

Design and implementation of a robust FNN-based adaptive sliding-mode
controller for pneumatic actuator systems[†]

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Abstract

An adaptive Fourier neural network sliding mode controller with H_x tracking performance (AFNN-SMC+ H_x) is applied for a Pneumatic actuator system (PAS) to overcome time-varying nonlinear dynamics and external disturbances. Benefiting from the use of orthogonal Fourier basis function, the proposed AFNN has fast estimated convergence speed; also, because AFNN has unique solution, it can avoid falling into the local minimum. The architecture of AFNN can also easily be determined by its clear physical meaning of the neurons. To attenuate the vibration of proportional directional control valve and the adaptive approximation error, the H_{∞} tracking design technique is incorporated into the proposed AFNN-SMC. Finally, practical experiments are successfully implemented in position regulation, trajectory tracking, and velocity control of the PAS, which illustrates the effectiveness of the proposed controller.

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Keywords: Adaptive control; Fourier neural network; *H*_{*} tracking performance; Pneumatic actuator system; Sliding mode control

1. Introduction

Over the past few years in industrial motion control applications, a Pneumatic actuator system (PAS) has become popular because of high speed, low energy consumption, and easy maintenance. In addition, due to their low-cost and high ratio of power to weight, a PAS is well-known for linear movement control and robotic manipulator control. Also, a PAS is clean, safe in operation, and no overheating problems. However, the PAS is limited in the difficulty of making stable and accurate motion control. The use of a PAS is traditionally called "end stop" motion, which is controlled by on/off valves. In the 1950s, researchers started attempting the continuous position control of a PAS. It is unfortunate that limited on low performance microprocessors and pneumatic components servo-controlled, a PAS was seldom utilized in industry in the last 10 years. In recent years, however, low-cost and high-
tive controller are synthesized via backstepping method for performance microprocessors and pneumatic components have been accessible, and more complicated control methods in pneumatic system control were possible to be realized [1].

Tracking control of nonlinear systems with uncertain parameters or unknown dynamics has attracted great attention for the last few decades. Adaptive control (AC) was proposed as a way of automatically adjusting the controller parameters

in case of the unknown time-varying parameters of a plant. Many adaptive approaches can also be found to take care of uncertainties in the fluid power servo system, which could be classified into two different domains, pneumatic servo system and hydraulic servo system [2-4]. However, the fluid power servo system usually contains much parameter uncertainty caused by the change of environmental temperature and working state. Hence, many researches have applied the AC to the fluid power servo system to cope with these uncertainties. Developing AC is required for a clear mathematical model of the controlled plant, but its robustness is limited by the persistent excitation, the slow time variation, and the strict positiveness. Therefore, researchers have developed ACs with few design limitations, simplified algorithms, which can save computational time, and better robustness. For example, a robust integral of the sign of the error controller and an adapmotion control of a hydraulic rotary actuator [5]. Under the assumption of which the disturbances are periodic-like, Yao et al. presented an adaptive repetitive control based on projection algorithm [6]. Based on a nonlinear system model, a discontinuous projection-based nonlinear adaptive robust backstepping controller was developed to take into account the particular nonlinearities and then a stable parameter adaptation was derived to eliminate the effect of unknown but constant para metric uncertainties [7].

Even though the PAS is simple and inexpensive now, it still

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cannot meet the demand of being accurate, versatile, and flexible due to the inborn nonlinearities associated with com pressibility of air and complex friction distributions along the cylinders. In addition, unknown time-varying parameters or disturbances have been known to be a challenge of control design. These inborn problems make the PAS hard to acquire high precision motion control. As a result, for the purpose of serving on more complicated motion-control tasks, many researchers [8-11] have explored the PAS for servo system control. Over the last ten years, the input-output feedback linearization technique has been adopted to the PAS [12]. The main idea is first to transform the PAS into a linear system by a nonlinear feedback, and second to use well-known and powerful linear design techniques to fulfill the control design. These techniques, however, can only be applicable to nonlinear systems when the system parameters are precisely known and are unlikely in a practical application. Nearly all kinds of robust control schemes [13] have been proposed to solve the control problems of the PAS, one of which is Sliding mode control (SMC) [14]. The SMC is robust against external disturbances and can provide a systematic approach to maintain stability in the face of modeling imprecision and uncertainty. Nonetheless, the design of traditional SMC has to be required for precise system model, so-called as a model-based control design; therefore, the system dynamic model and the value of uncertain bound for controller design are needed. In general, the SMC suffers from large control chattering that may excite unmodeled high frequency response of the systems so that it has a trade-off between chattering and robustness. Hence, various controllers incorporating SMC have been proposed [15-19] to reduce the chattering. Unfortunately, most of the existing methods require that nonlinear functions of the dynamical system are known, which is impractical in real applications. Furthermore, although SMC rejects uncertainties and disturbances, it suffers from an assumption of which the matching uncertainties or disturbances have to be bounded on norm and the bound should be available for design. Some parameters related the PAS, i.e., the tube length, the cylinder bore diameter, and the critical pressure ratio of chokes flow, are easily measured or obtained from the manufacturer or even can be calculated by extensively received formulas; however, the inner dynamics of the valve flow is still difficult to com prehend with the PAS. Besides, the friction [20, 21] and inter nal energy change [22], which are necessary in experiments, are quite hard to model and are not yet completely understood. Therefore, due to the complex structure of uncertainties, un certainty bounds may not be easily obtained, and the traditional SMC cannot be implemented for the PAS.

Lately, many theoretical and practical works have been done in the field of approximate-based control [23-29]. The functional approximation technique is applied to release this model-based requirement. Particularly, Approximate-based adaptive control (ABAC) has been verified and deemed useful to solve control problems of the PAS with time-varying function or unknown nonlinear parameters [30]. In ABAC, the unknown time-varying functions are usually modeled online by several commonly used function approximates, such as multi-layer Neural network (NN) [31], fuzzy neural network [32, 33], wavelet neural network [34], and radial basis function neural network [35, 36]. The NN-based control method has been successfully employed in many applications, but there still are some difficulties in selecting parameters of the activation function and determining network structures. Moreover, the approximation error is affected by the number of layers and neurons so that it is hard to decide specified approximation accuracy. If there are not enough layers and neurons in the network, assuring that the parameters converge to their optimal values may be a problem. On the other hand, having too many layers and neurons causes massive computational burden and inevitably degrades convergent speed. As a result, it deters the on-line control applications and practical implementation feasibility. Thus, for the NNs both the network topology and stability analysis are not easy to carry out [37]. These drawbacks prevent the NNs from being widely used in control design.

To solve the above-mentioned problems and avoid draw backs of the traditional SMC and NNs, we propose the $AFNN-SMC+H_{\infty}$ for the motion control design of the PAS with unknown and uncertain bounded parameters. In our proposed control strategy, the technique of input-output feedback linearization is first used to transform the PAS nonlinear model into a linear one. Subsequently, the powerful and universal AFNN approximator is used to approximate the uncertain nonlinear functions of the dynamical system, and the SMC is then applied to stabilize the whole system and attenu ate the bounded disturbance. However, the SMC has an inher ent limitation of which the matching uncertainties or distur bances have to be bounded for controller design. To release this limitation, the H_{∞} tracking design technique [32, 38] based on the relaxed assumption was incorporated into the SMC. Taking advantages of the orthogonal activation basis functions and the clear definition of Fourier series, AFNN has a clear physical meaning and easily determined structure, so that it is over conventional NNs; in addition, AFNN has been proven that there are no local minima for optimization problem [39-41]. Because of the orthogonality of the basis functions, the FNN provides fast convergent speed and it is therefore suitable for real-time implementation [39, 40]. To further decrease the effect caused by approximation error, time varying dynamics, and external disturbance, an adaptation technique and a compensator with H_{∞} tracking performance [32, 38] are included into the AFNN-SMC, named as the $AFNN-SMC+H_∞$. The weighting update algorithms of the AFNN-SMC+ H_{∞} are derived from the Lyapunov stabilizing theory. Compared with traditional SMC approaches, which generally require prior knowledge on the upper bound of the uncertainties, the proposed approach not only assures closedloop stability, but also guarantees the desired H_{∞} tracking performance for the overall system based on a much relaxed assumption. Moreover, control chattering happening in the

Fig. 1. Pneumatic actuator system schema.

traditional SMC can be greatly lowered by using the proposed approach. The experimental results show that the proposed controller has effective tracking performance despite uncertainties and time-varying payload.

The main contributions of this study are as follows: (1) Thanks to orthogonality of the basis functions, the FNN is not only easy to realize, but also highly improves the convergence speed and the local minimum avoidance. (2) The FNN has a clear physical meaning; therefore, the network topology and the parameter selection of the basis functions become convenient for real applications. (3) The constraint in demanding prior knowledge on upper bounds of the lumped uncertainties is removed. (4) The proposed AFNN-SMC+ H_{∞} significantly reduces the control chattering.

2. System description and modeling

2.1 System hardware

A schematic diagram of the PAS is shown in Fig. 1. Table 1 specifies the main components and the system hardware com prises a rodless pneumatic cylinder, a proportional directional control valve, a granite air cushion, and a PC-based control unit. The critical frequency with maximum spool stroke (Festo model MPYE-5-1/8-HF-010B) is 100 Hz and the rodless cylinder in our test rig (Festo model DGPL-25-700-PPV-A- HD40-GK-D2) is with 25 mm bore and 700 mm stroke. The exter proposed meant the restore is and the source pressure is regulated at 5 the state of the proposed AFNN-SMC+ H_a significantly rig. 2. Test rig of pneumatic actuator system, is removed at λ schematic diagram of th process. The rodless cylinder has a built-in linear slider. An optical encoder with a resolution of 20 nm is installed to measure the piston's position, and the default payload is set at 6 kg. The scale's measured signals are feedback to the controlling PC via a decoder IC, Agilent HCTL-2032, and a digital I/O converter. The PC-based control system is implemented on a Pentium III CPU with an ADVANTECH's PCL-726 IO card which contains D/A converters and digital I/O converter. The proposed control strategy is implemented in an interrupted service routine with 1ms sampling time under MS- DOS environment. The input voltage for the proportion servo valve comes from the controlling PC via D/A converters, and the control law is calculated by a 32-bit Open Watcom C Lan guage program.

Table 1. Main components' specifications of the test rig.

Components	Specifications
Pneumatic cylinder	Diameter: 25 mm Stroke: 600 mm
Proportional directional control valve	Valve function: 5/3-way Input: $-5 \sim 5$ V
Optical encoder	Decoder IC: HCTL-2032 Resolution: 20 nm
PC-based controller	Pentium III CPU RAM: 512 MB
A/D D/A cards	12 bit $A/D \times 16$ CH 12 bit $D/A \times 6$ CH D/I, $D/O \times 16 \text{ CH}$

Fig. 2. Test rig of pneumatic actuator system.

Fig. 3. Schematic drawing of a pneumatic actuator.

2.2 Pneumatic actuator system dynamic model

Fig. 2 shows the test rig of the PAS. In the rodless pneu matic servo system, the opening area of the proportional directional control valve's orifice depends on the control input to affect the air flow. As the air flows into the rodless pneumatic cylinder, the pressure difference between two cylinder cham bers results in the motion of the pneumatic cylinder. An analysis of the dynamic behavior of the PAS usually requires individual mathematical descriptions of the dynamics of the three component parts: the valve, the actuator and the load. Such an analysis is presented below with reference to the coordinate system illustrated in Fig. 3.

2.2.1 Flow relationships for the control valves

With the assumption of constant supply and exhaust pressures, the mass flow rates *m* across two control ports of the control valves can be regarded as a function of the valve displacement and the chamber pressure [42, 43] (*L.W. Lee and I-H. Li / Journal of Mechanical Science and Technology 30 (1) (2016)*
 C.*I* **Flow relationships for the control valves** where $P_s = 6 \times 10^5$ Pa is the

with the assumption of constant supply and exhaust pr

$$
m_a = q_a(X, P_a),\tag{1}
$$

$$
m_b = q_b(X, P_b),\tag{2}
$$

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(<i>L*, *How relationships for the control valves* where $P_s = 6 \times 10^5$ Pa is the

exhaust pressure, $T_s =$

exhaust pressure, $T_s =$
 where m_a and m_b are the mass flow rates into the chambers c A and B, X is the spool displacement of valves, P_a and $c_V \rho_c V T_s$, w P_b are the absolute pressures in the chambers A and B. According to the standard orifice theory, the mass flow rate through the valve orifice takes the form *L*. *W. Lee and <i>H. Lt. Journal of Mechanical Science and Technology 30 (1) (2016) 381

<i>LI* **Flow relationships for the control valves** where $P_z = 6 \times 10^5$ Pa is the sumption of constant supply and exhaust press-

wh

$$
m = \frac{C_d C_0 w X P_u \tilde{f}(p_r)}{T_u^{\frac{1}{2}}},
$$
\n(3)

where C_d is the discharge coefficient ($C_d = 0.8$), C_0 is the where C_p flow constant, *w* is the port width (m) , T_u is the up-stream P is the cylinder chamber temperature, $p_r = P_d / P_u$ is the ratio between the down-

where
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m_a
$$
 and m_b are the mass flow rates into the chambers
\nA and B, X is the spool displacement of values, P_a and $c_{\nu} \rho_c V T_s$, where V is the cylinder volume, ρ_c is
\n P_s are the absolute pressures in the chambers A and B. Ac-
\ncording to the standard orifice theory, the mass flow rate
\nthrough the valve orifice takes the form
\nthrough the valve orifice takes the form
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$$
m = \frac{C_a C_0 w X P_u \hat{J} (p_r)}{T_a^{\frac{1}{2}}},
$$
\n(3) $\frac{d}{dt} (c_{\nu} \rho_c V T_s) = \dot{m} C_p T_s - P \dot{V},$
\nwhere C_a is the discharge coefficient $(C_a = 0.8)$, C_0 is the
\ntemperature, $p_r = P_a / P_u$ is the ratio between the down-
\nstream and up-stream pressure at the orifice and
\nrate in Eq. (7) to obtain
\n
$$
\hat{J}(p_r) =\begin{cases}\n1, & \frac{P_{am}}{I} & \frac{P_{am}}{I} & \frac{P_{m}}{I} \\ 1, & \frac{P_{am}}{I} & \frac{P_{m}}{I} & \frac{P_{m}}{I} \end{cases}
$$
\n(4) $\dot{m} = \frac{P}{C_p T_s} \frac{dV}{dt} + \frac{1}{kRT_s} \frac{d}{dt} (PV)$
\nFor air $k = 1.4$, $C_r = 0.528$ and $C_k = 3.864$. It can be
\nshown that the function $\hat{J}(r)$ and its derivative are continu-
\nous with respect to p_r . For the convenience of the analysis, $\hat{J} = \frac{1}{R} - \frac{1}{R} + \frac{1}{kRT_s} \frac{d}{dt} (PV)$
\n
$$
\frac{1}{R} = \frac{1}{C_p} + \frac{1}{kR}
$$
\n(5) $\frac{1}{R} = \frac{1}{C_p} + \frac{1}{kR}$

For air $k = 1.4$, $C_r = 0.528$ and $C_k = 3.864$. It can be

where
$$
C_{\mu}
$$
 is the distance coefficient $(C_{\mu} = 0.8)$, C_{μ} is the wave of C_{μ} is the positive constant pressure,
flow constant, w is the part width (m), T_{μ} is the time-streum P is the cylinder chamber charge, $m\zeta, T_{\mu}$ is the critical
temperature, $p_{\mu} = P_{\mu}/P_{\mu}^*$ is the ratio between the down-
stream and up-stream pressure at the orifice and
stream and up-stream pressure at the orifice and
heterum and up-stream pressure at the orifice and

$$
\hat{f}(p_{\mu}) = \begin{bmatrix} 1 & \frac{P_{\mu\nu}}{P_{\nu}} < p_{\mu} < C, \\ 1 & \frac{P_{\mu\nu}}{P_{\nu}} < p_{\mu} < C. \end{bmatrix}
$$
 (4) $\frac{1}{P_{\mu}} = \frac{P_{\mu}}{C_{\mu}} \frac{V_{\mu}}{H} + \frac{1}{RRT_{\mu}} \frac{d}{d}(PV)$
(5) $\hat{f}(p_{\mu}) = \begin{bmatrix} 1 & \frac{P_{\mu\nu}}{P_{\nu}} < p_{\mu} < C, \\ 1 & \frac{P_{\mu\nu}}{P_{\nu}} < p_{\mu} < C. \end{bmatrix}$ (6) $\frac{1}{R} = \frac{P_{\mu}}{C_{\mu}} V + \frac{1}{RRT_{\mu}} \frac{d}{d}(PV)$
For air $k = 1.4$, $C_{\mu} = 0.528$ and $C_{k} = 3.864$. It can be
shown that the function $\tilde{f}(t)$ and its derivative are continuous.
where $k = \frac{C_{\mu}}{C_{\mu}}$ is the ratio of specific heats for air at the tem-
ous with respect to p_{μ} . For the convenience of the analysis, ρ parameter T_{μ} . For a perfect gas
the following functions are introduced:

