}) \leq 0 \\
 & \quad \quad g_{comf}(\bar{x}_{23}) \leq 0 \\
 & \quad \quad \bar{x}_{23} = [M_{body}^L, K_f^U, Tq_{max}, GR, R_{tire}] .
 \end{aligned} \tag{15}$$

Note that acceleration performance and comfortability are listed in constraints.

Lastly formulation for suspension spring model is configured as follows:

$$\begin{aligned}
 & \min_{\bar{x}_{34}} \pi(\bar{x}_{34}) \\
 & \text{s.t.} \quad g_{M_{spring}}(\bar{x}_{34}) \leq 0 \\
 & \quad \quad g_{C_{spring}}(\bar{x}_{34}) \leq 0 \\
 & \quad \quad \bar{x}_{34} = [K_f^L, d, D, N_a] .
 \end{aligned} \tag{16}$$

Mass of spring and spring index are considered as constraints. Fig. 5 shows 4 optimization formulations of each models and linking variables between models. To perform RTC evaluation of robustness based on statistical information should be shared between models [20, 21]. In this research, statistical moment, variance of firing angle is used as linking variable. Additionally, optimization formulation for All-at-once (AAO), one of the MDO technique, is performed as following equation to compare optimization result with RTC:

$$\begin{aligned}
 & \max_{\bar{x}} f_{accuracy}(\bar{x}) \\
 & \text{s.t.} \quad g_{longi}(\bar{x}) \leq 0 \\
 & \quad \quad g_{lat}(\bar{x}) \leq 0 \\
 & \quad \quad g_{acc}(\bar{x}) \leq 0 \\
 & \quad \quad g_{comf}(\bar{x}) \leq 0 \\
 & \quad \quad g_{M_{spring}}(\bar{x}) \leq 0 \\
 & \quad \quad g_{C_{spring}}(\bar{x}) \leq 0 \\
 & \quad \quad \bar{x} = [M_{body}, mp, mc, Tq_{max}, GR, R_{tire}, d, D, N_a] .
 \end{aligned} \tag{17}$$

Table 7. Convergence of linking variables.

Linking variables	Current	RTC	
Ballistic	$\sigma_{pitch}^{2,U}$	5.00e-4	1.84e-4
	$\sigma_{yaw}^{2,U}$	5.00e-8	5.95e-8
Firing stability	$\sigma_{pitch}^{2,L}$	5.50e-4	1.84e-4
	$\sigma_{yaw}^{2,L}$	5.50e-8	5.95e-8
	$M_{body}^U$	7000	7700
Mobility	$M_{body}^L$	7000	7700
	$K_f^U$	300000	300120
Suspension spring	$K_f^L$	300000	300120

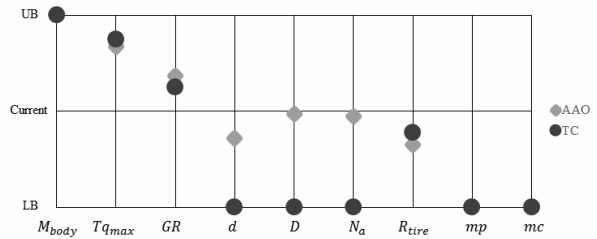


Fig. 6. Optimum points of AAO and RTC.

Objective function and constraints of above AAO formulation are identical to RTC formulation.

### 3.4 Results

RTC problem of combat vehicle is solved based on formulation in Sec. 3.3. Penalty weight update parameter is configured as  $\beta = 2.5$  and convergence tolerance is configured as  $\tau = 0.001$ .

Table 7 shows change of linking variables after RTC. In current design, linking variables,  $\sigma_{pitch}^2$  and  $\sigma_{yaw}^2$ , have inconsistencies. After performing optimization, however, these linking variables are converged to identical values. This result shows that RTC gives optimum design with feasibility. Table 8 and Fig. 6 show performances and design variables of RTC optimum and AAO optimum. Both optimum design converged to similar design. Objective function, firing accuracy, is converged to identical value and most of constraints and design variables converged to identical value also. In optimum design, firing accuracy is increased by 8.5 % while satisfying all constraints. However, in computational time, RTC consumed 3.69 s while AAO consumed 17.2 s.

### 4. Conclusion

In this research, optimization of combat vehicle is performed. TC is employed for hierarchical consideration of total system while saving time and cost for system integration process. The TC considers the complexity of combat vehicle and independence of design teams in reality. To improve fir-



Table 8. Results of performances in RTC and AAO.

