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Rotation and gravitational field effect on two-temperature thermoelastic material with voids and temperature dependent properties type III †

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Abstract

This paper studies the two dimensional problem of thermoelastic rotating material with voids under the effect of the gravity and the temperature dependent properties employing the two-temperature generalized thermoelasticity in the context of Green-Naghdi (G-N) theory of types II and III. The normal mode method is used to obtain the exact expressions for the physical quantities which have been shown graphically. The comparisons have been made in the presence and the absence of the rotation, the gravity, the temperature dependent properties and the two-temperature effect.

Keywords: Energy dissipation; Gravity; Green-naghdi theory; Normal mode analysis; Rotation; Temperature dependent; Thermoelasticity; Two-temperature; Voids

1. Introduction

Thermoelasticity is the change in the size and the shape of a solid object as the temperature of that object fluctuates, it used to design materials and objects that can withstand fluctuations in temperature without breaking. Materials that are more elastic will expand and contract more than those materials that are more inelastic. The generalized theory of thermo-elasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity that has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity theory stated by Biot [1]. Lord and Shulman [2] formulated a generalized theory of thermoelasticity with one thermal relaxation time, who obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [3] developed a theory of thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction. Green and Naghdi [4-6] proposed three models, which are subsequently referred to as (G-N) -I, II and III types. The linearized version of type-I corresponds to the classical thermoelastic Fourier's law for the heat conduction equation. In type II the internal rate of production of the entropy is taken to be identically zero implying no dissipation of thermal energy and known as thermoelasticity without energy dissipation. Type-III includes the previous two models as special cases and admits dissipation of energy. In Refs. [7, 8] Othman et al. used (G-N) theory of type III to study some models in thermoelasticity with energy dissipation. The classical theory of elasticity developed from the consideration that a solid is a continuum, appears to be inadequate for the study of the response of a solid to applied load when porous materials containing voids (such as geological materials like rock and soils and manufactured materials like ceramics and pressed powder) are considered. Cowin and Nunziato [9] developed general model to predict the mechanical behavior of the solid materials with voids, This liberalized theory of the elastic materials with voids is a generalization of the classical theory of elasticity and reduces to it when the voids are suppressed. Some basic investigation related to studying the thermoelastic materials with voids were discussed by Cowin [10]. Othman et al. [11, 12] used (G-N) theory to study two models of thermoelastic medium with voids. Recently Abbas and Kumar [13] studied the response of the initially stressed generalized thermoelastic solid with voids to thermal source. Keeping in view that the propagation of plane waves in a rotating media is important in many realistic problems, e.g., rotation of heavenly bodies and the moon. Schoenberg and Censor [14] established the propagation of the waves in a rotating, homogenous, isotropic, linear elastic medium for any orientation of the rotation axis with respect to free space taking into consideration the Coriolis and the Centripetal acceleration. Abo-Dahab et al.

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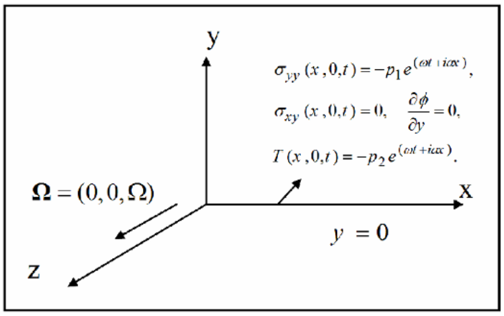

Fig. 1. Geometry of the problem.

[15] discussed the problem of rotating elastic media. The gravity effect was generally neglected in the classical theory of elasticity. Bromwich [16] established the effect of the gravity on the wave propagation of an elastic solid medium. Most of the studies were done under the assumption of the temperature independent material properties that is considered a limiting of the applicability of the obtained solutions certain ranges of the temperature. The thermal and mechanical properties of the materials vary with the temperature, so the temperature-dependent on the material properties must be taken into consideration in the thermal stress analysis of these elements. Othman [17] studied the generalized thermoelastic plane waves in a rotating media with thermal relaxation under the temperature dependent properties. Chen and Gurtin [18], Chen et al. [19] have formulated a theory of the heat conduction in deformable bodies, which depends upon two distinct temperatures, the thermodynamic temperature T, and the conductive temperature θ. Warren and Chen [20] investigated the wave propagation in the two-temperature theory of thermoelasticity. Youssef [21] established the two-temperature generalized thermoelasticity theory together with a general uniqueness theorem.

The aim of this work is to determine the distributions of the physical quantities for a homogenous, isotropic, thermoelastic material with voids analytically in the case of absence and presence of the rotation, the gravity, the temperature dependent and the two temperature effect using (G-N) theory of types II and III in terms of the normal modes to obtain the exact expressions for the physical quantities which represented graphically.

