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Strongly nonlinear free vibration of four edges simply supported stiffened plates with geometric imperfections †

Zhaoting Chen*, Ronghui Wang, Li Chen and Chunguang Dong

School of Civil Engineering and Transportation, South China University of Technology ,Guangzhou 510640, China

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Abstract

This article investigated the strongly nonlinear free vibration of four edges simply supported stiffened plates with geometric imperfections. The von Karman nonlinear strain-displacement relationships are applied. The nonlinear vibration of stiffened plate is reduced to a one-degree-of-freedom nonlinear system by assuming mode shapes. The Multiple scales Lindstedt-Poincare method (MSLP) and Modified Lindstedt-Poincare method (MLP) are used to solve the governing equations of vibration. Numerical examples for stiffened plates with different initial geometric imperfections are presented in order to discuss the influences to the strongly nonlinear free vibration of the stiffened plate. The results showed that: the frequency ratio reduced as the initial geometric imperfections of plate increased, which showed that the increase of the initial geometric imperfections of plate can lead to the decrease of nonlinear effect; by comparing the results calculated by MSLP method, using MS method to study strongly nonlinear vibration can lead to serious mistakes.

Keywords: Strongly nonlinear; Free vibration; Stiffened plate; Geometric imperfection

1. Introduction

A thin plate stiffened by ribs in one or two directions can achieve greater strength with relatively less material. Therefore, stiffened plates are widely used in ships, aircrafts, bridges, etc. Due to the extensive use of the stiffened plates, the vibration characteristics of stiffened plates are of considerable importance to mechanical and structural engineers.

The dynamic behaviour of stiffened plates has been the subject of intensive study for many years, numerous studies have been reported in open literature. Among the well-known solution techniques to the vibration of stiffened plates, there are grillage model [1], Rayleigh-Ritz method [2, 3], finite element method [4, 5], finite difference method [6, 7], differential quadrature method [8], meshless method [9] and other methods [10-12].

When the deflection of structure is under a great time-varying load which may lead to large deformations of structure, geometrical nonlinear vibrations will occur. At large deflection level, membrane stresses are produced which give additional stiffness to the structure, moreover the strain-displacement relationship becomes nonlinear in this range. Extensive researches on nonlinear vibration analysis of plates have been done. For example, see Haterbouch and Benamar [13], Nerantzaki and Katsikadelis [14], Celep and Guler [15]. Presently, the common analytical methods for the nonlinear vibration of plates include Ritz method, Galerkin method, perturbation method, successive approximation method, finite difference method, Runge-Kutta integration and so on [16-22].

Recently, Li et al. [23] obtained the analytic free vibration solutions of corner point supported rectangular plates by an up-to-date symplectic superposition method. Nikkhoo et al. [24] studied the vibration of a rectangular plate under multiple moving inertial loads by employing eigenfunction expansion method. Joshi et al. [25] studied the free vibration and geometrically linear thermal buckling phenomenon of a thin rectangular isotropic plate by an analytical model. Breslavsky et al. [26] investigated the static and dynamic analyses of hyperelastic plates by a method for building a local model, which approximates the plate behavior around a deformed configuration. Naghsh and Azhari [27] analyzed the large amplitude free vibration of point supported laminated composite skew plates by the Element-free Galerkin (EFG) method. Razavi and Shooshtari [28] studied the nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates with simply supported boundary condition, and solved the nonlinear ordinary differential equations by using the Galerkin method. Hassanabadi et al. [29] studied the vibration of a thin rectangular plate carrying a moving oscillator, the transverse vibration of a thin rectangular plate under a traveling mass-spring-damper system is revealed by eigenfunction ex-

*Corresponding author. Tel.: +86 13751788124
E-mail address: libertyczt@163.com
† Recommended by Associate Editor Ohseop Song

pansion method. Hassanabadi et al. [30] studied the dynamic behavior of a circular thin plate traversed by a moving mass, a series of the plate natural shape functions is used to discretize the spatial domain, and a novel effective computational optimization is developed. Viswanathan et al. [31] studied the free vibration of symmetric angle-ply laminated truncated conical shell by using the spline method.

Geometric imperfections have been recognized since a long time for having a major effect on the linear and non-linear characteristics of thin-walled structures such as plates and shells [32]. The influence of geometric imperfections on the behaviour of plates has also been reported by many researches, such as Amabili [33], Alijani [34], Hui [35], and so on.