Objective func.		Current	AAO	RTC	Unit
Firing accuracy		67.8	76.3	76.3	%
Constraint	Longi.	1.15e-2	7.29e-3	7.29e-3	deg
	Lat.	3.52e-1	2.81e-1	2.81e-1	deg
	Acc. perf.	1.84	1.84	1.84	sec
	Comfort.	13.8	6.95	9.50	m/s <sup>2</sup>
	$M_{spring}$	24.0	22.5	16.1	kg
	$C_{spring}$	5.33	5.47	5.33	-
CPU time		-	17.2	3.69	s
Function call		-	1,537	1,488	times

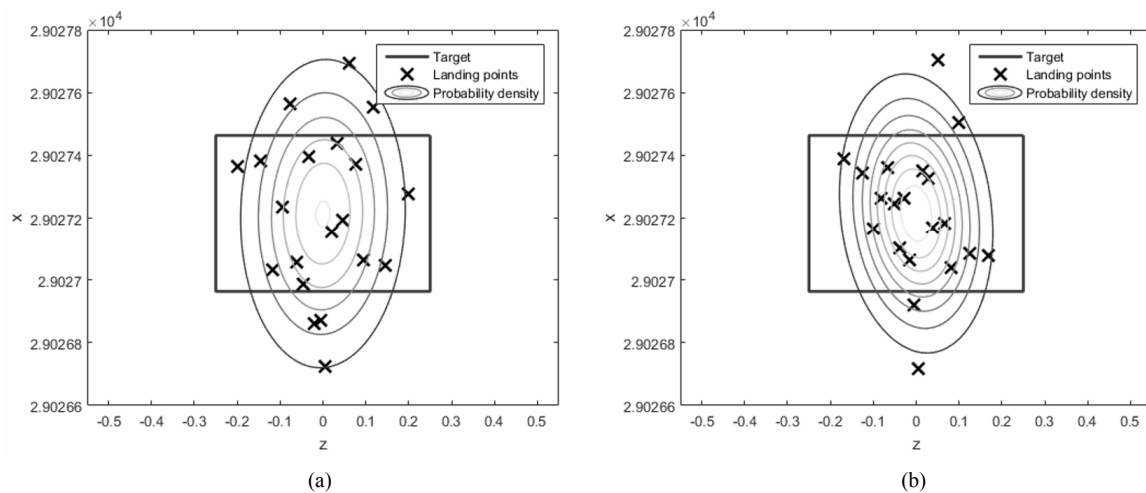


Fig. 7. Contours of PDF of landing point at (a) current design; (b) optimum design.

ing accuracy of combat vehicle, uncertainty of firing angle is considered and robust optimization concept is employed. Based on these two concepts, RTC for improving firing accuracy is carried out.

Modeling of combat vehicle was performed by benchmarking APD. 4 parts of combat vehicle are modeled using commercial software MATLAB and MapleSim. Based on input and output of 4 models, relationships between models are clarified and linking variables are defined. Each of 4 models are allocated to system, subsystem and component level to perform RTC. To evaluate robustness of objective function, statistical moment is used as linking variable in RTC.

Before performing RTC, ANOVA and surrogate modeling are employed to increase efficiency of optimization process. Through ANOVA, the number of design variables of firing stability model decreased to 3 from 15. In mobility model, 18 design variables are screened into 5. Based on ANOVA result, surrogate modeling of ballistic model, firing stability model and mobility model are performed. Sequential design of experiment using OLHD and MDD is employed to increase the accuracy of surrogate model. From these sample points, kriging surrogate models of 7 responses are constructed.

Based on hierarchical modeling of combat vehicle, ANOVA and surrogate modeling, RTC of combat vehicle is performed. Statistical information of firing angle is employed as linking variable to perform TC and robust optimization together. Statistical properties between system and subsystem level are propagated through linking variables and robustness of firing accuracy is evaluated in system level. Optimization formulations of subproblems are defined using quadratic penalty function. In addition, formulation of AAO is performed to verify the result of RTC.

The results show that optimum design of RTC has feasibility with respect to linking variables and is converged similarly to AAO. Computational time for RTC is required much shorter than that for AAO. It is noted that RTC is efficient for not only system integration process but also optimization process. Both result gives design with 8.5 % increased firing accuracy and feasibility of constraints. Fig. 7 shows change of PDF of landing point of shot. Scatter of landing points of optimum design is more concentrated on the center and corresponding PDF is more narrowed into target. The figure shows that robustness of landing points is increased and firing accuracy is improved.

## Acknowledgment

This work supported by a research program (The Specialized Research Center on the Future Ground System) funded by the Agency for Defense Development of Korea and we also appreciate Maplesoft for technical support.

