

Rotation and gravitational field effect on two-temperature thermoelastic material with voids and temperature dependent properties type III[†]

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Abstract

This paper studies the two dimensional problem of thermoelastic rotating material with voids under the effect of the gravity and the temperature dependent properties employing the two-temperature generalized thermoelasticity in the context of Green-Naghdi (G-N) theory of types II and III. The normal mode method is used to obtain the exact expressions for the physical quantities which have been shown graphically. The comparisons have been made in the presence and the absence of the rotation, the gravity, the temperature dependent properties and the two-temperature effect.

Keywords: Energy dissipation; Gravity; Green-naghdi theory; Normal mode analysis; Rotation; Temperature dependent; Thermoelasticity; Two-temperature; Voids

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1. Introduction

Thermoelasticity is the change in the size and the shape of a solid object as the temperature of that object fluctuates, it used to design materials and objects that can withstand fluctuations in temperature without breaking. Materials that are more elastic will expand and contract more than those materials that are more inelastic. The generalized theory of thermo-elasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity that has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity theory stated by Biot [1]. Lord and Shulman [2] formulated a generalized theory of thermoelasticity with one thermal relaxation time, who obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [3] developed a theory of thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction. Green and Naghdi [4-6] proposed three models, which are subsequently referred to as (G-N) -I, II and III types. The linearized version of type-I corresponds to the classical thermoelastic Fourier's law for the heat conduction equation. In type II the internal rate of production of the entropy is taken to be identically zero implying no dissipation of thermal energy and known as thermoelasticity without energy dissipation. Type-III includes the previous two models as special cases and admits dissipation of energy. In Refs. [7, 8] Othman et al. used (G-N) theory of type III to study some models in thermoelasticity with energy dissipation. The classical theory of elasticity developed from the consideration that a solid is a continuum, appears to be inadequate for the study of the response of a solid to applied load when porous materials containing voids (such as geological materials like rock and soils and manufactured materials like ceramics and pressed powder) are considered. Cowin and Nunziato [9] developed general model to predict the mechanical behavior of the solid materials with voids, This liberalized theory of the elastic materials with voids is a generalization of the classical theory of elasticity and reduces to it when the voids are suppressed. Some basic investigation related to studying the thermoelastic materials with voids were discussed by Cowin [10]. Othman et al. [11, 12] used (G-N) theory to study two models of thermoelastic medium with voids. Recently Abbas and Kumar [13] studied the response of the initially stressed generalized thermoelastic solid with voids to thermal source. Keeping in view that the propagation of plane waves in a rotating media is important in many realistic problems, e.g., rotation of heavenly bodies and the moon. Schoenberg and Censor [14] established the propagation of the waves in a rotating, homogenous, isotropic, linear elastic medium for any orientation of the rotation axis with respect to free space taking into consideration the Coriolis and the Centripetal acceleration. Abo-Dahab et al.

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Fig. 1. Geometry of the problem.

[15] discussed the problem of rotating elastic media. The gravity effect was generally neglected in the classical theory of elasticity. Bromwich [16] established the effect of the gravity on the wave propagation of an elastic solid medium. Most of the studies were done under the assumption of the temperature independent material properties that is considered a limiting of the applicability of the obtained solutions certain ranges of the temperature. The thermal and mechanical properties of the materials vary with the temperature, so the temperaturedependent on the material properties must be taken into consideration in the thermal stress analysis of these elements. Othman [17] studied the generalized thermoelastic plane waves in a rotating media with thermal relaxation under the temperature dependent properties. Chen and Gurtin [18], Chen et al. [19] have formulated a theory of the heat conduction in deformable bodies, which depends upon two distinct temperatures, the thermodynamic temperature *T*, and the conductive temperature θ . Warren and Chen [20] investigated the wave propagation in the two-temperature theory of thermoelasticity. Youssef [21] established the two-temperature generalized that $|(T-T_0)/T_0| \ll 1$, ϕ is the change in the volume fracthermoelasticity theory together with a general uniqueness theorem.

The aim of this work is to determine the distributions of the physical quantities for a homogenous, isotropic, thermoelastic material with voids analytically in the case of absence and presence of the rotation, the gravity, the temperature dependent and the two temperature effect using (G-N) theory of types II and III in terms of the normal modes to obtain the exact expressions for the physical quantities which represented graphically. remperature *ex*. warren ano Chen proposition in the wo-temperature theory of thermoelasticity.

