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MHD copper-water nanofluid flow and heat transfer through convergent-divergent channel †

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Abstract

This work is focused on the analytical solution of a nanofluid consisting of pure water with copper nanoparticle steady flow through convergent-divergent channel. The velocity and temperature distributions are determined by a novel method called Reconstruction of variational iteration method (RVIM). The effects of angle of the channel, Reynolds and Hartmann numbers on the nanofluid flow are then investigated. The influences of solid volume fraction and Eckert number upon the temperature distribution are discussed. Based on the achieved results, Nusselt number enhances with increment of solid volume fraction of nanoparticles, Reynolds and Eckert numbers. Also the fourth order Runge-Kutta method, which is one of the most relevant numerical techniques, is used to investigate the validity and accuracy of RVIM and good agreement is observed between the solutions obtained from RVIM and some known numerical results.

Keywords: Convergent-divergent channel; Nanofluid flow; MHD; Heat transfer enhancement; Analytical approximate solution; RVIM

1. Introduction

The study of Magneto-hydrodynamics (MHD) focuses on the interactions between the dynamics of electrically conducting fluids and the electromagnetic forces. Alfvén initiated the research field of magneto-hydrodynamics and received the Nobel Prize of physics in 1970 [1]. However, incompressible fluid magneto-hydrodynamics was initiated in 1937 by Hartmann [2] who studied the effect of a lateral magnetic field, which was considered to be external and uniform, upon the flow field of a viscous incompressible electrically conducting fluid between infinite parallel fixed plates (with the assumption of insulating properties).

One of the most important fields in power industries is fluid heating, which has also crucial role in manufacturing plants. There is an increasing demand for finding novel and effective cooling techniques to improve the efficiency of the cooling processes related to various types of high energy devices. In most cases, for this purpose, common fluids such as water, engine oil or ethylene glycol are used. But, because of many limitations in heat transfer capabilities of these fluids, it is better to find some alternative ways. Thermal conductivity values of most of metals are very high, and can be estimated to be around three-fold higher than the aforementioned fluids. Consequently, it might be desirable to use a uniform combination of metal and fluid to gain the benefits of both matters. Thus, by this technique one could achieve a proper heat transfer medium which could behave like a fluid and, at the same time, could have a high thermal conductivity (the same as metals).

Recently, a remarkable number of researches have been done in the field of nanofluids and their vast applications in industries have been studied. In addition, it has been demonstrated that they have certain superiorities (as a heat transfer agent) in comparison with common conventional fluids like water or engine oil [3-6].

An investigation on the flow field and heat transfer specifications of nanofluid between two horizontal parallel plates in a rotating system has been done [7]. As reported [7], for suction and injection, the rate of heat transfer at the surface would enhance by increment of the solid nanoparticle volume fraction, Reynolds number, and the parameters related to the injection/suction process. However, it was demonstrated that [7] the rate of heat transfer at the surface would reduce with increment in the power of rotation parameter, which represents the effect of rotation of system on the physics of the problem.

Azimi et al. [8] studied the effect of graphene oxide nanoparticle solid volume fraction, the moving parameter and the Eckert number on heat transfer characteristics of nanofluid flow between two moving parallel plates. In Azimi et al. [9], magneto-hydrodynamics Jeffery-Hamel flow of nanofluid containing graphene oxide nanoparticles for various Hartman number has been investigated. They concluded that the di-

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mensionless velocity for water-GO nanofluid is an increasing function of Re, in a divergent channel.

Heat transfer enhancement is an important achievement in heat transfer systems like heat exchangers because more heat transfer enhancement would lead to more heat fluxes and easier cooling processes [10].

Lihong and Hangming [11] investigated the distribution of thin copper wire in a chamber cross section (assuming the chamber to be isothermal) by employing topological techniques to achieve heat transfer enhancement from the wall to the chamber interior and to promote the isothermal specifications of the chamber in their study.

Moslehi and Saghafian [12] studied flow of a Newtonian electrically conducting fluid in laminar condition, in a vertical parallel micro-channel, considering the direct influence of an external uniform magnetic field as well as mixed convection heat transfer using a well known numerical method. Azimi and Riazi [13] used Galerkin optimal homotopy asymptotic method (GOHAM) to achieve an approximate solution of the unsteady magneto-hydrodynamics squeezing flow between two parallel disks. Their investigation depicted that the average Nusselt number can be increased by using higher concentration of graphene-oxide particles in base fluid as the nanoparticle between plates. Their results show that the same manner can be obtained by increasing Rayleigh number, while their calculations show that Nusselt number would be decreased if higher magnetic field would be used. So, the average Nusselt number has a reverse relation with the Hartmann number. Hamad et al. [14] investigated analytically the steady free convection boundary layer flow over a vertical semi-infinite flat plate embedded within water (containing nanoparticles) considering the effects of magnetic field. Their results show that more heat transfer enhancement can be achieved by using copper and silver nanoparticles in base fluid. The CVFEM (Control volume based finite element method) was employed as a novel numerical technique to study the MHD natural convection heat transfer of copper water nanofluid within a coaxial cold outer circular case together with a hot inner sinusoidal circular cylinder by Sheikholeslami et al. [15]. As reported, with negligible value of Hartman number, increment of Rayleigh number could lead to reduction of the enhancement ratio; however, in presence of a horizontal magnetic field, increase of the Rayleigh number could lead to increment of the enhancement ratio (the Jeffery-Hamel problem).