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\frac{1}{R} = \frac{1}{C_{\mu}} + \frac{1}{RRT_{\mu}}
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 by $\frac{1}{RRT_{\mu}}$ by $\frac{1}{R}$ (9)
 $\hat{f}(P_{\mu}, P_{\mu}, P_{\mu}) = \begin{bmatrix} \frac{\partial}{\partial T_{\mu$

and

$$
\int \frac{P_a f}{\sqrt{T_a}} \text{chamber B is a drive chamber}
$$
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$$
\dot{m} = \left(\frac{1}{C_P} + \frac{1}{kH}\right)
$$
\n
$$
= \frac{1}{RT_s} P \dot{V} +
$$
\nFor the cylinder

\n
$$
\hat{f}(P_b, P_s, P_e) = \begin{cases}\nP_b \tilde{f}(\frac{P_e}{P_b}) & \text{chamber A is a drive chamber} \\
\frac{P_b \tilde{f}(\frac{P_e}{P_b})}{\sqrt{T_b}} & \text{chamber B is a drive chamber} \\
\frac{P_s \tilde{f}(\frac{P_b}{P_s})}{\sqrt{T_s}} & \text{chamber B is a drive chamber} \\
\frac{P_s \tilde{f}(\frac{P_b}{P_s})}{\sqrt{T_s}} & \text{chamber B is a drive chamber}\n\end{cases}
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Figure and Technology 30 (1) (2016) 381~396
where $P_s = 6 \times 10^5$ Pa is the supply pressure, $P_e = 1 \times 10^5$ Pa is
the exhaust pressure, $T_s = 293$ K is the cylinder air tempera-
ture, T_a and T_b are the temperatures of c the exhaust pressure, $T_s = 293 K$ is the cylinder air temperature, T_a and T_b are the temperatures of chambers A and B, respectively.

2.2.2 Dynamic relationship within the control chambers

L.-W. Lee and I-H. Li / Journal of Mechanical Science and Technology 30 (1) (2016) 381-3
 ionships for the control valves

umption of constant supply and exhaust press-

the exhaust pressure, $T_z = 293 K$

flow rates *m* **Example 1.** $m_z = q_z(X, P_z)$, (1) $m_s = q_s(X, P_s)$, (1) $\frac{2.2.2 \text{ Dynamic relationship within } m_s = q_s(X, P_s)$, $m_s = q_s(X, P_s)$, (2) $\frac{2.2.2 \text{ Dynamic relationship within } m_s = q_s(X, P_s)$, $\frac{2.2.2 \text{ Dynamic relationship within } m_s = q_s(X, P_s)$, $\frac{2.2.2 \text{ Dynamic relationship within } m_s = q_s(X, P_s)$, $\frac{2.2.2 \text{ Dynamic relationship within } m_s = q_s(X, P_s)$, $\frac{2.2.2$ e m_a and m_b are the mass flow rates into the chambers

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to free control volume bounded b
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where $P_s = 6 \times 10^5$ Pa is the supply pressure, $P_c = 1 \times 10^5$ Pa is

the exhaust pressure, $T_s = 293$ K is the cylinder air tempera-

ture, T_a and T_b are the temperatures o $c_V \rho_c V T_s$, where V is the cylinder volume, ρ_c is the cylinder air density and c_v is the specific heat of air at constant volume. If the air flows into the cylinder it is assumed to be an adiabatic process of a perfect gas, the change in energy due to the mass transport equals form Rets. [42, 45], a moder for the mass flow mio cach of
cylinder chambers can be obtained from the energy con-
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Dynamic relationship within the contr ere $P_i = 6 \times 10^5$ Pa is the supply pressure, $P_e = 1 \times 10^5$ Pa is
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2. **Dynamic relationship w P** is the cylinder of example in the conference of examines of examines of examines of the expectively.
 P 2.2.2 Dynamic relationship within the control chambers

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From Refs. [42, 43], a model for the mass flow into each of

the cylinder chambers can be obtained from the energy con-

servation equation. The control volume energ *eylinder chambers can be obtained from the energy con-
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How is the specific heat of air at cons here *V* is the cylinder volume, ρ_C is the cylin-
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\frac{d}{dt}\left(c_{V}\rho_{C}VT_{s}\right)=\dot{m}C_{P}T_{s}-P\dot{V},\qquad(7)
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in the chambers A and B. Ac-
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of the minimal correct density and (3)
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(*m*), T_u is the up-stream *P* is the cylinder chamb

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corifice and Assuming is the spool displacement of valves, P_s and
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standard orifice theory, the mass flow rate airdinesty and B. According the standard orifice theory, the mass flow where C_p is the specific heat of air at a constant pressure, internal gas energy and $P\vec{V}$ is the work done by the cylinder. Assuming an ideal gas, $P = R\rho_c T_s$ and ρ_c can be eliminated in Eq. (7) to obtain P_{P} is the specific heat of air at a constant pressure,

cylinder chamber pressure, $mC_{P}T_{s}$ is the change in

gas energy and $P\dot{V}$ is the work done by the cylinder.

g an ideal gas, $P = R\rho_{C}T_{s}$ and ρ_{C} ca *z*, where \vec{r} is an eyalinear volume, p_c is an eyalinear volume,
density and c_v is the specific heat of air at constant
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ic process of a perfect gas, th at example the specific heat of air at a constant pressure,

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summing an ideal gas,
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P = R\rho_c T_s
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 and ρ_c can be eliminated in Eq. (7) to obtain
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\dot{m} = \frac{P}{C_p T_s} \frac{dV}{dt} + \frac{1}{kRT_s} \frac{d}{dt} (PV)
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\n
$$
= \frac{P}{C_p T_s} \dot{V} + \frac{1}{kRT_s} \dot{P} V + \frac{1}{kRT_s} P \dot{V},
$$
\n(8)
\nhere $k = \frac{C_p}{c_V}$ is the ratio of specific heats for air at the tem-
\nrature T_s . For a perfect gas
\n
$$
\frac{1}{R} = \frac{1}{C_p} + \frac{1}{kR},
$$
\n(9)
\n
$$
m = \left(\frac{1}{C_p} + \frac{1}{kR}\right) \frac{1}{T_s} P \dot{V} + \frac{1}{kRT_s} \dot{P} V
$$
\n
$$
= \frac{1}{RT_s} P \dot{V} + \frac{1}{kRT_s} \dot{P} V.
$$
\n(10)
\nFor the cylinder chambers A and B, the following equations
\nId:

where $k = \frac{C_p}{c_V}$ is the ratio of specific heats for air at the temperature *T^s* . For a perfect gas

$$
\frac{1}{R} = \frac{1}{C_P} + \frac{1}{kR},
$$
\n(9)

then

$$
\dot{m} = \frac{P}{C_p T_s} \frac{d}{dt} + \frac{P}{kRT_s} \frac{d}{dt} (PV)
$$
\n
$$
= \frac{P}{C_p T_s} \dot{V} + \frac{1}{kRT_s} \dot{P} V + \frac{1}{kRT_s} P \dot{V},
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\n(8)
\nhere $k = \frac{C_p}{c_V}$ is the ratio of specific heats for air at the tem-
\nrature T_s . For a perfect gas
\n
$$
\frac{1}{R} = \frac{1}{C_p} + \frac{1}{kR},
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\n(9)
\n
$$
\dot{m} = \left(\frac{1}{C_p} + \frac{1}{kR}\right) \frac{1}{T_s} P \dot{V} + \frac{1}{kRT_s} \dot{P} V
$$
\n
$$
= \frac{1}{RT_s} P \dot{V} + \frac{1}{kRT_s} \dot{P} V.
$$
\n(10)
\nFor the cylinder chambers A and B, the following equations
\n
$$
\dot{m}_a = \frac{1}{RT_s} \left(P_a \dot{V}_a + \frac{V_a \dot{P}_a}{k} \right),
$$
\n(11)
\n
$$
\frac{d}{dt} = \frac{1}{RT_s} \left(P_a \dot{V}_a + \frac{V_a \dot{P}_a}{k} \right),
$$
\n(12)

For the cylinder chambers A and B, the following equations hold:

$$
\dot{m}_a = \frac{1}{RT_s} \left(P_a \dot{V}_a + \frac{V_a \dot{P}_a}{k} \right),\tag{11}
$$

and

*L-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
\n
$$
\dot{m}_b = \frac{1}{RT_s} \left(P_b \dot{V}_b + \frac{V_b \dot{P}_b}{k} \right).
$$
\n(12) \n takes and components, $l(m)$ is the stroke and $x \neq$ rearranging Eqs. (18) and (19), the chambers vo and V_b can be rewritten as\n\n
$$
V_a = A(x + \Delta),
$$
\n
$$
\dot{P}_a = -\frac{kV_a}{V_a} P_a + \frac{k}{V_a} R T_s C_a C_0 w X f (P_a, P_s, P_e),
$$
\n(13) \n
$$
\dot{P}_b = -\frac{kV_b}{V_b} P_b + \frac{k}{V_b} R T_s C_a C_0 w X f (P_b, P_s, P_e),
$$
\n(14) \nwhere Δ can be considered as an equivalent extra the cylinder. Let $x_1 = x$, $x_2 = \dot{x}$, $x_3 = P_a$, $x_4 = P_a$ there V_a and V_b are the volumes of chambers A and B, respectively.\n\n
$$
\dot{x}_1 = x_2,
$$
\n
$$
\dot{x}_2 = \frac{1}{M} [A(x_3 - x_4) + F_i - F_r].
$$
\n2.3 *Load dynamics**

From Eqs. $(3)-(6)$, (11) and (12) , the following equations can be derived:

$$
\dot{P}_a = -\frac{k\dot{V}_a}{V_a}P_a + \frac{k}{V_a}RT_sC_aC_0wXf(P_a, P_s, P_e),
$$
\n(13)

$$
\dot{P}_b = -\frac{k\dot{V}_b}{V_b}P_b + \frac{k}{V_b}RT_sC_dC_0wXf(P_b, P_s, P_e),
$$
\n(14) th

where V_a and V_b are the volumes of chambers A and B, respectively.

2.2.3 Load dynamics

The moving mass, M, consists of the masses of the payload, the slide table and the piston. The force on the piston due to the air pressure and external load is expressed by

$$
F_p = A(P_a - P_b) + F_l,
$$
\n(15)

From Eqs. (3)-(6), (11) and (12), the following equations
 $\vec{r}_a = A(x + \Delta)$,

be derived:
 $\vec{P}_a = \frac{k\vec{V}_p}{V_a} P_a + \frac{k}{V_a} R T_c C_a C_0 w X f (P_a, P_s, P_e)$,
 $\vec{P}_a = \frac{k\vec{V}_p}{V_b} P_a + \frac{k}{V_a} R T_c C_a C_0 w X f (P_a, P_s, P_e)$,
 $\vec{P}_b = \frac{k\vec{V}_b}{V$ where F_i is external load force. The friction force is assumed to be modeled by the traditional combination of the stick-slip, Coulomb and viscous components. According to Newton's second law of motion, the pneumatic cylinder's motion can be described ere V_a and V_b are the volumes of chambers A and B,
 $\dot{x}_1 = x_2$,
 $\dot{x}_2 = \frac{1}{M} [A(x_3 - x_4) +$
 $\dot{x}_3 = \frac{R}{M} [A(x_3 - x_4) +$
 $\dot{x}_4 = \frac{R}{M} [A(x_3 - x_4) +$
 $\dot{x}_5 = \frac{R}{M} [A(x_3 - x_4) +$
 $\dot{x}_6 = \frac{R}{M} [A(x_3 - x_4) + \frac{R}{M} [A(x_3 - x$

$$
F_p - F_r = M\ddot{x}.\tag{16}
$$

The term F_r in Eq. (16) represents the sum of static and dynamic friction forces in the system, in which the static friction forces are unevenly distributed along the cylinders. This Δ uneven distribution of friction causes difficulties for modeling $K_{\nu f}$ and controlling the pneumatic cylinder actuators. The static P_{atm} component allows for stick-slip motion, as shown in Eq. (17). *^p p sf s* A **f** \therefore **Fiston a**
 shown in Eq. (16) represents the sum of static and $C_d = 0.8$ \therefore **Dischargement** friction forces in the system, in which the static fric-
 o forces are unevenly distributed along the c *F* $(P_x - P_x) + F_t$,
 $\begin{aligned}\n &\text{if } (P_x - P_x) + F_t\n \end{aligned}$
 $\begin{aligned}\n &\text{if } (P_x - P_x) + F_t\n \end{aligned}$
 $\begin{aligned}\n &\text{if } (P_x - P_x) = 0, \\
 &\text{if } (P_x - P_x) =$ **EVALUAT SURFALU CONDUCE THE friction force is assumed where** $x_i \in [0, l]$ **,** $x_i \in [P_e, P_s]$ **,** $x_i \in [P_e, P_s]$ **are horded by the traditional combination of the stick-slip, tem modeled by Eq. (22) can be considered as a subsystem** *F*_{*i*} is external load force. The friction force is assumed

where $x_i \in [0, l]$, $x_3 \in [P_\epsilon]$

deled by the traditional combination of the stick-slip,

we of motion, the pneumatic cylinder's motion can be

in model are *x R F_x* is external load force. The friction force is assumed where $x_i \in [0, 1]$, $x_i \in [P_i \quad P_i]$, $x_i \in [P_i \quad P_i]$ *x* $\{P_i \in [P_i \quad R_i] \}$, $x_i \in [P_i \quad R_i]$ and evisos components. According to Newton's necessive compone odeled by the traditional combination of the stick-slip,

tem modeled by Eq. (22)

band viscous components. According to Newton's

the first subsystem and

dal are listed below:

F_r = *Mi*.

(16)

emerric cylinder's mot

, 0 , 0 0 *cf vf* ï ⁼ ^í ⁺ ^ï > & & & & & & (17) , *V Ax ^a* = + D (18) () , *V A l x ^b* = - + D (19)

where F_{sf} is the stick-slip friction force, F_{cf} is the Coulomb friction force, and $K_{\nu f}$ is the coefficient of viscous *M* friction. Further analysis and description of the static and the P_{ν} friction. Further analysis and description of the static and the dynamic friction forces can be found in Refs. [44, 45].
The chamber volumes V_a and V_b are defined as

$$
V_a = Ax + \overline{\Delta},\tag{18}
$$

$$
V_b = A(l - x) + \overline{\Delta},\tag{19}
$$

where $\overline{\Delta}$ is the residual volume generated by the connecting

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* $\left(P_s V_s + \frac{V_s \dot{P}_s}{k}\right)$ *.

(12) tubes and components, <i>1* (*m*) is the stroke and $x \in \left[\begin{array}{c} R_s V_s + \frac{V_s \dot{P}_s}{k} \end{array}\right]$.

(1 L.W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
 $\left(P_s v_k^i + \frac{V_s v_k^i}{k}\right)$.

(12) the samd components, $I(m)$ is the stroke and $x \in \text{maxming B}$.