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Youssef [21] established the two-temperature generalized that Voisset [21] established the Wo-temperature generalized that $[(I - I_0)/I_0] < 1$, θ is the change in the volume of this work is to determine the distributions of the components of frain tensor, δ_{θ} is the Kroneck
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2. Formulation and solution of the problem

Consider a linear homogeneous isotropic thermoelastic mathe surface $z = 0$. For two dimensional problem assume the rotated with a uniform angular velocity *Ω* such that the time variable *t,* and of the coordinates *x* and *y.*

The equations of motion, the field equation and the heat

equation under the effect of the gravity field of rotating thermoelastic material with voids and the two-temperature theory in the case type III of the (G-N) theory (see Cowin and Nunziato [9], Green and Naghdi [5] and [21] due to Youssef) in the absence of body forces, heat sources and extrinsic equilibrated body force will be *and Technology 29 (9) (2015) 3739-3746*
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e absence of body forces, heat sources and extrinsic equili-
ated body force will be

$$
\mu \nabla^2 u + (\lambda + \mu) \frac{\partial e}{\partial x} + b \frac{\partial \phi}{\partial x} - \beta \frac{\partial T}{\partial x} + \rho g \frac{\partial v}{\partial x}
$$

$$
= \rho \left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t} \right], \qquad (1)
$$

$$
\mu \nabla^2 v + (\lambda + \mu) \frac{\partial e}{\partial y} + b \frac{\partial \phi}{\partial y} - \beta \frac{\partial T}{\partial y} - \rho g \frac{\partial u}{\partial x}
$$

$$
= \rho \left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t} \right], \qquad (2)
$$

$$
\alpha \nabla^2 \phi - b e - \xi_1 \phi - \omega_0 \frac{\partial \phi}{\partial t} + m T = \rho \psi \frac{\partial^2 \phi}{\partial t^2}, \qquad (3)
$$

$$
k \nabla^2 \theta + k^* \frac{\partial}{\partial t} \nabla^2 \theta - m T_0 \frac{\partial \phi}{\partial t} = \rho C_e \frac{\partial^2 T}{\partial t^2} + \beta T_0 \frac{\partial^2 e}{\partial t^2}, \qquad (4)
$$

$$
\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + 2\mu e_{ij} + b \phi \delta_{ij} - \beta T \delta_{ij}, \qquad (5)
$$

$$
e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \qquad (6)
$$

$$
T = (\theta - d \theta_{ii}). \qquad (7)
$$
Where, λ , μ are the Lame's constants, α , b , ξ_1 , ω_0 , m , ψ
the material constants due to presence of voids,
 $= (3\lambda + 2\mu)\alpha_i$, while α_i is the element, k is the

$$
\mu \nabla^2 v + (\lambda + \mu) \frac{\partial e}{\partial y} + b \frac{\partial \phi}{\partial y} - \beta \frac{\partial T}{\partial y} - \rho g \frac{\partial u}{\partial x}
$$

$$
= \rho \left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}\right],\tag{2}
$$

$$
\alpha \nabla^2 \phi - b \, e - \xi_1 \, \phi - \omega_0 \frac{\partial \phi}{\partial t} + m \, T = \rho \, \psi \, \frac{\partial^2 \phi}{\partial t^2},\tag{3}
$$

$$
k\nabla^2 \theta + k^* \frac{\partial}{\partial t} \nabla^2 \theta - m T_0 \frac{\partial \phi}{\partial t} = \rho C_e \frac{\partial^2 T}{\partial t^2} + \beta T_0 \frac{\partial^2 e}{\partial t^2},
$$
 (4)

$$
\sigma_{ij} = \lambda u_{k,k} \, \delta_{ij} + 2\mu \, e_{ij} + b \, \phi \, \delta_{ij} - \beta T \, \delta_{ij}, \tag{5}
$$

$$
e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),
$$
\n(6)

$$
T = (\theta - d \theta_{,ii}).
$$
\n(7)