2. Formulation and solution of the problem

Consider a linear homogeneous isotropic thermoelastic material with voids and a half-space $(y \geq 0)$ the rectangular Cartesian coordinate system (x, y, z) having originated on the surface $z = 0$. For two dimensional problem assume the dynamic displacement vector as $\boldsymbol{u} = (u, v, 0)$. The material is rotated with a uniform angular velocity $\boldsymbol{\Omega}$ such that $\boldsymbol{\Omega} = (0,0,\Omega)$. All quantities considered will be a function of the time variable t, and of the coordinates x and y.

The equations of motion, the field equation and the heat equation under the effect of the gravity field of rotating thermoelastic material with voids and the two-temperature theory in the case type III of the (G-N) theory (see Cowin and Nunziato [9], Green and Naghdi [5] and [21] due to Youssef) in the absence of body forces, heat sources and extrinsic equilibrated body force will be

$$\mu \nabla^2 u + (\lambda + \mu) \frac{\partial e}{\partial x} + b \frac{\partial \phi}{\partial x} - \beta \frac{\partial T}{\partial x} + \rho g \frac{\partial v}{\partial x} = \rho [\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t}], \tag{1}$$

$$\mu \nabla^2 v + (\lambda + \mu) \frac{\partial e}{\partial y} + b \frac{\partial \phi}{\partial y} - \beta \frac{\partial T}{\partial y} - \rho g \frac{\partial u}{\partial x} = \rho [\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}], \tag{2}$$

$$\alpha \nabla^2 \phi - b\, e - \xi_1 \phi - \omega_0 \frac{\partial \phi}{\partial t} + m\, T = \rho \psi \frac{\partial^2 \phi}{\partial t^2}, \tag{3}$$

$$k \nabla^2 \theta + k^* \frac{\partial}{\partial t} \nabla^2 \theta - m T_0 \frac{\partial \phi}{\partial t} = \rho C_e \frac{\partial^2 T}{\partial t^2} + \beta\, T_0 \frac{\partial^2 e}{\partial t^2}, \tag{4}$$

$$\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + 2\mu e_{ij} + b \phi \delta_{ij} - \beta T \delta_{ij}, \tag{5}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{6}$$

$$T = (\theta - d\, \theta_{,ii}). \tag{7}$$

Where, λ, μ are the Lame's constants, $\alpha, b, \xi_1, \omega_0, m, \psi$ are the material constants due to presence of voids, $\beta = (3\lambda + 2\mu)\alpha_t$, while α_t is the thermal expansion coefficient, ρ is the density, C_e is the specific heat, k is the thermal conductivity, k^* is the material constant characteristic of the theory, T_0 is the reference temperature chosen such that $\left|(T - T_0)/T_0\right| << 1$, ϕ is the change in the volume fraction field, σ_{ij} are the components of the stress tensor, e_{ij} are the components of strain tensor, δ_{ij} is the Kronecker delta, d is the two temperature parameter so that $d > 0$, T is the thermodynamic temperature and θ is the conductive temperature and g is the acceleration due to the gravity, when $k^* \to 0$, then Eq. (4) reduces to the heat conduction equation in the (G-N) theory (of type II).

To investigate the effect of the temperature dependent properties on thermoelastic material with voids, therefore we assume that

$$\lambda = \lambda_0 f(T), \quad \mu = \mu_0 f(T), \quad \beta = \beta_0 f(T), \quad \alpha = \alpha_0 f(T),$$
$$\omega_0 = \omega_{10} f(T), \quad \xi_1 = \xi_{10} f(T), \ \psi = \psi_0 f(T), \ m = m_0 f(T),$$
$$k = k_0 f(T), \quad b = b_0 f(T).$$

Since $\lambda_0, \mu_0, \beta_0, \alpha_0, \omega_{10}, \xi_{10}, \psi_0, m_0, k_0, b_0$ are constants, $f(T)$ is a given non-dimensional function of temperature, such that $f(T) = (1 - \alpha^* T_0)$, and α^* is the empirical material constant. The governing equation can be put into a more convenient form using the following non-dimensional variables

$$x' = \frac{\omega_1^*}{c_1}x,\quad y' = \frac{\omega_1^*}{c_1}y,\quad u' = \frac{\omega_1^*}{c_1}u,\quad v' = \frac{\omega_1^*}{c_1}v,\quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu_0},$$

$$T' = \frac{T}{T_0},\quad \theta' = \frac{\theta}{T_0},\quad \Omega' = \frac{\Omega}{\omega_1^*},\quad \phi' = \frac{\omega_1^{*2}\psi_0}{c_1^2}\phi,\quad t' = \omega_1^* t,$$

$$P_1' = \frac{P_1}{\mu_0},\quad P_2' = \frac{P_2}{T_0},\quad c_1^2 = (\frac{\lambda_0 + 2\mu_0}{\rho}),\quad \omega_1^* = \frac{\rho C_e c_1^2}{k}.$$