Recently, a considerable number of researches have been performed regarding the possibility of using analytical approaches to achieve proper solutions for high-order nonlinear differential equations. Some of these analytical methods are Homotopy perturbation method (HPM) [18], Reconstruction of variational iteration method (RVIM) [19], Differential transformation method (DTM) [20] and others [21-23].

Our purpose was to gain the analytic approximate solutions of the two-dimensional magneto-hydrodynamics copper water nanofluid flow between converging/diverging channels using RVIM and investigating the effect of different parameters upon both the flow and heat transfer characteristics.

2. Problem description

To investigate the problem of Jeffery-Hamel magneto-hydrodynamics flow analytically, a 2-D flow field of an incompressible, electrically conducting viscous fluid, considering the influence of an external transverse homogeneous magnetic field (see Fig. 1) was studied in the current research. As indicated in Fig. 1, the steady 2-D flow of an incompressible, electrically conducting viscous fluid with respect to a sink or source located at the intersection of two non-parallel plane walls is considered. We assumed that the velocity would be purely radial and a function of r and θ only. To control the flow field, we studied the influence of magnetic field on dynamic of flow. The generated electric field is assumed to be negligible, which may lead to the reduction of Ohm's law to $J=\sigma\left(V\times B\right)$, where V is the velocity vector. As a consequence, the Lorentz force would be simplified to $F_B=\left(\sigma\left(V\times B\right)\right)\times B$. Based on these assumptions, the mass, momentum and energy relations in polar coordinates could be written as:

$$\frac{\rho_{nf}}{r}\frac{\partial\left(ru\left(r,\theta\right)\right)}{\partial r}=0 \tag{1}$$

$$u\left(r,\theta\right)\frac{\partial u\left(r,\theta\right)}{\partial r}=-\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r} +v_{nf}\left(\frac{\partial^2 u\left(r,\theta\right)}{\partial r^2}+\frac{1}{r}\frac{\partial u\left(r,\theta\right)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u\left(r,\theta\right)}{\partial\theta^2}-\frac{u\left(r,\theta\right)}{r^2}\right) -\frac{\sigma B_0^2}{\rho_{nf}r^2}u\left(r,\theta\right) \tag{2}$$

$$-\frac{1}{\rho_{nf}r}\frac{\partial P}{\partial\theta}+\frac{2\mu_{nf}}{\rho_{nf}r^2}\frac{\partial u\left(r,\theta\right)}{\partial\theta}=0 \tag{3}$$

$$u\left(r,\theta\right)\frac{\partial T\left(r,\theta\right)}{\partial r}= \alpha_{nf}\left(\frac{\partial^2 T\left(r,\theta\right)}{\partial r^2}+\frac{1}{r}\frac{\partial T\left(r,\theta\right)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T\left(r,\theta\right)}{\partial\theta^2}\right) +\frac{\mu_{nf}}{\left(\rho C_p\right)_{nf}} \left(2\left(\frac{\partial u\left(r,\theta\right)}{\partial r}\right)^2+2\left(\frac{u\left(r,\theta\right)}{r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u\left(r,\theta\right)}{\partial\theta}\right)^2\right). \tag{4}$$

Due to the symmetric geometry, the boundary conditions will be:

$$\left(At\ the\ Channel\ Centerline\right): \left(\frac{\partial u\left(r,\theta\right)}{\partial\theta}\right)=0,\ \frac{\partial T\left(r,\theta\right)}{\partial\theta}=0,\ u\left(r,\theta\right)=U_{\max} \tag{5}$$

Table 1. Thermophysical properties of water and copper nanoparticle.

Material	Copper	Water
ρ [kg/m^3]	8933	997.1
c_p [J/(kg·K)]	385	4179
k [W/(m·K)]	1.67	21.0
β [K^{-1}] $\times 10^5$	401	0.613

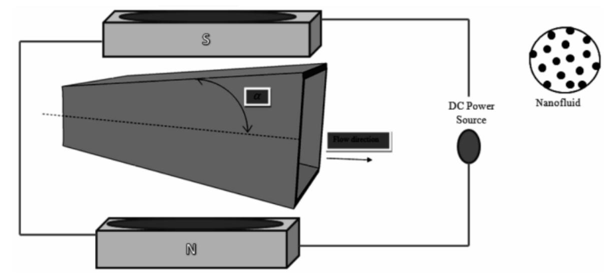

Fig. 1. Geometry of problem.