## References

- [1] H. M. Kim, N. F. Michelena, P. Y. Papalambros and T. Ji-ang, Target cascading in optimal system design, *Journal of Mechanical Design*, 125 (3) (2003) 474-480.
- [2] N. Kang, M. Kokkolaras, P. Y. Papalambros, S. Yoo, W. Na, J. Park and D. Featherman, Optimal design of commercial vehicle systems using analytical target cascading, *Structural and Multidisciplinary Optimization*, 50 (2014) 1103-1114.
- [3] H. M. Kim, D. G. Rideout, P. Y. Papalambros and J. L. Stein, Analytical target cascading in automotive vehicle design, *Journal of Mechanical Design*, 125 (2003) 481-489.
- [4] M. Kokkolaras, L. S. Louca, G. J. Delagrammatikas, N. F. Michelena, Z. S. Filipi, P. Y. Papalambros, J. L. Stein and D. N. Assanis, Simulation-based optimal design of heavy trucks by model-based decomposition: An extensive analytical target cascading case study, *International Journal of Heavy Vehicle Systems*, 11 (3) (2004) 403-433.
- [5] J. Allison, D. Walsh, M. Kokkolaras, P. Y. Papalambros and M. Cartmell, Analytical target cascading in aircraft design, *44<sup>th</sup> AIAA Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, USA (2006).
- [6] R. Choudhary, A. Malkawi and P. Y. Papalambros, Analytic target cascading in simulation-based building design, *Automation in Construction*, 14 (2005) 551-568.
- [7] S. H. Yoo, D. Y. Chung, M. Oh, N. Shin and S. H. Nam, Analysis of the estimation of the deflection and hit probability of a gun barrel of next infantry fighting vehicle, *Journal of the Korea Institute of Military Science and Technology*, 9 (3) (2006) 12-9.
- [8] W. J. Yeo, J. H. Lee and E. J. Choi, A study on variation of an accuracy rate as the gradient of a pistol, *Journal of the Korea Institute of Military Science and Technology*, 14 (2) (2011) 167-72.
- [9] T. L. DePhillips, *The Effect of Target Location Uncertainty upon Weapon System Evaluation*, Naval Postgraduate School, Monterey, California (1973).
- [10] D. E. Carlucci and S. S. Jacobson, *Ballistics: Theory and Design of Guns and Ammunition*, Second Ed., CRC Press, Florida, USA (2013).
- [11] R. L. McCoy, *Modern Exterior Ballistics: The Launch and Flight Dynamics of Symmetric Projectiles*, Second Ed., Schiffer, Pennsylvania, USA (2004).
- [12] N. Metropolis and S. Ulam, The Monte Carlo method, *Journal of the American Statistical Association*, 44 (247) (1949) 335-341.
- [13] B. Patnaik, American method and comparison of the different methods of internal ballistics, *Defence Science Journal*, 2 (1) (1952) 1-7.
- [14] S. Tosserams, L. F. P. Etman, P. Y. Papalambros and J. E. Rooda, An augmented Lagrangian relaxation for analytical target cascading using the alternating direction method of multipliers, *Structural and Multidisciplinary Optimization*, 31 (2006) 176-189.
- [15] D. C. Montgomery, *Design and analysis of experiments*, Fifth Ed., Wiley, New York, USA (2001).
- [16] J. S. Park, Optimal Latin-hypercube designs for computer experiments, *Journal of statistical Planning and Inference*, 39 (1) (1994) 95-111.
- [17] M. E. Johnson, L. M. Moore and D. Ylvisaker, Minimax and maximin distance designs, *Journal of Statistical Planning and Inference*, 26 (1990) 131-148.
- [18] J. Sacks, W. J. Welch, T. J. Mitchell and H. P. Wynn, Design and analysis of computer experiments, *Journal of the Institute of Mathematical Statistics*, 4 (4) (1989) 409-423.
- [19] H. Kwon, S. Yi and S. Choi, Numerical investigation for erratic behavior of Kriging surrogate model, *J. of Mechanical Science and Technology*, 28 (9) (2014) 3697-3707.
- [20] W. Chen, P. Y. Papalambros and H. M. Kim, Probabilistic analytical target cascading: a moment matching formulation for multilevel optimization under uncertainty, *Journal of Mechanical Design*, 128 (2006) 991-1000.
- [21] D. O. Kang, S. J. Heo, M. S. Kim, W. C. Choi and I. H. Kim, Robust design optimization of suspension system by using target cascading method, *International Journal of Automotive Technology*, 13 (1) (2011) 109-122.



disciplinary design optimization.



Tae Hee Lee is a Professor at the Department of Automotive Engineering, Hanyang University, Seoul, Korea, and serves currently as a President of Korean Society of Design Optimization and Executive Committee members of Asian Society of Structural and Multidisciplinary Optimization (ASSMO). He received Ph.D. degree at the University of Iowa in 1991. He received an award for excellence in academic achievement in 2013 from Korean Society for Mechanical Engineers. He was Plenary Lecturer of CJK-OSM8 in 2004 and WCCM in 2016. His research interests include design optimization, design and analysis of computer experiments, design under uncertainty, and surrogate model based optimization.