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New chatter-free sliding mode synchronization of steer-by-wire systems under chaotic conditions †

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Abstract

Vehicle safety issues and its systems dependence on electronics are rapidly increasing. In intelligent vehicles, rigid components are replaced with controlled x-by-wire systems including configurable electronic elements. In the paper, through complete modeling of Steer-by-wire (SBW) nonlinear dynamics, the chaotic motion of the system is elucidated by a new yaw stability control method. The designed chatter free Sliding mode controller (SMC) is used to synchronize chaotic motion of the SBW system. Following stability and robustness analysis, computer simulations show that the proposed control system is quite significant for nonlinear systems like the SBW system. Besides, the robustness of the SMC against exogenous disturbance protects vehicle from undesired slide and spin motions.

Keywords: Steer-by-wire; Yaw stability; Chaos synchronization; Sliding mode control

1. Introduction

Focusing on intelligent vehicle systems offers significantly enhanced safety and convenience [1]. Through In-vehicle-network (IVN) system as an important section of intelligent vehicles, electronic components are connected to the Electronic control unit (ECU) [2]. By replacing mechanical components with electronic types and digital communication networks, the application of IVN systems in real-time controlled components like brakes, throttle and steer systems are being developed [3].

Besides learning steering responses of a car under different road and operation conditions, a driver is normally responsible for judgment on detection of physical limits of various handling maneuvers. However, most drivers do not detect such limits before reaching by the vehicle. Nowadays, for emending steering safety, the current mechanical/ electrical steering systems of vehicles are replaced by the SBW kinds. The SBW provides the potential benefits of enhanced vehicle performance, energy saving, improved handling, and fully integrated control of vehicle. Furthermore, the SBW detects space and packaging benefits in the engine compartment through elimination of the steering wheel column, mechanical steering box and the intermediate shaft.

Ackermann developed a yaw rate controller in combination with active steering, in order to robustly decouple the yaw rate from the lateral acceleration. Hence, the generated yaw angle by braking on split friction roads is removed [4, 5].

Tajima et al. have established the control logics on steering properties based on study of driver behaviours in a steady-state cornering motion. However, the motion control in transient state was not considered [6, 7]. Mousavinejad et al. and Segawa et al. investigated the stability control of a SBW manipulated vehicle using variable structure and reactive torque methods, respectively [8, 9]. Following modelling the chaotic motion of SBW, Chang has proposed a continuous-time control method in which the chaotic motion of the SBW has been synchronized [10]. More recently, Zheng et al. devised a fault-tolerant SBW wheel control system. Using dual motor and microcontroller, the capability to tolerate single-point failures without degrading the control system performance has been obtained [11].

Using fuzzy logic and sliding mode controllers, Song has developed enhanced vehicle stability by active steering system. The stability and the controllability in every driving condition were compared to the ABS-equipped vehicle under different road conditions [12]. The sliding mode controller of chaos systems by Li et al. yields low chattering and fast response [13].

In current research, a novel SMC is proposed to drive yaw stability control based on SBW system. The main advantage of SMC as a robust nonlinear technique is robustness to parameter uncertainty and disturbances. However, the classic SMC mainly suffers from chattering effect on the outputs owing to the sign switching term. The chattering deteriorates

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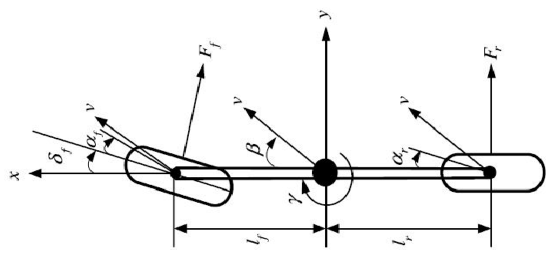

Fig. 1. 2DOF bicycle lateral model of vehicle.

the performance of control system and even may lead to failure in mechanical sections of SBW system.