However, even if a large number of theoretical studies on nonlinear vibration of plates are available in the scientific literature, results of stiffened plates considering geometric imperfections are very scarce.

The aim of this paper is to analyze the strongly nonlinear free vibration of four edges simply supported stiffened plate with initial geometric imperfections. In the paper, the initial geometric imperfections of the plate taken into account are considered only in z direction. The nonlinear vibration of stiffened plate is reduced to a one-degree-of-freedom nonlinear system by assuming mode shapes. The Multiple scales Lindstedt-Poincare method (MSLP) and Modified Lindstedt-Poincare method (MLP) are used to solve the governing equations of vibration. Some numerical example are presented to demonstrate how the initial geometric imperfections of plate influence the strongly nonlinear free vibration of stiffened plate.

2. Derivation of the governing equation

Consider a stiffened plate in Fig. 1, which is composed of x-stiffeners, y-stiffeners and a plate. In order to simplify the problem, material nonlinearity is not considered in this paper. In the following computation, the plate and its stiffeners are made of the same material, and the material of stiffened plate is isotropic. The nomenclature is as follows: a and b are the lengths in x-direction and y-direction, respectively; ρ, E and μ are the mass density, Young's modulus and Poisson ratio of the stiffened plate, respectively; N_x and N_y are the numbers of x-stiffeners and y-stiffeners, respectively; the coordinates of the ith x-stiffener and y-stiffener are $y = y_i$ and $x = x_i$, respectively; b_1 and a_1 are the distances between neighboring x-stiffeners and y-stiffeners, respectively; A_x and A_y are the cross-section areas of x-stiffeners and y-stiffeners, respectively; EI_x and EI_y are the flexural rigidity of x-stiffeners and y-stiffeners with respect to the neutral surface, respectively; h is the thickness of plate; d_x is the distance between the neutral plane of x-section and the bending neutral plane of the compound section; d_y is the distance between the neutral plane of y-section and the bending neutral plane of the compound section; the flexural rigidity of plate is: $D = Eh^3/[12(1-\mu^2)]$.

A stiffened plate with four edges simply supported is investigated in this paper.

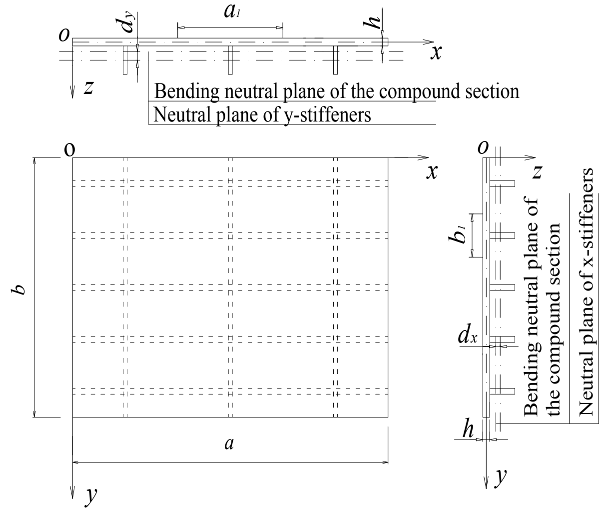

Fig. 1. Structure of the stiffened plate.

u and v denote the displacements of the middle surface of the plate along the x-direction and the y-direction, respectively, and w denotes the deflection of the plate. Taking geometric nonlinearity and initial geometric imperfections of the plate into consideration, according to von Karman's theory, the middle surface strain-displacement relationships and changes in the curvature and torsion are given by [36]:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w^0}{\partial x} \tag{1a}$$

$$\varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial w^0}{\partial y} \tag{1b}$$

$$\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w^0}{\partial y} + \frac{\partial w^0}{\partial x}\frac{\partial w}{\partial y} \tag{1c}$$

where the initial geometric imperfections of the plate associated with zero initial tension are denoted by out-of-plane displacement w^0.