(18) and (19), the chambers volume and *ence and Technology 30 (1) (2016) 381-396* 385
tubes and components, *l* (*m*) is the stroke and $x \in [0, l]$. By
rearranging Eqs. (18) and (19), the chambers volumes V_a
and V_b can be rewritten as
 $V_a = A(x + \Delta)$, (20)
 V rearranging Eqs. (18) and (19), the chambers volumes V_a and V_b can be rewritten as *e and Technology 30 (1) (2016) 381-396* 385

bes and components, $l(m)$ is the stroke and $x \in [0, l]$. By

pranging Eqs. (18) and (19), the chambers volumes V_a
 $V_a = A(x + \Delta)$, (20)
 $V_b = A(l - x + \Delta)$, (21)

ere Δ can be con *e* and *Technology* 30 (1) (2016) 381-396 385

bes and components, $l(m)$ is the stroke and $x \in [0, l]$. By

parranging Eqs. (18) and (19), the chambers volumes V_a
 $V_a = A(x + \Delta)$, (20)
 $V_b = A(l - x + \Delta)$, (21)

here Δ can b the cylinder. Let *x*₁ *z x x* = *i x* + *i x* = *i x* + *i x* = *i x* + *x* + *x* + *x* + *x* + *x* +

$$
V_a = A(x + \Delta),\tag{20}
$$

$$
V_b = A(l - x + \Delta),\tag{21}
$$

where Δ can be considered as an equivalent extra length to

$$
\dot{x}_1 = x_2,\tag{22a}
$$

$$
\dot{x}_2 = \frac{1}{M} \Big[A(x_3 - x_4) + F_l - F_r \Big],
$$
\n(22b)

ce and Technology 30 (1) (2016) 381~396 385
\nbees and components, *l* (*m*) is the stroke and *x* ∈ [0,*l*]. By
\narranging Eqs. (18) and (19), the chambers volumes *V_a*
\nd *V_b* can be rewritten as
\n
$$
V_a = A(x + Δ),
$$
 (20)
\n
$$
V_b = A(l - x + Δ),
$$
 (21)
\nhere Δ can be considered as an equivalent extra length to
\ne cylinder. Let $x_1 = x$, $x_2 = \dot{x}$, $x_3 = P_a$, $x_4 = P_b$, $u = X$,
\n $\dot{x}_1 = x_2$, (22a)
\n
$$
\dot{x}_2 = \frac{1}{M} [A(x_3 - x_4) + F_t - F_r],
$$
 (22b)
\n
$$
\dot{x}_3 = \frac{-k \left[x_2 x_3 - \frac{RT_s}{A} C_a C_0 w \hat{f}(x_3, P_s, P_e) u\right]}{x_1 + Δ},
$$
 (22c)

ce and Technology 30 (1) (2016) 381-396 385
\nbees and components, 1 (*m*) is the stroke and *x* ∈ [0,*l*]. By
\narranging Eqs. (18) and (19), the chambers volumes *V_a*
\nand *V_b* can be rewritten as
\n
$$
V_a = A(x + Δ),
$$
\n(20)
\nHere Δ can be considered as an equivalent extra length to
\ne cylinder. Let *x₁* = *x*, *x₂* = *x̄*, *x₃* = *P_a*, *x₄* = *P_b*, *u* = *X*,
\nthen a state-space system model is obtained
\n
$$
\dot{x_1} = x_2,
$$
\n(22a)
\n
$$
\dot{x_2} = \frac{1}{M} [A(x_3 - x_4) + F_1 - F_r],
$$
\n(22b)
\n
$$
\dot{x_3} = \frac{-k \left[x_2 x_3 - \frac{RT_s}{A} C_a C_0 w \hat{f}(x_3, P_s, P_e) u \right]}{x_1 + Δ},
$$
\n(22c)
\n
$$
\dot{x_4} = \frac{k \left[x_2 x_4 - \frac{RT_s}{A} C_a C_0 w \hat{f}(x_4, P_s, P_e) u \right]}{1 - x_1 + Δ},
$$
\n(22d)
\nthere
$$
x_1 \in [0, 1], x_3 \in [P_e, P_s], x_4 \in [P_e, P_s].
$$
\nThe system modeled by Eq. (22) can be considered as a cascade con-
\nmonedled by Eq. (22) can be considered as a cascade con-
\n
$$
x_1 \in [0, 1], x_2 \in [P_e, P_s], x_4 \in [P_e, P_s].
$$
\nThe system modeled by Eq. (22) can be considered as a cascade con-

load force. The friction force is assumed

where $x_1 \in [0, l]$, $x_3 \in [P_c, P_3]$, $x_4 \in [P_c, I$

traditional combination of the stick-slip,

tem modeled by Eq. (22) can be considered as

somponents. According to Newton's

me (20)

(21)

onsidered as an equivalent extra length to
 $\begin{pmatrix}\n x_1 = x, & x_2 = \dot{x}, & x_3 = P_a, & x_4 = P_b, & u = X,\n\end{pmatrix}$

stem model is obtained

(22a)
 $\begin{pmatrix}\n x_1 + F_1 - F_r\n\end{pmatrix}$, (22b)
 $\begin{pmatrix}\n x_3 + C_d C_0 w \hat{f}(x_3, P_s, P_e) u \\
 x_1 + \Delta\n\end{$ and components, \mathbf{r} , \mathbf{w} , \mathbf{y} are stocked that $\mathbf{x} \in [0, 1]$. D_y
 \mathbf{r}_b can be considered as an equivalent extra length to
 $\mathbf{r}_a = A(x + \Delta)$, (20)
 Δ a can be considered as an equivalent extra le (20)

(21)

(22)

(22)

(22)

(22b)

(22b)
 $x_1 + E_i - F_r$]

(22b)

(22b)
 $x_1 + \Delta$

(22c)

(22c)
 $x_1 + \Delta$

(22c)

(22c)
 $\frac{\sum_{i=1}^{n} C_a C_0 w_i^2 ($ $V_b = A(I - x + \Delta),$ (21)

where Δ can be considered as an equivalent extra length to

the cylinder. Let $x_1 = x$, $x_2 = \lambda$, $x_3 = P_a$, $x_4 = P_b$, $u = X$,

then a state-space system model is obtained
 $\dot{x}_1 = x_2$, (22a)
 $\dot{x}_2 = \$ tem modeled by Eq. (22) can be considered as a cascade con nection of two nonlinear subsystems, Eqs. (22a) and (22b) for the first subsystem and Eqs. (22c) and (22d) for the second subsystem. The parameters and values used in the system model are listed below: $k_4 = \frac{k \left[x_2 x_4 - \frac{RT_s}{A} C_a C_0 w \hat{f}(x_4, P_s, P_s) u \right]}{1 - x_1 + \Delta}$, (22d)

where $x_1 \in [0, l]$, $x_3 \in [P_e, P_5]$, $x_4 \in [P_e, P_5]$. The sys-

there $x_1 \in [0, l]$, $x_3 \in [P_e, P_5]$, $x_4 \in [P_e, P_5]$. The sys-

nem modeled by Eq. (22 *R*₂ = $\frac{V}{L} = \frac{V}{L}$ / $R_1 + \Delta$ / (220)

where $x_i \in [0, 1]$, $x_i \in [P_{\epsilon}, P_{\bar{S}}]$, $x_i \in [P_{\epsilon}, P_{\bar{S}}]$. The sys-

em modeled by Eq. (22) can be considered as a cascade con-

encition of two nonlinear subsystems, E

A : Piston area (m^2) , C_d = 0.8 : Discharge coefficient, C_0 : Flow constant, : The general residual chamber volume, *Kvf* : Viscous frictional coefficient, *Patm* : Atmospheric pressure (Pa),

0 $T_s = 293$: Cylinder air temperature (*K*)

 $k = 1.4$: Specific heat constant

V : Volume (m^3)

w : Port width (*m*)

M : Payload (*kg*)

P^u : Up-stream pressure (Pa)

P_d: Down-stream pressure (Pa)

3. Description of Fourier neural networks

The features of fault tolerance, parallelism, and adaptation suggest that NNs make good candidates for the control of nonlinear systems, and the NN [25, 46, 47] can present a com plex nonlinear function. A compact matrix form is shown as Eq. (23).

Fig. 4. Structure of a SISO FNN.

$$
f(x) = \mathbf{W}^T \boldsymbol{\sigma}(x) + \varepsilon(x),\tag{23}
$$

where $x \in \Re$ and $f \in \Re$ are the input and output, respectively, $\mathbf{W} = \begin{bmatrix} W_1 & \cdots & W_n \end{bmatrix}^T \in \mathbb{R}^n$ is the network weig $\begin{bmatrix} \sigma_1(x) & \dots & \sigma_n(x) \end{bmatrix}^T \in \overline{\mathfrak{R}}^n$, $\sigma_i(\cdot), i = 1, 2, \dots, n$, denotes the acti-1... W. Lee and *I-H. Li* /Journal of Mechanical Science and Technology 30 (1) (2016) 381-39

1... W. Lee and *I-H. Li* /Journal of Mechanical Science and Technology 30 (1) (2016) 381-39

1. Then they exactly the control **Example 12.4.** *I. W. Lee and H. Li / Journal of Mechanical Science and Technology 30 (1) (2016) 581-396*
 16.6. 16.6. 16.6. 16.6. 16.6. 16.6. 16.6. 16.1. 17.1. 18.1. 18.1. 18.1. 18.1. 18.1. Finden hyer

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variable called the sime cosine for

variable of the sime called the sime cosine for

variable of the sime called the sime called the ing algorithm for Eq. (23) are chosen properly, the estimation **Example layer**

Figure 1 and the set of a SISO FNN.

Fig. 4. Structure of a SISO FNN problems, however, restrict the traditional NNs to the real applications, such as the selection of the network structure, the bility analysis of the closed-loop system.

Many abstracts and general situations have adopted extended concept of Fourier analysis, which is particularly used to decompose a signal into its component frequencies with different amplitudes and phases. The FNN is proposed in the view of the Fourier analysis and the neural network theory, which instead of using common Gaussian functions or sigwhere $x \in \Re$ and $f \in \Re$ are the input and output, respectively, $\mathbf{W} = \{w_1, w_2, \ldots, w_n\}$ is the network weight, $\sigma(x) = x$, $\sigma(x)$, $\sigma(x) = \sigma(x)$, $\sigma(x) = \sigma(x)$, $\sigma(x) = \sigma(x)$, $\sigma(x) = \int \sigma(x)$, $\sigma(x) = \int \sigma(x)$, $\sigma(x) = \int \sigma(x)$, $\$ the family of complex Fourier functions. Due to Fourier analysis's excellent performance in nonlinear function modeling and decomposition, unlike ordinary NN, that employing the Fourier basis function may result in much higher availability of rates of convergence for the approximation. The structure of a Single-input-single-output (SISO) FNN is illustrated in Fig. 4. Both the input layer and the output layer only have one node. Note that the chosen node number in the hidden layer tends to be based upon the system bandwidth. In Fig. 4, *x* but provien speed, the provien or occur minimum and the state $\rightarrow x \rightarrow x \rightarrow y \rightarrow z$. Hence, $f(x)$ and $\rightarrow \infty$. Hence, $f(x)$ and $\rightarrow \infty$. Hence, $f(x)$ and $\rightarrow \infty$ and $\rightarrow \infty$. Hence, $f(x)$ and $\rightarrow \infty$ and $\rightarrow \infty$ and $\rightarrow \infty$ Many anasys of ne coosecutions have adopted ex-

Many abstracts and general situations have adopted ex-
 x^2 in is large enough, with an error sate

lended concept of Fourier analysis, which is particularly used

decomp Many abstracts and general sutuations have adopted ex-

tended concept of Fourier analysis, which is particularly used

to decompose a signal into its component frequencies with

different amplitudes and phases. The FNN i integer based upon the system bandwidth, and the total num of excompacts a spant in the store of the proposition of the hidden layer composed in the store of the FNN is proposed in the FNN I functions, the activation function $\sigma_i(\cdot)$ is selected as
series and provides a specific lin

flamity of complex Fourier functions. Due to Fourier cintes
for $\sigma_i(\cdot)$ is the Fourier theories and the Fourier transformat the input layer and the output layer only have
 e that the chosen node number in the hidden
 e based upon the system bandwidth. In Fig. 4,

respectively, represent the input and output;
 $f,...,M$) are the basis functio Note that the chosen node number in the historic state of the system bandwidth. In Fi
 i s to be based upon the system bandwidth. In Fi
 $f(x)$, respectively, represent the input and ou
 $= -M, ..., M$) are the basis function id functions, the activation function $\sigma_i(\cdot)$ is selected as
series and provides a pecifical
family of complex Fourier functions. Due to Fourier circulas and the Fourier trans
flysis's excellent performance in nonlinear *y* of rates of convergence for the approximation. In estruction
and Caster of a Single-input-isingle-output (SISO) FNN is illustrated Thus, FNN can
are of a Single-input-isingle-output (SISO) FNN is illustrated Thus, FNN

$$
f(x) = \sum_{i=-M}^{M} c_{0,i} e^{jw_ix}.
$$
 (24) no

The family of complex Fourier functions $e^{j w_n x}$ has orthogonality, where j is the imaginary unit that meets $x^2 = -1$, $w_n = \frac{2n\pi}{T}$ is the *n*th harmonic of the function f $=\frac{2n\pi}{\pi}$ is the *n*th harmonic of the function *f* where $Q - \frac{w_n}{\pi}$

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with $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is
based on Euler's formula. In real implementation, Eq. (24) is
often represented as the sine/cosine form
 $f(x) = \mathbf{W}^T \math$ 96
 $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

implementation, Eq. (24) is

e form

(25) based on Euler's formula. In real implementation, Eq. (24) is often represented as the sine/cosine form · ·

$$
f(x) = \mathbf{W}^T \mathbf{q}(x) \tag{25}
$$

and Technology 30 (1) (2016) 381~396
 $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

d on Euler's formula. In real implementation, Eq. (24) is

represented as the sine/cosine form
 $(x) = \mathbf{W}^T \mathbf{q}(x)$ (25)
 $\mathbf{$ *f x x* ⁼ **W q** (25) where 1 1 () 1, cos , sin ,...,cos , sin *^T M M x w x w x w x w x* ^º é ù ë û **^q** is the *ence and Technology 30 (1) (2016) 381-396*

with $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

based on Euler's formula. In real implementation, Eq. (24) is

often represented as the sine/cosine form
 $f(x) = \mathbf{W}$ *ence and Technology 30 (1) (2016) 381-396*

with $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

based on Euler's formula. In real implementation, Eq. (24) is

often represented as the sine/cosine form
 $f(x) = \mathbf{W}$ family of activation functions, and $\mathbf{W} = \begin{bmatrix} W_0 & W_1 & \dots & W_{2M-1} & W_{2M} \end{bmatrix}^T$ is the vector of network weights. Substituting the activation function *of x* **3** *(1) (2016) 381–396*

with *n* = 0,±1,±2,...,±M, and *e*^{*n*₆x} = cos *w_n*x + *i*sin *w_n*x is

based on Euler's formula. In real implementation, Eq. (24) is

often represen *ence and Technology 30 (1) (2016) 381-396*

with $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_0 x} = \cos w_n x + i \sin w_n x$ is

based on Euler's formula. In real implementation, Eq. (24) is

often represented as the sine/cosine form
 $f(x) = \mathbf{W$ and Technology 30 (1) (2016) 381-396
 $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

d on Euler's formula. In real implementation, Eq. (24) is

represented as the sine/cosine form
 $(x) = \mathbf{W}^T \mathbf{q}(x)$ (25)
 $\mathbf{$ *f f <i>x* = *x <i>x* + *i* shows the *n* = 0, ±1, ±2,..., ±M, and *e*^{*n*_xx</sub>} = cos *w_{_ix}x* + *i* sin *w_ix* is based on Euler's formula. In real implementation, Eq. (24) is often represented as the sine/cosine f $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

d on Euler's formula. In real implementation, Eq. (24) is

represented as the sine/cosine form
 $(x) = W^T q(x)$ (25)
 $q(x) = \begin{bmatrix} q(x) \equiv [1, \cos w_n x, \sin w_n x, ..., \cos w_n x, \sin w_n x] \end{bmatrix}$ i *i* $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{i\omega_n x} = \cos w_n x + i \sin w_n x$ is

ed on Euler's formula. In real implementation, Eq. (24) is

an represented as the sine/cosine form
 $f(x) = \mathbf{W}^T \mathbf{q}(x)$ (25)

ere $\mathbf{q}(x) = [1, \cos w_i x, \sin w_i x, ..., \cos w$ $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{jw_n x} = \cos w_n x + i \sin w_n x$ is

on Euler's formula. In real implementation, Eq. (24) is

epresented as the sine/cosine form
 $\mathbf{q} \cdot \mathbf{x} = \mathbf{W}^T \mathbf{q}(x)$ (25)
 $\mathbf{q}(x) = \begin{bmatrix} 1, & \cos w_1 x, & \sin w_1 x,$ From $f(x) = \mathbf{W}^T \mathbf{q}(x)$ (25)

nere $\mathbf{q}(x) = [1, \cos w_i x, \sin w_i x, \dots, \cos w_i x, \sin w_i x]^T$ is the

mily of activation functions, and $\mathbf{W} = [W_0 W_1 \dots W_{1M-1} W_{2M}]^T$ is the vector of network weights. Substituting

activation funct *f* xx = **w q**(x) = [1, cos w_ix, sin w_ix,..., cos w_{ix}x, sin w_i xx^T is the carrival of activation functions, and **w** = $\left[H_0^r \ W_1 \right]$...
 F x_{2x}, *H* y_x^T is the vector of network weights. Substituting where $\mathbf{q}(x) = \begin{bmatrix} 1, & \cos w_1 x, & \sin w_1 x, ..., \cos w_M x, & \sin w_M x \end{bmatrix}^T$ is the *L.W. Lee and HI. Li / Journal of Mechanical Science and Technology 30 (1) (2016) 381-396*