In the paper, first a planar bicycle model of vehicle is considered in order to approach mathematical formulation of the vehicle's yaw characteristics together with a nonlinear tire force model. Second, as the main purpose, through synchronization of chaotic motion in the designed control algorithm, the handling and steering performance of the vehicle is improved during hard maneuvers. Based on integral differential sliding surface, the stability of the SMC system is examined by Lyapunov direct method. Following dynamical modelling and chaos analysis of the SBW system, the analytical design, stability and simulation of the proposed SMC in chaotic condition are emphasized in the paper. In comparison with the recently documented research works on SBW system [10], our newly designed controller though a simple structure yields reduced chatter effect and high robustness against uncertainties. Besides, the new SMC shows a fast response compared with those of the recent publishments in autonomous vehicles field.

2. Model of SBW system

2.1 Vehicle dynamical model

The lateral dynamics of a car vehicle in horizontal plane is represented by the single track bicycle model of Fig. 1 which includes yaw and lateral motions and neglects the roll motion.

The lateral motion is specified by slip angle, β at the centre of gravity, yaw rate, γ, slip angle of front tires, α_f, slip angle of rear tires, α_r front steer angle, δ_f and the vehicle velocity v. The lateral motion equations can now be written as:

$$mv\left(\dot{\beta}+\gamma\right)=F_f+F_r \tag{1}$$

$$v=2\pi RN/6000 \tag{2}$$

where m is the vehicle mass; F_f and F_r stand for cornering force of front and rear tires, respectively; R (cm) and N show the radius and the rotational speed of wheels. Assuming small front steer angle, δ_f the vehicle's yaw motion is determined by:

$$I_z\dot{\gamma}=\left(l_f F_f-l_r F_r\right) \tag{3}$$

where I_z is inertial moment along yaw axis and l_f, l_r denote distance of front and rear axle to the centre of gravity.

Table 1. Coefficients of magic formula.

Road	Wheel	B_f, B_r	C_f, C_r	D_f, D_r	E_f, E_r
High friction	Front	11.275	1.5600	2574.7	1.9990
	Rear	18.631	1.5600	1749.7	1.7908
Low friction	Front	6.7651	1.3000	6436.8	1.9990
	Rear	9.0051	1.3000	5430.0	1.7908

Small slide angles, α_f, α_r may be geometrically computed as:

$$\alpha_f=\beta+\tan^{-1}\left(\frac{l_f}{v}\gamma\cos\beta\right)-\delta_f \tag{4}$$

$$\alpha_r=\beta-\tan^{-1}\left(\frac{l_r}{v}\gamma\cos\beta\right). \tag{5}$$

2.2 Tire model

The nonlinear Magic formula of tire forces is used as [14]:

$$F_f=D_f\sin\left[C_f\tan^{-1}\left\{B_f\left(1-E_f\right)\alpha_f+E_f\left(\tan^{-1}\left(B_f\alpha_f\right)\right)\right\}\right] \tag{6}$$

$$F_r=D_r\sin\left[C_r\tan^{-1}\left\{B_r\left(1-E_r\right)\alpha_r+E_r\left(\tan^{-1}\left(B_r\alpha_r\right)\right)\right\}\right]. \tag{7}$$

Following the assumption of small slide angels α_f and α_r and neglecting higher order terms, Eqs. (6) and (7) are linearized in terms of parameters, y_r and y_f as follows.

$$F_f\left(\alpha_f\right)\approx D_f\left(y_f-\frac{1}{6}y_f^3\right) \tag{8}$$

$$F_r\left(\alpha_r\right)\approx D_r\left(y_r-\frac{1}{6}y_r^3\right) \tag{9}$$

$$y_f\approx C_fB_f\alpha_f-\left(\frac{E_fC_fB_f}{3}+\frac{B_fC_f}{3}\right)\alpha_f^3 \tag{10}$$

$$\mathrm{y_r}\approx \mathrm{C_rB_r\alpha_r}-\left(\frac{\mathrm{E_rC_rB_r}}{3}+\frac{\mathrm{B_rC_r}}{3}\right)\mathrm{\alpha_r^3}\,. \tag{11}$$

In Table 1, the values of coefficients B_i, C_i, D_i and E_i ($i=f, r$) are given under different road conditions.