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and comparation P , and the conductive cent, ρ is the density, ζ_c is the special
reaction in the two-temperature θ . Warren and Chen [20] investigated the wave
thermal conductivity, k^2 is the Where, ^l ^m, are the Lame's constants, 1 0 ^a ^x ^w ^y , , , , , *b m* are the material constants due to presence of voids, $\mu \nabla^2 v + (\lambda + \mu) \frac{\partial e}{\partial y} + b \frac{\partial \phi}{\partial y} - \beta \frac{\partial T}{\partial y} - \rho g \frac{\partial u}{\partial x}$
 $= \rho \left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2 \Omega \frac{\partial u}{\partial t} \right],$ (2)
 $\alpha \nabla^2 \phi - b e - \xi_1 \phi - \omega_0 \frac{\partial \phi}{\partial t} + m T = \rho \psi \frac{\partial^2 \phi}{\partial t^2},$ (3)
 $k \nabla^2 \theta + k^* \frac{\partial}{\partial t} \nabla^2 \theta$ while α_t is the thermal expansion coefficient, ρ is the density, C_e is the specific heat, k is the thermal conductivity, k^* is the material constant characteristic of the theory, T_0 is the reference temperature chosen such $= \rho \left[\frac{\partial^2 V}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}\right],$ (2)
 $\alpha \nabla^2 \phi - b e - \xi_1 \phi - \omega_0 \frac{\partial \phi}{\partial t} + m T = \rho \psi \frac{\partial^2 \phi}{\partial t^2},$ (3)
 $k \nabla^2 \theta + k^* \frac{\partial}{\partial t} \nabla^2 \theta - m T_0 \frac{\partial \phi}{\partial t} = \rho C_e \frac{\partial^2 T}{\partial t^2} + \beta T_0 \frac{\partial^2 e}{\partial t^2},$ (4)
 $\sigma_{ij} = \lambda u_{$ tion field, σ_{ij} are the components of the stress tensor, e_{ij} are the components of strain tensor, δ_{ij} is the Kronecker delta, *d* is the two temperature parameter so that $d > 0$, *T* is the thermodynamic temperature and θ is the conductive temperature and *g* is the acceleration due to the gravity, when $k^* \rightarrow 0$, then Eq. (4) reduces to the heat conduction equation in the (G-N) theory (of type II). where, x_2 *M* are the Lamin s outstants, u , v_1 , w_0 , m , ψ is the material constants due to presence of voids,
 $=(3\lambda + 2\mu)\alpha_i$, while α_i is the thermal expansion coeffi-

nt, ρ is the density, C_e is or the menoty, I_0 is the reterned temperature chosen such
at $[(T - T_0)/T_0] < 1$, ϕ is the change in the volume frac-
n field, σ_{ij} are the components of the stress tensor, e_{ij} are
e components of strain tensor, aat $|(I - I_0)/I_0| \ll 1$, φ is the chappe in the volume fraction field, σ_{ij} are the components of the stress tensor, e_{ij} are he components of strain tensor, δ_{ij} is the Kronecker delta, I is the two temperatur *f₃* are the components of the stress tensor, e_{ij} are
hents of strain tensor, δ_{ij} is the Kronecker delta,
wo temperature parameter so that $d > 0$, *T* is the
mic temperature and θ is the conductive tem-
d *g*

To investigate the effect of the temperature dependent properties on thermoelastic material with voids, therefore we assume that

$$
\lambda = \lambda_0 f(T), \quad \mu = \mu_0 f(T), \quad \beta = \beta_0 f(T), \quad \alpha = \alpha_0 f(T),
$$

\n
$$
\omega_0 = \omega_{10} f(T), \quad \xi_1 = \xi_{10} f(T), \quad \psi = \psi_0 f(T), \quad m = m_0 f(T),
$$

\n
$$
k = k_0 f(T), \quad b = b_0 f(T).
$$

such that $f(T) = (1 - \alpha^* T_0)$, and α^* is the empir rial constant. The governing equation can be put into a more convenient form using the following non-dimensional variables

M. I. A. Othman and M. I. M. Hilal / Journal of Mechanical Science and Technology 29 (9) (2015) 3739-3746
\n
$$
x' = \frac{\omega_1^*}{c_1} x, \quad y' = \frac{\omega_1^*}{c_1} y, \quad u' = \frac{\omega_1^*}{c_1} u, \quad v' = \frac{\omega_1^*}{c_1} v, \quad \sigma'_0 = \frac{\sigma'_0}{\mu_0},
$$
\n
$$
\Gamma = \frac{\omega_1^*}{\tau_0}, \quad \theta' = \frac{\theta_1}{\theta_0}, \quad \theta' = \frac{\omega_1^* \omega_0}{\mu_0} \phi, \quad t' = \omega_1^* t,
$$
\n
$$
P'_1 = \frac{\mu_1}{\mu_0}, \quad P'_2 = \frac{\mu_2}{\tau_0}, \quad \sigma'_1 = \frac{(\lambda_1 + 2\mu_0)}{\rho}, \quad \omega_1^* = \frac{\rho C_0 c_1^2}{k}.
$$
\n
$$
P'_1 = \frac{\mu_1}{\mu_0}, \quad P'_2 = \frac{\mu_2}{\tau_0}, \quad c_1^2 = \frac{(\lambda_1 + 2\mu_0)}{\rho}, \quad \omega_1^* = \frac{\rho C_0 c_1^2}{k}.
$$
\n
$$
P'_2 = \frac{\mu_2}{\tau_0}, \quad c_1^2 = \frac{(\lambda_1 + 2\mu_0)}{\rho}, \quad \omega_1^* = \frac{\rho C_0 c_1^2}{k}.
$$
\n
$$
\Gamma = \frac{\rho C_0 c_1^2}{k}.
$$
\n
$$
\Gamma = \frac{\rho C_0 c_1}{k}.
$$
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$$
\Gamma = \frac
$$

