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Research on the design of non-traditional dynamic vibration absorber for damped structures under ground motion †

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Abstract

An analytical approach is presented to investigate the optimal problem of non-traditional type of Dynamic vibration absorber (DVA) for damped primary structures subjected to ground motion. Different from the standard configuration, the non-traditional DVA contains a linear viscous damper connecting the absorber mass directly to the ground instead of the main mass. There have been many studies on the design of the non-traditional DVA for undamped primary structures. Those studies have shown that the non-traditional DVA produces better performance than the standard DVA does. When damping is present at the primary system, there are very few works on the non-traditional dynamic vibration absorber. To the best of our knowledge, there is no study on the design of non-traditional DVA for damped structures under ground motion. We propose a simple method to determine the approximate analytical solutions of the non-traditional DVA when the damped primary structure is subjected to ground motion. The main idea of the study is based on the criterion of the equivalent linearization method to replace approximately the original damped structure by an equivalent undamped one. Then the approximate analytical solution of the DVA's parameters is given by using known results for the undamped structure obtained. Comparisons have been done to validate the effectiveness of the obtained results.

Keywords: Dynamic vibration absorber; Non-traditional; Damped structure; Weighted dual criterion; Analytical solutions

1. Introduction

A Dynamic vibration absorber (DVA) is a passive control device installed in primary structures to reduce harmful vibration. The DVA without damping element was first introduced by Frahm [1] in 1909, but only useful in a narrow range of frequencies very close to the DVA's frequency. In 1928, Ormondroyd and Den Hartog [2] determined that the DVA containing a viscous damper was effective to an extended range of frequencies. This type DVA is now known as the standard DVA, where a spring element and a viscous element are arranged in parallel. The principal parameters of the DVA are its tuning ratio and damping ratio because the mass ratio of the DVA to the primary structures is usually fixed to be few percent.

There have been many optimal criteria given to design the standard DVA for undamped primary structures. Three typical criteria are H_∞ optimization, H_2 optimization, and stability maximization. The H_∞ optimization was proposed by Ormondroyd and Den Hartog [2] when the primary structure was subjected to harmonic excitation. The purpose was to minimize the maximum amplitude magnification factor of the primary structure. The optimum tuning ratio of the DVA was first derived by Hahnkamm [3], and later Brock [4] gave the optimum damping ratio. The optimal parameters of the DVA then were introduced by Den Hartog [5]. The optimal tuning ratio and damping ratio of the standard DVA determined by using the fixed-point method are not exact, because some approximations are taken when they are derived. However, when Nishihara and Asami [6] proposed the exact solutions and compared those with the results given by Den Hartog, they found that both optimal tuning ratio and damping ratio presented by Den Hartog were very close to the exact solutions. Therefore, the fixed-point theory provided a very good approximation of the exact solutions for the H_∞ optimization in practice because the exact solution was too complicated to use. The H_2 optimization was suggested by Crandall and Mark [7] in 1963 when the primary structure is subjected to random excitation. The purpose was to minimize the area under the frequency response curve of the system (i.e., total vibration energy of the structure over all frequencies). After that, Iwata [8] and Asami [9] gave the optimal parameters of a DVA us-

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ing the H_2 optimization. The stability maximization criterion and the exact solutions of the optimum parameters of a DVA were first given by Yamaguchi [10] in 1988 where the aim was to improve the transient vibration of the structure. In summary, all optimal criteria of the standard DVA have been already solved analytically for undamped primary structures.

When the primary structure takes into account damping, however, it is difficult to obtain analytical solutions for the optimum parameters of the standard DVA. Ioi and Ikeda [11] gave the empirical formulae based on the numerical method. Randall et al. [12] suggested numerical optimization procedures for evaluating the optimum DVA's parameters. Thompson [13] proposed a method where the tuning ratio was optimized numerically and the optimum damping ratio of the DVA was determined analytically using the optimum value obtained for the tuning ratio. Warburton [14] performed a detailed numerical study for a lightly damped structure subject to both harmonic and random excitation with DVA, and then the optimal parameters of the DVA were presented in the form of design tables. Fujino and Abe [15] used a perturbation technique to derive formulae for the DVA's optimal parameters, which may be used with good accuracy for the mass ratio less than 2 percent and for very low values of the structural damping ratio. In 1997, Nishihara and Matsuhisa [16] gave the exact solution for the stability maximization criterion. Pennestrì [17] proposed a min-max design of a DVA where a min-max objective function subject to six constraint equations with seven unknown variables was found. In 2002, Asami et al. [18] presented a series solution for the H_∞ optimization and an exact solution for the H_2 optimization but their solution was extremely complicated. Based on an approximate assumption of the existence of two fixed points, Ghosh and Basu [19] gave a closed-form expression for the optimal tuning ratio of the DVA. Brown and Singh [20] developed a minimax procedure to design a DVA in the presence of uncertainties in the forcing frequency range. Anh and Nguyen [21, 22] suggested approximate analytical solutions of the optimal tuning ratio of the DVA by using the idea of the equivalent linearization method for the H_∞ optimization. Tigli [23] proposed the exact optimum design parameters of the DVA for the H_2 criterion in the case of minimizing the variance of the velocity and approximate solutions in the displacement and acceleration cases.