2.3 Steer by wire model

In novel SBW systems, the steering parameters commanded by driver are detected by a set of steering angle sensors and a torque sensor. The data of sensors are transmitted to the control system, which uses the driver's data and the sensing information to control the front-wheel angle by a steering actuator. The steering actuator is selected based on the maximum torque and speed required in the steering of vehicle under critical driving conditions.

The dynamical equation of SBW system of Fig. 2 is:

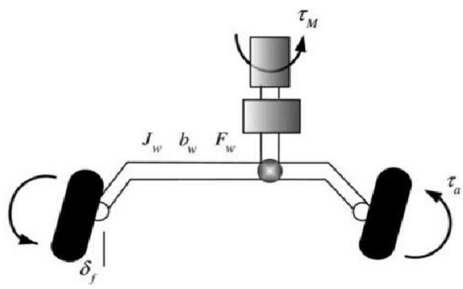

Fig. 2. Steer system in SBW dynamics control [10].

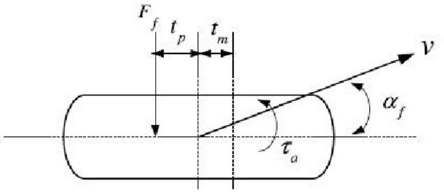

Fig. 3. Tire force model.

$$J_w \ddot{\delta}_f + b_w \dot{\delta}_f + \tau_f + \tau_a = r_s \tau_M \tag{12}$$

where J_w (Nms^2/rad), b_w (Nms/rad) and τ_f (Nm) are the total moment of inertia, viscous damping and coulomb friction, respectively. The tire self-aligning moment, τ_a (Nm) depends on the steering geometry, in particular the caster angle and the manner in which the tire deforms to generate the lateral forces. The total aligning, τ_a and servo motor torque, τ_M are given as:

$$\tau_a = \left(t_p + t_m\right) \tag{13}$$

$$\tau_M = k_m \; i_m e_f \; . \tag{14}$$

k_m, r_s, i_m and e_f are the steering ratio, motor constant, current and the motor efficiency. As Fig. 3 shows, F_f is the cornering force of the front tires. The pneumatic trail, t_p is the distance between the resultant application point of the lateral force and the geometric centre of the tire. t_m is the mechanical trail, which is the distance between the centre of tire and the steering axis. v shows the velocity of gravity centre of tire. The coulomb friction torque, τ_f is given by:

$$\tau_f = 9.8 t_p \, \mu F_w \, \mathrm{sgn}(\delta) \tag{15}$$

where, μ is the coefficient of friction and F_w (kg) is the weight on the front wheels. The state space equations of considered SBW system are expressed as:

$$\dot{x}_1 = \frac{3000\left(F_f + F_r\right)}{\pi m R N} - x_2$$

$$\dot{x}_2 = \frac{l_f F_f - l_r F_r}{I_z}$$

$$\dot{x}_3 = x_4 \tag{16}$$

$$\dot{x}_4 = -\frac{b_w x_4}{J_w} + \frac{r_s \tau_m}{J_w} - \frac{\tau_f}{J_w} - \frac{\tau_a}{J_w} + u,$$

$$x_1 = \beta, \quad x_2 = \gamma, \quad x_3 = \delta_f \,, \quad x_4 = \dot{\delta}_f \;\text{ and }\; u = \frac{r_s}{J_w} \tau_M \,.$$

According to Eq. (12), the unit of control input, u is rad/ s^2.