The boundary conditions of the stiffened plate with four edges simply supported are: $u = v = w = M_x = \partial^2 w/\partial x^2 = 0$ at $x = 0, a$; $u = v = w = M_y = \partial^2 w/\partial y^2 = 0$ at $y = 0, b$. Then, for single mode analysis, the displacement can be assumed as:

$$u(x,y,t) = u_{mn}^t(t)\sin\frac{2m\pi x}{a}\sin\frac{n\pi y}{b} \tag{2a}$$

$$v(x,y,t) = v_{mn}^t(t)\sin\frac{m\pi x}{a}\sin\frac{2n\pi y}{b} \tag{2b}$$

$$w(x,y,t) = \left[w_{mn}^t(t) + w_{ij}^0\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{2c}$$

where m and n are the numbers of half-waves in x and y directions, respectively.

According to Eqs. (1a)-(1c) and (2a)-(2c), using the method of Ref. [37] which based on the Lagrange equation and the energy principle, ignoring the effect of damping, then, the (m, n) th nonlinear free vibration differential coefficient equation can be obtained as follows:

$$\ddot{\bar{w}}_{mn} + \hat{\alpha}_1 \bar{w}_{mn} + \hat{\alpha}_2 \left(\bar{w}_{mn}\right)^2 + \hat{\alpha}_3 \left(\bar{w}_{mn}\right)^3 = 0 \tag{3}$$

where $\dot{\bar{w}}_{mn}$ is the derivative of $\bar{w}_{mn}$ with respect to τ, $\ddot{\bar{w}}_{mn}$ is the second derivative of $\bar{w}_{mn}$ with respect to τ, $\bar{w}_{mn}$ and τ are the dimensionless quantities of w_{mn} and t, which based on the following transformation: $\tau = t\sqrt{D/h\rho a^2 b^2}$, $\bar{w}_{mn} = w_{mn}/h$, respectively. The dimensionless coefficient $\hat{\alpha}_1 \sim \hat{\alpha}_3$ are given as follows:

$$\hat{\alpha}_1 = \rho a^2 b^2 \left(hZ + \bar{w}_{mn}^0 \bar{w}_{mn}^0 G_1\right) / D\chi \tag{4a}$$

$$\hat{\alpha}_2 = \rho a^2 b^2 \left(\bar{w}_{mn}^0 G_2 + 3h^2 \delta_3 / 4\right) / D\chi \tag{4b}$$

$$\hat{\alpha}_3 = \rho a^2 b^2 \left(G_3 + h^3 \zeta_4 / 4\right) / D\chi \tag{4c}$$

where

$$Z = \frac{\pi^4 abD}{8}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 + \frac{m^4\pi^4}{2a^3}\delta_1 + \frac{n^4\pi^4}{2b^3}\delta_2 \tag{5a}$$

$$\chi = ab\rho h/8 + a\rho A_x \beta_1^x/4 + b\rho A_y \beta_1^y/4 \tag{5b}$$

$$\delta_1 = \left(EA_x d_x^2 + EI_x\right)\beta_1^x/2 \tag{5c}$$

$$\delta_2 = \left(EA_y d_y^2 + EI_y\right)\beta_1^y/2 \tag{5d}$$

$$\delta_3 = EA_x d_x \beta_2^x \gamma_1 + EA_y d_y \beta_2^y \gamma_2 \tag{5e}$$

$$\beta_1^x = \sum_{i=1}^{N_x} \sin^2 \frac{n\pi y_i}{b} \tag{5f}$$

$$\beta_1^y = \sum_{i=1}^{N_y} \sin^2 \frac{m\pi x_i}{a} \tag{5g}$$

$$\beta_2^x = \sum_{i=1}^{N_x} \sin^3 \frac{n\pi y_i}{b} \tag{5h}$$

$$\beta_2^y = \sum_{i=1}^{N_y} \sin^3 \frac{m\pi x_i}{a} \tag{5i}$$

$$\beta_3^x = \sum_{i=1}^{N_x} \sin^4 \frac{n\pi y_i}{b} \tag{5j}$$

$$\beta_3^y = \sum_{i=1}^{N_y} \sin^4 \frac{m\pi x_i}{a} \tag{5k}$$

$$\gamma_1 = \frac{m^3\pi^3}{3a^3}\left[\left(-1\right)^m - 1\right] \tag{5l}$$

$$\gamma_2 = \frac{n^3\pi^3}{3b^3}\left[\left(-1\right)^n - 1\right] \tag{5m}$$

$$G_1 = \frac{Eh}{2\left(1-\mu^2\right)}\left(2ah\phi_u \zeta_1 + 2bh\phi_v \zeta_2 + h^3\zeta_3\right) \tag{5n}$$