A non-traditional type DVA was proposed by Ren [24] and Liu and Liu [25]. Different from the traditional configuration, the non-traditional DVA contains a linear viscous damper connecting the absorber mass directly to ground instead of the main mass. There have been some studies on the design of the non-traditional DVA but mainly for undamped primary structures. Liu and Liu [25] gave the optimum parameters of the non-traditional DVA for the case of a primary structure subjected to excitation force. Cheung and Wong [26] studied the non-traditional DVA by minimizing the maximum vibration velocity response. Wong and Cheung [27] considered a case when the primary structure is subjected to ground motion by minimizing the absolute displacement of the primary mass. Cheung and Wong [28, 29] designed the non-traditional DVA by using the H_∞ optimization and H_2 optimization. When the primary structure is damped, there are very few studies on the design of the non-traditional DVA. Liu and Coppola [30] proposed an approximate solution based on Ghosh and Basu's method [19] and employed two numerical methods to obtain the optimal parameters of the non-traditional DVA attached to damped structures.

Equivalent linearization is a common approach to approximate analysis of dynamical systems. The original linearization for deterministic systems was proposed by Krylov and Bogoliubov [31]. Then Caughey [32, 33] expanded the method for stochastic systems. Thenceforth, there have been some extended versions of the equivalent linearization method. Recently, Anh et al. [34, 35] proposed a so-called dual criterion for the equivalent linearization method. Anh and Nguyen [36] suggested a new criterion called the weighted dual criterion where the conventional criterion and dual criterion can be obtained from the weighted dual criterion as special cases. Based on the idea of the weighted dual criterion, the authors give an analytical approach to the design of the non-traditional DVA for damped structures under ground motion by replacing the original damped structure by an equivalent undamped structure. Comparisons have been done to validate the accuracy of the obtained results.

2. Weighted dual equivalent linearization criterion

To describe the weighted dual equivalent linearization technique, we consider a single degree of freedom system as follows:

$$\ddot{x} + 2h\dot{x} + \omega_0^2 x + g(x,\dot{x}) = f(t), \tag{1}$$

where h and ω_0 are two constants, $g(x,\dot{x})$ is a nonlinear function of two arguments x and $\dot{x}$.

Linearizing Eq. (1), we obtain an equation in the linear form as

$$\ddot{x} + (2h+b)\dot{x} + (\omega_0^2 + k)x = f(t), \tag{2}$$

where the linearization coefficients b,k are determined by an optimal criterion. The most extensively used criterion is the mean square error criterion. This criterion requires that the mean square of the error $e(x,\dot{x}) = g(x,\dot{x}) - b\dot{x} - kx$ between the original Eq. (1) and its linearized Eq. (2) is minimal

$$\langle e^2(x,\dot{x})\rangle = \langle (g(x,\dot{x}) - b\dot{x} - kx)^2 \rangle \to \min\ (b,k), \tag{3}$$

where the operator $\langle . \rangle$ is defined as the mean value on a period or a part of the period in the case of deterministic systems and the expectation operator in the case of stochastic systems.

Criterion Eq. (3) is called the classical criterion. In the case of the major nonlinearity the solution error using the classical criterion Eq. (3) may be unacceptable. To reduce the solution error, Anh et al. [34, 35] proposed a dual approach to the equivalent linearization method as follows:

$$\left\langle \left(g\left(x,\dot{x}\right)-b\dot{x}-kx\right)^2\right\rangle+\left\langle \left(b\dot{x}+kx-\lambda g\left(x,\dot{x}\right)\right)^2\right\rangle, \quad \rightarrow \min\ (b,k,\lambda) \tag{4}$$

where the first term describes the conventional replacement and the second term is its dual replacement.

Afterwards, the authors [36] suggested a so-call weighted dual criterion in the following form

$$\rho\left\langle \left(g\left(x,\dot{x}\right)-b\dot{x}-kx\right)^2\right\rangle+ \left(1-\rho\right)\left\langle \left(b\dot{x}+kx-\lambda g\left(x,\dot{x}\right)\right)^2\right\rangle \rightarrow \min_{b,k,\lambda} \tag{5}$$

where ρ is an weighted parameter varying in the interval $0\le\rho\le1$. When $\rho=1$ and $\rho=1/2$ we obtain the conventional and dual criteria, respectively, as special cases. Using criterion Eq. (5) we get the local linearization coefficients b, k as functions of ρ corresponding to a given local value ρ

$$b=b\left(\rho\right), \quad k=k\left(\rho\right). \tag{6}$$

The global values of the coefficients of the linearization are to be obtained as the averaging values of $b\left(\rho\right)$, $k\left(\rho\right)$ over the interval $0\le\rho\le1$

$$b=\int_0^1 b\left(\rho\right)d\rho, \quad k=\int_0^1 k\left(\rho\right)d\rho. \tag{7}$$

Using the idea of the replacement of the equivalent linearization, the authors suggest the general replacement

$$\rho\left\langle A-\alpha B\right\rangle+\left(1-\rho\right)\left\langle \alpha B-\beta A\right\rangle\rightarrow \min_{\alpha,\beta}. \tag{8}$$

When A is a nonlinear system and B is a linear system, we have the equivalent linearization method. When A is a damped structure and B is an undamped structure, we have the problem considered in the paper.

In the following, we use the idea of the above criteria in the design problem of non-traditional DVA for damped primary structures under ground motion. Namely, we will replace the damped primary structure by an equivalent undamped one and then use known results for undamped structures to give approximate analytical solutions of the non-traditional DVA's parameters.