Assuming the potential functions $\psi_1(x,y,t)$ and $\psi_2(x,y,t)$ in dimensionless form

$$u = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y},\quad \text{and}\quad v = \frac{\partial\psi_1}{\partial y} - \frac{\partial\psi_2}{\partial x}. \tag{8}$$

In terms of non-dimensional quantities defined and the potential functions the system reduced to (drop the prime for convenience)

$$[b_1\nabla^2 - a_5\frac{\partial^2}{\partial t^2} + a_5\Omega^2]\psi_1 - [a_4\frac{\partial}{\partial x} + 2a_5\Omega\frac{\partial}{\partial t}]\psi_2 + a_2\phi - a_3(1 - a_{12}\nabla^2)\theta = 0, \tag{9}$$

$$[a_4\frac{\partial}{\partial x} + 2a_5\Omega\frac{\partial}{\partial t}]\psi_1 + [\nabla^2 - a_5\frac{\partial^2}{\partial t^2} + a_5\Omega^2]\psi_2 = 0, \tag{10}$$

$$-a_6\nabla^2\psi_1 + [\nabla^2 - a_7 - a_8\frac{\partial}{\partial t} - a_{10}\frac{\partial^2}{\partial t^2}]\phi + a_9(1 - a_{12}\nabla^2)\theta = 0, \tag{11}$$

$$-\varepsilon_1\frac{\partial^2}{\partial t^2}\nabla^2\psi_1 - a_{11}\frac{\partial\phi}{\partial t} + (\varepsilon_3 + \varepsilon_2\frac{\partial}{\partial t})\nabla^2\theta - \frac{\partial^2}{\partial t^2}(1 - a_{12}\nabla^2)\theta = 0. \tag{12}$$

The solution of the considered physical quantities can be decomposed in terms of the normal modes as the following form

$$[\psi_1,\psi_2,\phi,\theta,T,\sigma_{ij}](x,y,t) = [\psi_1^*,\psi_2^*,\phi^*,\theta^*,T^*,\sigma_{ij}^*](y)\exp(\omega t + i\,a\,x). \tag{13}$$

Where, $[\psi_1^*,\psi_2^*,\phi^*,\theta^*,T^*,\sigma_{ij}^*](y)$ are the amplitude of the physical quantities, ω is the angular frequency, $\mathrm{i} = \sqrt{-1}$ and a is the wave number in the x - direction.

Using Eq. (13) then Eqs. (9)-(12) take the form

$$[\mathrm{D}^2 - b_2]\psi_1^* - b_3\psi_2^* + b_4\phi^* - [b_5 - b_6(\mathrm{D}^2 - a^2)]\theta^* = 0, \tag{14}$$

$$b_7\psi_1^* + [\mathrm{D}^2 - b_8]\psi_2^* = 0, \tag{15}$$

$$-a_6[\mathrm{D}^2 - a^2]\psi_1^* + [\mathrm{D}^2 - b_9]\phi^* + [a_9 - b_{10}(\mathrm{D}^2 - a^2)]\theta^* = 0, \tag{16}$$

$$-b_{14}[\mathrm{D}^2 - a^2]\psi_1^* - b_{15}\phi^* + [\mathrm{D}^2 - b_{16}]\theta^* = 0. \tag{17}$$

Eliminating ψ_1^*, ψ_2^*, ϕ^* and θ^* between Eqs. (14)-(17), we obtain the differential equation.

$$[\mathrm{D}^8 - A\mathrm{D}^6 + B\mathrm{D}^4 - C\mathrm{D}^2 + E]\{\psi_1^*(y),\psi_2^*(y),\phi^*(y),\theta^*(y)\} = 0. \tag{18}$$

Eq. (32) can be factored as

$$[(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)]\{\psi_1^*(y),\psi_2^*(y),\phi^*(y),\theta^*(y)\} = 0. \tag{19}$$

Where, $k_n^2 (n = 1,2,3,4)$ are the roots of the characteristic equation of the Eq. (19) and A, B, C, E can be obtained from elimination the functions between Eqs. (14)-(17).