$$G_2 = \frac{Eh}{2\left(1-\mu^2\right)}\left[3ah\zeta_1\phi_u + 3bh\zeta_2\phi_v + \frac{3}{2}h^3\zeta_3\right] \tag{5o}$$

$$G_3 = \frac{Eh}{2\left(1-\mu^2\right)}\left[ah\zeta_1\phi_u + bh\zeta_2\phi_v + \frac{1}{4}h^3\zeta_3\right] \tag{5p}$$

$$\zeta_1 = \left(-\frac{m^3\pi^2 b}{3na^2} - \frac{mn\pi^2}{12b} + \frac{mn\pi^2}{4b}\mu\right)\left[\left(-1\right)^n - 1\right] \tag{5q}$$

$$\zeta_2 = \left(-\frac{n^3\pi^2 a}{3mb^2} - \frac{mn\pi^2}{12a} + \frac{mn\pi^2}{4a}\mu\right)\left[\left(-1\right)^m - 1\right] \tag{5r}$$

$$\zeta_3 = \frac{9m^4\pi^4 b}{64a^3} + \frac{m^2n^2\pi^4}{32ab} + \frac{9n^4\pi^4 a}{64b^3} \tag{5s}$$

$$\zeta_4 = \frac{3m^4\pi^4}{8a^3}EA_x\beta_3^x + \frac{3n^4\pi^4}{8b^3}EA_y\beta_3^y \tag{5t}$$

$$\phi_u = \frac{2bh^2\zeta_1\psi_2 - bh^2\zeta_2\varphi}{ab\varphi^2 - 4ab\psi_1\psi_2} \tag{5u}$$

$$\phi_v = \frac{2ah^2\zeta_2\psi_1 - ah^2\zeta_1\varphi}{ab\varphi^2 - 4ab\psi_1\psi_2} \tag{5v}$$

$$\varphi = \frac{4}{9}\left[\left(-1\right)^m - 1\right]\left[\left(-1\right)^n - 1\right]\left(1+\mu\right) \tag{5w}$$

$$\psi_1 = \frac{m^2\pi^2 b}{a} + \frac{n^2\pi^2 a}{8b}\left(1-\mu\right) \tag{5x}$$

$$\psi_2 = \frac{n^2\pi^2 a}{b} + \frac{m^2\pi^2 b}{8a}\left(1-\mu\right). \tag{5y}$$

3. Solution procedure

For Eq. (3), if the value of $\hat{\alpha}_2/\hat{\alpha}_1$ or $\hat{\alpha}_3/\hat{\alpha}_1$ is far less than 1, the Eq. (3) will be considered as the weakly nonlinear equation. Otherwise, it turns into the case of strong nonlinearity.

This paper considers the Eq. (3) as the strongly nonlinear case, and solves it through the Multiple scales Lindstedt-Poincare method (MSLP).

According to MSLP method, a parameter ε should be introduced so that the linear and nonlinear terms appear in the same perturbation equation. Suppose $\hat{\alpha}_2 = \varepsilon\alpha_2$, $\hat{\alpha}_3 = \varepsilon\alpha_3$, $\hat{\tau} = \bar{\Omega}\tau$, and substituting them into Eq. (3), we obtain:

$$\bar{\Omega}^2 \ddot{\bar{w}}_{mn} + \bar{\omega}_0^2 \bar{w}_{mn} + \varepsilon\alpha_2\left(\bar{w}_{mn}\right)^2 + \varepsilon^2\alpha_3\left(\bar{w}_{mn}\right)^3 = 0 \tag{6}$$

where $\bar{\omega}_0^2 = \hat{\alpha}_1$, $\bar{\omega}_0$ is the natural frequency, $\bar{\Omega}$ is the nonlinear frequency, $\ddot{\bar{w}}_{mn}$ is the second derivative of $\bar{w}_{mn}$ with respect to $\hat{\tau}$.