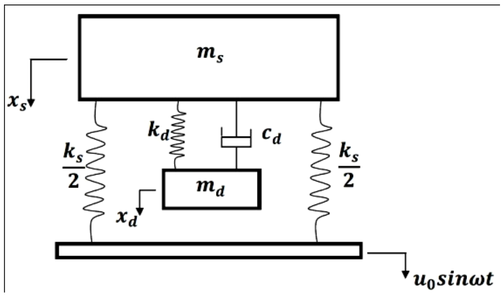


Fig. 1. The standard DVA attached to an undamped primary structure.

3. Parameters of dynamic vibration absorber for undamped structures

3.1 Standard dynamic vibration absorber

Fig. 1 describes a standard DVA attached to an undamped primary structure under ground motion. When minimizing the absolute displacement of the primary structure, the transmissibility is given by [2]

$$A_S=\left|\frac{x_s}{u_0}\right|=\sqrt{\frac{\left(\alpha^2-\beta^2\right)^2+\left(2\alpha\beta\xi_d\right)^2}{\left[\alpha^2-\left(1+\alpha^2+\mu\alpha^2\right)\beta^2+\beta^4\right]^2+\left[2\alpha\beta\xi_d\left(1-\beta^2-\mu\beta^2\right)\right]^2}} \tag{9}$$

where

$$\mu=\frac{m_d}{m_s},\ \omega_s=\sqrt{\frac{k_s}{m_s}},\ \omega_s=\sqrt{\frac{k_d}{m_d}}, \quad \xi_d=\frac{c_d}{2m_d\omega_d},\ \alpha=\frac{\omega_d}{\omega_s},\ \beta=\frac{\omega}{\omega_s}. \tag{10}$$

As explained earlier, the purpose of the H_∞ criterion was to minimize the maximum amplitude magnification factor of the primary structure, i.e.

$$\min_{\alpha,\xi_d}\left(\max_{\beta} A_s\right). \tag{11}$$

Using the fixed-points method, Den Hartog proposed the optimal parameters of the standard DVA for H_∞ optimization as [5]

$$\alpha=\frac{1}{1+\mu};\ \xi_d=\sqrt{\frac{3\mu}{8\left(1+\mu\right)}}. \tag{12}$$

When considering the relative displacement, the transmissibility is [14]

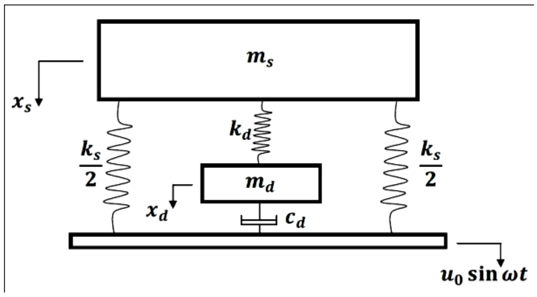


Fig. 2. The non-traditional DVA attached to an undamped primary structure.

$$R_S = \left| \frac{x_s - u(t)}{u_0} \right| = \sqrt{\frac{\left[\alpha^2(1+\mu) - \beta^2\right]^2 + \left[2\alpha\beta\xi_d(1+\mu)\right]^2}{\left[\alpha^2 - \left(1+\alpha^2+\mu\alpha^2\right)\beta^2 + \beta^4\right]^2 + \left[2\alpha\beta\xi_d\left(1-\beta^2-\mu\beta^2\right)\right]^2}} \quad (13)$$

and the optimal parameters of the DVA are derived as follows [14]:

$$\alpha = \frac{\sqrt{1-\mu/2}}{1+\mu}; \; \xi_d = \sqrt{\frac{3\mu}{8(1+\mu)(1-\mu/2)}} \, . \quad (14)$$

3.2 Non-traditional dynamic vibration absorber

Fig. 2 shows a non-traditional DVA installed in an undamped structure under ground motion. In the case of absolute displacement of the primary structure, the transmissibility is [27]

$$A_N = \left| \frac{x_s}{u_0} \right| = \sqrt{\frac{\left(\alpha^2 - \beta^2\right)^2 + \left[2\alpha\beta\xi_d\left(1+\mu\alpha^2\right)^2\right]^2}{\left[\alpha^2 - \left(1+\alpha^2+\mu\alpha^2\right)\beta^2 + \beta^4\right]^2 + \left[2\alpha\beta\xi_d\left(1-\beta^2+\mu\alpha^2\right)\right]^2}} \quad (15)$$

and the non-traditional DVA's coefficients are presented as [27]

$$\alpha = \frac{1}{\sqrt{1-\mu}}; \; \xi_d = \sqrt{\frac{\mu(3-\mu)}{8}} \, . \quad (16)$$

When minimizing the relative displacement of the primary mass, to the best of our knowledge, there does not exist any result in the literature. Therefore, the parameters of non-traditional DVA in this case are proposed in the paper. The transmissibility is of the form

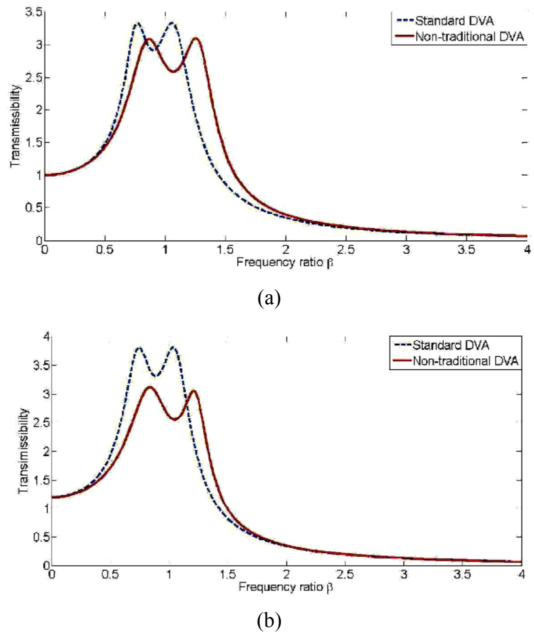


Fig. 3. Comparison of two DVA models in the case of (a) absolute displacement; (b) relative displacement.