$$
u = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}, \quad \text{and} \quad v = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}.
$$
 (8)

In terms of non-dimensional quantities defined and the potential functions the system reduced to (drop the prime for convenience)

$$
P_{1}' = \frac{P_{1}}{\mu_{0}}, P_{2}' = \frac{P_{2}}{T_{0}}, c_{1}^{2} = (\frac{\lambda_{0} + 2\mu_{0}}{\rho}), o_{1}^{*} = \frac{\rho C_{e}c_{1}^{2}}{k}.
$$

\n
$$
[(D^{2} - k_{1}^{2})(D^{2} - k_{2}^{2})(D^{2} - k_{2}^{2})(D^{2} - k_{2}^{2})]
$$
\n
$$
\{w_{1}'(y), w_{2}^{*}(y), \phi^{*}(y), \theta^{*}(y), \theta^{*}(y)\} = 0.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial x} + \frac{\partial w_{2}}{\partial y}, \text{ and } v = \frac{\partial w_{1}}{\partial y} - \frac{\partial w_{2}}{\partial x}.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial x} + \frac{\partial w_{2}}{\partial y}, \text{ and } v = \frac{\partial w_{1}}{\partial y} - \frac{\partial w_{2}}{\partial x}.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial y}, \text{ and } v = \frac{\partial w_{1}}{\partial y} - \frac{\partial w_{2}}{\partial x}.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial y}, \text{ and } v = \frac{\partial w_{1}}{\partial y} - \frac{\partial w_{2}}{\partial x}.
$$
\n
$$
(8)
$$
\n
$$
u = \frac{\partial v_{1}}{\partial y} + \frac{\partial v_{2}}{\partial y} + \frac{\partial v_{2}}{\partial y} + \frac{\partial v_{2}}{\partial y}.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial y} + \frac{\partial w_{2}}{\partial y}.
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\n
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u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial y}.
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\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial z}.
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\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial z}.
$$
\n
$$
u = \frac{\partial w_{1}}{\partial y} + \frac{\partial w_{2}}{\partial z}.
$$
\n
$$
u
$$

$$
[a_4 \frac{\partial}{\partial x} + 2a_5 \Omega \frac{\partial}{\partial t}] \psi_1 + [\nabla^2 - a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2] \psi_2 = 0, \qquad (10) \qquad \theta(x, y, t)
$$

$$
-a_6 \nabla^2 \psi_1 + [\nabla^2 - a_7 - a_8 \frac{\partial}{\partial t} - a_{10} \frac{\partial^2}{\partial t^2}] \phi
$$

+ a_9 (1 - a_{12} \nabla^2) \theta = 0, (11) $\sigma_{xx}(x, y, t) =$

$$
P_{1}^{\prime} = \frac{P_{1}}{\mu_{0}}, P_{2}^{\prime} = \frac{P_{2}}{I_{0}}, c_{1}^{2} = (\frac{\lambda_{0} + 2\mu_{0}}{\rho}), \omega_{1}^{\star} = \frac{\rho C_{e}c_{1}^{2}}{\rho},
$$

\nAssuming the potential functions $\psi_{1}(x, y, t)$ and $\psi_{2}(x, y, t)$
\nin dimensions $\psi_{1}(x, y, t)$ and $\psi_{2}(x, y, t)$
\nin dimensions $\psi_{1}(x, y, t)$ and $\psi_{2}(x, y, t)$
\nin dimensions $\psi_{2}(x, y, t)$
\n $u = \frac{\partial \psi_{1}}{\partial x} + \frac{\partial \psi_{2}}{\partial y},$ and $v = \frac{\partial \psi_{1}}{\partial y} - \frac{\partial \psi_{2}}{\partial x}.$
\nIn terms of non-dimensional quantities defined and the pro-
\nentential functions the system reduced to (drop the prime for
\nconvenience)
\n
$$
[b_{1} \nabla^{2} - a_{2} \frac{\partial^{2}}{\partial t^{2}} + a_{3} \Omega^{2}] \psi_{1} - [a_{4} \frac{\partial}{\partial x} + 2a_{5} \Omega \frac{\partial}{\partial t}] \psi_{2}
$$