The general solution of the Eq. (19), which are bound at $y \to \infty$, is given by

$$u(x,y,t) = \sum_{n=1}^{4} G_{1n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{20}$$

$$v(x,y,t) = \sum_{n=1}^{4} M_{1n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{21}$$

$$\phi(x,y,t) = \sum_{n=1}^{4} H_{2n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{22}$$

$$\theta(x,y,t) = \sum_{n=1}^{4} H_{3n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{23}$$

$$T(x,y,t) = \sum_{n=1}^{4} H_{4n}R_n\exp(-k_n y + \omega t + i\,a\,x). \tag{24}$$

$$\sigma_{xx}(x,y,t) = \sum_{n=1}^{4} H_{5n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{25}$$

$$\sigma_{yy}(x,y,t) = \sum_{n=1}^{4} H_{6n}R_n\exp(-k_n y + \omega t + i\,a\,x), \tag{26}$$

$$\sigma_{xy}(x,y,t) = \sum_{n=1}^{4} H_{8n}R_n\exp(-k_n y + \omega t + i\,a\,x). \tag{27}$$

Since $R_n (n = 1,2,3,4)$ being some coefficients and H_{1n} to H_{4n} can be obtained from elimination the functions between Eqs. (14)-(17).

3. Boundary conditions

Consider the boundary conditions to determine the coefficients $R_n (n = 1,2,3,4)$, and suppress the positive exponentials to avoid the unbounded solutions at infinity. Then the non-dimensional boundary conditions at the surface of the material are given at $y = 0$ as follows:

(1) The mechanical boundary conditions are

(i) The normal stress condition (mechanically stressed by constant force), so that

$$\sigma_{yy} = -p_1\exp(\omega t + i\,a\,x), \tag{28}$$

Where p_1 is the magnitude of the applied force in the half-space.

(ii) The tangential stress condition (stress free), then

$$\sigma_{xy} = 0, \tag{29}$$

(2) The condition of the voids (the volume fraction field is constant in y-direction). This implies that

$$\frac{\partial \phi}{\partial y} = 0. \tag{30}$$

(3) The thermal condition (the half-space subjected to thermal shock applied to the boundary). This leads to

$$T = p_2 \exp(\omega\, t + i\, a\, x). \tag{31}$$

p_2 is applied constant temperature to the boundary.

Substituting the expressions of the considered quantities in the above boundary conditions, to obtain the parameters. After applying the inverse of matrix method, one can get the values of the four constants $R_n (n = 1,2,3,4)$.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} = \begin{pmatrix} H_{61} & H_{62} & H_{63} & H_{64} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ -k_1 H_{21} & -k_2 H_{22} & -k_3 H_{23} & -k_4 H_{24} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \\ p_2 \end{pmatrix}. \tag{32}$$

Hence obtain the expressions for the physical quantities of the plate surface.

4. Numerical results and discussion

In order to illustrate the obtained theoretical results in the preceding section, following Dhaliwal and Singh in Ref. [22] the Magnesium crystal-like thermoelastic material with voids was chosen for purposes of numerical evaluations. All the units of parameters used in the calculation are given in SI units. The constants of the problem were taken as

$\lambda = 2.17 \times 10^{10}\, N/m^2$, $\mu = 3.278 \times 10^{10}\, N/m^2$,
$k = 1.7 \times 10^2\, W/m \cdot K$, $\rho = 1.74 \times 10^3\, kg/m^3$, $T_0 = 298\, K$,
$\beta = 2.68 \times 10^6\, N/m^2 \cdot K$, $C_e = 1.04 \times 10^3\, J/kg \cdot K$,
$\omega_1^* = 3.58 \times 10^{11}\, /s$, $\alpha_t = 1.78 \times 10^{-5}\, N/m^2$,
$\psi = 1.753 \times 10^{-15}\, m^2$, $\alpha = 3.688 \times 10^{-5}\, N$,
$\xi_1 = 1.475 \times 10^{10}\, N/m^2$, $b = 1.13849 \times 10^{10}\, N/m^2$,
$m = 2 \times 10^6\, N/m^2 \cdot K$, $\omega_0 = 0.0787 \times 10^{-3}\, N/m^2\, s$,
$p_1 = 1\, N/m^2$, $p_2 = 0.5\, K$, $k^* = 85\, W/m \cdot K$,
$a = 0.001\, m$, $\omega = \eta + i\eta_1$, $\eta = -0.5\, rad/s$,
$\eta_1 = 0.1\, rad/s$, $x = 0.5\, m$, $t = 0.2\, s$ and $0 \le y \le 10\, m$.

These numerical values used to obtain the distribution of the real part of the displacement u the temperature T, the stress σ_{xx} and the change in the volume fraction field ϕ with the distance y for (G-N) theory of types II and III. In each graph the solid and dashed lines represent the solu tion in the context of the (G-N) theory of type II and the solid

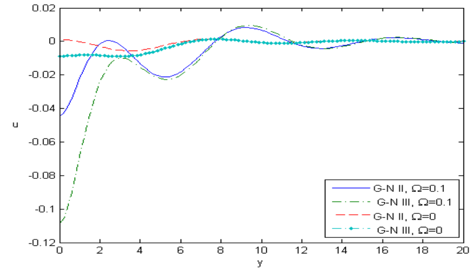

Fig. 2. The distribution of u against y while $\Omega = 0.1\, rad/s, 0$.