$\left(At\ \ the\ \ Plates \right)$:

$T = T_w,\ \ u\left(r,\theta\right) = 0$.

Here B_0 is the electromagnetic induction, $u(r)$ is the velocity along radial direction, P is the fluid pressure, σ is the conductivity of the fluid, ρ_{nf} is the density of fluid and v_{nf} is the coefficient of kinematic viscosity. The physical properties of Cu-water nanofluid are provided in Table 1.

By assuming ϕ as the solid volume fraction, the density of fluid, dynamic viscosity, the fluid kinematic viscosity, thermal diffusivity and thermal conductivity of nanofluid could be considered as the following relations [13]:

$$\begin{cases} \rho_{nf} = \rho_f\left(1-\phi\right) + \rho_s\phi \\ \mu_{nf} = \dfrac{\mu_f}{\left(1-\phi\right)^{2.5}} \\ v_{nf} = \dfrac{\mu_f}{\rho_{nf}} \\ \alpha_{nf} = \dfrac{k_{nf}}{\left(\rho C_p\right)_{nf}} \\ \dfrac{k_{nf}}{k_f} = \dfrac{\left(k_s + 2k_f\right) - 2\phi\left(k_s - k_f\right)}{\left(k_s + 2k_f\right) + \phi\left(k_s - k_f\right)}. \end{cases} \tag{6}$$

From Eq. (2) we can easily obtain: $F(\theta) = ru(r,\theta)$. Using $\eta = \dfrac{\theta}{\alpha}$ as the dimensionless degree, the dimensionless form of the velocity parameter can be obtained by dividing to its maximum values as $F(\eta) = \dfrac{F(\theta)}{F_{\max}}$ where $F_{\max} = rU_{\max}$. Introducing $\zeta = \dfrac{T}{T_w}$ as the dimensionless form of temperature, and substituting these dimensionless parameters into Eqs. (1)-(5) and eliminating the pressure term implies the following nonlinear third-order boundary value problems:

$$\begin{aligned} &F''' + 2\alpha\,\mathrm{Re}\left[\left(\left(1-\varphi\right) + \frac{\rho_s}{\rho_f}\varphi\right)\left(1-\varphi\right)^{2.5}\right]FF' \\ &+\left(4 - \left(1-\varphi\right)^{1.25}H\right)\alpha^2 F' = 0 \end{aligned} \tag{7}$$

$$\frac{\left[\dfrac{k_{nf}}{k_f}\zeta'' + \dfrac{\Pr Ec}{\left(1-\phi\right)^{2.5}}\left(4\alpha^2F^2 + F'^2\right)\right]}{\left[1 - \phi + \phi\dfrac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]} = 0 \tag{8}$$

where $\Pr = \dfrac{\mu_f\left(C_p\right)_f}{k_f}$ is the Prandtl number, $\mathrm{Re} = \dfrac{\alpha U_{\max}}{\nu}$ is Reynolds number, $Ec = \dfrac{U^2}{\left(C_p\right)_f T_w}$ is Eckert number and $H = \sqrt{\dfrac{\sigma B_0^2}{\rho\nu}}$ is the Hartmann number. With the following boundary conditions:

$$\begin{aligned} &F\left(0\right) = 1,\ \ F\left(1\right) = 0,\ \ F'\left(0\right) = 0, \\ &\zeta\left(1\right) = 1,\ \ \zeta'\left(0\right) = 0. \end{aligned} \tag{9}$$

Note that in the case of $\alpha = 0$, the flow acts very much like that of a plane Poiseuille flow between two parallel plates. The skin friction coefficient, local Nusselt number, heat transfer rate and shear stress can be defined as:

$$\begin{aligned} &c_f = \frac{\tau_w}{\rho_f U^2},\ \ Nu = \frac{r\,q_w|_{\theta=\alpha}}{k_f T_w}, \\ &q_w = -k_{nf}\nabla T,\ \ \tau_w = \mu_{nf}\left(\frac{1}{r}\left(\frac{\partial u\left(r,\theta\right)}{\partial\theta}\right)\right). \end{aligned} \tag{10}$$

Substitution of Eq. (10) into Eqs. (8) and (9), gives:

$$c_f = \frac{1}{\mathrm{Re}\left(1-\phi\right)^{2.5}}F'\left(1\right),\ \ Nu = -\frac{1}{\alpha}\frac{k_{nf}}{k_f}\zeta'\left(1\right). \tag{11}$$

3. Solution procedure

To recognize the best value of the Lagrange multiplier, the procedure of Laplace transform can be employed.