\n $[\psi_{1}^{\star}(y, y, t)] = \sum_{n=1}^{4} \frac{1}{\lambda_{1n}} R_{n} \exp(-k_{n}y + \omega t + i a x),$
\n $[\psi_{1} \nabla^{2} - a_{2} \frac{\partial^{2}}{\partial t^{2}} + a_{5} \Omega^{2}] \psi_{1} - [a_{4} \frac{\partial}{\partial x} + 2a_{5} \Omega \frac{\partial}{\partial t}] \psi_{2}$
\n $[\psi_{2} \nabla^{2} - a_{5} \frac{\partial^{2}}{\partial t^{2}} + a_{5} \Omega^{2}] \psi_{1} - [a_{4} \frac{\partial}{\partial x} + 2a_{5} \Omega \frac{\partial}{\partial t}] \psi_{2}$
\n $[\psi_{1} \nabla_{2} - a_{5} \Omega_{1}^{2} - a_{5} \Omega^{2} + a_{5} \Omega^{2}] \psi_{1} = 0,$
\n $[\psi_{1} \nabla_{2}$

The solution of the considered physical quantities can be decomposed in terms of the normal modes as the following

$$
[\psi_1, \psi_2, \phi, \theta, T, \sigma_{ij}](x, y, t) = [\psi_1^*, \psi_2^*, \phi^*, \theta^*, T^*, \sigma_{ij}^*](y)
$$

exp($\omega t + i a x$). (13)

in terms of the normal modes as the following H_{4n}

Eqs. (
 $\phi, \theta, T, \sigma_{ij}](x, y, t) = [\psi_1^*, \psi_2^*, \phi^*, \theta^*, T^*, \sigma_{ij}^*](y)$
 $\exp(\omega t + i \alpha x).$ (13)
 $[\psi_1^*, \psi_2^*, \phi^*, \theta^*, T^*, \sigma_{ij}^*](y)$ are the amplitude of the cients

quantities, **3. Boundary co**
 $\exp(\omega t + i a x)$. (13)

Where, $[\psi_1^*, \psi_2^*, \phi^*, \sigma^*, T^*, \sigma_{ij}^*](y)$ are the amplitude of the

sical quantities, ω is the angular frequency, $i = \sqrt{-1}$ and

is the wave number in the x - direction.

Jsing Eq

$$
[D2 - b2] \psi1* - b3 \psi2* + b4 \phi* - [b5 - b6 (D2 - a2)] \theta* = 0, \qquad (14)
$$

$$
b_{7}\psi_{1} + [\mathbf{D}^{2} - b_{8}]\psi_{2} = 0, \qquad (15)
$$

$$
- a_{6}[\mathbf{D}^{2} - a^{2}]\psi_{1}^{*} + [\mathbf{D}^{2} - b_{9}]\phi^{*}
$$

$$
+[a_9 - b_{10} (D^2 - a^2)]\theta^* = 0,
$$
\n(16)

$$
-b_{14}[D^2 - a^2]\psi_1^* - b_{15}\phi^* + [D^2 - b_{16}]\theta^* = 0.
$$
 (17)

Eliminating ψ_1^* , ψ_2^* , ϕ^* and θ^* between Eqs. (14)-(17), we obtain the differential equation.

1. 1. M. Hilal / Journal of Mechanical Science and Technology 29 (9) (2015) 3739-3746
\n
$$
v' = \frac{\omega_1^*}{c_1} v
$$
, $\sigma'_{ij} = \frac{\sigma_{ij}}{\mu_0}$, $[D^8 - AD^6 + BD^4 - CD^2 + E]$
\n $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (18)
\n $\frac{\omega_1^{*2}\psi_0}{c_1^2} \phi$, $t' = \omega_1^* t$,
\n $\omega_1^* = \frac{\rho C_e c_1^2}{k}$.
\n $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
\n $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (19)
\n (x, y, t) and $\psi_2(x, y, t)$
\nWhere, $k_n^2(n = 1, 2, 3, 4)$ are the roots of the characteristic equation of the Eq. (19) and *A*, *B*, *C*, *E* can be obtained from elimination the functions between Eqs. (14)-(17)