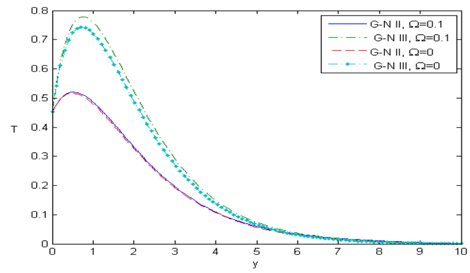

Fig. 3. The distribution of T against y while $\Omega = 0.1\, rad/s, 0$.

with dots and dashed with dots represent the solution using (G-N) theory of type III. Figs. 2-5 represent the change in the behavior of the physical quantities against distance y in 2D in the context of both types II and III of (G-N) theory during $g = 9.8\, m/s^2$, $\alpha^* = 0.00051/K$ and $d = 10^{-15}$ for $\Omega = 0.1\, rad/s, 0$. Figs. 6-9 show the behavior of the physical quantities against distance y in 2D in the context of both types II and III of (G-N) theory during $\Omega = 0.1\, rad/s$, $\alpha^* = 0.00051/K$ and $d = 10^{-15}$ for $g = 9.8\, m/s^2, 0$. Figs. 10-13 depict the distribution of the physical quantities against distance y in 2D in the context of both types of (G-N) theory during $\Omega = 0.1\, rad/s, g = 9.8\, m/s^2$ and $d = 10^{-15}$ for $\alpha^* = 0.00051/K, 0$. While Figs. 14 and 15 explain the distribution of the physical quantities against distance y in 2D in the context of both types of (G-N) theory at $\Omega = 0.1\, rad/s$, $g = 9.8\, m/s^2$ and $\alpha^* = 0.00051/K$ for $d = 10^{-15}, 0$.

Fig. 2 shows the distribution of the displacement component u; it noticed that in the case of (G-N) of type II the distribution of u decreased in the intervals $0 \le y \le 1.8$, $3.4 \le y \le 8$ and $12 \le y \le 15$, but increased in $1.8 \le y \le 3.5$, $8 \le y \le 12$ and $15 \le y \le 20$, while u distribution in the case of type III of (G-N) decreased in the intervals $0 \le y \le 8$ and $12 \le y \le 15$ while it increased in $8 \le y \le 12$ and $15 \le y \le 20$ with the increase of the rotation value.

Fig. 3 explains that the distribution of the temperature increasing with the increase of the rotation for in both types of (G-N) theory as increasing the rotation for $y > 0$.

Fig. 4 depicts that the distribution of the stress σ_{xx} increasing for both types II and III of (G-N) theory with the increasing of the rotation for $y > 0$.

Fig. 5 expresses that the distribution of ϕ increasing for both types II and III of (G-N) theory with the increase of the rotation for $y > 0$. It explained that the rotation has an effec-

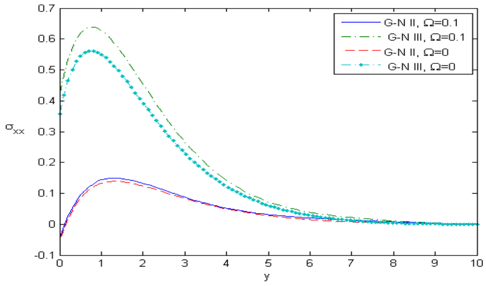

Fig. 4. Distribution of σ_{xx} against y while $\Omega = 0.1\ rad/s$, 0.

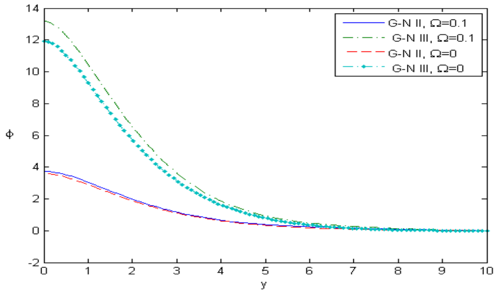

Fig. 5. The distribution of ϕ against y while $\Omega = 0.1\ rad/s$, 0.

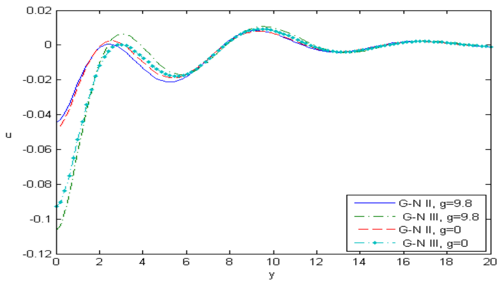

Fig. 6. The distribution of u against y while $g = 9.8\ m/s^2$, 0.