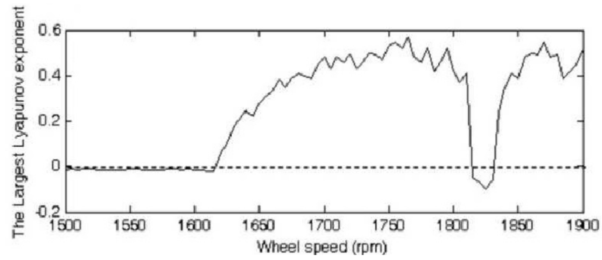

Fig. 4. Lyapunov exponent of SBW system.

3. Chaos identification using Lyapunov exponents

According to Chang research [10], the largest Lyapunov exponent of our nonlinear SBW system is positive for $1645(rpm) < N < 1825(rpm)$ and $1855(rpm) < N$ intervals which depicts the chaotic behaviour. Here, the chaotic type signal is characterized by a phase portrait and a frequency spectrum as Figs. 5 and 6. The frequency spectrum of Fig. 6 contains a broad band chaotic motion.

4. Design of SMC

The affine tracking error dynamics of SBW system Eq. (16) can be rewritten as follows [13]:

$$e_i(t) = x_i(t) - R_i(t) \qquad (i = 1,2,3,4) \tag{17}$$

$$\dot{e} = Ae + F(t,e) + d(t) + u(t) \tag{18}$$

where $e = [e_1,\, e_2, \ldots\ldots \, e_n{}^T$, $(n = 4)$ shows the tracking error vector with respect to reference signal, $R_i(i = 1:4)$; $d_{n*1}(t)$ and $F_{(n*1)}(t,e)$ stands for external disturbance and nonlinear vector of variables and parameters, not embedded in matrix *A*. Control input vector is denoted by $u_{n*1}(t)$.

Assumption 1: $\underset{t \to \infty}{limit}\, e(t) = 0$

In which $\| \; \|$ denotes Euclidean norm of states vector. The sliding mode variable with integral operator is defined as:

$$s_i(t) = k_i \int_0^t e_i(\tau) d\tau + e_i(t) \,. \tag{19}$$

Eq. (19) and positive $\lambda_i \left(i \div 1 \;\, n\right)$, yield the sliding surface as:

$$\sigma_i = \dot{s}_i(t) + \lambda_i s_i(t) \tag{20}$$

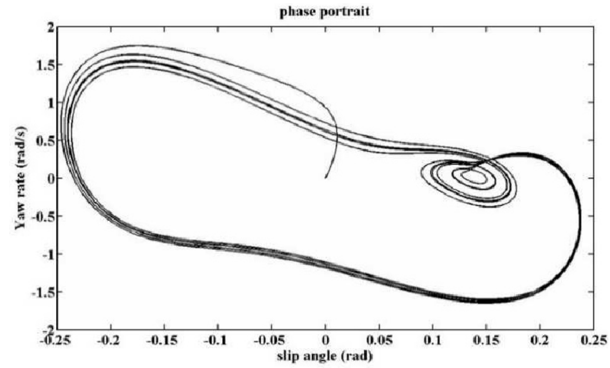

Fig. 5. Phase portrait of SBW system.

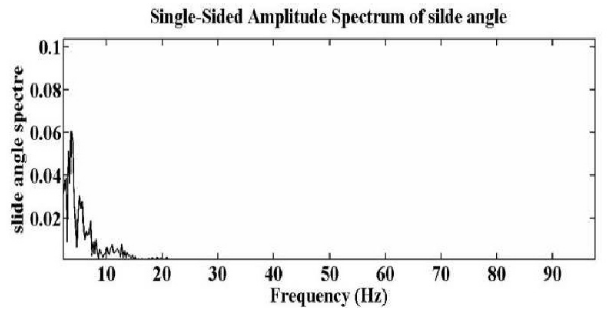

Fig. 6. Spectrum of chaotic sliding angle.