$$R_N = \left| \frac{x_s - u(t)}{u_0} \right| = \sqrt{\frac{\left[\alpha^2(1+\mu) - \beta^2\right]^2 + \left(2\alpha\beta\xi_d\right)^2}{\left[\alpha^2 - \left(1+\alpha^2+\mu\alpha^2\right)\beta^2 + \beta^4\right]^2 + \left[2\alpha\beta\xi_d\left(1-\beta^2+\mu\alpha^2\right)\right]^2}} \, . \quad (17)$$

By using the fixed-point method, the non-traditional DVA's parameters in this case are determined by the following set of equations:

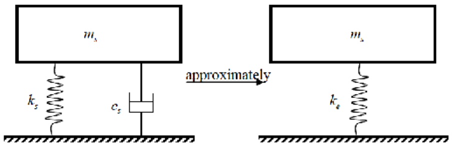
$$\frac{\partial R_N}{\partial \xi_d} = 0; \; \frac{\partial R_N}{\partial \beta} = 0; \; R_N\big|_{\beta_P} = R_N\big|_{\beta_Q} \quad (18)$$

where P, Q are two fixed points of the graph R_N versus the frequency ratio β. Solving Eq. (18) yields

$$\alpha = \sqrt{\frac{2}{2-\mu}}; \; \xi_d = \sqrt{\frac{\mu\left(-\mu^2+8\mu+12\right)}{8\left(\mu^2+2\mu+4\right)}} \, . \quad (19)$$

Fig. 3 illustrates comparisons between the standard DVA and non-traditional DVA where the mass ratio μ is equal to 0.2.

We can observe that the peaks of the transmissibilities when using the non-traditional DVA are significantly lower than those in the case of standard DVA. So the non-traditional DVA provides a greater vibration reduction than the standard DVA does.



(a) (b)

Fig. 4. An approximation of primary structure.

4. Equivalent undamped structure

The main idea of the present work is to use the dual and weighted dual criteria presented in Sec. 2 to replace approximately the original damped structure as Fig. 4(a) by an equivalent undamped structure as shown in Fig. 4(b).

In Fig. 4(a) with the original damped structure, the equation of motion is given as follows:

$$\ddot{x}_s + 2\xi_s\omega_s\dot{x}_s + \omega_s^2 x_s = 0 . \quad (20)$$

In Fig. 4(b) with the equivalent undamped structure, the motion equation has of the form

$$\ddot{x}_s + \omega_e^2 x_s = 0 , \quad (21)$$

where ω_e is an equivalent frequency that is denoted as

$$\omega_e^2 = \gamma + \omega_s^2 . \quad (22)$$

Using the idea of the weighted dual criterion Eq. (8), we replace $2\xi_s\omega_s\dot{x}_s$ with γx_s; thereby the term γ is determined by the following criterion:

$$W = \rho\left\langle\left(2\xi_s\omega_s\dot{x}_s - \gamma x_s\right)^2\right\rangle_D + (1-\rho)\left\langle\left(\gamma x_s - 2\lambda\xi_s\omega_s\dot{x}_s\right)^2\right\rangle_D \to \min_{\gamma,\lambda} \quad (23)$$

in which the operator $\langle . \rangle_D$ is defined as

$$\langle . \rangle_D = \frac{1}{D}\int_0^D (.)\,dt \quad (24)$$

with D is a given integral region. Criterion Eq. (23) leads to a set of equations in terms of two coefficients γ and λ as follows:

$$\frac{\partial W}{\partial \gamma} = 0; \ \frac{\partial W}{\partial \lambda} = 0 . \quad (25)$$

Substituting the expression of the function W in the criterion Eq. (23) into the set of Eq. (25), we obtain

$$\begin{aligned} &\langle x_s^2\rangle_D\,\gamma - 2(1-\rho)\xi_s\omega_s\langle x_s\dot{x}_s\rangle_D\,\lambda - 2\rho\xi_s\omega_s\langle x_s\dot{x}_s\rangle_D = 0 \\ &2\xi_s\omega_s\langle \dot{x}_s^2\rangle_D\,\lambda - \langle x_s\dot{x}_s\rangle_D\,\gamma = 0 . \end{aligned} \quad (26)$$

Solving Eq. (26) with respect to two unknown constants γ and λ yields

$$\begin{aligned} \gamma &= 2\xi_s\omega_s\frac{\rho\langle \dot{x}_s^2\rangle_D\langle x_s\dot{x}_s\rangle_D}{\langle x_s^2\rangle_D\langle \dot{x}_s^2\rangle_D - (1-\rho)\langle x_s\dot{x}_s\rangle_D^2} \\ \lambda &= \frac{\rho\langle x_s\dot{x}_s\rangle_D^2}{\langle x_s^2\rangle_D\langle \dot{x}_s^2\rangle_D - (1-\rho)\langle x_s\dot{x}_s\rangle_D^2} \end{aligned} . \quad (27)$$

