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Storage reliability estimation of one-shot systems using accelerated destructive degradation data†

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Abstract

A gamma process model is widely used to estimate reliability of general systems using non-destructive degradation data. However, the daunting challenge is the application of the model to one-shot systems given that recursive measurements are unavailable and degradation trends over time are nonlinear. This study shows a storage reliability estimation method of one-shot systems using accelerated destructive degradation data when information of either chemical reaction or degradation trend is unknown. An n-th kinetic model, which is a Physics-based model, was used to evaluate the degradation phenomenon of one-shot systems using destructive degradation data. The accelerated degradation model was constructed to estimate storage reliability for normal storage temperature. The proposed method was applied to estimate storage reliability of IR flare using the stabilizer contents that were destructively measured over time at three temperature levels. The real application results show applicability of the proposed method.

Keywords: Accelerated destructive degradation data; IR flare; *n*-th order kinetic model; One-shot system; Storage reliability

1. Introduction

A one-shot system employed in fire extinguishers, missiles, ammunitions, medicine, and space-launch vehicles completes its mission after one use, making it impossible to test the system again after initial testing for reliability estimation [1]. One-shot systems in general spend their whole life in either standby or storage state unlike operating systems [2]. The two types of data used to estimate storage reliability of one-shot systems in open literature are (a) quantal-response data expressed as a number of the system succeeded for total number of tests at each measurement time and (b) degradation or aging data of performance characteristics over inspecting times based on destructive tests, such as the effective illumination time of an illuminating projectile because its response stands for its performance [3]. The illumination times for different projectiles at each measurement time are measured as shown in Fig. 1(a). This measurement is called destructive degradation test.

Gamma processes were used to model destructive degradation data [1]. In process modeling, only data of performance characteristics measured at an arbitrary time denoted as $Y(t_1)$ or $Y(t_1')$ in Fig. 1(b), were used to estimate parameters of the gamma process. Hence, (a) to evaluate the degradation increments over a series of measurement times would be difficult using the destructive degradation data, and (b) the initial value $Y(t_0)$ would not be identical. Performance characteristics at a series of times from the destructive degradation tests are generally measured to estimate reliability accurately, as shown in Fig. 1(a). In the gamma process, no physical models were considered to model degradation, and cumulative degradation has been assumed to be linear over time such that nonlinear degradation trends should be definitely linearized before applying a gamma process [1]. Thus, application of a reliability estimation method based on gamma process to destructive

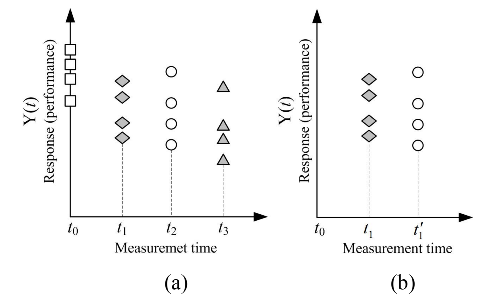

Fig. 1. Types of destructive degradation data: (a) General data; (b) special data for a gamma process model.

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degradation data measured at multiple times would be limited.

This study proposes a storage reliability estimation method of one-shot systems using accelerated destructive degradation tests. Application results of the proposed method to a one-shot system will be discussed in the next sections.

2. Degradation modeling based on kinetics of a reaction

Responses of a one-shot system as physically measurable characteristics would tend to either increase or decrease over a storage-time under stresses, such as temperature and relative humidity. These characteristics could be examples of degradation phenomenon. Let the response under stresses **S** at a time t be $Y(t, \mathbf{S})$ with the initial content, $Y_0 = Y(t = 0)$. Then for a decreasing response over time, the fraction degradation, $\alpha(t, \mathbf{S})$ compared to Y_0 has a form [4]

$$\alpha(t,\mathbf{S}) = \frac{Y_0 - Y(t,\mathbf{S})}{Y_0}. \tag{1}$$

The rate of the fraction degraded over time could be determined by both reaction rate coefficient $K(\mathbf{S})$ and n-th order kinetics from kinetics of a reaction. Therefore, the rate could be expressed as [5]

$$\frac{d}{dt}\left[\alpha(t,\mathbf{S})\right] = K(\mathbf{S})(1-\alpha(t,\mathbf{S}))^n. \tag{2}$$

From the integration of Eq. (2) with the initial condition $\alpha(t = 0, \mathbf{S}) = 0$, the fraction degraded over time could be rewritten using $K(\mathbf{S})$ and n as

$$\alpha(t,\mathbf{S}) = 1 - \left[(n-1)K(\mathbf{S})t + 1\right]^{1/(1-n)}. \tag{3}$$

Substituting Eq. (3) into Eq. (1) and arrangement provide the response at time t as

$$Y(t,\mathbf{S}) = Y_0\left[(n-1)K(\mathbf{S})t + 1\right]^{1/(1-n)}. \tag{4}$$

Eq. (4) agrees with the n-th order kinetic model of the stabilizer consumption of NC (nitrocellulose) propellant in Ref. [6].