Introducing the time scales: $T_0 = \hat{\tau}$, $T_1 = \varepsilon\hat{\tau}$, $T_2 = \varepsilon^2\hat{\tau}$. An approximate solution can be given by

$$\bar{w}_{mn}\left(\hat{\tau}, \varepsilon\right) = U_0\left(T_0, T_1, T_2\right) + \varepsilon U_1\left(T_0, T_1, T_2\right) + \varepsilon^2 U_2\left(T_0, T_1, T_2\right). \tag{7}$$

Suppose:

$$\bar{\Omega}^2 = \bar{\omega}_0^2 + \varepsilon\bar{\Omega}_1 + \varepsilon^2\bar{\Omega}_2. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6), and equating the coefficients of ε^0, ε^1 and ε^2 in both sides of the equation, we obtain

$$\bar{\Omega}^2 D_0^2 U_0 + \bar{\Omega}^2 U_0 = 0 \tag{9}$$

$$\bar{\Omega}^2 D_0^2 U_1 + \bar{\Omega}^2 U_1 = -2\bar{\Omega}^2 D_0 D_1 U_0 + \bar{\Omega}_1 U_0 - \alpha_2 U_0^2 \tag{10}$$

$$\begin{aligned}\bar{\Omega}^2 D_0^2 U_2 + \bar{\Omega}^2 U_2 &= -2\bar{\Omega}^2 D_0 D_1 U_1 - 2\bar{\Omega}^2 D_0 D_2 U_0 \\ &\quad - \bar{\Omega}^2 D_1^2 U_0 + \bar{\Omega}_1 U_1 + \bar{\Omega}_2 U_0 \\ &\quad - 2\alpha_2 U_0 U_1 - \alpha_3 U_0^3 .\end{aligned} \tag{11}$$

The complex solution of Eq. (9) can be written as

$$U_0 = A\left(T_1, T_2\right) e^{iT_0} + \bar{A}\left(T_1, T_2\right) e^{-iT_0} \tag{12}$$

where A is unknown complex function, and $\bar{A}$ is the conjugate complex of A.

Substituting Eq. (12) into Eq. (10) yields:

$$\begin{aligned}\bar{\Omega}^2 D_0^2 U_1 + \bar{\Omega}^2 U_1 &= -2i\bar{\Omega}^2 D_1 A e^{iT_0} + \bar{\Omega}_1 A e^{iT_0} \\ &\quad - \alpha_2 A^2 e^{2iT_0} - \alpha_2 A\bar{A} + CC\end{aligned} \tag{13}$$

where CC is the complex conjugate part to the preceding terms. The secular terms are eliminated by:

$$-2i\bar{\Omega}^2 D_1 A e^{iT_0} + \bar{\Omega}_1 A e^{iT_0} = 0 . \tag{14}$$

$D_1 A = 0$ is selected, which implies $A = A\left(T_2\right)$ and $\bar{\Omega}_1 = 0$. Therefore the solution of Eq. (13) is:

$$U_1 = \frac{\alpha_2}{3\bar{\Omega}^2}\left(A^2 e^{2iT_0} + \bar{A}^2 e^{-2iT_0} - 6A\bar{A}\right). \tag{15}$$

Substituting Eqs. (12) and (15) into Eq. (11) yields:

$$\begin{aligned}&\bar{\Omega}^2 D_0^2 U_2 + \bar{\Omega}^2 U_2 \\ &= -\left[2i\bar{\Omega}^2 D_2 A - \bar{\Omega}_2 A + \left(3\alpha_3 - \frac{10\alpha_2^2}{3\bar{\Omega}^2}\right) A^2 \bar{A}\right] e^{iT_0} \\ &\quad + CC + NST\end{aligned} \tag{16}$$

where CC is the complex conjugate part to the preceding terms, and NST stands for non-secular terms. The secular terms are eliminated by:

$$2i\bar{\Omega}^2 D_2 A - \bar{\Omega}_2 A + \left(3\alpha_3 - \frac{10\alpha_2^2}{3\bar{\Omega}^2}\right) A^2 \bar{A} = 0 . \tag{17}$$

In order to solve Eq. (17), The complex function A can be expressed as

$$A = \frac{1}{2}\lambda\left(T_2\right) e^{i\theta(T_2)} \tag{18}$$

where $\lambda\left(T_2\right)$ and $\gamma\left(T_2\right)$ are real functions with respect to T_2. Substituting Eq. (18) into Eq. (17), and separating the real and imaginary part, respectively, then we obtain:

$$\bar{\Omega}^2 D_2 \lambda = 0 \tag{19}$$

$$\bar{\Omega}^2 D_2 \theta = -\frac{1}{2}\bar{\Omega}_2 + \left(\frac{9\alpha_3\bar{\Omega}^2 - 10\alpha_2^2}{24\bar{\Omega}^2}\right)\lambda^2 . \tag{20}$$