Consider x, t as two independent variables, where *x, t* are the principal and secondary variables, respectively. Assuming $w(x,t)$ as a function of two variables (i.e., x and t), the

Laplace transform can be defined as:

$$L\left(w(x,t)\ ;s \right) = \int_0^\infty e^{-st} w(x,t)\,dt. \tag{12}$$

We can use some preliminary information as follows:

$$L\left(\frac{\partial w}{\partial t};s \right) = \int_0^\infty e^{-st} \frac{\partial w}{\partial t} dt = sW(x,s) - w(x,0) \tag{13}$$

$$L\left(\frac{\partial^2 w}{\partial t^2};s \right) = s^2 W(x,s) - sW(x,s) - w_t(x,0) \tag{14}$$

$$W(x,s) = L\left(w(x,t)\ ;s \right). \tag{15}$$

There are certain functions that could not be considered as the transform of some known functions but, at the same time, they could possibly be assumed as a product of two functions. Suppose the given functions as $W(x,s)$, $Z(x,s)$, where $W(s)$ and $Z(s)$ are considered as the transform of the functions $w(x,t)$, $z(x,t)$, respectively. $W(x,s)$, $Z(x,s)$ are the Laplace Transform of $\int_0^t w(x,t-\varepsilon) z(x,\varepsilon) d\varepsilon$:

$$L^{-1}\left(W(x,s), Z(x,s) \right) = \int_0^t w(x,t-\varepsilon) z(x,\varepsilon) d\varepsilon. \tag{16}$$

To explain the difference between classical variational iteration method and Reconstruction of variational iteration method (RVIM), we introduce the novel nonlinear or linear function as $h\left(w(x,t)\right) = q(x,t) - N\left(w(x,t)\right)$ and supposing the novel equation, write $h\left(w(x,t)\right) = q(x,t) - N\left(w(x,t)\right)$ again as:

$$L\left(w(t,x) \right) = h(t,x,w). \tag{17}$$

Using a novel idea based on Laplace transform theorem and applying the mentioned transform to the both sides of Eq. (17), we can implement the correctional function of variational iteration method; therefore, the artificial initial conditions can be fixed to zero for the current problem and the left hand side of Eq. (17), after applying Laplace transformation, might be rewritten as follows:

$$L\left[L\left\{ w(x,t) \right\} \right] = W(x,s) B(s). \tag{18}$$

Note that $B(s)$ is a polynomial and the degree of this arbitrary function is the highest order derivative of linear operator, which can be seen as follows:

$$L\left[L\left\{ w(x,t) \right\} \right] = W(x,s) B(s) = L\left[h\left\{ (x,t,w) \right\} \right] \tag{19}$$

$$W(x,s) = \frac{L\left[h\left\{ (x,t,w) \right\} \right]}{B(s)}. \tag{20}$$

Suppose that $D(s) = 1/B(s)$; we can use the convolution theorem to find the initial solution. By applying the inverse Laplace function on right side and left side of Eq. (19), we will achieve,

$$w(x,t) = w_0(x,t) + \int_0^t d(t-\varepsilon) h(x,\varepsilon,w) d\varepsilon. \tag{21}$$

Here, $w_0(x,t)$ is the initial solution and it could contain unknown parameters or not. Without unknown parameters, the initial boundary conditions must be satisfied by $w_0(x,t)$.

According to above-mentioned explanations of the RVIM and using initial guess as $f_0(\eta) = a\eta^2 + 1$ where a is constant, the following relation can be obtained:

$$f_{n+1}(\eta) = f_0(\eta) + \int_0^s \left(\begin{array}{l} 2\alpha \,\mathrm{Re}\left[\left((1-\varphi) + \dfrac{\rho_s}{\rho_f}\varphi \right) (1-\varphi)^{2.5} \right] ff' \\ + \left(4 - (1-\varphi)^{1.25} H \right) \alpha^2 f' \end{array} \right) \cdot \frac{(\eta - s)^2}{2} ds. \tag{22}$$

Using Eq. (22) the analytical solutions can be achieved.

4. Results and discussion

The obtained results of copper-water nano-fluid flow between converging-diverging plates considering various amounts of solid volume fraction, Eckert, Reynolds and Hartman numbers is studied in this section.

The effect of increment of Reynolds number on fluid velocity is demonstrated in Fig. 2(a), assuming a constant Hartmann number of $H = 500$ along with $\alpha = \pi/18, \phi = 0.1$. Note that the occurrence of back flow could be possible in large Reynolds numbers, as depicted in Fig. 2(a), for the case of diverging channels.

Fig. 2(b) illustrates the effect of variation of Hartmann number on velocity profile for a diverging channel. Based on the results of Fig. 2(b), one could suggest that by increment of Hartmann number the rate of momentum transport would considerably be decreased. This is because the variation of Hartmann number would lead to the change of the Lorentz force (considering the effect of magnetic field) and, as a consequence, the modified Lorentz force may result in a higher resistance to the transport of flow.