A distribution-based method to model degradation data is based on assumption that degradation data measured at each time would follow identical distribution with time-variant distribution parameters. The distribution parameters over time, $\mathbf{p}(t, \mathbf{S})$ would be a function of $K(\mathbf{S})$ and n from Eq. (4), that is $\mathbf{p}(t,\mathbf{S}) = \mathbf{p_0} \times f(t, K(\mathbf{S}), n)$ for $\mathbf{p_0} = \mathbf{p}(t = 0)$ [7]. Assume that degraded responses measured at definite time intervals follow normal distributions. Then, distribution parameters are defined as nominal values and standard deviations, $\mathbf{p}(t, \mathbf{S}) = [\mu(t, \mathbf{S}), \sigma(t, \mathbf{S})]$. From Eq. (4), the nominal value for **S** could be expressed for $[\beta_0(\mathbf{S}), \beta_1(\mathbf{S}), \beta_1(\mathbf{S})] = [\mu_0, (n\text{-}1), K(\mathbf{S})]$ as

$$\mu(t,\mathbf{S}) = E[Y(t,\mathbf{S})] = \beta_0(\mathbf{S})\left[\beta_1(\mathbf{S}) \times \beta_2(\mathbf{S}) \times t + 1\right]^{-1/\beta_1(\mathbf{S})}. \tag{5}$$

In Eq. (5), n is considered dependent on **S** because different stresses could cause different kinetics of reaction. The reaction rate coefficient in Eq. (5) is a function of **S**, and its relation to **S** can be determined using life-stress models, such as Arrhenius or Peck. For temperature stress T in Kelvin, the reaction rate coefficient has a form for Arrhenius model [8] as

$$\beta_2(\mathbf{S}) = A_1 \exp\left(-\frac{E_a}{k_B T}\right), \tag{6}$$

where A_1 stands for material dependent constant, E_a for activation energy, and k_B for Boltzmann's constant.

The standard deviation of responses degraded at a time is assumed to increase over time such that the increasing standard deviation at a time could be determined by $\mathbf{S}, \sigma = \sigma(t, \mathbf{S})$. For temperature-induced degradation, the standard deviation from Einstein relation and inverse power model could be given as [9]

$$\sigma(t,T) = \beta_3(T)\left(1 + \sqrt{\beta_4(T) \times T \times t}\right), \tag{7}$$

where β_3 represents initial standard deviation σ_0.

3. Storage reliability estimation method

Let us define $\mathbf{Y}(t_i, T_q)$ as degradation data of all units (n_i) at time t_i for temperature level q. To estimate $\boldsymbol{\beta}_q = [\beta_0(q), \beta_1(q), \beta_2(q), \beta_3(q), \beta_4(q)]$ in Eqs. (5) and (7), the likelihood function of the data at each temperature is expressed as

$$L(\boldsymbol{\beta}_q \mid \mathbf{Y}(t_i, T_q)) = \prod_{i=1}^{r}\prod_{j=1}^{n_i}\left\{\frac{1}{\sqrt{2\pi}\sigma(t_i,T_q)}\exp\left(-\frac{1}{2}\left(\frac{Y_j(t_i,T_q) - \mu(t_i,T_q)}{\sigma(t_i,T_q)}\right)^2\right)\right\}. \tag{8}$$

The ML (Maximum likelihood) estimators to the likelihood function are obtained from the optimization problem

$$\hat{\boldsymbol{\beta}}_q = \max_{\boldsymbol{\beta}_q}\left\{L(\boldsymbol{\beta}_q \mid \mathbf{Y}(t_i, T_q))\right\}. \tag{9}$$