For the steady response, it demands $D_2\lambda = 0$ and $D_2\gamma = 0$, thus:

$$\bar{\Omega}_2 = \left(\frac{18\alpha_3\bar{\Omega}^2 - 20\alpha_2^2}{24\bar{\Omega}^2}\right)\lambda^2 . \tag{21}$$

Substituting Eq. (21) into Eq. (8), the nonlinear frequency can be solved as follows:

$$\bar{\Omega} = \bar{\omega}_0 \sqrt{\frac{4\bar{\omega}_0^2 + 3\varepsilon^2\alpha_3\lambda^2}{8\bar{\omega}_0^2} + \sqrt{\left(\frac{4\bar{\omega}_0^2 + 3\varepsilon^2\alpha_3\lambda^2}{8\bar{\omega}_0^2}\right)^2 - \frac{5\varepsilon^2\alpha_2^2\lambda^2}{6\bar{\omega}_0^4}}} . \tag{22}$$

4. Numerical results and discussion

4.1 Example A

The fundamental parameters of the stiffened plate with four edges simply supported are as follows: a = 3 m, b = 2 m, E = 2.06×10^{11} Pa, $\rho = 7.85\times10^3$ kg/m^3, h = 0.014 m, $N_x = N_y = 1$, μ = 0.3, $A_x = 7.2\times10^{-4}$ m^2, $A_y = 6.0\times10^{-4}$ m^2, $EI_x = 1.78\times10^5$ N·m^2, $EI_y = 1.03\times10^5$ N·m^2.

According to the analysis of parameters mentioned above, the (2, 2)th mode of the stiffened plate is of strong nonlinearity, so this paper investigated the influence of the initial geometric imperfections of plate through the (2, 2)th mode. Four cases are analyzed including: $w_1^0 = 0$, $w_2^0 = 0.001\,m$, $w_3^0 = 0.002\,m$ and $w_4^0 = 0.003\,m$.

Meanwhile, for verifying the accuracy of the method of this paper, the results calculated by the Modified Lindstedt-Poincare method (MLP) which is suitable for application in strongly nonlinear case [38] are listed. The results calculated by the Multiple scales method (MS) which is suitable for application in weakly nonlinear case [39] are also listed for comparison.

The following Figs. 2-7 are about the amplitude λ-frequency ratio f curve, where frequency ratio f is $\bar{\Omega}/\bar{\omega}_0$ which calculated by Eq. (22).

Figs. 2-5 shows that the results calculated by MSLP method are very close to MLP method, and the Eq. (22) deduced by this paper is correct. However, the results calculated by the MS method are very different from MSLP and MLP method. It shows that the MS method can not be applied to calculate strongly nonlinear case.

From Fig. 6, it can be observed that the initial geometric imperfections of plate affect the frequency ratio. Given the changes of imperfection within 0 to 0.003 m, the frequency

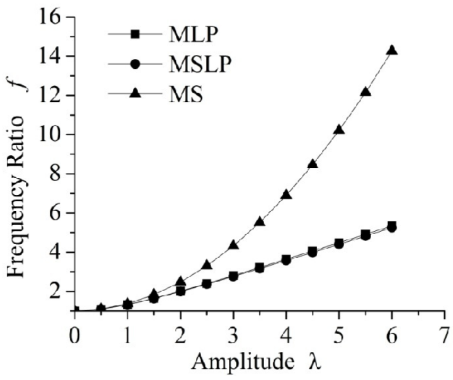

Fig. 2. The λ-f curve ($w_1^0 = 0$).

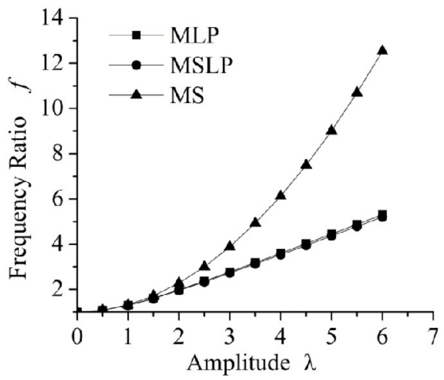

Fig. 3. The λ-f curve ($w_2^0 = 0.001\ m$).