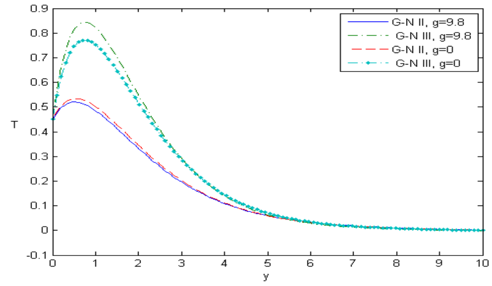

Fig. 7. The distribution of T against y while $g = 9.8\ m/s^2$, 0.

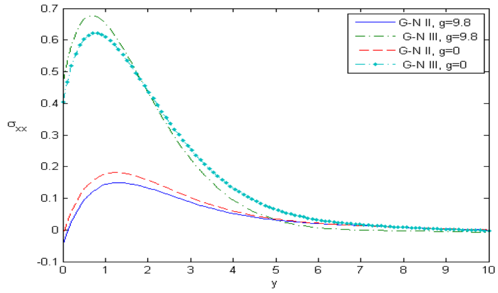

Fig. 8. The distribution of σ_{xx} against y while $g = 9.8\ m/s^2$, 0.

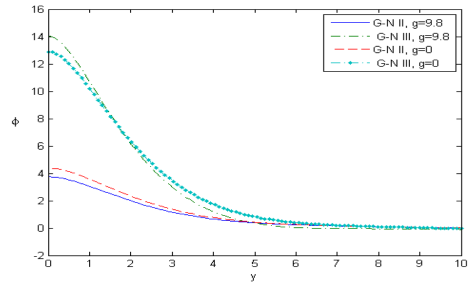

Fig. 9. The distribution of ϕ against y while $g = 9.8\ m/s^2$, 0.

tive role in the distribution of all physical quantities of the problem for the both types II and III of (G-N) theory since the distribution of these quantities varying (increasing or decreasing) with the increase of the rotation value while other physical operators (the gravity, the temperature dependent and the two temperature effect) in the problem are present.

Fig. 6 shows that the distribution of the displacement u increased in $0 \le y \le 2$ then decreased in $2 \le y \le 7$ then returns to increasing in $7 \le y \le 20$ for the case of type II of (G-N) theory, but the distribution of u in the case of type III decreased in $0 \le y \le 2$ then increased in $2 \le y \le 7$ then returns to decreasing in $7 \le y \le 20$ with the increase of the gravity value.

Fig. 7 determines that the distribution of T decreasing for type II and increasing for type III of (G-N) theory for $y > 0$ with the increase of the gravity value.

Fig. 8 shows that the distribution of the stress σ_{xx} for type II of (G-N) theory is decreasing for $y > 0$. While in the case of (G-N) of type III they increased in $0 \le y \le 2$ then decreased in $2 \le y \le 10$ with the increase of the gravity value.

Fig. 9 explains that the distribution of the change in the volume fraction field ϕ decreases in the case of type II and increased in $0 \le y \le 2$ then decreased in $2 \le y \le 10$ for (G-N) theory of type III with the increase of the gravity value. It observed that the gravity has a great effect on the distribution of all physical quantities in the case of both types II and III of (G-N) theory and the distribution of the physical quantities changing (increasing or decreasing) with the increase of the gravity value while other physical operators (the rotation, the temperature dependent and the two temperature effect) in the problem are available.

Fig. 10 shows the distribution of the displacement component u; it noticed that the distribution of u decreased in the intervals $0 \le y \le 6$ and $10 \le y \le 14$, but increased in $6 \le y \le 10$ and $14 \le y \le 20$ for both types II and III of (G-N) theory.

Fig. 11 explains that the distribution of T increasing with the increase of the rotation for in both types of (G-N) theory as increasing of α^* value for $y > 0$.

Fig. 12 depicts that the distribution of the stress σ_{xx} in-

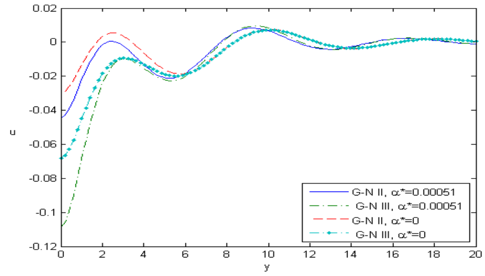

Fig. 10. The distribution of u against y while $\alpha^{*} = 0.00051/K, 0$.

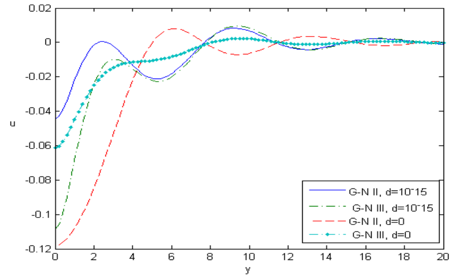

Fig. 14. The distribution of u against y while $d = 10^{-15}, 0$.