(dden have

with $n = 0, \pm 1, \pm 2, ..., \pm M$, and $e^{j\pi_0 x} = \cos w_n x + i \sin w_n x$ is

based on Euler's formula. In real implementation, Eq. (2 L.-W. Lee and I-H. Li / Journal of Mechanical Science and Technology 30 (1) (2016) 381-396

don hayer

with $n = 0, \pm 1, \pm 2, ..., \pm N$, and $e^{fn_x x} = \cos w_n x + i \sin w_n x$ is

based on Euler's formula. In real implementation, Eq. (24)

$$
f(x) = \mathbf{W}^T \mathbf{q}(x) + \varepsilon_n(x) \tag{26}
$$

satisfies

$$
\left|\varepsilon_{n}\right| \leq \sum_{i>n} \left(|w_{i}| + |w_{i+1}|\right).
$$
 (27)

convergent speed, the problem of local minimum and the sta-
bility analysis of the closed-loop system
 $n \to \infty$. Hence, $f(x)$ can be approximated as follows as long possible mean square approximation to the function as *n* is large enough, with an error satisfying Eq. (27): T_{2M} I is ue vector of network weights. Substituting
 $(x) = \mathbf{W}^T \mathbf{q}(x)$, we can rewrite Eq. (23) as
 $(x) = \mathbf{W}^T \mathbf{q}(x) + \varepsilon_n(x)$ (26)
 $(x) = \mathbf{W}^T \mathbf{q}(x) + \varepsilon_n(x)$ (26)
 $\in \varepsilon_n(t)$, $n = 2M + 1$, shows the approx *M*-1 *P*_{2M} J is to vector of network weigns. Substituting
activation function $\mathbf{g}(x)$ be orthogonal Fourier activa-
function $\mathbf{g}(x)$, we can rewrite Eq. (23) as
 $f(x) = \mathbf{W}^T \mathbf{q}(x) + \varepsilon_n(x)$ (26)
 $f(x) = \mathbf{W}^T \math$

$$
f(x) \cong \mathbf{W}^T \mathbf{q}(x). \tag{28}
$$

FNN can be regarded as a particular case of NN. It maintains the same universal approximation property as a Fourier series and provides a specific link between the network coefficients and the Fourier transform. The nature of the ideal network weights of the FNN is the spectra of the approximated function. This clear physical picture indicates that when we employ the FNN in real applications, the selection of the term number *M* can be established on the system bandwidth. Thus, FNN can accomplish the same quality of approximation with a network of reduced size. Remark 1 expresses the idea of which the Fourier series is applied to approximate the non periodic nonlinear functions. *f* $\rightarrow \infty$. Fielding, $f(x)$ can be approximated as to toows as only s n is large enough, with an error satisfying Eq. (27):
 $f(x) \equiv W^T q(x)$. (28)

FNN can be regarded as a particular case of NN. It main-

finitials functi $f(x) \cong W^T \mathbf{q}(x)$. (28)

SNN can be regarded as a particular case of NN. It main-

is the same universal approximation property as a Fourier

es and provides a specific link between the network coeffi-

its and the Four FNN can be regarded as a particular case of NN. It main-
tains the same universal approximation property as a Fourier
series and provides a specific link between the network coeffi-
cients and the Fourier transform. The n tion. This clear physical picture indicates that when we
loy the FNN in real applications, the selection of the term
ber *M* can be established on the system bandwidth.
i, FNN can accomplish the same quality of approximat crion. This clear physical picture indicates that when we
ploy the FNN in real applications, the selection of the term
there *M* can be established on the system bandwidth.
Is, FNN can accomplish the same quality of appro function. This clear physical picture indicates that when we
employ the FNN in real applications, the selection of the term
number *M* can be established on the system bandwidth.
Thus, FNN can accomplish the same quality

Remark 1 [39]: Take into account of a nonlinear function

$$
f(t) = \int_{-\infty}^{\infty} F(w)e^{jwt} dw,
$$
 (29)

c_{1,i} and *c_{0,i} f*(*t*) as a non-periodical funct

is a positive

the total num-
 f(*t*) = $\int_{-\infty}^{\infty} F(w)e^{jwt} dw$,

Therefore,

where $F(w) = \int_{-\infty}^{\infty} f(t)e^{-jwt} dt$.

in the frequency domain, Eq. (24)

for-periodical $=\int_{-\infty}^{\infty} f(t)e^{-jwt}dt$. By applying Shannon's theory in the frequency domain, Eq. (29) is equivalent to partition the non-periodical function with an appropriate window in the time domain. Then, it can be achieved in discrete form, Take into account of a nonlinear function
odical function. We come to
 $f(t)e^{-jwt}dt$. By applying Shannon's theory
main, Eq. (29) is equivalent to partition the
etion with an appropriate window in the
it can be achieved in di

$$
f(t) = \int_{-\infty}^{\infty} F(w)e^{jwt} dw = \sum_{n=-M}^{M} X(n\Omega)e^{jn\Omega t} \Delta w,
$$
 (30)

w. From an engineering point of view, the control objective is in which $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vec- $\Delta w = \frac{2\pi}{T}$. *T L*-*W. Lee and I-H. Li/Journal of Mechanical Science and Technology

w. From an engineering point of view, the control objective is in which* $f(\mathbf{x})$ *,

often defined at a limited time interval [0,<i>T*], and conse-

up to

4. Design of control algorithms and stability analysis

The objective of this study was to develop an AFNN-based SMC with H_{∞} tracking design technique for the motion control of a rodless PAS, in which the H_{∞} tracking design technique is introduced in the SMC to handle the function approximation errors, un-modeled dynamics and disturbances. Before developing the controller, the feedback linearization should be done as in the following.

4.1 Feedback linearization of the input-output map

According to the proposed linearization strategy in Ref. [49], the model of PAS, as shown in Eq. (22), can be linearized by differentiating its output. The static and Coulomb friction forces as well as external load are considered as uncertainties so that we neglect it at the beginning while linearizing. By and applying the feedback linearization theory to a pneumatic The objective of this study was to develop an AFNN-based

SMC with H_x tracking design

control of a rodess PAS, in which the H_x tracking design

technique is introduced in the SMC to handle the function

periyonizatio developing the controller, the receivance interarization

developing the controller, the receivance interarization

ording to the proposed linearization strategy in Ref. [49],

del of PAS, as shown in Eq. (22), can be lin ld be done as in the following.

Feedback linearization of the input-output map
 $= f(x,t) + \overline{g}(x,t)u$

where

ccording to the proposed linearization strategy in Ref. [49],

model of PAS, as shown in Eq. (22), can be lineari Feedback linearization of the input-output map

According to the proposed linearization strategy in Ref. [49],

model of PAS, as shown in Eq. (22), can be linearized by

Ferentiating its output. The static and Coulomb fri

$$
\dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, t)u(t),
$$

\n
$$
y = h(\mathbf{x}, t) = x_1,
$$
\n(31)

where the state vector **x** and control input *u* are

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad u(t) = X,\tag{32}
$$

scribed as

rvo system Eq. (22), the system can be expressed as
\n
$$
\bar{g}(x,t) = \frac{kRT_1C_0C_0u(\hat{f}(x_1,P_2,P_1)-x_1+\Delta)+\hat{f}(x_4,P_2)}{M(x_1+\Delta)(I-x_1+\Delta)}
$$
\nHere the state vector x and control input u are
\nthere the state vector x and control input u are
\nvector, $\bar{f}(x,t)$ is the function of state variable
\nthe control gain, and $u(t)$ is the control input w
\nserved v. Note that $\bar{f}(x,t)$ and $\bar{g}(x,t)$ are
\nfunction with unknown variation bound.
\n
$$
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, u(t) = X,
$$
\n(32) However, the nonlinear time-varying behavior
\ntan boundary function bounded
\ndin the corresponding vector field $f(x)$ and $g(x)$ are de-
\ndesign. Therefore, the AFNN is proposed here to
\ndisign. Therefore, the AFNN is proposed here to
\n $\bar{g}(x,t)$ (to be discussed in the unknown function)
\ntracking control is introduced to compensate the
\n $\bar{g}(x,t)$ (to be discussed in the unknown function)
\ntrace the computational load.
\n $f(x,t) = \begin{bmatrix} x_2 \\ \frac{1}{M} \left[-K_yx_2 + A(x_3 - x_4) \right] \\ -\frac{k(x_2x_3)}{x_1+\Delta} \\ 0 \\ \frac{kRT_3C_4C_0w\hat{f}(x_3,P_2)}{x_1+\Delta} \end{bmatrix},$ \n(33) *4.2 Design of an AFNN-SMC*
\nTransforming the system dynamics Eq. (35) in
\nsystem, we have the following equation:
\n $y^{(n)} = \bar{f}(x,t) + \bar{g}(x,t)u(t) + d(x,t)$
\n $g(x,t) = \begin{bmatrix} 0 \\ \frac{kRT_3C_4C_0w\hat{f}(x_3,P_2)}{A(x_1+\Delta)} \\ 0 \\ \frac{-kRT_3C_4C_0w\hat{f}(x_3,P_1,P_2)}{A(x_1+\Delta)} \end{bmatrix},$ \n(34) there $x = [y(t) \ \hat{y}(t) \dots y^{(n-1)}(t)]^T \in \Re^n$ is the
\nand the unmodeled static and Coulomb fire
\nparameter uncertainty. Without

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<i>w*. From an engineering point of view, the control objective is in which $f(x, t)$ and $g(x, t)$ are unknown and smooth of

often quently the base frequency can be simply selected as $L_f h$ and $L_g h$ be the Lie derivatives of h with respect to f and g. Since $L_g L_f h(\mathbf{x}, t) = 0$ for all $k < 3$, and *ence and Technology 30 (1) (2016) 381–396* 387

in which *f* (**x**,*t*) and *g*(**x**,*t*) are unknown and smooth vector functions on the set $\Omega \subset \mathbb{R}^4$; $y = h(\mathbf{x}, t) \in (0, l) \subset \mathbb{R}$. Let $L_f h$ and $L_g h$ be the *Lie der ence and Technology 30 (1) (2016) 381∼396* 387

in which $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vector functions on the set $\Omega \subset \mathbb{R}^4$; $y = h(\mathbf{x},t) \in (0, l) \subset \mathbb{R}$. Let $L_f h$ and $L_g h$ be the *Lie derivat Let and Technology 30 (1) (2016) 381~396* 387
 L n which $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vector functions on the set $\Omega \subset \mathbb{R}^4$; $y = h(\mathbf{x},t) \in (0,t) \subset \mathbb{R}$. Let $L_f h$ and $L_g h$ be the *Lie derivati* and *Technology* 30 (1) (2016) 381~396

hich $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vec-

unctions on the set $\Omega \subseteq \mathfrak{R}^4$; $y = h(\mathbf{x},t) \in (0,t) \subset \mathfrak{R}$. Let

and $L_g h$ be the *Lie derivatives* of h with respe $L_f h$ and $L_c h$ be the *Lie derivatives* of *h* with respect to *f ence and Technology 30 (1) (2016) 381∼396* 387

in which $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vector functions on the set Ω ⊂ \mathbb{R}^4 ; $y = h(\mathbf{x},t) \in (0,t) \subset \mathbb{R}$. Let $L_f h$ and $L_g h$ be the *Lie derivati* mce and Technology 30 (1) (2016) 381~396

1 which $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vector functions on the set $\Omega \subset \mathbb{R}^4$; $y = h(\mathbf{x},t) \in (0,l) \subset \mathbb{R}$. Let
 $L_l h$ and $L_g h$ be the *Lie derivatives* of h gree of the system is 3. Then, we obtain [50] $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vec-

ns on the set $\Omega \subset \mathbb{R}^4$; $y = h(\mathbf{x},t) \in (0, l) \subset \mathbb{R}$. Let
 h be the *Lie derivatives* of *h* with respect to *f*

Since $L_g L_f^k h(\mathbf{x},t) = 0$ for all $k < 3$, and

$$
c e
$$
 and Technology 30 (1) (2016) 381-396\n387\nwhich $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown and smooth vec-
\nr functions on the set Ω ⊂ η⁴; $y = h(\mathbf{x},t) \in (0,t) \subset \mathfrak{R}$. Let f_h and $L_g h$ be the Lie derivatives of h with respect to f
\nd g . Since $L_g L_f^k h(\mathbf{x},t) = 0$ for all $k < 3$, and k^2 , $k^2 h(\mathbf{x},t) \neq 0$ for all $k \geq 3$ for all $x(t)$, the relative de-
\nee of the system is 3. Then, we obtain [50]
\n
$$
y^{(3)} = L_f^3 h(\mathbf{x},t) + L_g L_f^2 h(\mathbf{x},t)u(t)
$$
\n
$$
= \begin{cases} \frac{K_{\varphi}^2 x_2 - K_{\varphi} A(x_3 - x_4)}{M^2} - \frac{A k x_2 [x_3 (l - x_1 + \Delta) + x_4 (x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)} \end{cases} + \begin{cases} \frac{k R_f c_g C_0 u_1 f(x_3, P, P_s)(l - x_1 + \Delta) + f(x_4, P, P_s)(x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)} \end{cases} h
$$
\nthere
\nwhere
\n
$$
\overline{f}(\mathbf{x},t) = \frac{K_{\varphi}^2 x_2 - K_{\varphi} A(x_3 - x_4)}{M^2} - \frac{A k x_2 [x_3 (l - x_1 + \Delta) + x_4 (x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)} \end{cases}
$$
\nand
\n
$$
\overline{g}(\mathbf{x},t) = \frac{k R_f C_g C_0 u_1 f(x_3, P, P_s)(l - x_1 + \Delta) + f(x_4, P, P_s)(x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)}
$$
\nand
\n
$$
\overline{g}(\mathbf{x},t) = \frac{k R_f C
$$

where

$$
= f(\mathbf{x},t) + \overline{g}(\mathbf{x},t)u(t),
$$
\n(35)
\nwhere
\n
$$
\overline{f}(\mathbf{x},t) = \frac{K_{y}^{2}x_{2} - K_{y}A(x_{3} - x_{4})}{M^{2}} - \frac{A k x_{2}[x_{3}(l - x_{1} + \Delta) + x_{4}(x_{1} + \Delta)]}{M(x_{1} + \Delta)(l - x_{1} + \Delta)}
$$
\nand
\n
$$
\overline{g}(\mathbf{x},t) = \frac{k R T_{s} C_{d} C_{0} M_{t} \hat{f}(x_{3}, P_{s}, P_{e})(l - x_{1} + \Delta) + \hat{f}(x_{4}, P_{s}, P_{e})(x_{1} + \Delta)]}{M(x_{1} + \Delta)(l - x_{1} + \Delta)}
$$
\nIn Eq. (35), *y* is the piston displacement, **x** is the state

$$
\overline{g}(\mathbf{x},t) = \frac{kRT_sC_dC_0M\hat{f}(x_3,P_s,P_e)(l-x_1+\Delta) + \hat{f}(x_4,P_s,P_e)(x_1+\Delta)}{M(x_1+\Delta)(l-x_1+\Delta)}
$$