Furthermore, the parameter of nanoparticle volume fraction could play a key role on behavior of the flow field. Particularly, it has a significant effect on both the velocity and temperature fields. The effect of variation of solid volume fraction for the copper nanoparticles upon temperature profile, considering the condition of $H = 300, R = 50, Ec = 0.2, \Pr = 7, \alpha = \pi/12$ is

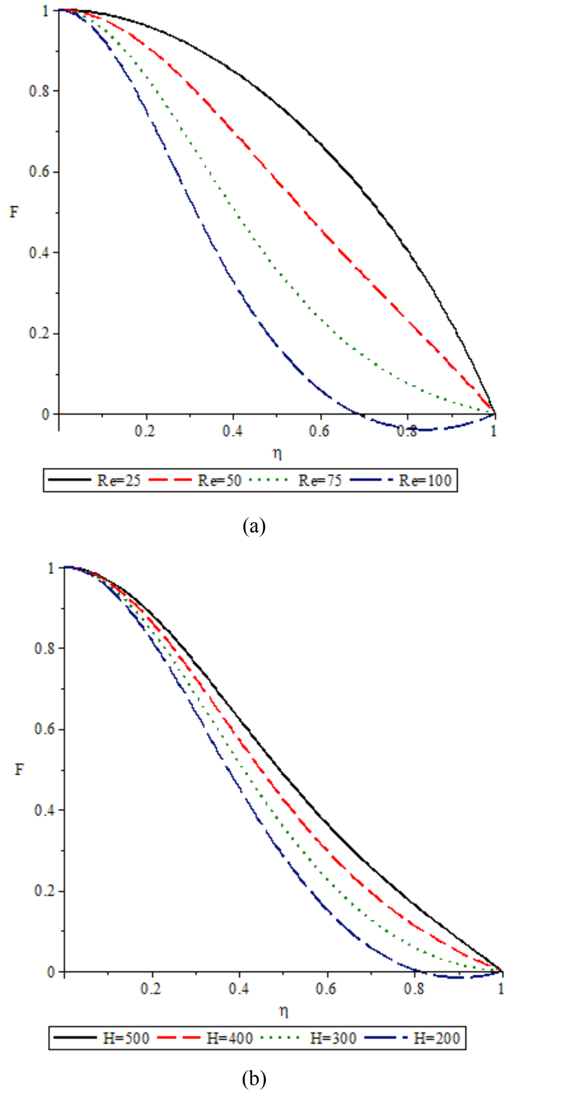

Fig. 2. Influence of Reynolds and Hartmann numbers upon the radial velocity profile for the case of $\alpha = \pi/18$ $\phi = 0.1$: (a) $H = 500$; (b) $\mathrm{Re} = 60$.

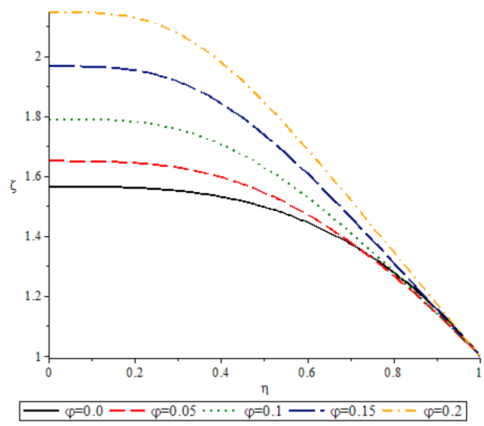

Fig. 3. Effect of variation in the volume fraction of Nanoparticle upon the temperature profile for the case of $H = 300$, $R = 50$, $Ec = 0.2$, $\mathrm{Pr} = 7$, $\alpha = \pi/12$.