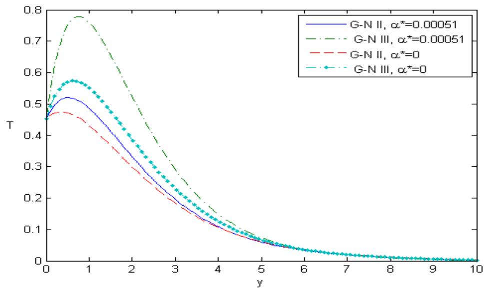

Fig. 11. The distribution of T against y while $\alpha^{*} = 0.00051/K, 0$.

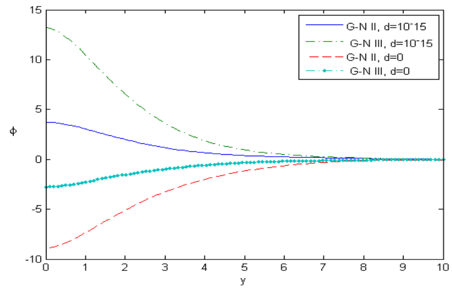

Fig. 15. The distribution of ϕ against y while $d = 10^{-15}, 0$.

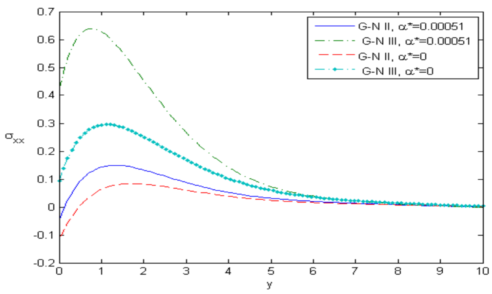

Fig. 12. Distribution of σ_{xx} against y while $\alpha^{*} = 0.00051/K, 0$.

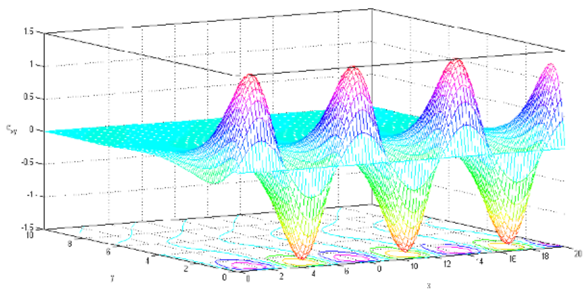

Fig. 16. 3D Curve of σ_{xy} versus the components of distance.

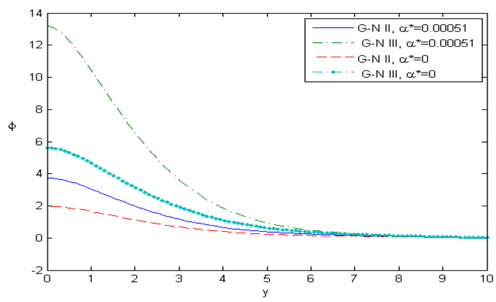

Fig. 13. Distribution of ϕ against y while $\alpha^{*} = 0.00051/K, 0$.

creasing for both types II and III of (G-N) theory with the increasing of α^{*} value for $y > 0$.

Fig. 13 expresses that the distribution of ϕ increasing for both types II and III of (G-N) theory with the increase of α^{*} value for $y > 0$. It explained that the temperature dependent properties have a significant role in the distribution of all physical quantities of the problem for the both types II and III of (G-N) theory; as the distribution of the physical quantities having an alteration (increasing or decreasing) with the increase of the temperature dependent properties while other physical operators (the rotation, the gravity and the two temperature effect) in the problem are attending.

Fig. 14 shows that the distribution of u increased in the ranges $0 \le y \le 4$, $8 \le y \le 12$ $14 \le y \le 20$, while it decreased in $4 \le y \le 8$ and $12 \le y \le 14$ for the case of type II of (G-N) theory, but the distribution of u in the case of type III decreased in the ranges $0 \le y \le 2$, $4 \le y \le 8$ and $12 \le y \le 14$, while it increased in intervals $2 \le y \le 4$, $8 \le y \le 12$, $14 \le y \le 20$, with the increase of d.

Fig. 15 explains that the distribution of the change in the volume fraction field ϕ increases in the case of types II and III of (G-N) theory with the increase of d. It observed that all the curves, continuous and converges to zero, and the two temperature effect has a great effect on the distribution of all physical quantities in the case of both types II and III of (G-N) theory; as the distribution of the physical quantities changing (increasing or decreasing) with the increase of two temperature effect while other physical operators(the rotation, the gravity and the temperature dependent properties) in the problem exists. 3D curve is representing the complete relation between σ_{xy} against both components of the distance as shown in Fig. 16 where $\Omega = 0.1\, rad/s$, $\alpha^* = 0.00051/K$ at $t = 0.2\, s$ and $d = 10^{-15}$ in the absence of the gravity effect; under (G-N) theory of type III. This figure is very important to show that the functions are moving in wave propagation.