 $\frac{4(x_3 - x_4)}{M} - \frac{A k x_2 [x_3(l - x_1 + \Delta) + x_4(x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)}$
 $\hat{f}(x_3, P_s, P_e)(l - x_1 + \Delta) + \hat{f}(x_4, P_s, P_e)(x_1 + \Delta)]$
 $M(x_1 + \Delta)(l - x_1 + \Delta)$

the piston displacement, **x** is the state

the function of state variables, \over In Eq. (35) , *y* is the piston displacement, **x** is the state $\int_{t}^{t} \frac{kRT_{x}C_{x}C_{0}w\hat{i}f(x_{x},P_{x},P_{z})(I-x_{1}+\Delta)+\hat{f}(x_{x},P_{x},P_{z})(x_{1}+\Delta)}{M(x_{1}+\Delta)(I-x_{1}+\Delta)} du$
 $=\overline{f}(\mathbf{x},t)+\overline{g}(\mathbf{x},t)u(t),$ (35)

where
 $\overline{f}(\mathbf{x},t)=\frac{K_{y}^{2}x_{2}-K_{y}A(x_{3}-x_{4})}{M^{2}}-\frac{Akx_{2}[x_{3}(I-x_{1}+\Delta)+x_{4}(x_{1}+\Delta)]}{M(x_{$ $\frac{\left\{kR T_{s}C_{\theta}C_{0}w\left(\hat{f}(x_{s},P_{s},P_{e})(I-x_{1}+\Delta)+\hat{f}(x_{s},P_{s},P_{e})(x_{1}+\Delta)\right)}{M(x_{1}+\Delta)(I-x_{1}+\Delta)}\right\}u}{\left(\hat{f}(x,t)=\frac{K_{y}^{2}x_{2}-K_{y}A(x_{3}-x_{4})}{M^{2}}-\frac{Akx_{3}[x_{3}(I-x_{1}+\Delta)+x_{4}(x_{1}+\Delta)]}{M(x_{1}+\Delta)(I-x_{1}+\Delta)}\right.$

and

and
 $\overline{g}(x,t)=\frac{kR T$ $M(x_1 + \Delta)(l - x_1 + \Delta)$ (35)
 $= \overline{f}(\mathbf{x}, t) + \overline{g}(\mathbf{x}, t)u(t),$ (35)

where
 $\overline{f}(\mathbf{x}, t) = \frac{K_y^2 x_2 - K_y A(x_3 - x_4)}{M^2} - \frac{A k x_2 [x_2 (l - x_1 + \Delta) + x_4 (x_1 + \Delta)]}{M(x_1 + \Delta)(l - x_1 + \Delta)}$

and

and
 $\overline{g}(\mathbf{x}, t) = \frac{k R T_i C_a C_b w_i \hat{f}(x_3, P_i, P$ functions with unknown variation bound.

and the corresponding vector field $f(x)$ and $g(x)$ are interesting in the corresponding vector field $f(x)$ and $g(x)$ are de-
the control of the corresponding vector field $f(x)$ and $x = f(x, t) + g(x, t)u(t)$,
so the vector field (stem can be expressed as
 $\overline{g}(x,t) = \frac{kRT_cC_0w_1\hat{f}(x_3,P_c,P_c)(1-x_1+\Delta)+\hat{f}(x_2+\Delta)+\hat{f}(x_3-P_c)}{M(x_1+\Delta)(1-x_1+\Delta)}$

(31) In Eq. (35), y is the piston displacement,

vector, $\overline{f}(x,t)$ is the function of state varial

the contro 22), the system can be expressed as
 $\frac{1}{g(x,t)} = \frac{kRT_cC_cwt\hat{f}(x_1, P_1, P_c)(I-x_1+\Delta) + \hat{f}}{M(x_1+\Delta)(I-x_1+\Delta)}$

(31) In Eq. (35), y is the piston displacement

vector, $\overline{f}(x,t)$ is the piston displacement

vector, $\overline{f}(x,t)$ is regieto in a the vegining wine interacting the strategy is the spectral of the spectral of the system can be expressed as
 $\overline{g}(x,t) = \frac{kRT_cC_c\omega_1(\hat{f}(x_x,P_x,P_x)(1-x_1+\Delta)+\hat{f}(x_x, P_x)}{M(x_1+\Delta)(1-x_1+\Delta)}$
 $\overline{g}(x,t) = \frac{kRT_cC_c\omega_1(\hat{f}(x$ e feedback linearization theory to a pneumatic
 $E(x,t) = \frac{kRT_cC_e\sqrt{t}}{(x_1R_c + \Delta)(t-x_1+\Delta)} + g(x_1\lambda\mu(t))$
 $= x_1$,

(31) In Eq. (35), y is the piston displacement,

atte vector x and control input u are
 $\frac{\sec(x) + \tan(x) + \sec(x) + \tan(x) + \tan(x)$ However, the nonlinear time-varying behavior with uncertain bounds of the system dynamics makes it difficult to obtain an accurate dynamic model for a model-based controller design. Therefore, the AFNN is proposed here to approximate $f(\mathbf{x},t) = \frac{\kappa R T_c C_s C_0 w_t \hat{f}(x_3,P_c)(1-x_1+\Delta) + \hat{f}(x_4,P_c)(x_1+\Delta)}{M(x_1+\Delta)(1-x_1+\Delta)}$

and

and
 $\overline{g}(\mathbf{x},t) = \frac{kRT_c C_s C_0 w_t \hat{f}(x_3,P_c)(1-x_1+\Delta) + \hat{f}(x_4,P_c)(x_1+\Delta)}{M(x_1+\Delta)(1-x_1+\Delta)}$.

In Eq. (35), y is the piston displacement, **x** is and
 $\overline{g}(x,t) = \frac{kRT_sC_sC_0w[\hat{f}(x_3, P_s, P_e)(-x_1 + \Delta) + \hat{f}(x_4, P_s, P_e)(x_1 + \Delta)]}{M(x_1 + \Delta)(1 - x_1 + \Delta)}$

In Eq. (35), y is the piston displacement, x is the state

vector, $\overline{f}(x,t)$ is the function of state variables, $\overline{g}(x,t)$ tracking control is introduced to compensate the approximation error and thus improve the control performance and reduce the computational load. control gam, and $u(t)$ is the control miput votage of the
coincid gam, and $u(t)$ is the control miput votage of the
citions with unknown variation bound.
However, the nonlinear time-varying behavior with uncer-
hounds of flunctions with unknown variation bound.

However, the nonlinear time-varying behavior with uncer-

tain bounds of the system dynamics makes it difficult to ob-

tain an accurate dynamic model for a model-based controller However, the nonlinear time-varying behavior with uncertain bounds of the system dyaminics makes it difficult to obtain an accurate dynamic model for a model-based controller design. Therefore, the AFNN is proposed here t tan bounds of the system dynamics makes it difficult to ob-
tain an accurate dynamic model for a model-based controller
design. Therefore, the AFNN is proposed here to approximate
the unknown functions $\overline{f}(\mathbf{x},t)$ (to

4.2 Design of an AFNN-SMC

Transforming the system dynamics Eq. (35) into a general system, we have the following equation:

$$
y^{(n)} = \overline{f}(\mathbf{x},t) + \overline{g}(\mathbf{x},t)u(t) + d(\mathbf{x},t)
$$
\n(36)

and the unmodeled static and Coulomb friction forces, **f** the multion functions $\overline{f}(x, t)$ (to be discussed later) and $\overline{g}(x, t)$ to dispose of model dependency. In addition, H_x racking control is introduced to compensate the approximation error and thus improve the co the unknown functions $f(x, t)$ (to be discussed later) and
 $\overline{g}(x, t)$ to dispose of model dependency. In addition, H_x

tracking control is introduced to compensate the approxima-

tion error and thus improve the contr

be assumed to be strictly positive, i.e., $\bar{g}(x,t) \ge g' > 0$. As-

be assumed to be strictly positive, i.e., $\bar{g}(x,t) \ge g' > 0$. As-

suming that the solution of the system exists and the order of

the system is known, we c suming that the solution of the system exists and the order of the system is known, we can rewrite Eq. (36) as

$$
y^{(n)} = F(\mathbf{x}, t) + \overline{g}(\mathbf{x}, t)u(t)
$$
\n(37)

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assumed to be strictly positive, i.e., $\overline{g}(\mathbf{x},t) \ge g^t > 0$. As-

nomial in which λ is a Laplace ope

ning that the solution 388 *L.W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396*

be assumed to be strictly positive, i.e., $\overline{g}(x, t) \ge g' > 0$. As-

suming that the solution of the system exists and the orde **g** the summed to be strictly positive, i.e., $\overline{g}(x,t) \geq g' > 0$. As uncoming that the solution of the system exists and the order of that the system is known, we can rewrite Eq. (36) as $e^{(n-1)} = -a_1e - y^{(n)} = F(x,t) + \overline{g}(x$ use AFNN to approximate them for the controller design in this study. The adaptive laws of the coefficient vector of network weight can be acquired from the Lyapunov stability theorem. We make the following assumption to obtain a gen eralized result. 388
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be assumed to be strictly positive, i.e.,* $\overline{g}(x, t) \ge g' > 0$ *. As-

suming that the solution of the system exists and the or*

Assumption: (i) The nonlinear system expressed in Eq. (31) lable region U_c , and (iii) the internal dynamics of the system with the following AFNN-SMC are stable.

show the system is known of the system control of the system is known, we can rewrite Eq. (36) as
 $y^{(n)} = F(\mathbf{x}, t) + \overline{g}(\mathbf{x}, t)u(t)$ (37)

where $F(\mathbf{x}, t) = \overline{f}(\mathbf{x}, t) + d(\mathbf{x}, t)$. Hence, with $F(\mathbf{x}, t)$ and will become
 cause the design of tests rig and restricted control outputs *y*^(a) = $F(x, t) = \overline{f}(x, t) + d(x, t)$. Hence, with $F(x, t)$ and $\overline{g}(x, t)$ in Eq. (37)

Where $F(x, t) = \overline{f}(x, t) + d(x, t)$. Hence, with $F(x, t)$ and will become

suse AFNN to approximate them for the controller design in

this where $F(x,t) = \overline{f}(x,t) + d(x,t)$. Hence, with $F(x,t)$ and $W = [e,e,...,e^{x-1}]$ $= [e,e,...,e_{n-1}]$, $\overline{g}(x,t)$ to approximate them for the controller design in
this study. The adaptive laws of the coefficient vector of net-
this study. so they can be indicated by the FNN as shown in Eq. (26). Hence, we have states and the following and the control of paster and the control of paster and the control of the c n. We make the following assumption to obtain a gen-
 $\text{where } \mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} \end{bmatrix}$.

Elative degree of *n*, (ii) the control Assumption: (i) The nonlinear system expressed in Eq. (31)

has a relative degree of n , (ii) the control u appears linear binds.

with respect to $y^{(n)}$ and $\bar{g}(\mathbf{x},t) \neq 0$ for \mathbf{x} in some control-

lable reg with respect to $y^{(n)}$ and (\bar{y}_{x} , and (\bar{y}_{x} , and (\bar{y}_{x} , and (\bar{y}_{x} , and (\bar{y}_{x}), and (\bar{y}_{x}) and \bar{y}_{x}),

$$
y^{(n)} = (\mathbf{W}_F^T \mathbf{q}_F(t) + \varepsilon_F(t)) + (\mathbf{W}_\mathbf{g}^T \mathbf{q}_\mathbf{g}(t) + \varepsilon_\mathbf{g}(t))u
$$

=
$$
\mathbf{W}_F^T \mathbf{q}_F(t) + \mathbf{W}_\mathbf{g}^T \mathbf{q}_\mathbf{g}(t)u + w_t
$$
 (38)

activation function vector, W_F and $W_{\overline{g}}$ are the coefficient I lumped uncertainty. To facilitate the design process of the controller, the lumped uncertainty is generally assumed to have an upper bound.

Assumption: (iv) There exists a positive constant w_t^u , such being elen

cause the desiring of tests rig and restricted control outputs derivations in Refs. [32, 51], we can acquire a control law for
velocity of piston x_3 , and the acceleration or piston x_4 . the Eq. (38) by applying the s anticipated to produce an optimal mean square approximation Therefore $F(x, t)$ and $\overline{g}(x, t)$ meet the Dirichlet conditions,

Hence, we have
 $y^{(n)} = (\mathbf{W}_r^T \mathbf{q}_F(t) + \mathbf{W}_g^T \mathbf{q}_F(t) + \mathbf{w}_f^T \mathbf{q}_F(t))$
 $= \mathbf{W}_r^T \mathbf{q}_F(t) + \mathbf{W}_g^T \mathbf{q}_F(t) + \mathbf{w}_f^T \mathbf{q}_F(t) + \mathbf{w}_f^T \mathbf{$ Hence, we have $x = \frac{-\mathbf{w}_F \mathbf{q}_F(t) - \sum_{i=1}^n a_i e_{i+1} - \sum_{$ $y^{(n)} = (\mathbf{W}_k^T \mathbf{q}_F(t) + \varepsilon_F(t)) + (\mathbf{W}_g^T \mathbf{q}_{\bar{g}}(t) + \varepsilon_{\bar{g}}(t))u$
 $= \mathbf{W}_F^T \mathbf{q}_F(t) + \mathbf{W}_g^T \mathbf{q}_{\bar{g}}(t)u + w_i$ (38)

where $\mathbf{q}_F(t)$ and $\mathbf{q}_{\bar{g}}(t)$ are the family orthogonal Fourier respectively. Cho as = $\mathbf{w}_F \mathbf{q}_F(t) + \mathbf{w}_{\bar{g}} \mathbf{q}_{\bar{g}}(t)u + w$,

ere $\mathbf{q}_F(t)$ and $\mathbf{q}_{\bar{g}}(t)$ are the family orthogonal Fourier respectively. Choosis

ivation function vector, \mathbf{W}_F and \mathbf{W}_g are the coefficient Lyapun of network weight, and $w_i = \varepsilon_i (y) + \varepsilon_i (y)$ is the $\mathbf{A}_i^T \mathbf{P} + \mathbf{P} \mathbf{A}_i = -\mathbf{Q}$

uncertainty. To facilitate the design process of the $\mathbf{A}_i^T \mathbf{P} + \mathbf{P} \mathbf{A}_i = -\mathbf{Q}$

er, the lumped uncertainty is gen *n* or of retwork weight, and $w_i = x_r(t) + x_s(t)dt$ is the $\mathbf{a} \times y_i = \mathbf{a}_i e + a_i e^i + ... + a_i e^{i\pi + 1}$, $a_n = 1$

are a upper bound.
 $\mathbf{a} \times y_i = y_i - y_{i\pi}$, and $\mathbf{b} \times y_i = 0$ and $\mathbf{b} \times y_i = 0$ and $\mathbf{b} \times y_i = 0$ and $\mathbf{b} \$

$$
e = y - y_m. \tag{39}
$$

A switch surface can be defined as

$$
s = a_1 e + a_2 \dot{e} + \dots + a_n e^{(n-1)}, \ a_n = 1 \tag{40}
$$

where a_i are chosen such that $\sum_{i=1} a_i \lambda^{i-1}$ is a Hurwitz poly- $\sum_{i=1}^{n} a_i \lambda^{i-1}$ is a Hurwitz poly-

ivative of Eq.

nomial in which λ is a Laplace operator. Eq. (40) indicates that

$$
e^{(n-1)} = -a_1 e - a_2 \dot{e} - \dots - a_{n-1} e^{(n-2)} + s.
$$
 (41)

and Technology 30 (1) (2016) 381~396

iial in which λ is a Laplace operator. Eq. (40) indicates

⁽ⁿ⁻¹⁾ = -a₁e - a₂e - ... - a_{n-1}e⁽ⁿ⁻²⁾ + s. (41)
 i e = [e, e, ..., e⁽ⁿ⁻²⁾]^T = [e₁, e₂, ..., e_{n-1}] *e and Technology 30 (1) (2016) 381-396*

mial in which λ is a Laplace operator. Eq. (40) indicates
 $e^{(n-1)} = -a_1 e - a_2 e^2 - ... - a_{n-1} e^{(n-2)} + s.$ (41)