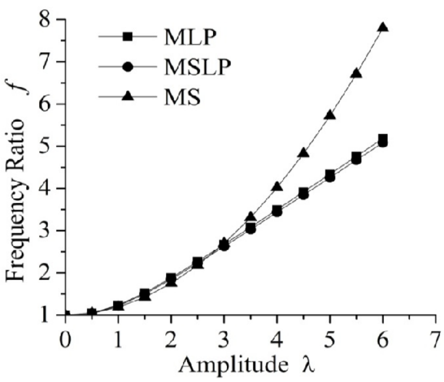

Fig. 4. The λ-f curve ($w_3^0 = 0.002\ m$).

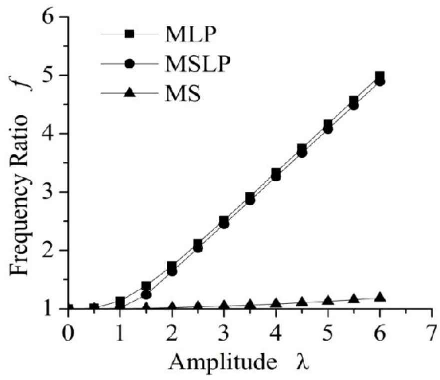

Fig. 5. The λ-f curve ($w_4^0 = 0.003\ m$).

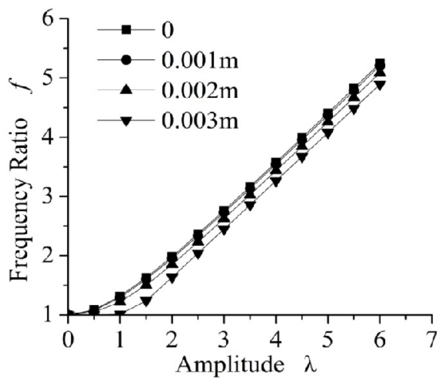

Fig. 6. The results calculated by MSLP method.

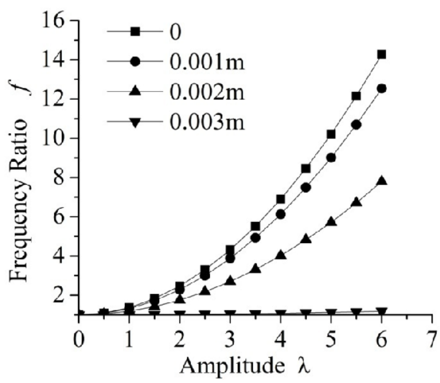

Fig. 7. The results calculated by MS method.

ratio reduced as the initial geometric imperfections of plate increased, and the ratio declined about 4% at most. In other words, the increase of the initial geometric imperfections of plate can lead to the decrease of nonlinear effect.

From Fig. 7, the results calculated by MS method also show that the increase of the initial geometric imperfections of plate will lead to the decrease of nonlinear effect. However, with the imperfection changes from 0 to 0.003 m, the frequency ratio reaches declined about 85% at most. Comparing the results calculated by MSLP method, it is easy to see that using MS method to study strongly nonlinear vibration can lead to serious mistakes.

4.2 Example B

The fundamental parameters of the stiffened plates with four edges simply supported are as follows: $a = 3$ m, $b = 2$ m, $E = 2.06 \times 10^{11}$ Pa, $\rho = 7.85 \times 10^3$ kg/m^3, $h = 0.014$ m, $N_x = N_y = 3$, $\mu = 0.3$, $A_x = 4.8\times10^{-4}$ m^2, $A_y = 3.6\times10^{-4}$ m^2, $EI_x = 5.27 \times 10^4$ N·m^2, $EI_y = 2.22 \times 10^4$ N·m^2.

According to the analysis of parameters mentioned above, the (1, 2)th mode of this stiffened plate is of strong nonlinearity, so the (1, 2)th mode is investigated. Similarly, four cases are analyzed including: $w_1^0 = 0$, $w_2^0 = 0.001\,m$, $w_3^0 = 0.002\,m$ and $w_4^0 = 0.003\,m$.