Using a transformation of variable $\varphi = \omega_e t$, we have

$$\langle . \rangle_D = \langle . \rangle_\Phi = \frac{1}{\Phi}\int_0^\Phi (.)\,d\varphi \quad \text{with}\ \Phi = \omega_e D . \quad (28)$$

Therefore, Eq. (27) can be rewritten in the form

$$\begin{aligned} \gamma &= 2\xi_s\omega_s\frac{\rho\langle \dot{x}_s^2\rangle_\Phi\langle x_s\dot{x}_s\rangle_\Phi}{\langle x_s^2\rangle_\Phi\langle \dot{x}_s^2\rangle_\Phi - (1-\rho)\langle x_s\dot{x}_s\rangle_\Phi^2} \\ \lambda &= \frac{\rho\langle x_s\dot{x}_s\rangle_\Phi^2}{\langle x_s^2\rangle_\Phi\langle \dot{x}_s^2\rangle_\Phi - (1-\rho)\langle x_s\dot{x}_s\rangle_\Phi^2} \end{aligned} . \quad (29)$$

Using Eq. (21), we obtain the solution

$$x_s = a\cos\varphi,\ \varphi = \omega_e t + \varphi_0 . \quad (30)$$

Introducing Eq. (30) into Eq. (28) yields

$$\begin{aligned} \langle x_s^2\rangle_\Phi &= \frac{a^2}{2\Phi}\left(\Phi + \frac{1}{2}\sin 2\Phi\right) \\ \langle x_s\dot{x}_s\rangle_\Phi &= \frac{a^2\omega_e}{4\Phi}\left(\cos 2\Phi - 1\right) \\ \langle \dot{x}_s^2\rangle_\Phi &= \frac{a^2\omega_e^2}{2\Phi}\left(\Phi - \frac{1}{2}\sin 2\Phi\right) \end{aligned} . \quad (31)$$

Substituting Eq. (31) into the first equation of Eq. (29) and combining with Eq. (22), after simplifying we get

$$\omega_e^2 + \frac{2\rho(1-\cos 2\Phi)(2\Phi - \sin 2\Phi)}{4\Phi^2 - \sin^2 2\Phi - (1-\rho)(1-\cos 2\Phi)^2}\xi_s\omega_s\omega_e - \omega_s^2 = 0 . \quad (32)$$

Eq. (32) is a quadratic equation in terms of ω_e. Solving this equation, we easily obtain

$$\omega_e = \omega_s\left(\begin{aligned} &\sqrt{1 + \left[\frac{\rho(1-\cos 2\Phi)(2\Phi - \sin 2\Phi)}{4\Phi^2 - \sin^2 2\Phi - (1-\rho)(1-\cos 2\Phi)^2}\right]^2\xi_s^2} \\ &-\frac{\rho(1-\cos 2\Phi)(2\Phi - \sin 2\Phi)}{4\Phi^2 - \sin^2 2\Phi - (1-\rho)(1-\cos 2\Phi)^2}\xi_s \end{aligned}\right) . \quad (33)$$

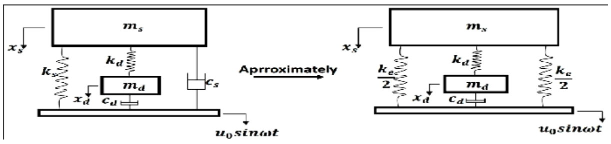


Fig. 5. A non-traditional DVA attached to a damped structure under ground motion and the equivalent system.

Eq. (33) is the general case where the integration domain is arbitrarily chosen. We take the mean value over a quarter of the period of the primary system [21], i.e., $\Phi = \pi/2$. Putting the value $\Phi = \pi/2$ into Eq. (33) yields

$$\omega_e = \omega_s \left(\sqrt{1 + \left[\frac{2\rho\pi}{\pi^2 - 4(1-\rho)} \right]^2 \xi_s^2} - \frac{2\rho\pi}{\pi^2 - 4(1-\rho)} \xi_s \right). \quad (34)$$

When $\rho = 1/2$ we have the result using the dual criterion

$$\omega_{e\_dua} = \omega_s \left(\sqrt{1 + \left(\frac{\pi}{\pi^2 - 2} \right)^2 \xi_s^2} - \frac{\pi}{\pi^2 - 2} \xi_s \right). \quad (35)$$

Using the weighted dual criterion, we obtain

$$\omega_{e\_wei} = \int_0^1 \omega_s \left(\begin{array}{l} \sqrt{1 + \left[\frac{2\rho\pi}{\pi^2 - 4(1-\rho)} \right]^2 \xi_s^2} \\ -\frac{2\rho\pi}{\pi^2 - 4(1-\rho)} \xi_s \end{array} \right) d\rho. \quad (36)$$

Integrating Eq. (36) leads to

$$\begin{aligned} \frac{\omega_{e\_wei}}{\omega_s} &= 1 - \frac{\pi\xi_s}{2} + \frac{\pi\xi_s^2}{\sqrt{\pi^2 + 4\xi_s^2} + \pi} + \frac{\pi(\pi^2 - 4)\xi_s}{8} \ln \frac{\pi\left(\sqrt{\pi^2 + 4\xi_s^2} + 2\xi_s\right)}{\pi^2 - 4} \\ &- \frac{\pi^2(\pi^2 - 4)\xi_s^2}{8\sqrt{4 + \pi^2\xi_s^2}} \ln \frac{2\pi^2(1 + \xi_s^2) + \pi\sqrt{(4 + \pi^2\xi_s^2)(\pi^2 + 4\xi_s^2)}}{(\pi^2 - 4)\left(2 + \sqrt{4 + \pi^2\xi_s^2}\right)}. \end{aligned} \quad (37)$$

Using the dual and weighted dual criteria, we have replaced the damped original structure by an equivalent undamped structure where the approximate frequencies ω_{e_dua} and ω_{e_wei} are presented in Eqs. (35) and (37), respectively. In the next section, we use these results to give the parameters of the non-traditional DVA attached to a damped linear structure under ground motion.