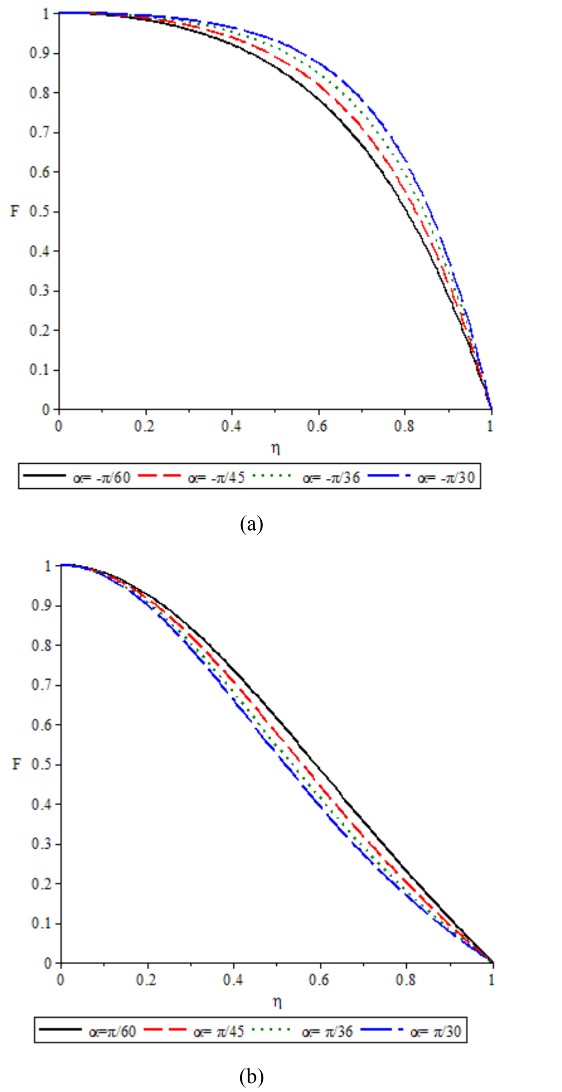

Fig. 4. Influence of change of angle between plates on Velocity profile when $H = 600$ $R = 50$, $Ec = 0.1$, $\mathrm{Pr} = 7$, $\phi = 0.15$ for (a) converging channel; (b) diverging channel.

indicated in Fig. 3.

By addition of solid volume fraction of nanofluid the thermal boundary layer thickness would reduce. Hence, the increase of Cu-nanoparticles volume fraction could result in reduction of dimensionless temperature due to increase of the heat transfer rate. Two considerable properties of nanofluids are usually noticed by the energy related communities. The first characteristic is the higher thermal conductivity of nanofluids, which can lead to increment of heat transfer. The other important property of nanofluids is their absorption characteristic. In this study, the absorption properties of Cu-Water nanofluid have been assumed to be negligible.

By keeping the Reynolds and Hartman numbers constant, the influence of channel half-angle upon velocity profile for (a) converging channel and (b) diverging channel is illustrated in Figs. 4(a) and (b), respectively. It is obvious that the dimen-

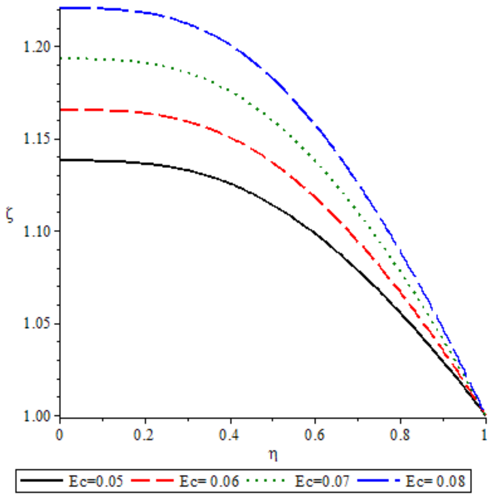

Fig. 5. Effect of change of Eckert number upon the temperature profile for the case of $H = 300$, $R = 25$, $\phi = 0.2$, $\Pr = 7$, $\alpha = \pi / 15$.

sionless velocity is a decreasing function of α, whereas this dimensionless variable (dimensionless velocity) grows when α is added in convergent channel. So, by increment of the channel half-angle (for converging channels) due to the increase of the effect of favorable pressure gradient, the fluid elements near the wall would accelerate. The influence of Eckert number variation upon dimensionless temperature profiles is indicated in Fig. 5. Based on the results (see Fig. 5), one could suggest that by increment of Eckert number, due to the related effects of frictional heating, the heat energy would be stored in the liquid and, consequently, the temperature would be increased.

The variation of Nusselt number and the skin friction coefficients with the parameters such as solid volume fraction, Eckert number, Reynolds and Hartman numbers are presented in Tables 2 and 3, respectively.

Based on the calculations discussed earlier, (Eq. (6)) and Table 2, one can thoroughly consider that generally, by addition of nanoparticles to the working fluid, the value of the Nusselt number would be increased. Moreover, the other result from Table 2 is that the variation in the magnitude of Eckert number has certain influences on the change of Nusselt number. Note that the viscous dissipation effects (related to the increase of Eckert number) would lead to the increment of fluid temperature between the two nonparallel plates. Consequently, by increase of the Eckert number, the magnitude of the Nusselt number would be enhanced.

The effects of channel half-angle, Reynolds and Hartmann numbers upon the coefficient of skin friction are given in Table 3. The results show that by addition of Reynolds number and the amount of opening angle, the coefficient of skin friction enhances. In contrast, increment of the Hartmann number would result in reduction of the coefficient of skin friction. To study the effect of concentration of nanoparticle upon the increased heat transfer performance of nanofluid, the Cu-water nanofluid with two different particle volume fractions

Table 2. Nusselt number for the case of $H = 50$ and $\alpha = 15^o$.