Eq. (32) can be factored as

$$
[(D2 - k12)(D2 - k22)(D2 - k32)(D2 - k42)]
$$

{ $\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)$ } = 0. (19)

Cience and Technology 29 (9) (2015) 3739-3746

[D⁸ - AD⁶ + BD⁴ - CD² + E]

{ $\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)$ } = 0. (18)

Eq. (32) can be factored as

[(D² - k_1^2)(D² - k_2^2)(D² - k_3^2)(D² - k_4^2) nd Technology 29 (9) (2015) 3739-3746

1 1 2 (19) (2015) 3739-3746

1 4 1 0⁶ + B 1 ⁹ - C 1 0² + E]

1 ($y_1^*(y), y_2^*(y), \phi^*(y), \theta^*(y)$) = 0.

1 (18)

1 ($y_1^*(y), y_2^*(y), \phi^*(y), \theta^*(y)$) = 0.
 ${y_1^*(y), y_2^*(y), \phi^*(y), \theta^*(y$ Where, $k_n^2(n=1,2,3,4)$ are the roots of the characteristic equation of the Eq. (19) and A, B, C, E can be obtained from (*operator) n* = 3741
 $\int_{0}^{5} + BD^{4} - CD^{2} + E$]
 $\int_{0}^{5} + BC^{2} + C^{2} + C^{2} + C^{2} + E$
 $\int_{0}^{5} + BC^{2} + C^{2} + C^{2} + C^{2} + E$
 $\int_{0}^{5} + BC^{2} + C^{2} + C^{2} + C$ al Science and Technology 29 (9) (2015) 3739-3746 3741
 $[D^8 - AD^6 + BD^4 - CD^2 + E]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (18)

Eq. (32) can be factored as
 $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta$ elimination the functions between Eqs. (14)-(17). *^y* ® ¥ , is given by ⁴ ence and Technology 29 (9) (2015) 3739-3746

3741
 $S^8 - AD^6 + BD^4 - CD^2 + E$]
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)\} = 0.$ (18)

1. (32) can be factored as
 $D^2 - k_1^2 \{(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_1^2)\}$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\}$ *icience and Technology 29 (9) (2015) 3739-3746* 3741
 $[D^8 - AD^6 + BD^4 - CD^2 + E]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (18)
 $Eq. (32)$ can be factored as
 $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta$ Technology 29 (9) (2015) 3739-3746
 3741
 $29^6 + BD^4 - CD^2 + E$
 $(y), \psi_2^*(y), \phi^*(y), \theta^*(y)) = 0.$ (18)

an be factored as
 $\phi^2((D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2))$
 $\phi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)) = 0.$ (19)
 $\phi_n^2(n = 1, 2, 3, 4)$ are ience and Technology 29 (9) (2015) 3739-3746
 $D^8 - AD^6 + BD^4 - CD^2 + E$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)\} = 0.$ (18)
 q . (32) can be factored as
 $D^2 - k_1^2 (D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)\} = 0.$ *zience and Technology 29 (9) (2015) 3739-3746* 3741
 $[D^8 - AD^6 + BD^4 - CD^2 + E]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (18)
 $Eq. (32)$ can be factored as
 $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*($ $2^6 + BD^4 - CD^2 + E$]
 $(y), \psi_2^*(y), \phi^*(y), \theta^*(y)) = 0.$ (18)

an be factored as
 $\psi_2(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)$]
 $\psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y)) = 0.$ (19)
 $\psi_n^2(n = 1, 2, 3, 4)$ are the roots of the characteristic

the Eq. (1 $D^8 - AD^6 + BD^4 - CD^2 + E$]

{*w*₁⁺(*y*),*w*₂⁺(*y*),*φ*⁴ (*y*),*φ*⁴(*y*),*θ*⁴(*y*),*θ*⁴(*y*)) = 0. (18)

4. (32) can be factored as
 $D^2 - k_1^2$)($D^2 - k_2^2$)($D^2 - k_3^2$)($D^2 - k_4^2$)]

{*w*₁⁺(*y*),*w*₂ $[D^8 - AD^6 + BD^4 - CD^2 + E]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (18)

Eq. (32) can be factored as
 $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (19)