5. Parameters of nontraditional dynamic vibration absorber for damped linear structures

5.1 Absolute displacement

A non-traditional DVA attached to a damped primary structure under ground motion is shown in Fig. 5. When minimizing the absolute displacement of the primary structure, the transmissibility is

$$A_N = \left| \frac{x_s}{u_0} \right| = \sqrt{ \frac{ \left(\alpha^2 - \beta^2 - 4\xi_s\xi_d\alpha\beta^2 \right)^2 + \left[2\beta\left(\alpha\xi_d + \mu\alpha^3\xi_d + \xi_s\alpha^2 - \xi_s\beta^2 \right) \right]^2 }{ \left[\alpha^2 - \left(1 + \alpha^2 + \mu\alpha^2 + 4\alpha\xi_s\xi_d \right)\beta^2 + \beta^4 \right]^2 + \left[2\alpha\beta\xi_d\left(1 - \beta^2 + \mu\alpha^2 \right) + 2\xi_s\beta\left(\alpha^2 - \beta^2 \right) \right]^2 } } \quad (38)$$

where $\xi_s = c_s/2m_s\omega_s$ is the structural damping ratio.

Using the result Eq. (16) for the equivalent undamped structure, we have

$$\alpha_e = \frac{1}{\sqrt{1-\mu}}; \ \xi_{de} = \sqrt{\frac{\mu(3-\mu)}{8}}. \quad (39)$$

Note that

$$\alpha_e = \frac{\omega_d}{\omega_e}; \ \alpha = \frac{\omega_d}{\omega_s}. \quad (40)$$

Utilizing the dual criterion with ω_{e_dua} in Eq. (35), we obtain the non-traditional DVA's parameters as follows:

$$\begin{aligned} \alpha_{dua} &= \frac{1}{\sqrt{1-\mu}} \left(\sqrt{1 + \left(\frac{\pi}{\pi^2 - 2} \right)^2 \xi_s^2} - \frac{\pi}{\pi^2 - 2} \xi_s \right). \\ \xi_{ddua} &= \sqrt{\frac{\mu(3-\mu)}{8}} \end{aligned} \quad (41)$$

Meanwhile using the weighted dual criterion Eq. (37) leads to

$$\begin{aligned} \alpha_{wei} &= \frac{1}{\sqrt{1-\mu}} \left(\begin{array}{l} 1 - \frac{\pi\xi_s}{2} + \frac{\pi\xi_s^2}{\sqrt{\pi^2 + 4\xi_s^2} + \pi} \\ + \frac{\pi(\pi^2 - 4)\xi_s}{8} \ln \frac{\pi\left(\sqrt{\pi^2 + 4\xi_s^2} + 2\xi_s\right)}{\pi^2 - 4} \\ - \frac{\pi^2(\pi^2 - 4)\xi_s^2}{8\sqrt{4 + \pi^2\xi_s^2}} \times \\ \times \ln \frac{2\pi^2(1 + \xi_s^2) + \pi\sqrt{(4 + \pi^2\xi_s^2)(\pi^2 + 4\xi_s^2)}}{(\pi^2 - 4)\left(2 + \sqrt{4 + \pi^2\xi_s^2}\right)} \end{array} \right) \\ \xi_{dwei} &= \sqrt{\frac{\mu(3-\mu)}{8}}. \end{aligned} \quad (42)$$

Eqs. (41) and (42) are the approximate analytical solutions for the parameters of the non-traditional DVA attached to the primary damped structure under ground motion when optimizing the absolute displacement.

5.2 Relative displacement

In the case of relative displacement, the transmissibility is given as follows:

$$R_N = \left|\frac{x_s - u(t)}{u_0}\right| = \sqrt{\frac{\left[\alpha^2(1+\mu)-\beta^2\right]^2 + \left(2\alpha\beta\xi_d\right)^2}{\left[\alpha^2 - \left(1+\alpha^2+\mu\alpha^2+4\alpha\xi_s\xi_d\right)\beta^2+\beta^4\right]^2 + \left[2\alpha\beta\xi_d\left(1-\beta^2+\mu\alpha^2\right)+2\xi_s\beta\left(\alpha^2-\beta^2\right)\right]^2}}\ . \tag{43}$$

Using Eq. (19) and the result of dual criterion Eq. (35) yields

$$\alpha_{dua} = \sqrt{\frac{2}{2-\mu}}\left(\sqrt{1+\left(\frac{\pi}{\pi^2-2}\right)^2\xi_s^2} - \frac{\pi}{\pi^2-2}\xi_s\right)$$
$$\xi_{ddua} = \sqrt{\frac{\mu\left(-\mu^2+8\mu+12\right)}{8\left(\mu^2+2\mu+4\right)}}\ . \tag{44}$$