$$\begin{aligned}\dot{\sigma}_i &= \ddot{s}_i(t) + \lambda_i \dot{s}_i(t)\\ &= \ddot{e}_i + (\lambda_i + k_i)\dot{e}_i(t) + \lambda_i k_i e_i(t)\\ &= A_i \dot{e}_i(t) + \sum_{j=1}^{n} \frac{\partial F_i(t,e)}{\partial e_j}\dot{e}_j + \frac{\partial F_i(t,e)}{\partial t} + \dot{d}_i(t) + \dot{u}_i\\ &\quad + (\lambda_i + k_i)\big(A_i e + F_i(t,e) + d_i(t) + u_i(t)\big) + \lambda_i k_i e_i(t).\end{aligned} \tag{21}$$

In aforementioned equations, the index, i denotes the $i-th$ row of matrix/vector. Using Eqs. (16) and (18) the nonlinear terms F_i are explicit algebraic functions of state variable and the term $\partial F_i(t,e)/\partial e_j$ represents Jacobin matrix of vector. As described previously, the desired control input u causes the dynamical sliding surfaces Eq. (20) converge to zero while the sliding condition is satisfied $(\sigma_i = 0)$.

Before developing the design of sliding mode controller and stability analysis, the following assumptions are introduced.

Assumption 2. The uncertain term $d_i(t)$ in Eq. (18) is assumed to be bounded, i.e., there exists a positive bounded function B_i satisfying the following inequalities:

$$|d_i(t)| \le B_i(e) \qquad \forall e \in R^n \quad (i = 1,2,3,\ldots,n). \tag{22}$$

Assumption 3. The first derivative of uncertain term $\dot{d}_i(t)$ is assumed to be bounded. Hence, there exists a positive bounded function, $\overline{B}_i(e)$ satisfying the following inequalities:

$$|\dot{d}_i(t)| \le \overline{B}_i(e) \qquad \forall e \in R^n \quad (i = 1,2,3,\ldots,n). \tag{23}$$

Assumption 4. There exists a positive constant ε_i satisfying the following inequalities.

$$\varepsilon_i > \overline{B}_i(e) + (\lambda_i + k_i)B_i(e) \quad \forall e \in R^n \quad (i = 1,2,\ldots,n). \tag{24}$$

Considering $\dot{\sigma} \to 0$ in Eq. (21), the dynamical sliding mode control law is designed as follows.

$$\begin{aligned}\dot{u}_i &= -A_i e_i(t) - \sum_{j=1}^{n} \frac{\partial F_i(t,e)}{\partial e_j}\dot{e}_j - \frac{\partial F_i(t,e)}{\partial t}\\ &\quad -(\lambda_i + k_i)\big(A_i e + F_i(t,e) + d_i(t) + u_i(t)\big)\\ &\quad -\lambda_i k_i e_i(t) - \epsilon_i\, sign(\sigma_i) \qquad (i = 1,2,3,\ldots,n).\end{aligned} \tag{25}$$

From Eqs. (16) and (18), the explicit nonlinear terms F_i are obtained and therefore, simple mathematical manipulations yield partial first order terms.

4.1 Stability analysis

To analysis the stability of the sliding mode controller, let us define a discrete Lyapunov function as [15]:

$$V = 0.5\sum_{i=1}^{n}\sigma_i^{2}. \tag{26}$$

The first time derivative of V gives:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\sigma_i\dot{\sigma}_i\\ &= \sum_{i=1}^{n}\sigma_i(\dot{d}_i(t) + (\lambda_i + k_i)d_i(t) - \epsilon_i\, sign(\sigma_i)\\ &= \sum_{i=1}^{n}\sigma_i(\dot{d}_i(t) + (\lambda_i + k_i)\sigma_i d_i(t) - \epsilon_i|\sigma_i|\\ &\le \sum_{i=1}^{n}\Big(\sigma_i\big(\overline{B}_i + (\lambda_i + k_i)B_i\big)\Big) - \epsilon_i|\sigma_i|\\ &= -\sum_{i=1}^{n}\Big(\epsilon_i - \big(\overline{B}_i + (\lambda_i + k_i)B_i\big)\Big))|\sigma_i|.\end{aligned} \tag{27}$$

Considering the aforementioned assumptions, the derivative of Lyapunov function yields: $\dot{V} \le 0$.