Where, $k_2^2(n = 1, 2, 3, 4)$ are the ro (16)

an be factored as

an be factored as
 $\gamma_1^2(y), \gamma_2^2(y), \gamma_3^3(y), \sigma^3(y) = 0.$ (19)
 $\gamma_1^2(y), \gamma_2^2(y), \phi^*(y), \phi^*(y) = 0.$ (19)
 $\gamma_1^2(x), \gamma_2^2(y), \phi^*(y), \phi^*(y) = 0.$ (19)
 $\gamma_2^2(n = 1, 2, 3, 4)$ are the roots of the characte 1. (32) can be factored as
 $D^2 - k_1^2 (D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)$
 $\{\psi_1'(y), \psi_2'(y), \phi'(y), \phi'(y)\} = 0.$ (19)
 $\{(\psi_1'(y), \psi_2'(y), \phi'(y), \phi'(y))\} = 0.$ (19)
 $\{(\psi_1'(y), \psi_2'(y), \phi'(y), \phi'(y))\} = 0.$ (19)

there, $k_n^2 (n = 1, 2, 3, 4)$ *i* $v_1 \rightarrow v_2 \rightarrow v_3$, $v_3 \rightarrow v_4 \rightarrow v_5$. (13)

Eq. (32) can be factored as
 $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{w_1^*(y), w_2^*(y), \phi^*(y), \phi^*(y)\} = 0.$ (19)

Where, $k_0^2 (n = 1, 2, 3, 4)$ are the roots of the characteri an be lactored as
 $f^{2}(D^{2} - k_{2}^{2})(D^{2} - k_{3}^{2})(D^{2} - k_{4}^{2})$
 $f_{1}^{*}(y), \psi_{2}^{*}(y), \phi^{*}(y), \phi^{*}(y)) = 0.$ (19)
 $f_{n}^{*}(n=1,2,3,4)$ are the roots of the characteristic

the Eq. (19) and A, B, C, E can be obtained from

the f $D^2 - k_1^2 (D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)$
 $\{\psi_1^*(y), \psi_2^*(y), \phi^*(y), \theta^*(y)\} = 0.$ (19)

here, $k_n^2 (n = 1, 2, 3, 4)$ are the roots of the characteristic

trion of the Eq. (19) and A, B, C, E can be obtained from

ination th $[(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)]$
 $\{ \psi_1^*(y), \psi_2^*(y), \phi^*(y), \phi^*(y) \} = 0.$ (19)

Where, $k_n^2(n = 1, 2, 3, 4)$ are the roots of the characteristic

nation of the Eq. (19) and A, B, C, E can be obtained from

minat $\int_{0}^{2} (D - k_2)(D - k_3)(D - k_4) J$
 $\int_{0}^{2} (y), \psi_2^*(y), \phi^*(y), \phi^*(y) \} = 0.$ (19)
 $\int_{0}^{2} (n-1, 2, 3, 4)$ are the roots of the characteristic

the Eq. (19) and A, B, C, E can be obtained from

the functions between Eqs. (14) here, $k_a^2(n=1,2,3,4)$ are the roots of the characteristic
tion of the Eq. (19) and A, B, C, E can be obtained from
ination the functions between Eqs. (14)-(17).
le general solution of the Eq. (19), which are bound at
 \in Where, $k_n^2(n = 1, 2, 3, 4)$ are the roots of the characteristic
 nation of the Eq. (19) and *A*, *B*, *C*, *E* can be obtained from

mination the functions between Eqs. (14)-(17).

The general solution of the Eq. (19), (*n* = 1, 2, 3, 4) are the roots of the characteristic

e Eq. (19) and *A*, *B*, *C*, *E* can be obtained from

e functions between Eqs. (14)-(17).

I solution of the Eq. (19), which are bound at

en by
 $\sum_{n=1}^{4} G_{1n} R$

The general solution of the Eq. (19), which are bound at

$$
u(x, y, t) = \sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i \alpha x),
$$
 (20)

$$
v(x, y, t) = \sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i \alpha x),
$$
 (21)

$$
\phi(x, y, t) = \sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i a x),
$$
 (22)

$$
\theta(x, y, t) = \sum_{n=1}^{4} H_{3n} R_n \exp(-k_n y + \omega t + i a x),
$$
 (23)

$$
T(x, y, t) = \sum_{n=1}^{4} H_{4n} R_n \exp(-k_n y + \omega t + i a x).
$$
 (24)

$$
\sigma_{xx}(x, y, t) = \sum_{n=1}^{4} H_{5n} R_n \exp(-k_n y + \omega t + i a x),
$$
 (25)

$$
\sigma_{yy}(x, y, t) = \sum_{n=1}^{4} H_{6n} R_n \exp(-k_n y + \omega t + i \alpha x), \tag{26}
$$

equation of the Eq. (19) and *A*, *B*, *C*, *E* can be obtained from
\n*imination the functions between Eqs. (14)-(17).*