And using Eq. (19) and the weighted dual criterion Eq. (37), we obtain

$$\alpha_{wei} = \sqrt{\frac{2}{2-\mu}}\left(\begin{aligned}&1-\frac{\pi\xi_s}{2}+\frac{\pi\xi_s^2}{\sqrt{\pi^2+4\xi_s^2}+\pi}\\&+\frac{\pi\left(\pi^2-4\right)\xi_s}{8}\ln\frac{\pi\left(\sqrt{\pi^2+4\xi_s^2}+2\xi_s\right)}{\pi^2-4}\\&-\frac{\pi^2\left(\pi^2-4\right)\xi_s^2}{8\sqrt{4+\pi^2\xi_s^2}}\ln\frac{2\pi^2\left(1+\xi_s^2\right)+\pi\sqrt{\left(4+\pi^2\xi_s^2\right)\left(\pi^2+4\xi_s^2\right)}}{\left(\pi^2-4\right)\left(2+\sqrt{4+\pi^2\xi_s^2}\right)}\end{aligned}\right)$$
$$\xi_{dwei} = \sqrt{\frac{\mu\left(-\mu^2+8\mu+12\right)}{8\left(\mu^2+2\mu+4\right)}}\ . \tag{45}$$

Eqs. (44) and (45) are solutions when minimizing the relative displacement of the primary structure. The effectiveness of above results will be verified in next section.

6. Comparisons

As far as we know, there has been no study on the non-traditional dynamic vibration absorber when the damped primary structure is subjected to ground motion. Hence, there are no exact formulas for optimal parameters of non-traditional DVA available in the literature. In this section, the results Eqs. (40), (41), (43) and (44) proposed in the paper are compared with results obtained by numerical methods. The numerical optimizations are done by using the *fminsearch* command in MATLAB.

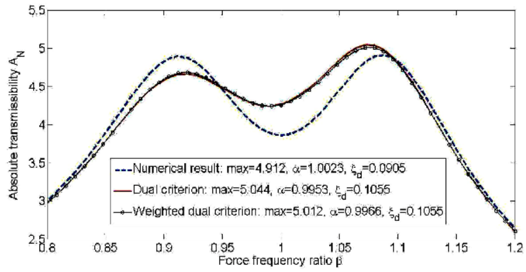


Fig. 6. Comparison of the absolute transmissibility A_N where $\mu = 0.03$ and $\xi_s = 0.05$.

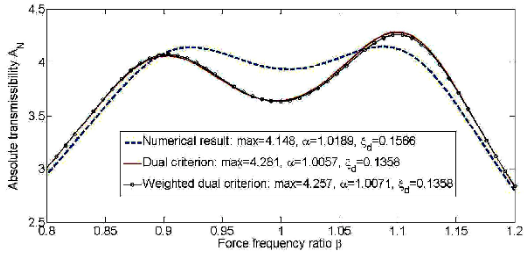


Fig. 7. Comparison of the absolute transmissibility A_N where $\mu = 0.05$ and $\xi_s = 0.05$.

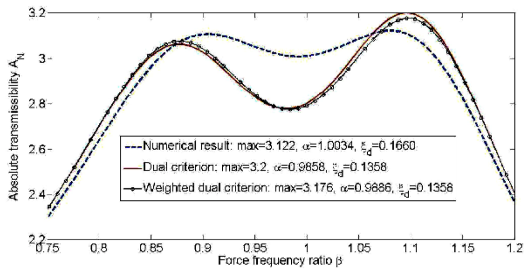


Fig. 8. Comparison of the absolute transmissibility A_N where $\mu = 0.05$ and $\xi_s = 0.1$.

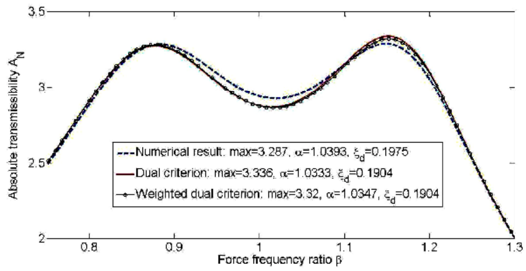


Fig. 9. Comparison of the absolute transmissibility A_N where $\mu = 0.1$ and $\xi_s = 0.05$.

6.1 Absolute displacement

To validate the effectiveness of the results Eqs. (41) and (42) presented in this study, these expressions are compared with the numerical results as shown in Figs. 6-10. Fig. 6 describes the graph of the transmissibility A_N where $\mu = 0.03$ and $\xi_s = 0.05$. Figs. 7-9 present three cases where $\mu = 0.05$, $\xi_s = 0.05$; $\mu = 0.05$, $\xi_s = 0.1$ and $\mu = 0.1$, $\xi_s = 0.05$, respectively. Fig. 10 shows the comparison of the maximum

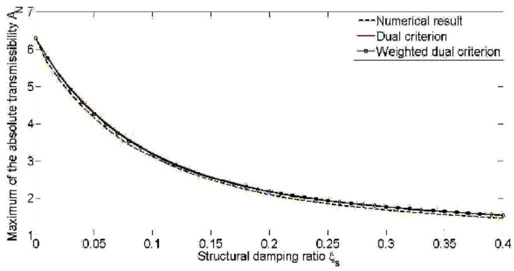


Fig. 10. Comparison of the maximum of absolute transmissibility $A_{\rm N}$ where $\mu = 0.05$.