If $\mathbf{e} = [e, \dot{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dynamics *ce and Technology 30 (1) (2016) 381-396*

anial in which λ is a Laplace operator. Eq. (40) indicates

at
 $e^{(n-1)} = -a_1e - a_2e^2 - ... - a_{n-1}e^{(n-2)} + s.$ (41)

If $\mathbf{e} = [e, \dot{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dy *nd Technology 30 (1) (2016) 381~396*
 ial in which λ is a Laplace operator. Eq. (40) indicates
 $n^{-1} = -a_1e - a_2e^2 - ... - a_{n-1}e^{(n-2)} + s.$ (41)
 $\mathbf{e} = [e, \dot{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dynamics

become If $e = [e, \dot{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dynamics will become

$$
\dot{\mathbf{e}} = \mathbf{A}_1 \mathbf{e} + [0, \dots, 0, s]^{\mathrm{T}} \tag{42}
$$

\n Hence and Technology 30 (1) (2016) 381~396
\n nominal in which λ is a Laplace operator. Eq. (40) indicates that\n

\n\n
$$
e^{(n-1)} = -a_1e - a_2e - \dots - a_{n-1}e^{(n-2)} + s.
$$
\n (41)\n

\n\n If $\mathbf{e} = [e, \dot{e}, \dots, e^{(n-2)}]^T = [e_1, e_2, \dots, e_{n-1}]^T$, the error dynamics will become\n

\n\n
$$
\dot{\mathbf{e}} = \mathbf{A}_1 \mathbf{e} + [0, \dots, 0, s]^T
$$
\n

\n\n where $\mathbf{A}_1 = \begin{bmatrix}\n 0 & 1 & 0 & \cdots & 0 \\
 0 & 0 & 1 & \cdots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1}\n \end{bmatrix}$ \n

\n\n Theorem 1: Take the finite bandwidth system into account with unknown nonlinear time-varying functions *F*(**x**,*t*) and\n

In our experimental system, the unknown time-varying $\bar{g}(x,t)$ in Eq. (37), which is approximated as Eq. (28). Sup*n* λ is a Laplace operator. Eq. (40) indicates
 ${}_{1}e-a_{2}e^{-}...-a_{n-1}e^{(n-2)}+s.$ (41)
 $...e^{(n-2)}I^{T} = [e_{1},e_{2},...,e_{n-1}]^{T}$, the error dynamics
 $0,...,0,sI^{T}$ (42)
 $0 \t 0 \t 1 \t ... \t 0$
 $\vdots \t i \t \vdots \t \ddots \t i$
 $-a_{1} - a_{2} - a_{$ blogy 30 (1) (2016) 381-396

iich λ is a Laplace operator. Eq. (40) indicates
 $e^{(\alpha-2)}e^{-\lambda} - a_{n-1}e^{(n-2)} + s.$ (41)
 \cdots , $e^{(n-2)}\big)^T = [e_1, e_2, \dots, e_{n-1}]^T$, the error dynamics

0,..., 0, s]^T (42)
 $0, \dots, 0, s$]^T **Theorem 1**: Take the finite bandwidth system into account nomial in which λ is a Laplace operator. Eq. (40) indicates
that
that
 $e^{(n-1)} = -a_1e - a_2e^2 - \dots - a_{n-1}e^{(n-2)} + s.$ (41)
If $e = [e, \dot{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dynamics
will become
 $\dot{e} = A_1e + [0, ..., 0, s]^T$ **hat**
 a *g*^(*n*-1) = -*a*_{*g*} *e* -*a*_{*g*}^{*g*} -... -*a*_{*n*-1}*g*^(*n*-2) + *s*. (41)
 If c = [*e*,*e*,...,*e*^(*n*-2)]^{*T*} = [*e*₁, *e*₂,...,*e*_{*n*-1}]^{*T*}, the error dynamics
 cill become
 c = **A** pose assumptions (i)-(iv) are satisfied and following similar derivations in Refs. [32, 51], we can acquire a control law for Eq. (38) by applying the sliding-mode control method, the **Theorem 1:** Take the finite bandwidth system into account
with unknown nonlinear time-varying functions $F(\mathbf{x},t)$ and
 $\overline{g}(\mathbf{x},t)$ in Eq. (37), which is approximated as Eq. (28). Sup-
pose assumptions (i)-(iv) are sa ystem into account
ctions $F(\mathbf{x},t)$ and
d as Eq. (28). Sup-
l following similar
re a control law for
ontrol method, the
 $\frac{(n)}{m} - k_p \text{sgn}(s)$ **em 1**: Take the finite bandwidth system into account
nown nonlinear time-varying functions $F(\mathbf{x},t)$ and
n Eq. (37), which is approximated as Eq. (28). Sup-
mptions (i)-(iv) are satisfied and following similar
ns in Ref ..., 0, s]^T (42)
 $\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} \end{bmatrix}$.

Take the finite bandwidth system into account

nonlinear time-varying functions $F(\mathbf{x}, t)$ and

(37), whic () *n* **n** *i* $-a_2 - a_3$ $\cdots -a_{n-1}$ *j*
 n n : Take the finite bandwidth system into account

bown nonlinear time-varying functions $F(\mathbf{x},t)$ and

Eq. (37), which is approximated as Eq. (28). Sup-

protions (i)-(iv) a e the finite bandwidth system into account
linear time-varying functions $F(\mathbf{x},t)$ and
, which is approximated as Eq. (28). Sup-
i)-(iv) are satisfied and following similar
[32, 51], we can acquire a control law for
ng t *g*-varying functions $F(\mathbf{x}, t)$ and
approximated as Eq. (28). Sup-
satisfied and following similar
we can acquire a control law for
ding-mode control method, the
 $\sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - k_p \text{sgn}(s)$
 $\sum_{i=1}^{n} \frac{p$ $\dot{\mathbf{e}} = \mathbf{A}_1 \mathbf{e} + [0,...,0,s]^T$ (42)
 $\text{etc. } \mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} \end{bmatrix}$
 Theorem 1: Take the finite bandwidth system into account

th unknown no *ta*₂ - -*a*₃ ... - -*a*_{*n*-1}]

the finite bandwidth system into account

mear time-varying functions $F(\mathbf{x},t)$ and

which is approximated as Eq. (28). Sup-
 $F(\mathbf{x},t)$ and

(which is approximated as Eq. (28). Sup-Finite bandwidth system into account
time-varying functions $F(\mathbf{x},t)$ and
th is approximated as Eq. (28). Sup-
are satisfied and following similar
51], we can acquire a control law for
e sliding-mode control method, the
 A₁e + [0,...,0,*s*]^T (42)
 $A_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} \end{bmatrix}$.
 orem 1: Take the finite bandwidth system into account
 orem 1: Take the finite bandwi and Technology 30 (1) (2016) 381-396

ial in which λ is a Laplace operator. Eq. (40) indicates

which λ is a Laplace operator. Eq. (40) indicates
 $\mathbf{e} = [e, \hat{e}, ..., e^{(n-2)}]^T = [e_1, e_2, ..., e_{n-1}]^T$, the error dynamics
 0 ... 0 ... 0
 \therefore ... 0 ... 0
 \therefore ... = ...

inte bandwidth system into account

me-varying functions $F(\mathbf{x},t)$ and

is approximated as Eq. (28). Sup-

are satisfied and following similar

1, we can acquire a contr **Theorem 1:** Take the finite bandwidth system into account
with unknown nonlinear time-varying functions $F(\mathbf{x},t)$ and
 $\bar{g}(\mathbf{x},t)$ in Eq. (37), which is approximated as Eq. (28). Sup-
pose assumptions (i)-(iv) are sat

pose assumptions (i)-(iv) are satisfied and following similar
derivations in Refs. [32, 51], we can acquire a control law for
Eq. (38) by applying the sliding-mode control method, the
control input is chosen as

$$
u = \frac{-\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - k_p \text{sgn}(s)}{\hat{\mathbf{W}}_g^T \mathbf{q}_{\overline{s}}(t)}
$$
(43)
where $\hat{\mathbf{W}}_F^T$ and $\hat{\mathbf{W}}_g^T$ are the estimates of \mathbf{W}_F^T and $\mathbf{W}_{\overline{g}}^T$,
respectively. Choosing $\mathbf{P} > 0$, $\mathbf{P} \in R^{(n-1)\times(n-1)}$, satisfies the
Lyapunov matrix equation
 $\mathbf{A}_1^T \mathbf{P} + \mathbf{P} \mathbf{A}_1 = -\mathbf{Q}$ (44)
with *s* being the sliding surface defined in Eq. (40); $p_{(n-1)i}$
being elements of **P** in Eq. (44); $\mathbf{Q} > 0$ being given and
the adaptive laws being chosen as
 $\hat{\mathbf{W}}_F = \mathbf{\Gamma}_1 s \mathbf{q}_F(t)$
 $\hat{\mathbf{W}}_{\overline{g}} = \mathbf{\Gamma}_2 s \mathbf{q}_{\overline{g}}(t) u$ (45)
where $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ ($\mathbf{\Gamma}_1 > 0$ and $\mathbf{\Gamma}_2 > 0$) are the adaptation
gain matrix, the following result holds for *s* and **e**: $s \rightarrow 0$ and
 $\mathbf{e} \rightarrow 0$ as $t \rightarrow \infty$.
Proof:

where $\hat{\mathbf{W}}_F^T$ and $\hat{\mathbf{W}}_{\overline{g}}^T$ are the estimates of \mathbf{W}_F^T and $\mathbf{W}_{\overline{g}}^T$, $u = \frac{1}{\mathbf{w}_g^T \mathbf{q}_g(t)}$ (43)

where $\hat{\mathbf{w}}_f^T$ and $\hat{\mathbf{w}}_g^T$ are the estimates of \mathbf{W}_f^T and \mathbf{W}_g^T ,

respectively. Choosing **P** > 0, **P** $\in R^{(n-1)\times(n-1)}$, satisfies the

Lyapunov matrix equation
 where $\hat{\mathbf{W}}_k^T$ and $\hat{\mathbf{W}}_k^T$ are the estimates of \mathbf{W}_k^T and \mathbf{W}_k^T , respectively. Choosing $\mathbf{P} > 0$, $\mathbf{P} \in R^{(n-1)\times(n-1)}$, satisfies the Lyapunov matrix equation
 $\mathbf{A}_1^T \mathbf{P} + \mathbf{P} \mathbf{A}_1 =$

$$
\mathbf{A}_1^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{A}_1 = -\mathbf{Q} \tag{44}
$$

the adaptive laws being chosen as

$$
\dot{\hat{\mathbf{W}}}_F = \Gamma_1 s \mathbf{q}_F(t) \n\dot{\hat{\mathbf{W}}}_g = \Gamma_2 s \mathbf{q}_{\overline{g}}(t) u
$$
\n(45)

where Γ_1 and Γ_2 ($\Gamma_1 > 0$ and $\Gamma_2 > 0$) are the adaptation gain matrix, the following result holds for *s* and **e**: $s \rightarrow 0$ and $\mathbf{W}_F = \mathbf{\Gamma}_1 \mathbf{S} \mathbf{q}_F(t)$
 $\dot{\mathbf{W}}_{\overline{g}} = \mathbf{\Gamma}_2 \mathbf{S} \mathbf{q}_{\overline{g}}(t) u$ (45)

here $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ ($\mathbf{\Gamma}_1 > 0$ and $\mathbf{\Gamma}_2 > 0$) are the adaptation

in matrix, the following result holds for *s* and e: apunov matrix equation
 $A_1^T P + PA_1 = -Q$ (44)
 $h \ s \text{ being the sliding surface defined in Eq. (40); } p_{(a-1)i}$

ong elements of **P** in Eq. (44); $Q > 0$ being given and

adaptive laws being chosen as
 $\dot{\mathbf{W}}_F = \mathbf{\Gamma}_1 s \mathbf{q}_F(t)$
 $\dot{\mathbf{W}}_g = \mathbf{\Gamma}_2 s \mathbf{$ + **PA**₁ = -**Q** (44)

being the sliding surface defined in Eq. (40); $p_{(n-1)i}$

lelements of **P** in Eq. (44); **Q** > 0 being given and

tive laws being chosen as
 $=\Gamma_1 s \mathbf{q}_F(t)$ (45)
 Γ_1 and Γ_2 ($\Gamma_1 > 0$ and $\Gamma_1 s q_F(t)$
 $\Gamma_2 s q_{\overline{g}}(t) u$ (45)
 Γ_1 and Γ_2 ($\Gamma_1 > 0$ and $\Gamma_2 > 0$) are the adaptation

ix, the following result holds for *s* and **e**: $s \to 0$ and
 $s t \to \infty$.

se the Lyapunov function as
 $s^2 + \frac{1}{2} \mathbf{$ ctively. Choosing **P** > 0, **P** ∈ R^{ccc}^{1,2} ω³, satisfies the

unov matrix equation

s being the sliding surface defined in Eq. (40); $p_{(n-1)j}$

s leight ends of **P** in Eq. (44); Q > 0 being given and

daptive laws with *s* being the sliding surface defined in Eq. (40); $p_{(n-1)k}$
being elements of **P** in Eq. (44); **Q** > 0 being given and
the adaptive laws being chosen as
 $\hat{\mathbf{W}}_F = \mathbf{\Gamma}_1 \mathbf{s} \mathbf{q}_F(t)$
 $\hat{\mathbf{W}}_g = \mathbf{\Gamma}_2 \mathbf{s} \$

$$
\mathbf{V} = \frac{1}{2} s^2 + \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{e} + \frac{1}{2} \tilde{\mathbf{W}}_F^T \mathbf{\Gamma}_1^{-1} \tilde{\mathbf{W}}_F + \frac{1}{2} \tilde{\mathbf{W}}_{\overline{g}}^T \mathbf{\Gamma}_2^{-1} \tilde{\mathbf{W}}_{\overline{g}}
$$
(46)

rivative of Eq. (46) becomes

L-H. *Lee and LH. Li/bound of Methodanical Science and Technology 30 (1) (2016) 381-396*
\n
$$
\dot{V} = s\dot{s} + \frac{1}{2} \epsilon^3 P \epsilon + \frac{1}{2} \epsilon^2 P \epsilon + \frac{1}{2} \epsilon^3 P \epsilon + \hat{W}_\mu^2 T \gamma^4 \hat{W}_\mu + \hat{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{s} + \frac{1}{2} [\epsilon^3 A_1^2 + 10, ..., 0, s]] Pe + \frac{1}{2} \epsilon^3 P [\Delta_1 e + 10, ..., 0, s]^T]
$$
\n
$$
= \dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu
$$
\n
$$
= s\dot{W}_\mu^2 T \gamma^4 \hat{W}_\mu = s\dot{W}_\mu^2 T \gamma^4 \hat
$$

Applying Eqs. (43) and (45)-(47) and letting $k_p = k_1 + w_t^u$,

$$
\dot{\mathbf{V}} \le -k_p |s| - \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e} + s w_t \le -k_1 |s| - \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e} \le 0.
$$
 (48)

According to Barbalat's lemma, Eq. (48) suggests that when

system unknown nonlinear time-varying functions, and an adaptive control law is employed to adjust the network weights to improve the convergence speed. Whenever there is = s(W_r q_r(x) + W_r q_r (y) + w_r y⁻ x² + 2^{*n*} error, the competence of *y* = V_r + W_r² (o) + ¹ $\frac{1}{2}p^2$ is used to guaran-

Applying Eqs. (43) and (45)-(47) and letting k_p = k_t + w_r².
 tee the closed-loop asymptotic stabilization and convergence of the overall system. Because the network can automatically adjust its weighting values, an effective adaptation is achieved for practical control applications.