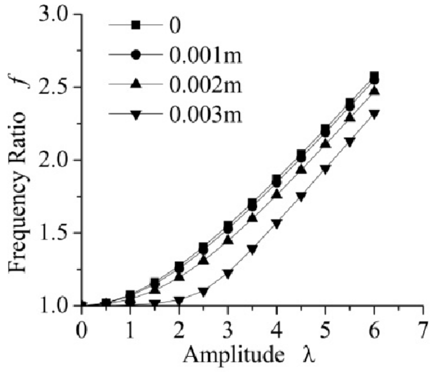

Fig. 8. The results calculated by MSLP method.

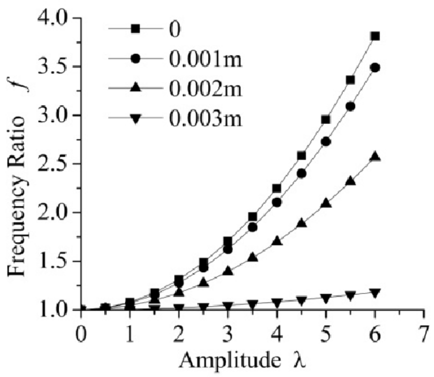

Fig. 9. The results calculated by MS method.

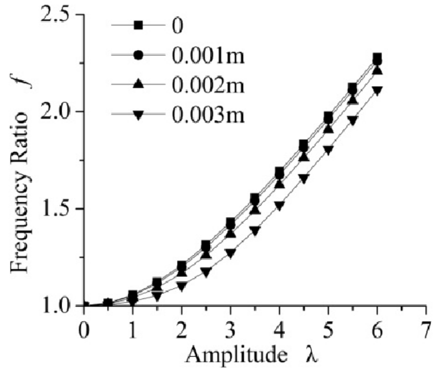

Fig. 10. The results calculated by MSLP method.

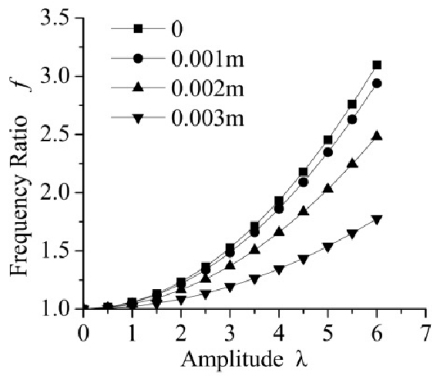

Fig. 11. The results calculated by MS method.

As the same as Fig. 6, it can be observed from Fig. 8 that the initial geometric imperfections of plate affect the frequency ratio. The frequency ratio reduces about 10% with the imperfection changes from 0 to 0.003 m.

Similarly, it can be observed from Figs. 8 and 9 that using MS method to study strongly nonlinear vibration can lead to serious mistakes.

4.3 Example C

Using the fundamental parameters of example B, and setting $N_x = N_y = 5$. The (1, 2)th mode is investigated.

As the same as Figs. 6 and 8, it can be observed from Fig. 10 that the initial geometric imperfections of plate affect the frequency ratio. The frequency ratio reduces about 7% with the imperfection changes from 0 to 0.003 m.

From Figs. 10 and 11, it can be observed that there is a huge difference among the results by using MSLP method and MS method, which indicates that serious mistakes can be occurred if using MS method to study strongly nonlinear vibration.

5. Conclusions

This paper investigated the strongly nonlinear free vibration of four edges simply supported stiffened plate with initial geometric imperfections. The Multiple scales Lindstedt-Poincare method (MSLP) and the Modified Lindstedt-Poincare method (MLP) are used to solve the governing equations of vibration. Numerical examples for stiffened plates with different initial geometric imperfections are presented in order to discuss the influences to the strongly nonlinear free vibration of the stiffened plate, which yields the following conclusions:

(1) The results calculated by MSLP method are very close to MLP method. It supported that the equations deduced by this paper is correct.

(2) The frequency ratio reduced as the initial geometric imperfections of plate increased, which showed that the increase of the initial geometric imperfections of plate can lead to the decrease of nonlinear effect.

(3) There is a huge difference among the results by using MSLP method and MS method. It indicates that using MS method to study strongly nonlinear vibration can lead to serious mistakes.

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Zhaoting Chen is a doctoral candidate pursuing a Ph.D. degree in Bridge and Tunnel Engineering at South China University of Technology, China. His current research area is mainly on the nonlinear dynamics analysis of stiffened plate using analytical method.