5. Concluding remarks

(1) The rotation, the gravity and the temperature dependent properties having great role in the distribution of the physical quantities, since these quantities varying with the increase of the physical operators.

(2) The value of all physical quantities converges to zero with an increase in the distance y and all functions are continuous.

(3) The deformation of a body depends on the nature of the applied forces as well as the type of boundary conditions.

(4) The two temperature theory has an important effect on many problems in thermoelasticity and on the distribution of the considered physical quantities.

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Appendix

$$a_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad a_2 = \frac{b_0 c_1^2}{\omega_1^{*2} \psi_0 \mu_0}, \quad a_3 = \frac{\beta_0 T_0}{\mu_0},$$

$$a_4 = \frac{\rho g c_1^2}{\mu_0 f(T)}, \quad a_5 = \frac{\rho c_1^2}{\mu_0 f(T)}, \quad a_6 = \frac{b_0 \psi_0}{\alpha_0},$$

$$a_7 = \frac{\xi_{10} c_1^2}{\alpha_0 \omega_1^{*2}}, \quad a_8 = \frac{\omega_{10} c_1^2}{\alpha_0 \omega_1^*}, \quad a_9 = \frac{m_0 T_0 \psi_0}{\alpha_0},$$

$$a_{10} = \frac{\rho c_1^2 \psi_0}{\alpha_0}, \quad a_{11} = \frac{m_0 c_1^2 f(T)}{\rho C_e \psi_0 \omega_1^{*3}}, \quad a_{12} = \frac{d\, \omega_1^{*2}}{c_1^2},$$

$$\varepsilon_1 = \frac{\beta_0 f(T)}{\rho C_e}, \quad \varepsilon_2 = \frac{k^* \omega_1^*}{\rho C_e c_1^2}, \quad \varepsilon_3 = \frac{k_0 f(T)}{\rho C_e c_1^2},$$

$$A^* = \frac{1}{f(T)} = \frac{1}{(1 - \alpha^* T_0)}, \quad a_{13} = \frac{\lambda_0 f(T)}{\mu_0}, \quad a_{14} = \frac{b_0 c_1^2 f(T)}{\mu_0 \psi_0 \omega_1^{*2}},$$

$$a_{15} = \frac{\beta_0 T_0 f(T)}{\mu_0}, \quad a_{16} = f(T), \quad b_1 = a_1 + 1,$$

$$b_2 = a^2 + \frac{a_5(\omega^2 - \Omega^2)}{b1},\ b_3 = \frac{i\,a\,a_4 + 2\,a_5\,\omega\Omega}{b_1},\ b_4 = \frac{a_2}{b_1},$$
$$b_5 = \frac{a_3}{b_1},\ b_6 = \frac{a_3\,a_{12}}{b_1},\ b_7 = i\,a\,a_4 + 2\,a_5\,\omega\Omega,$$
$$b_8 = a^2 - a_5(\omega^2 - \Omega^2),\ b_9 = a^2 + a_7 + a_8\,\omega + a_{10}\,\omega^2,$$
$$b_{10} = a_9\,a_{12},\ b_{11} = \varepsilon_3 + \varepsilon_2\omega,\ b_{12} = a_{12}\,\omega^2,\ b_{13} = b_{11} + b_{12},$$
$$b_{14} = -\frac{\varepsilon_1\,\omega^2}{b_{13}},\ b_{15} = -\frac{a_{11}\,\omega}{b_{13}},\ b_{16} = a^2 + \frac{\omega^2}{b_{13}},\ \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}y},$$
$$G_{1n} = (i\,a - k_n\,H_{1n}),\ M_{1n} = -(k_n + i\,a\,H_{1n}),\quad n = 1,2,3,4.$$
$$H_{5n} = a_{13}(G_{1n} - k_n\,M_{1n}) + 2\mathrm{i}\,a\,a_{16}\,G_{1n} + a_{14}H_{2n} - a_{15}H_{4\mathrm{n}},$$
$$H_{6n} = a_{13}(G_{1n} - k_n\,M_{1n}) - 2\,k_n\,a_{16}\,M_{1n} + a_{14}H_{2n} - a_{15}H_{4\mathrm{n}},$$
$$H_{8\mathrm{n}} = \mathrm{a}_{16}(-k_n\,G_{1n} + \mathrm{i}\,a\,M_{1n}),\quad n = 1,2,3,4.$$

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