4.3 H¥ *tracking performance design*

To reduce the adverse effects resulting from approximate errors, un-modeled dynamics and disturbances prior, we com bine the technique of H_{∞} tracking design and the AFNN approximation with the sliding mode control method. To put it

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 $\vec{\bf{e}} + \tilde{\bf{W}}_F^T \mathbf{\Gamma}_1^{-1} \dot{\vec{\bf{W}}}_F + \tilde{\bf{W}}_g^T \mathbf{\Gamma}_2^{-1} \dot{\vec{\bf{W}}}_g$ into practice, however, the exact upper bound *L.W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396*
 $T = s\dot{s} + \frac{1}{2}\dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} + \mathbf{W}_F^T\mathbf{T}_i^{-1}\dot{\mathbf{W}}_F + \mathbf{W}_g^T\math$ *d I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381~396* 38
 $\hat{\mathbf{X}}_F + \tilde{\mathbf{W}}_{\overline{g}}^T \mathbf{\Gamma}_2^{-1} \hat{\mathbf{W}}_{\overline{g}}$ into practice, however, the exact upper bound w_t^u for the lumped uncertainty c L-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
 $\dot{x} + \frac{1}{2} \dot{\mathbf{e}}^T \mathbf{P} \mathbf{e} + \frac{1}{2} \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}} + \mathbf{W}_F^T \mathbf{\Gamma}_1^{-1} \dot{\mathbf{W}}_F + \mathbf{W}_g^T \mathbf{\Gamma}_2^{-1} \dot{\mathbf{W}}_g$ into $\tilde{\mathbf{W}}_F^T \mathbf{\Gamma}_1^{-1} \dot{\mathbf{W}}_F$ posed. According to the assumption of which the lumped un-L-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
 $+\frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \mathbf{W}_k^T\mathbf{\Gamma}_1^{-1}\mathbf{\hat{W}}_k + \mathbf{\hat{W}}_k^T\mathbf{\Gamma}_2^{-1}\mathbf{\hat{W}}_k$ into practice, ho the upper bound w_t^u can be chosen so as to attenuate the *L*₋*W*. Lee and *LH*. *Li*/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396

= $s\dot{s} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \mathbf{W}_F^T\mathbf{\Gamma}_1^{-1}\dot{\mathbf{W}}_F + \mathbf{W}_E$ *s into* process the constraint of $\vec{W}_e = \vec{P} \cdot \vec{P} \cdot \vec{W}_e$ $\vec{W}_e = \vec{P} \cdot \vec{P} \cdot \vec{W}_e$ and be shown as
 $\vec{P} \cdot \vec{P} = \vec{P} \cdot \vec{P} \cdot \vec{P} \cdot \vec{W}_e$ and $\vec{P} \cdot \vec{P} \cdot \vec{W}_e$ into practice, however, the exact upper *L.W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396*
 e^TPé + $\tilde{\mathbf{W}}_k^T \mathbf{\Gamma}_1^{-1} \dot{\mathbf{W}}_k + \tilde{\mathbf{W}}_k^T \mathbf{\Gamma}_2^{-1} \dot{\mathbf{W}}_k^-$ **into practice, however, the exact upper bound w** L.-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
 $\dot{\mathbf{V}} = s\dot{s} + \frac{1}{2}\dot{\mathbf{e}}^{\mathbf{T}}\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^{\mathbf{T}}\mathbf{P}\dot{\mathbf{e}} + \tilde{\mathbf{W}}_{k}^{T}\mathbf{T}_{l}^{-1}\dot{\mathbf{W}}_{k} + \tilde{\mathbf{W}}_{k}^{T}\math$ L-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-396
 $\sinh \frac{1}{2} \mathbf{c}^T \mathbf{P} \mathbf{e} + \frac{1}{2} \mathbf{c}^T \mathbf{P} \mathbf{e} + \hat{\mathbf{W}}_k^T \mathbf{\Gamma}_1^{-1} \hat{\mathbf{W}}_k + \hat{\mathbf{W}}_k^T \mathbf{\Gamma}_2^{-1} \hat{\mathbf{W}}_k$
 $\frac{$ *a e* into practice, however, the exact upper bound w_t^u for the lumped uncertainty cannot be obtained in general. Given that lumped uncertainty, large control chattering nevertheless occurs. To release the constraint of $k_p > w_t^u$, a new control law *p* $\frac{389}{4}$
 p the under w_t^u for the under in general. Given that
 p seems so as to attenuate the

chattering nevertheless oc-
 $k_p > w_t^u$, a new control law
 g design technique is pro-
 p on of which the lu developed from the H_{∞} tracking design technique is proence and Technology 30 (1) (2016) 381-396

into practice, however, the exact upper bound w_i^u for the

lumped uncertainty cannot be obtained in general. Given that

the upper bound w_i^u can be chosen so as to attenuat into practice, however, the exact upper bound w_i^u for the
lumped uncertainty cannot be obtained in general. Given that
the upper bound w_i^u can be chosen so as to attenuate the
lumped uncertainty, large control chatt bound w_t^u for the
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ng nevertheless oc-
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ich the lumped un-
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the control input
 $\frac{(n)}{m} - \frac{s}{2\rho^2}$. (49) t be obtained in general. Given that

n be chosen so as to attenuate the

control chattering nevertheless oc-

zaint of $k_p > w_i^u$, a new control law

tracking design technique is pro-

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incertainty, large control chattering nevertheless oc-

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However, the exact upper bound w_i^u for the

ainty cannot be obtained in general. Given that

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control chattering nevertheless oc-
int of $k_p > w_i^u$, a new control law
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sumption of which the lumped un-
 $T \in [0,\infty)$ [32], the control input
 $T = \sum_{i=1}^{n-1$ *the and Technology 30 (1) (2016) 381-396* 389

2 p p ractice, however, the exact upper bound w_i^u for the apped uncertainty cannot be obtained in general. Given that

tupper bound w_i^u can be chosen so as to attenuat wer, the exact upper bound w_i^u for the
cannot be obtained in general. Given that
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e control chattering nevertheless oc-
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actice, however, the exact upper bound w_i^u for the

d uncertainty cannot be obtained in general. Given that

per bound w_i^u can be chosen so as to attenuate the

d uncertainty, *md Technology 30 (1) (2016) 381-396* 389

practice, however, the exact upper bound w_i^* for the

bed uncertainty cannot be obtained in general. Given that

apper bound w_i^* can be chosen so as to attenuate the

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traint of $k_p > w_i^u$, a new control law
tracking design t

$$
u = \frac{-\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - \frac{s}{2\rho^2}}{\hat{\mathbf{W}}_g^T \mathbf{q}_{\bar{g}}(t)}.
$$
 (49)

 $\sum a_i e_{i+1}$ with ρ ($\rho > 0$) being the design constant for the attenuation $\sum p_{(n-1)i} e_i - \tilde{\mathbf{W}}_i^T \mathbf{\Gamma}_1^{-1} \tilde{\mathbf{W}}_F - \tilde{\mathbf{W}}_i^T \mathbf{\Gamma}_2^{-1} \tilde{\mathbf{W}}_{\overline{g}}$ $p_{(n-1)i}$ being elements of **P** in Eq. (44). Therefore, we can **Example shown as**
 $u = \frac{-\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - \frac{s}{2\rho^2}}{\hat{\mathbf{W}}_S^T \mathbf{q}_{\bar{g}}(t)}$. (49)

In Eq. (49), the adaptive laws can be represented as Eq. (45)

with ρ (ρ In Eq. (49), the adaptive laws can be represented as Eq. (45) level, *s* being the sliding surface defined in Eq. (40) and (a) $u = \frac{-\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - \frac{s}{2\rho}}{\hat{\mathbf{W}}_g^T \mathbf{q}_{\bar{g}}(t)}$

In Eq. (49), the adaptive laws can be represented

with $\rho(\rho > 0)$ being the design constant for the
 guarantee an H_{∞} tracking performance for overall without knowledge on the upper bound w_t^u of the lumped uncertainty. tainty is $w_i \in L_2[0,T]$, $\forall T \in [0,\infty)$ [32], the control input
be shown as
 $u = \frac{-\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} \alpha_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)} e_i + y_m^{(n)} - \frac{s}{2\rho^2}}{\hat{\mathbf{W}}_g^2 \mathbf{q}_g(t)}$ (49)
n Eq. (49), the adaptive laws *T* $\mathbf{w}_k = L_2[0, T]$, $\nabla I \in [0, \infty)$ [32], the control input
 $\mathbf{w}_k = \mathbf{w}_k \mathbf{q}_k(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - \frac{s}{2\rho^2}$. (49)
 $\hat{\mathbf{W}}_k^T \mathbf{q}_k(t)$

Eq. (49), the adaptive laws can be $=\frac{\hat{\mathbf{W}}_F^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} e_i + y_m^{(n)} - \frac{s}{2\rho^2}}{\hat{\mathbf{W}}_g^T \mathbf{q}_g(t)}$ (49)

Eq. (49), the adaptive laws can be represented as Eq. (45)
 $\rho(\rho > 0)$ being the design constant for the a *g* $\mathbf{w} = \mathbf{w}_k^T \mathbf{q}_F(t) - \sum_{i=1}^{n-1} a_i e_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)} e_i + y_m^{(n)} - \frac{s}{2\rho^2}$. (49)
 $\mathbf{w}_k^T \mathbf{q}_k(t)$
 \mathbf{n} Eq. (49), the adaptive laws can be represented as Eq. (45)
 \mathbf{h} ρ ($\rho > 0$) being t $\mathbf{\hat{w}}_g^T \mathbf{q}_g(t)$

In Eq. (49), the adaptive laws can be represented as Eq. (45)

with $\rho(\rho > 0)$ being the design constant for the attenuation

level, s being the silding surface defined in Eq. (40) and
 $p_{(n-1)u}$

 $\sum a_i e_{i+1}$ (49) ensures that the overall system Eq. (22) satis-**Theorem 2:** Under assumptions (i)-(iii), the proposed confies the H_{∞} tracking performance.

$$
\sum_{g=1}^{n} \sum_{g=1}^{n} \mathbf{w}_{g}
$$
\n
$$
V_{(n-1)i}
$$

47) where $\tilde{\mathbf{W}}_F = \mathbf{W}_F - \hat{\mathbf{W}}_F$ and $\tilde{\mathbf{W}}_g = \mathbf{W}_g - \hat{\mathbf{W}}_g$.

^u, Select the same Lyapunov function as Eq. (46) in accordance with the same procedure of Eq. (47). Then substituting Eqs. (45) and (49) into Eq. (47), we get

$$
\dot{\mathbf{V}} = -\frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e} - \frac{1}{2} \left(\frac{s}{\rho} - \rho w_t \right)^2 + \frac{1}{2} \rho^2 w_t^2 \le -\frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e} + \frac{1}{2} \rho^2 w_t^2.
$$
\n(51)

with ρ(ρ > 0) being the design constant for the attenuation
vel, *s* being then sliding surface defined in Eq. (44). Therefore, we can
parameter an *H_∞* tracking performance for overall without
nowledge on the upper bound *w_∗* of the lumped uncertainty.
Theorem 2: Under assumptions (i)-(iii), the proposed con-
ol law Eq. (49) ensures that the overall system Eq. (22) satisfies
the *H_∞* tracking performance.

$$
\frac{1}{2} \int_0^r e^T (r) \mathbf{Q} \mathbf{e}(\tau) d\tau \leq \frac{1}{2} s^2 (0) + \frac{1}{2} \mathbf{e}^T (0) \mathbf{P} \mathbf{e}(0)
$$

$$
+ \frac{1}{2} \tilde{\mathbf{W}}_g^T (0) \mathbf{\Gamma}_2^{-1} \tilde{\mathbf{W}}_g (0) + \frac{1}{2} \rho^2 \frac{1}{2} \int_0^T w_i^2 (\tau) d\tau
$$
(50)
there
$$
\tilde{\mathbf{W}}_F = \mathbf{W}_F - \hat{\mathbf{W}}_F
$$
 and
$$
\tilde{\mathbf{W}}_g = \mathbf{W}_g - \hat{\mathbf{W}}_g
$$
.
Proof:
Seler the same Lyapunov function as Eq. (46) in accordance with the same procedure of Eq. (47). Then substituting
qs. (45) and (49) into Eq. (47), we get

$$
\dot{\mathbf{V}} = -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} - \frac{1}{2} \left(\frac{s}{\rho} - \rho w_i \right)^2 + \frac{1}{2} \rho^2 w_i^2 \leq -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \frac{1}{2} \rho^2 w_i^2.
$$
(51)
Integrating Eq. (51) from $t = 0$ to $t = T$, we obtain

$$
\int_0^T \dot{\mathbf{V}}(\tau) d\tau \leq -\frac{1}{2} \int_0^T \mathbf{e}^T(\tau) \mathbf{Q} \mathbf{e}(\tau) d\tau + \frac{1}{2} \rho^2 \int_0^T w_i^2 d\tau
$$

$$
\Rightarrow \mathbf{V
$$

Substituting Eq. (46) into Eq. (52), we achieve a H_{∞}

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\nL-W. Lee and I-H. Li/Journal of Mechanical Science and Technology 30 (1) (2016) 381-
\ntracking performance satisfying:
\n
$$
\frac{1}{2}\int_0^T e^T(\tau)Qe(\tau)d\tau \le \frac{1}{2}s^2(0) + \frac{1}{2}e^T(0)Pe(0) + \frac{1}{2}\tilde{W}_F^T(0)\Gamma_1^{-1}\tilde{W}_F(0)
$$
\n
$$
+\frac{1}{2}\tilde{W}_g^T(0)\Gamma_2^{-1}\tilde{W}_g(0) + \frac{1}{2}\rho^2\int_0^T w_t^2d\tau.
$$
\n(53)
\nTherefore, the H_∞ tracking performance is achieved with
\n(a) Members

Therefore, the H_{∞} tracking performance is achieved with a prescribed. This completes the proof.

In Theorem 2, if the design constant, ρ , serving as an attenuation level, needs to be pre-specified during the design process, the constraint on setting an upper bound w_t^u for the unknown lumped uncertainties in Eq. (43) is, thus, removed. Furthermore, the chattering effect of the control input is substantially reduced with this method because the term sgn() *^p k s* related to the control chattering in Eq. (43) is recontrol law of Eq. (49). en 2, if the design constant, ρ , secreting as an at-

evel, needs to be pre-specified during the design

evel, needs to be pre-specified during the design

e, the chattering effect of the control inqut is sub-

equalit

Remark 3: If a set of the initial conditions of **e** , *s*, **W**_F and **W**_g (e(0) = 0, s(0) = 0, $\hat{\mathbf{W}}_F(0) = \mathbf{W}_F(0)$ and $\hat{\mathbf{W}}_{\bar{g}}(0) =$ $W_{\overline{g}}(0)$ is available, and $Q = I$, the overall system's control performance satisfies placed by a lined shootlet lefth $s/(2p^r)$ in the defined
control law of Eq. (49).
 Remark 3: If a set of the initial conditions of **e**, *s*,
 W_F and $W_{\overline{g}}$ (e(0) = 0, $s(0) = 0$, $\hat{W}_F(0) = W_F(0)$ and $\hat{W}_{\overline{g}}$ Let a smoother term $s/(2\rho^2)$ in the derived
 T **a** set of the initial conditions of **e**, *s*,
 $0) = 0$, $s(0) = 0$, $\hat{\mathbf{W}}_F(0) = \mathbf{W}_F(0)$ and $\hat{\mathbf{W}}_g(0) =$

lable, and $\mathbf{Q} = I$, the overall system's control i

$$
\frac{\|\mathbf{e}\|_{2}}{\|w_{i}\|_{2}} \leq \rho,\tag{54}
$$

words, an arbitrary attenuation level can be obtained, if ρ is adequately chosen.

In real applications, the implementation of the AFNN- SMC+H-infinity algorithm, in general, can be formed by sine/cosine function. The detailed steps are presented here.

matrix \mathbf{A}_1 is a Hurwitz matrix.

Step 2) Choose appropriate **Q** to solve the Lyapunov matrix Eq. (44).

Step 3) Refer to the system bandwidth to establish AFNN to **g** $\|\mathbf{e}\|_{\infty} \le \rho$, $\text{posed control stra}$

where $\|\mathbf{e}\|_{\infty}^2 = \int_0^r \mathbf{e}^T(\mathbf{r})\mathbf{e}(\mathbf{r})d\mathbf{r}$, $\|\mathbf{w}_i\|_{\infty}^2 = \int_0^r w_i^2(\mathbf{r})d\mathbf{r}$. In other $\text{podesed control stra}$

words, an arbitrary attenuation level can be obtai $\overline{g}(\mathbf{x},t)$. Determine initial values of network weight $\hat{\mathbf{W}}_F$ and $\hat{\mathbf{W}}_{\overline{g}}$, respectively.

and Γ ₂ to establish the Lyapunov function.