Solid volume fraction	Eckert number	Reynolds number	Nusselt number
0.1	0.1	50	4.9010
0.2	0.1	50	7.0706
0.1	0.2	50	9.8024
0.2	0.2	50	14.141
0.1	0.1	100	9.4166
0.2	0.1	100	13.8571
0.1	0.2	100	18.8331
0.2	0.2	100	27.713

Table 3. Skin friction coefficient in case $\phi = 0.15$.

Angle between plates	Hartmann number	Reynolds number	Skin friction coefficient
5 degree	50	25	-0.1771
10 degree	50	25	-0.2173
5 degree	100	25	-0.1802
10 degree	100	25	-0.2282
5 degree	50	50	-0.1049
10 degree	50	50	-0.1377
5 degree	100	50	-0.1063
10 degree	100	50	-0.1424

Table 4. Comparison between RVIM, DTM and NM (Ganji, Azimi, 2013) regarding the calculation of velocity for the case of $H = 250$, $\mathrm{Re} = 25$, $\alpha = \pi / 36$, $\phi = 0.0$.

η	Numerical solution	RVIM	DTM [17]	%Error (DTM)	% Error (RVIM)
0	1.0000	1.0000	1.0000	0.00	0.00
0.2	0.9547	0.9502	0.9412	1.42	0.45
0.4	0.8213	0.8083	0.7725	4.51	0.13
0.6	0.6148	0.5921	0.5744	5.53	0.22
0.8	0.3398	0.3197	0.3003	6.21	0.21
1	0.0000	0.0000	0.0000	0.00	0.00

($\phi = 0.1$, $\phi = 0.2$) are employed in the current investigation. The relationship between Eckert number and Nusselt number cannot be generally formulated. This may be due to the complications of the proposed model. Nevertheless, the present study provides a good base for further research.

The following relation introduces the percentage of error as:

$$\%Error = \left| \frac{F(\eta)_{NM} - F(\eta)_{RVIM}}{F(\eta)_{NM}} \right| \times 100 . \tag{23}$$

Table 4 provides the comparison of numerical results, considering the DTM (Ganji, and, Azimi, 2013) and RVIM solutions, regarding the calculation of velocity for different values of η . Table 4 shows that the results of reconstruction of itteration of variational method are in good agreement with those of the numerical method. This demonstrates the capability of RVIM for dealing with such problems and solving them in engineering and related fields. Based on the error analysis, RVIM can be more useful to find better solutions than DTM.

5. Conclusions

This investigation deals with the analysis of heat transfer and MHD viscous copper water nanofluid flow between two non-parallel walls for both converging/ diverging cases.

Based on the achieved results, increment of the intensity of magnetic field could lead to a considerable stabilizing effect on the growth of boundary layer for both geometries of diverging and converging channels.

The results show that the non-dimensional parameters have strong influences on the temperature profile. The main findings are summarized below:

- Increase of the solid volume fraction of nanofluid could lead to reduction of the thickness of thermal boundary layer (due to the higher heat transfer rate that is created).

- Nusselt number is enhanced by increment of Reynolds number, Ec (the Eckert number) and the amount of solid volume fraction.

- Skin friction coefficient would increase with addition of Reynolds number and the amount of opening angle. However, by increment of Hartmann number, the skin friction coefficient would be reduced.

- Comparing the analytical results with numerical calculations (which were achieved by fourth order Runge-Kutta method), one could suggest that the RVIM could be considered as a simple, powerful and efficient technique that can be employed regarding the analytical solutions of non-linear differential equations in various scientific and engineering related applications.

Nomenclature

B_0 : Magnetic field [wb/m^2]
Ec : Eckert number
$F(\eta)$: Dimensionless velocity
Ha : Hartmann number
P : Pressure term
Pr : Prandtl number
Re : Reynolds number
r, θ : Cylindrical coordinates
T : Temperature
T_w : Wall temperature
U_{max} : Maximum value of velocity
u, v : Velocity components along x, y

Greek symbols

α : Semi-Angle between two plates
η : Dimensionless angle
θ : Angle
ρ : Density
ϕ : Nanoparticle volume fraction
μ : Dynamic viscosity
ν : Kinematic viscosity
ζ : Dimensionless temperature
β : Thermal expansion coefficient

Subscript

∞ : Condition at infinity
nf : Nanofluid
f : Base fluid
S : Solid particles

References

[1] H. Alfvén, Existence of electromagnetic-hydrodynamic waves, *Nature*, 150 (1942) 405-406.

[2] J. Hartmann, Theory of the laminar flow of an electrically conducting liquid in a homogeneous magnetic field, *Matematisk-Fysiske Meddelelser Konglige Danske Videnskabernes Selskab*, 15 (1937) 1-27.