The reaction rate coefficient for T can be estimated using Arrhenius model if (a) $\beta_1(q)$ representing the reaction order for different temperature levels is statistically identical and, if (b) linearity of $\log_e(\beta_2(q))$ versus $(1/T_q)$ for all q is satisfied. For the same reaction order with temperatures satisfying linearity requirement, we could estimate degradation characteristics for normal storage temperature based on Arrhenius relationship. Only degradation data for different temperatures, which provide appropriateness of Arrhenius relationship, should be combined to estimate storage reliability at storage temperature. From Eqs. (5) and (7), the application of the Arrhenius model denoted as Eq. (6) for the statistically identical $\hat{n}$ provides the distribution parameters for T as

$$\mu(t,T) = \gamma_0 \left[(\hat{n}-1) \times \gamma_1 \times \exp\left(-\frac{\gamma_2}{T}\right) \times t + 1 \right]^{-1/(\hat{n}-1)}, \tag{10}$$

$$\sigma(t,T) = \gamma_3 \left(1 + \sqrt{\gamma_4 \times T \times t}\right), \tag{11}$$

where $\gamma_0 = E[Y_0], \gamma_1 = A, \gamma_2 = E_a/k, \gamma_3 = \sqrt{V[Y_0]}$.
To estimate $\boldsymbol{\gamma} = [\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4]$ in Eqs. (10) and (11), the likelihood function of the data for the temperature levels providing appropriateness of Arrhenius relationship, $\mathbf{Y}(t_i, \mathbf{T})$ can be expressed as

$$L(\boldsymbol{\gamma} \mid \mathbf{Y}(t_i, \mathbf{T})) = \prod_{q=1}^{w} \prod_{i=1}^{r_q} \prod_{j=1}^{n_i} \left\{ \frac{1}{\sqrt{2\pi}\sigma(t_i,T_q)} \exp\left(-\frac{1}{2} \left(\frac{Y_j(t_i,T_q) - \mu(t_i,T_q)}{\sigma(t_i,T_q)} \right)^2 \right) \right\}. \tag{12}$$

The ML estimators are defined as

$$\hat{\boldsymbol{\gamma}} = \max_{\boldsymbol{\gamma}} \left\{ L(\boldsymbol{\gamma} \mid \mathbf{Y}(t_i, \mathbf{T})) \right\}. \tag{13}$$

A sequential Quadratic Programming method was used to solve the formulated optimization problems in Eqs. (9) and (13).

For the distribution-based degradation model with $\hat{\boldsymbol{\gamma}}$ and pre-defined failure threshold LSL, storage reliability for a storage temperature T_s at time t could be estimated as

$$\hat{R}(t, \hat{\boldsymbol{\gamma}}, T_s) = \Phi\left(\frac{\hat{\mu}(t, \hat{\boldsymbol{\gamma}}, T_s) - LSL}{\hat{\sigma}(t, \hat{\boldsymbol{\gamma}}, T_s)} \right). \tag{14}$$

For the confidence level of interest, the one-sided lower bound of estimated storage reliability for T_s is evaluated as

$$\hat{R}_L(t, T_s) = \hat{R}(t, T_s) \Big/ \exp\left\{ \frac{Z_{C.L}\sqrt{V[\hat{R}(t,T_s)]}}{\hat{R}(t,T_s)} \right\}, \tag{15}$$

where $Z_{C.L}$ is 1.2816 and 1.6449 for confidence levels, 90% and 95%, respectively. The variance in Eq. (15) was evaluated using Fisher's information matrix for $\hat{\boldsymbol{\gamma}}$ [1, 10].

4. Storage reliability estimation of IR flare

IR flare is a one-shot system for jets and helicopters. The flare is dispensed from aircrafts and helicopters to provide protection from heat-seeking missiles or to break missile locks. The proposed storage reliability estimation method in this paper is applied to the IR flare, where the stabilizer content of the flare is considered as a response. To estimate storage reliability at 25°C, samples of the flare were subjected to accelerated aging at three temperature levels, 55°C (Case 1), 65°C (Case 2) and 75°C (Case 3). Fig. 2 shows data of the stabilizer contents measured at definite time intervals.

Table 1. Estimated parameters for the three cases.

	Case 1	Case 2	Case 3
β_0	2.39964	2.39744	2.39933
β_1	0.00005	0.00945	0.00000
β_2	0.01251	0.02085	0.05434
β_3	0.00776	0.00704	0.00684
β_4	0.00370	0.01162	0.01144

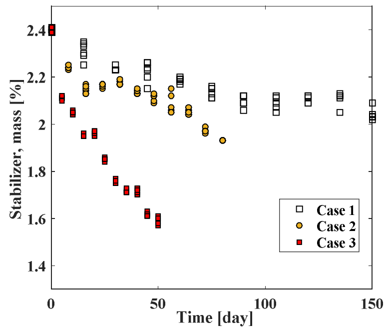

Fig. 2. Stabilizer contents at three different temperature levels.