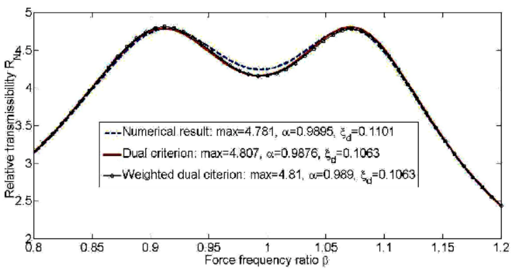


Fig. 11. Comparison of the relative transmissibility $R_{\rm N}$ where $\mu = 0.03$ and $\xi_s = 0.05$.

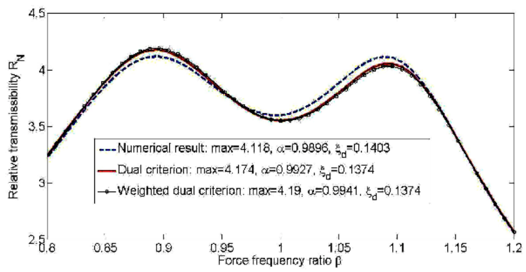


Fig. 12. Comparison of the relative transmissibility $R_{\rm N}$ where $\mu = 0.05$ and $\xi_s = 0.05$.

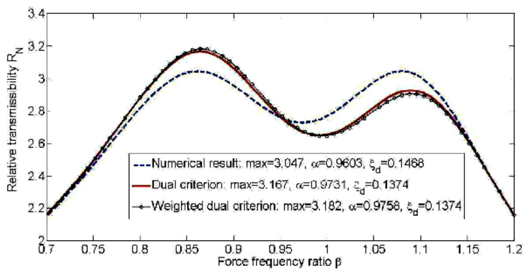


Fig. 13. Comparison of the relative transmissibility $R_{\rm N}$ where $\mu = 0.05$ and $\xi_s = 0.1$.

of absolute transmissibility $A_{\rm N}$ when the structural damping ratio $\xi_{\rm s}$ is changed.

6.2 Relative displacement

In the case of relative displacement, the two results Eqs. (44) and (45) are compared with numerical results in Figs. 11-16. Fig. 11 illustrates the case $\mu = 0.03$ and $\xi_{\rm s} = 0.05$. Figs.

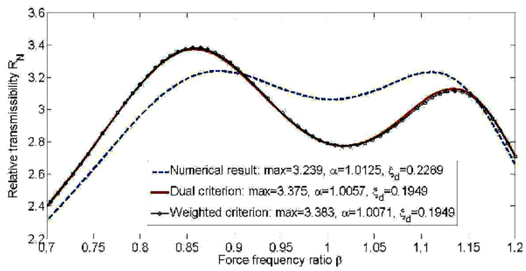


Fig. 14. Comparison of the relative transmissibility $R_{\rm N}$ where $\mu = 0.1$ and $\xi_s = 0.05$.

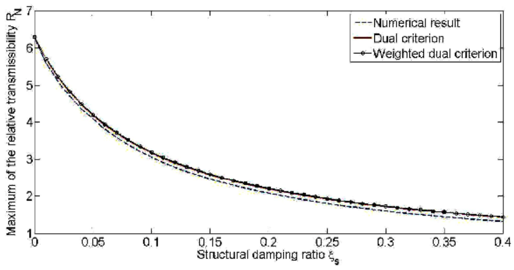


Fig. 15. Comparison of the maximum of relative transmissibility $R_{\rm N}$ where $\mu = 0.05$.

12-14 depict three cases: $\mu = 0.05$, $\xi_{\rm s} = 0.05$; $\mu = 0.05$, $\xi_{\rm s} = 0.1$ and $\mu = 0.1$, $\xi_{\rm s} = 0.05$, respectively. Fig. 15 describes the comparison of the maximum of relative transmissibility $R_{\rm N}$ when the structural damping ratio $\xi_{\rm s}$ is changed.

From the comparisons in Secs. 6.1 and 6.2, some remarks can be drawn. First, the results Eqs. (41), (42), (44) and (45) agree quite well with numerical results. Therefore, the analytical expressions for DVA's optimal parameters suggested in this paper are useful in practice. Second, in the case of absolute displacement, result Eq. (42) using the weighted dual criterion is better than result Eq. (41) using the dual criterion. Conversely, in the case of relative displacement, result Eq. (44) using the dual criterion is better than result Eq. (45) using the weighted dual criterion. However, the difference between two criteria is quite small.

7. Conclusions

Previous studies in the literature showed that the non-traditional type dynamic vibration absorber designed for undamped primary structure procedures had better performance than the standard model DVA does. However, to our best knowledge, there has been no study on the non-traditional dynamic vibration absorber for damped primary structures under ground motion. This paper proposes a simple approach to design the non-traditional DVA when the primary structure is damped and subjected to ground motion. The main idea is using the dual and weighted dual criteria of equivalent linearization method to replace approximately the damped primary structure with an equivalent undamped structure. Then, the parameters of the non-traditional DVA are obtained by using the known results for undamped structure. The fundamental findings in this study can be summarized below.

- Optimal parameters of a non-traditional DVA for undamped structures under ground motion are found when minimizing the relative displacement.
- Approximate analytical expressions of the non-traditional DVA's parameters are presented for damped structures under ground motion in both absolute and relative cases.
- The results presented in this study are compared with the numerical results . Our results agree quite well with the results obtained by using numerical optimizations. Thus, the analytical expressions proposed in this paper are useful in practice.

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