[3] P. C. Mukesh Kumar, J. Kumar, R. Tamilarasan, S. Sendhil Nathan and S. Suresh, Heat transfer enhancement and pressure drop analysis in a helically coiled tube using Al2O3 / water nanofluid, *J. of Mechanical Science and Technology*, 28 (5) (2014) 1841-1847.

[4] Y. C. Kim, Effect of surface roughness on pool boiling heat transfer in subcooled water-CuO nanofluid, *J. of Mechanical Science and Technology*, 28 (8) (2014) 3371-3376.

[5] P. C. Mukesh Kumar, K. Palanisamy, J. Kumar, R. Tamilarasan and S. Sendhilnathan, CFD analysis of heat transfer and pressure drop in helically coiled heat exchangers using Al2O3 / water nanofluid, *J. of Mechanical Science and Technology*, 29 (2) (2015) 697-705.

[6] M. Muthtamilselvan, D. Prakash and D. H. Doh, Effect of thermal non-equilibrium on transient hydromagnetic flow over a moving surface in a nanofluid saturated porous media, *J. of Mechanical Science and Technology*, 28 (9) (2014) 3709-3718.

[7] M. Sheikholeslami, H. R. Ashorynejad, G. Domairry and I. Hashim, Flow and Heat Transfer of Cu-Water Nanofluid between a Stretching Sheet and a Porous Surface in a Rotating System, *Journal of Applied Mathematics*, 35 (2012) 1-20.

[8] M. Azimi, A. Azimi and M. Mirzaei, Investigation of the unsteady Graphene oxide nanofluid flow between two moving plates, *Journal of Computational and Theoretical Nanoscience*, 11 (10) (2014) 2104-2109.

[9] M. Azimi, A. Azimi and M. Mirzaei, Analytical Investigation of MHD Jeffery Hamel Problem with Graphene Oxide

Nanoparticles using GOHAM, *Journal of Computational and Theoretical Nanoscience*, 12 (6) (2015) 991-995.
[10] S. Eiamsa-ard and W. Changcharoen, Flow structure and heat transfer in a square duct fitted with dual/quadruple twisted-tapes: Influence of tape configuration, *J. of Mechanical Science and Technology*, 29 (8) (2015) 3501-3518.
[11] Y. Lihong and S. Hangming, Study on thin copper-wire distribution for heat transfer enhancement from wall to interior of an isothermal chamber, *J. of Mechanical Science and Technology,* 29 (4) (2015) 1377-1382.
[12] M. Moslehi and M. Saghafian, MHD mixed convection slip flow in a vertical parallel plate microchannel heated at asymmetric and uniform heat flux, *J. of Mechanical Science and Technology,* 29 (12) (2015) 5317-5322.
[13] M. Azimi and R. Riazi, Heat transfer analysis of GO-water nanofluid flow between two parallel disks, *Propulsion and Power Research*, 4 (2015) 23-30.
[14] M. A. A. Hamad, I. Pop and A. I. MdIsmail, Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate, *Nonlinear Analysis: Real World Applications*, 12 (2011) 1338-1346.
[15] M. Sheikholeslami, S. Soleimani, M. Gorji-Bandpy, D. D. Ganji and S. M. Seyyedi, Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field, *International Communications in Heat and Mass Transfer*, 39 (9) (2012) 1435-1443.
[16] G. B. Jeffery, The two-dimensional steady motion of a viscous fluid, *Philosophical Magazine*, 6 (172) (1915) 455-465.
[17] G. Hamel, Spiralformige Bewgungen Zaher Flussigkeiten, *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 25 (1917) 34-60.
[18] M. Azimi and A. Azimi, Investigation on the Film Flow of a Third Grade Fluid on an Inclined Plane Using HPM, *Mechanics and Mechanical Engineering*, 18 (1) (2014) 251-256.
[19] A. Azimi and M. Azimi, Analytical Investigation on 2-D unsteady MHD Viscoelastic flow between Moving Parallel Plates Using RVIM and HPM, *Walailak J. of Science and Technology*, 11 (11) (2014) 955-963.
[20] D. D. Ganji and M. Azimi, Application of DTM on MHD Jeffery Hamel Problem with Nanoparticles, *U.P.B. Scientific Bulletin Series D*, 75 (1) (2013) 223-230.
[21] D. D. Ganji, M. Azimi and M. Mostofi, Energy Balance Method and amplitude frequency formulation based of strongly nonlinear oscillator, *Indian J. of Pure & Applied Physics*, 50 (2012) 670-675.
[22] D. D. Ganji and M. Azimi, Application of Max min Approach and Amplitude Frequency Formulation to the nonlinear oscillation systems, *The Scientific Bulletin Series A*, 3 (2012) 131-140.
[23] M. Azimi and R. Riazi, Analytical simulation of mixed convection between two parallel plates in presence of time dependent magnetic field, *Indian J. of Pure & Applied Physics,* 54 (2016) 327-332.

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