The sliding surface, s_i is integrable and continuous on time interval $[0\ +\infty]$. Therefore using Barbalat lemma we have $\sigma_i \to 0$ as $t \to \infty$, and the sliding mode surface is globally asymptotically stable at its equilibrium point. Therefore, Eq. (20) is equivalent to:

$$\dot{s}_i(t) = -\lambda_i s_i(t) \tag{28}$$

$$k_i\int_0^t e_i(\tau)d\tau = -e_i(t) \quad (i = 1,2,3,\ldots,n). \tag{29}$$

Eq. (28) signifies that $s_i \to 0$ as $t \to \infty$, and Eq. (19) means that by converging s_i to zero, $e_i(t)$ will be as:

Table 2. Specifications of SBW system.

System parameter	Value	Unit
m	500	Kg
R	30	Cm
I_z	2000	kg.m^2
l_f	1.2	m
l_r	1.3	m
j_w	10	Nms^2/rad
b_w	200	Nms^2/rad
r_s	30	Nm/A
k_m	0.078	-
e_f	0.7	-
t_p	0.012	m
t_m	0.010	m
f_w	200	Kg

$$\dot{e}_i = -k_i e_i(t), \quad (i = 1,2,3,\ldots,n). \tag{30}$$

Accordingly, the state tracking error vector $e(t)$ converges asymptotically to zero. Therefore, the global stability of the sliding mode control law is fully guaranteed.

5. Simulation result

The chaotic SBW system needs to be transformed into periodic motion in order to improve the performance of the steering system. Now, the possibility, effectiveness and robustness of the proposed SBW control system is assessed by simulations. Based on the considered SMC structure, the controller is applied to synchronize the SBW system for wheel's rotational speeds near $1880(rpm)$. The reference input signal, $R_i(t)$ is generated by the SBW dynamics with $1700(rpm)$ rotational speed of wheels. The control parameters are chosen as $k_i = 2.2$, $l_i = 2$, $\varepsilon_i = 5.6$ $(i = 1:3)$ which satisfy Eq. (24). The specifications of the SBW system considered in the simulation are given in Table 2.

As shown by Fig. 7, the synchronization errors, e_1, e_2, e_3 and e_4 of the chaotic SBW system subjected to SMC converge to about zero. Besides, the control input, u_1 depicted in Fig. 8 for example, is without chattering effect. Hence, the effectiveness of control method for improving the handling and steering characteristics is vivid. The sliding surface in Fig. 9 shows the stability and the significant transient response properties by the SMC system. Besides, the synchronization error of SBW system is stable within a limited range between -0.005 and 0.005.

To verify the effectiveness of the proposed control scheme in lowering disturbance effects, the modelling uncertainty, $0.01\sin(100t)$ is imposed on the SBW dynamic system.

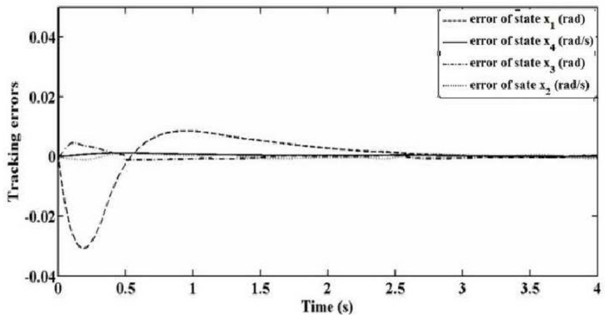

Fig. 7. Synchronization error of SBW without uncertainty.