Step 5) Obtain the update laws from Eq. (45), and the control laws from Eq. (43) or Eq. (49), respectively, depending on different assumptions on the lumped uncertainties.

5. Real-time implementation and experimental results

The objective of this study was to implement an AFNN based sliding-mode controller to improve the tracking performance for the PAS. To investigate the control performance of the proposed controller, we present the test results of tracking control and velocity control in a PAS. In the experimental

Fig. 5. Membership functions of the s and u_{fs} .

implementation, position regulation, trajectory tracking, and velocity control of the PAS are chosen for servo control. In addition, robustness tests are also performed to verify the proposed control strategy.

5.1 Experimental setup

To reduce the cost of the system set-up, the velocity and acceleration of the piston, as shown in Fig. 1, are calculated by a filtered differentiation of the measured position with a cut-off frequency at 350 Hz. The digital filter expressed is as follows:

$$
y_{out}(i) = -0.047 y_{out}(i-1) + 0.524[y_{in}(i) + y_{in}(i-1)],
$$
 (55)

Stephen and W_0 is \mathbb{R}_2 . S. Mombership functions of the s and u_A .
 Remark 3: If a set of the initial conditions of e, s,
 \mathbb{R}_2 is available, and Q = 1, the overall system's control
 $\left\| \mathbf{w} \right\|_2 = \int_0^$ Step 1) Select control parameters $a_1, a_2, ..., a_{n-1}$ such that where $y_{out}(t)$ represents the filter's output signal, and $y_{in}(t)$ addition, robustness tests are also performed to verify the pro-
 $\frac{\|\mathbf{e}\|_1}{\|\mathbf{v}\|_1} \leq \rho$, (54) posed control strategy.

where $\|\mathbf{e}\|_1^2 = \int_0^r e^x(\mathbf{r})e(\mathbf{r})d\mathbf{r}$. In other To reduce the cost of the syste Step 4) Choose an appropriate adaptation gain matrix Γ_1 trixes are set as constant matrices $\Gamma_1 = 83$ [I] and $\Gamma_2 =$ -1 -0.66 -0.33 0 -0.33 066 1

(b) Membership $M(u_{\beta})$

Fig. 5. Membership functions of the *s* and u_{β} .

implementation, position regulation, trajectory tracking, and

velocity control of the PAS are chosen for represents the measured piston position. To evaluate the control performance of the proposed AFNN-SMC+ H_{∞} , the following experiments are performed. The attenuation level is set at $\rho = 0.2$ for the control law Eq. (48), the sliding surface is velocity control of the PAS are chosen for servo control. In
addition, robustness tests are also performed to verify the pro-
posed control strategy.
5.*I Experimental setup*
To reduce the cost of the system set-up, the v stem set-up, the velocity and ac-

own in Fig. 1, are calculated by a

measured position with a cut-off

ital filter expressed is as follows:

0.524[$y_m(i) + y_m(i-1)$], (55)

filter's output signal, and $y_m(i)$

on position. T so performed to verify the pro-
em set-up, the velocity and ac-
n in Fig. 1, are calculated by a
assured position with a cut-off
f filter expressed is as follows:
 $524[y_m(i) + y_m(i-1)],$ (55)
ther's output signal, and $y_m(t)$
pos chosen as $s = \ddot{e} + 2\dot{e} + 6e$, $Q = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, and the gain ma-5.1 Experimental setup

To reduce the cost of the system set-up, the velocity and ac-

ecleration of the piston, as shown in Fig. 1, are calculated by a

filtered differentiation of the measured position with a cut-off

f $1.25 \times 10^{-4} [I]$. perimental setup

educe the cost of the system set-up, the velocity and ac-

educe the cost of the system set-up, the velocity and ac-

ion of the piston, as shown in Fig. 1, are calculated by a

differentiation of the me

To verify the feasibility, the Single-input fuzzy sliding mode controller (SFSMC), as proposed by Kim and Lee [53], was used to compare with the proposed AFNN-SMC and $AFNN-SMC+H_∞$ in terms of position regulation and trajectory tracking control performance. The sliding surface *s* and the control law u_{κ} , are, respectively, chosen as the input and output of SFSMC, and their membership functions are shown in Figs. 5(a) and (b). *NB*, *NM* , *NS*, *ZR*, *PS*, *PM* , and *PB*. In addition, the product inference, center-averaging, and singleton fuzzification are used in the fuzzy logic system. The

control parameters and the rule base of SFSMC are listed in Tables 2 and 3, respectively.

5.2 Test result of position regulation

The PAS plays an important role in industrial applications because it can easily and smoothly move a payload from one position to another. Hence, planning a smooth moving trajectory is vital. Moreover, because of the mechanical limitations, the maximum acceleration and velocity should be considered
error; (c) error zoom out; (d) control input. in the trajectory design. In our experiments, the fifth-order polynomial continuous function is adopted to be the moving trajectory, which is a trajectory function of the continuous b) parameters and the rule base of SFSMC are listed in
 $\sum_{i=1}^{8} \frac{1}{1}$ and 3, respectively.

Let $\sum_{i=1}^{8} \frac{1}{1}$ and 3, respectively.

Let $\sum_{i=1}^{8} \frac{1}{1}$ and smooth regulation
 $\sum_{i=1}^{8} \frac{1}{1}$ and smooth is vital. Moreover, because of the mechanical limitations,

maximum acceleration and velocity should be considered

interval of 15

maximum acceleration and velocity should be considered

momial continuous function is ado **and 3, respectively.**
 htt of position regulation

plays an important role in industrial applications

an easily and smoothly move a payload from one

another. Hence, planning a smooth moving trajec-

Moreover, becau

$$
y_m(t) = \begin{cases} h \left[10 \left(\frac{t}{t_f} \right)^3 - 15 \left(\frac{t}{t_f} \right)^4 + 6 \left(\frac{t}{t_f} \right)^5 \right], & 0 \le t < t_f \\ h, \quad t_f \le t \end{cases}
$$
 (56)

where h is the desired stroke; t_f denotes the desired duration; y_m indicates the desired path and t is the time which is set to 0 at the beginning of each tracking cycle. Then, we can reduce discontinuous shock and wear-out and fatigue of the hardware components when the payload is moving.

For the position regulation, the piston is first moved to the end of the cylinder and this position is then assumed to be the is set to $y_d = 150$ mm. A performance comparison between AFNN-SMC, SFSMC, and AFNN-SMC+ H_{∞} is shown in $\frac{9}{5}$ -Figs. 6-8. Figs. 6(b), 7(b) and 8(b) illustrate that the position errors of the three control methods are well converged and bounded with around ± 1.8 mm (AFNN-SMC), ± 2.2 mm (SFSMC), and ± 0.9 mm (AFNN-SMC+ H_{∞}). Figs. 6(c), 7(c) Fig. 7. Ex and 8(c) show the steady errors of AFNN-SMC, SFSMC, and AFNN-SMC+ H_{∞} are around 0.02 mm, 0.06 mm and

Fig. 6. Experimental results of AFNN-SMC for position regulation with position of 150 mm: (a) position control response; (b) control

Fig. 7. Experimental results of SFSMC for position regulation with position of 150 mm: (a) position control response; (b) control error; (c) error zoom out; (d) control input.

Fig. 8. Experimental results of AFNN-SMC+ H_{∞} for position regulation with position of 150 mm: (a) position control response; (b) control error; (c) error zoom out; (d) control input.

0.04 mm, respectively. Observing the control input in Figs. 6(d), 7(d) and 8(d), we can find that the amplitude and chattering of AFNN-SMC are much worse than that of SFSMC and AFNN-SMC+ H_{∞} . That is because a bigger controller factor W $k_p = 21000$ of AFNN-SMC is applied to the system to ensure the tracking performance. Note that the controller factor k_n of Eq. (43) is a trade-off parameter between the robustness and tracking performance. This undesired chattering may wear out the servo valve and can incur unstable system dynamics. In general, the term sgn() *^p k s* in control law Eq. (43) is applied for compensating the lumped uncertainties, but it may result in serious control chattering. Moreover, although AFNN-SMC has better tracking effects and lower steady state error when compared with them with SFSMC, the effect of restricted chattering of SFSMC is much superior to it of AFNN-SMC, because of use of fuzzy design. Figs. 8(c) and (d) show that introducing H_{∞} technique into AFNN-SMC \qquad $(AFNN-SMC+H_∞)$ can gain better tracking performance and chattering-reduction, as we compare it with AFNN-SMC and SFSMC. In Fig. 9, the integral time absolute error ITAE $k_p = 21000$ of AFNN-SMC is applied to the system to ensure
the tracking performance. Note that the controller factor k_p and SFSMC and Aft
of Eq. (43) is a trade-off parameter between the robustness
and tracking performa $\int |t|e(t)|dt$ is used to measure the tracking error of the position regulation for AFNN-SMC, SFSMC, and AFNN- SMC+ H_{∞} . It is obvious that the proposed AFNN-SMC+ H_{∞} AFNN-SMC+ H_{∞} has can perform much lower ITAE than others. Thus, the AFNN-

Fig. 9. Comparison of ITAE of tracking error for position regulation of AFNN-SMC, SFSMC and AFNN-SMC+ H_{∞} .

 $SMC+H_∞$ has superior control performance in controlling the PAS.

5.3 Test results of trajectory tracking

Case 1. Sinusoidal trajectory

The desired sinusoidal trajectory is defined as

$$
y_m(t) = 50 + h \left[1 - \sin\left(\frac{t}{T} \cdot 2\pi\right) \right],\tag{57}
$$

where the amplitude $h = 100$ mm and the period $T = 4$ sec are given in this paper. A comparison between AFNN-SMC, and SFSMC and AFNN-SMC+ H_{∞} is shown in Figs. 10-12. From Figs. 10(b), 11(b) and 12(b), the tracking errors for those controllers are, respectively, around \pm 1.8 mm, \pm 2.0 mm, and \pm 1.0 mm. Again, although the desired tracking performance of AFNN-SMC, as shown in Fig. 10(c), can be achieved, the chattering of the control input is still serious, which may result in high frequency switching of the servo valve and further reduce the servo valve's life duration. Com pared to SFSMC with AFNN-SMC, as shown in Figs. 10 and 11, AFNN-SMC has better tracking effects and lower steady state error, but control chattering is very serious. That is, the SFSMC can effectively prevent chattering by using fuzzy approach. Fig. 12 shows the experimental results of AFNN- SMC+ H_{∞} control, in which Fig. 12(a) shows the tracking response, where the dotted line indicates the target trajectory and the solid line denotes the tracking results. Fig. 12(b) shows the maximum tracking error is only about 1 mm, and Fig. 12(c) shows the control input, in which the chattering is significantly reduced. Compared to AFNN-SMC and SFSMC, $AFNN-SMC+ H_{\infty}$ has better tracking performance and smooth control input.

Fig. 10. Experimental results of AFNN-SMC for sinusoidal trajectory: (a) position control response; (b) control error; (c) control input.

Fig. 11. Experimental results of SFSM for sinusoidal trajectory: (a) position control response; (b) control error; (c) control input.

Case 2. Exponentially decreasing sinusoidal trajectory

To verify the AFNN-SMC+ H_{∞} control, we next test the performance of the exponentially decreasing 0.5 Hz sinusoidal trajectory covering 50% of the actuator stroke. The position responses, control efforts, and tracking errors, shown in Fig. 13, clarify that the AFNN-SMC+ H_{∞} control can achieve ex-
fa cellent control performance with small chattering for the ex ponentially decreasing 0.5 Hz sinusoidal trajectory tracking control. Fig. 13(b) shows that the control error is bounded.

Fig. 12. Experimental results of AFNN-SMC+ H_{∞} for sinusoidal trajectory: (a) position control response; (b) control error; (c) control input.

Fig. 13. Experimental results of AFNN-SMC+ H_{∞} for exponentially decreasing sinusoidal trajectory: (a) position control response; (b) control error; (c) control input.

5.4 Test results of trapezoidal velocity trajectory

In most servo PASs the control purpose is expressed in terms of driving the cylinder piston to follow a predefined velocity profile to achieve the desired velocity control. In almost all industrial applications, a trapezoidal velocity profile is employed. For example, a constant velocity trajectory is required to apply paint or other coatings evenly across a surface. Another example would be to lay down a smooth bead when welding. The trapezoidal velocity profile consists of three distinct operating phases. The first phase is when the piston begins moving from rest and attains the desired velocity.

Fig. 14. Experimental results of AFNN-SMC+ H_{∞} for trapezoidal is rectory with the velocity profile: (a) velocity control response; (b) control error; (c) control input.

During the second phase the piston must maintain the desired velocity. During the third phase the piston must be decelerated so that all motion will stop prior to the piston reaching the end of the cylinder. Fig. 14 shows the experimental result of the AFNN-SMC+ H_{∞} for the 120 mm/sec trapezoidal velocity profile. According to the experimental results, once the piston reaches the target velocity, the AFNN-SMC+ H_{∞} appears to $\frac{1}{2}$ be able to provide adequate control performance. However, the controller does not perform well during the initial phase of the trapezoidal velocity profile. During the initial phase, when the pressure difference between the two chambers is large enough, the friction force will be overcome by the valve. Since the piston does not begin to move during this time, no enough error is produced between the desired and actual velocity. Therefore, the result is an initial lag in tracking the desired velocity.

5.5 Robustness tests

Robustness is very important for a practical control system. The concept of robustness is different from that of generality. Robustness is the ability of a control system to be insensitive to the variation of the plant parameters when using the nominal controller designed based on the nominal plant model, while generality means the control strategy design method can be applied to control systems with different dynamic. The nominal control strategies were designed for the Festo rodless PAS with a moving mass $M = 6$ kg. To test the robustness to the variation of the moving mass, we increased the moving mass to 13 kg without altering the nominal control strategies. The robustness of the proposed AFNN-SMC+ H_{∞} was tested reduced by 63^o

Fig. 15. Experimental results of AFNN-SMC+ H_{∞} for sinusoidal trajectory with the payload changed from 6 kg to 13 kg: (a) position control response; (b) control error; (c) control input.

Fig. 16. Comparison of ITAE of tracking error for periodic sinusoidal tracking control of SFSMC, AFNN-SMC, AFNN-SMC+ H_{∞} and AFNN-SMC+ H_{∞} with payloads changed from 6 kg to 13 kg.

under different mass payload disturbances. Fig. 15 shows the trajectory tracking response, control effort and tracking error of the sinusoidal trajectory with payloads changed from 6 kg, as in Fig. 12, to 13 kg. With this change of the mass of the payload, only a slight increase in the tracking error occurs, and the tracking error can still keep within ± 1.5 mm. The test results verify that the robustness of the proposed the AFNN- $SMC+H_{\infty}$ is satisfied. In Fig. 16, the integral time absolute Fig. 16. Comparison of ITAE of tracking error for periodic sinusoidal

Fig. 16. Comparison of ITAE of tracking error for periodic sinusoidal

tracking control of SFSMC, AFNN-SMC, AFNN-SMC+ H_x and

AFNN-SMC+ H_x with pa error ITAE = $\int t |e(t)| dt$ is used to measure the tracking error of the sinusoidal trajectory tracking for AFNN-SMC, SFSMC, AFNN-SMC+ H_{∞} , and AFNN-SMC+ H_{∞} (payload is changed from 6 kg to 13 kg). Experimental results demon strated that the AFNN-SMC+ H_{∞} is very effective and ITAE is reduced by 63% on average.

6. Conclusions

We have developed the AFNN-SMC+ H_{∞} and successfully of a p applied it to the position regulation, trajectory tracking, and velocity control of the rodless PAS under different loading con ditions. We chose AFNN as the identification technique because it has the elegant property of consisting of a family of orthogonal Fourier functions. That is, the structure of AFNN can be easily decided according to clear physical meaning. The proposed AFNN-SMC+ H_{∞} has the following advantages: (1) It can reduce the serious chattering phenomenon and (2) based on the H_{∞} tracking design technique, it can attenuate the uncertainties caused by the un-modeled dynamics, the approximation error and the external disturbance. Compared with AFNN-SMC, the AFNN-SMC+ H_{∞} can result in a high tracking precision and reduce sensitivity to disturbance. The experimental results show our proposed $AFNN-SMC+H_{\infty}$ can overcome the AFNN-SMC [in tracking and robust performances.

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