The distribution of the stabilizer contents at any measurement time followed a normal distribution with time-variant mean and standard deviation. From Eqs. (5) and (7), the mean and standard deviation for a temperature T over time with the power transformation of time are written as

$$\mu(t,T) = \beta_0(T)\left[\beta_1(T) \times \beta_2(T) \times \sqrt{t} + 1\right]^{-1/\beta_1(T)} \tag{16}$$

$$\sigma(t,T) = \beta_3(T)\left(1 + \sqrt{\beta_4(T) \times T \times \sqrt{t}}\right). \tag{17}$$

The estimated parameters maximizing the likelihood function denoted as Eq. (9) are presented in Table 1. Degradation of the stabilizer content could be characterized by the reaction rate of the first order given that $\beta_1 \cong 0$ for all cases in Table 1. The estimated reaction order agrees with the reaction order for general double-base propellants in Ref. [6]. Thus, the proposed physics-based model denoted as Eqs. (16) and (17) with the parameters in Table 1 accurately describes the degradation data over time in Fig. 2. From the estimated first-order reaction model, the stabilizer content over time could be expressed as

$$Y(t,T) = Y_0 \exp\left(-K(T)\sqrt{t}\right). \tag{18}$$

The appropriateness of Arrhenius relationship can be evaluated statistically using χ^2 statistics [8]. For the three temperature levels (Cases 1, 2 and 3), the calculated χ^2 statistics had a p-value of 0. Thus, the reaction rate coefficients at 25°C were estimated using the data for temperatures 55°C and 65°C. The

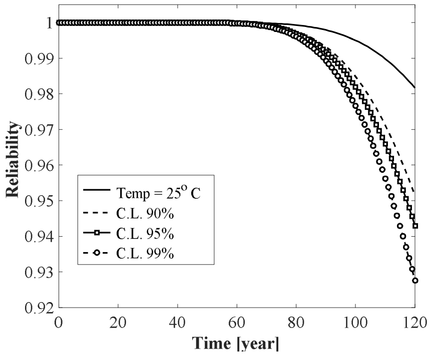

Fig. 3. Estimated storage reliability at the temperature of 25°C.

reaction rate coefficient of NC indicated a different trend over temperatures greater than 65°C [11]. From the Arrhenius model, the nominal value and standard deviation for stabilizer content degraded over time could be written as

$$\mu(t,T) = \gamma_0 \exp\left(-\gamma_1 \times 10^9 \times \exp\left(-\gamma_2 \frac{1}{T}\right)\sqrt{t}\right) \tag{19}$$

$$\sigma(t,T) = \gamma_3 (1 + \sqrt{\gamma_4 \times T \times \sqrt{t}}\,)\,. \tag{20}$$

The estimators from Eq. (13) using measurement data from temperatures 55°C and 65°C were used to estimate storage reliability for temperature T = 25°C and LSL = 1.2 in Eq. (14). Estimated storage reliability over time is shown in Fig. 3. Estimated reliability values for three different confidence levels using Eq. (15) are given in Fig. 3. The proposed degradation model provided the minimum sum of squared errors as compared to other degradation models, such as linear or quadratic model, etc. Therefore, the proposed method could provide accurate reliability estimation.

5. Conclusions

This paper proposed a storage reliability estimation method for one-shot systems based on the *n*-th kinetic model by using accelerated destructive degradation data. Integrating distribution-based degradation modeling with the kinetic model could provide estimations for the reaction order in the chemical reaction and parameters of Arrhenius model. Thus, accelerated degradation model from the integration could be used to estimate storage reliability for normal storage temperature.

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Nomenclature

$Y(t, \mathbf{S})$: Response (performance) under stresses **S** at time t
$Y_j(t_i, T_q)$: Degradation data of unit j at a time t_i for level q
$\mu(t_i,T_q)$: Nominal value of $Y_j(t_i, T_q)$ for $j = 1, 2, \ldots n_i$
$\sigma(t_i,T_q)$: Standard deviation of $Y_j(t_i, T_q)$ for $j = 1, 2, \ldots n_i$
exp( ) : Exponential function
$E[X]$ and $V[X]$: Expected value and variance of X
$\Phi(z)$: Cumulative distribution function of standard normal variable z

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