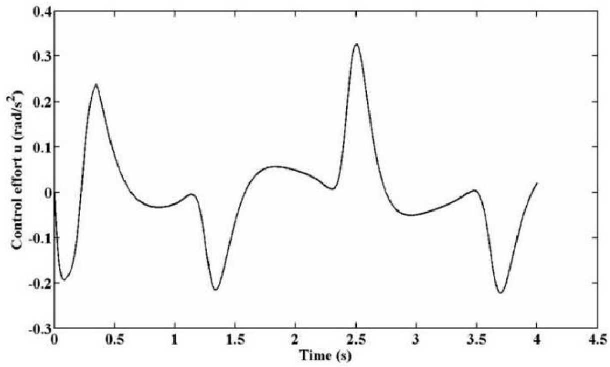

Fig. 8. Control input of SBW system without uncertainty.

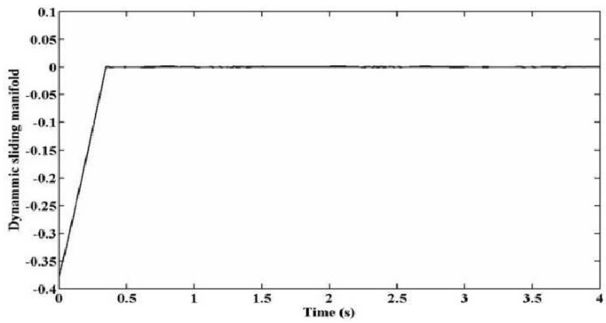

Fig. 9. Sliding surface without uncertainty.

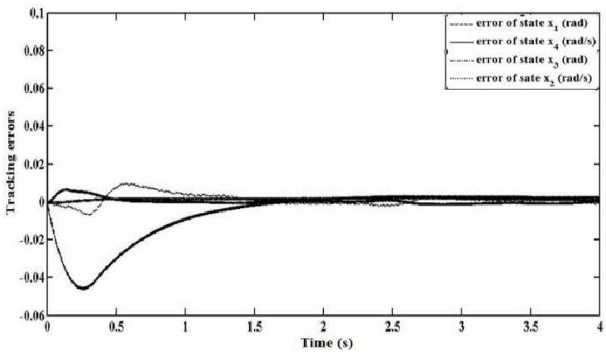

Fig. 10. Synchronization error of SBW with uncertainty.

Fig. 10 shows the synchronization errors, e_1, e_2, e_3 and e_4 of controlled chaotic SBW system subjected to the uncertainty. According Fig. 10, the disturbance is diminished under

0.001 of input sinusoidal signal. As shown by simulations, the designed SMC system yields fast transient response and about zero synchronisation errors along the tracked paths. Moreover, low control input compared with previous chaos control of SBW system [10], shows the effectiveness of the proposed SMC system in real-time applications.

6. Conclusion

Along with robust feedback control of chaotic SBW system as an EPS, a novel SMC system to drive yaw stability has been completely designed. In comparison with the recently documented works, the designed controller though a simple structure yields reduced chattering effect, high robustness to uncertainties and small transient response times.

Based on the dynamic SMC designed to remove the chaotic motions of SBW system, the handling performance of car vehicles and the stability of steering system in steady state turning maneuvers are improved. According to Lyapunov's theory, the global stability of SMC was investigated. The dynamic control of the SBW system considered merely in Ref. [10] yields about 30 seconds synchronization time which makes the control system not suitable for vehicle yaw control in steering maneuvers. Through our robust SMC system, the response time decreases to less than 1 second .

In common, non-smooth and non-differentiable control input of a SMC system results in chattering which may limit the application of sliding mode controller in mechanical chaos systems. However, the designed SMC with a simple structure, reduced chattering effect even under disturbances and small response times can be used as a real-time controller of autonomous vehicular systems.

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