\nThe general solution of the Eq. (19), which are bound at
$$
\rightarrow \infty
$$
, is given by

\n
$$
u(x, y, t) = \sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i a x), \qquad (20)
$$
\n
$$
v(x, y, t) = \sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i a x), \qquad (21)
$$
\n
$$
\phi(x, y, t) = \sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i a x), \qquad (22)
$$
\n
$$
\theta(x, y, t) = \sum_{n=1}^{4} H_{3n} R_n \exp(-k_n y + \omega t + i a x), \qquad (23)
$$
\n
$$
T(x, y, t) = \sum_{n=1}^{4} H_{4n} R_n \exp(-k_n y + \omega t + i a x). \qquad (24)
$$
\n
$$
\sigma_{xx}(x, y, t) = \sum_{n=1}^{4} H_{5n} R_n \exp(-k_n y + \omega t + i a x), \qquad (25)
$$
\n
$$
\sigma_{yy}(x, y, t) = \sum_{n=1}^{4} H_{6n} R_n \exp(-k_n y + \omega t + i a x), \qquad (26)
$$
\n
$$
\sigma_{xy}(x, y, t) = \sum_{n=1}^{4} H_{8n} R_n \exp(-k_n y + \omega t + i a x). \qquad (27)
$$
\nSince $R_n (n = 1, 2, 3, 4)$ being some coefficients and H_{1n} to A_n can be obtained from elimination the functions between

ie general solution of the Eq. (19), which are bound at ∞ , is given by
 $(x, y, t) = \sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (20)
 $(x, y, t) = \sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (21)
 $(x, y, t) = \sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (The general solution of the Eq. (19), which are bound at
 $x \to \infty$, is given by
 $u(x, y, t) = \sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i a x)$, (20)
 $v(x, y, t) = \sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i a x)$, (21)
 $\phi(x, y, t) = \sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i a$ If solution of the Eq. (19), which are bound at

then by
 $\sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (20)
 $\sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (21)
 $\sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i \alpha x)$, (22)
 $\sum_{n=1}^{4} H_{3n} R_n \exp(-k_n y + \omega t + i \alpha x)$ $u(x, y, t) = \sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i \alpha x),$ (20)
 $v(x, y, t) = \sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i \alpha x),$ (21)
 $\phi(x, y, t) = \sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i \alpha x),$ (22)
 $\theta(x, y, t) = \sum_{n=1}^{4} H_{3n} R_n \exp(-k_n y + \omega t + i \alpha x),$ (23)
 $T(x, y, t) = \sum_{n=1}^{4}$ = $\sum_{n=1}^{4} G_{1n} R_n \exp(-k_n y + \omega t + i a x),$ (20)

= $\sum_{n=1}^{4} M_{1n} R_n \exp(-k_n y + \omega t + i a x),$ (21)

= $\sum_{n=1}^{4} H_{2n} R_n \exp(-k_n y + \omega t + i a x),$ (22)

= $\sum_{n=1}^{4} H_{3n} R_n \exp(-k_n y + \omega t + i a x),$ (23)

= $\sum_{n=1}^{4} H_{4n} R_n \exp(-k_n y + \omega t + i a x).$ (24)

= $\sum_{$ Since $R_n(n=1,2,3,4)$ being some coefficients and H_{1n} to H_{4n} can be obtained from elimination the functions between Eqs. (14)-(17).

3. Boundary conditions

 $[A_1 \nabla^2 - a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2] \psi_1 - [a_4 \frac{\partial}{\partial t^2} + 2a_5 \Omega \frac{\partial}{\partial t}] \psi_2$
 $+ a_2 \phi - a_5 (1 - a_{15} \nabla^2) \theta = 0,$
 $[a_4 \frac{\partial}{\partial t} + 2a_5 \Omega \frac{\partial}{\partial t}] \psi_1 + [\nabla^2 - a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2] \psi_2 = 0,$
 $- a_6 \nabla^2 \psi_1 + [\nabla^2 - a$ = 0,

(9)
 $a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2 |y_2 = 0$, (10)
 $a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2 |y_2 = 0$, (10)
 $\theta(x, y, t) = \sum_{n=1}^4 H_{3n} R_n \exp(-k_n y + \omega t + i \alpha x),$
 $\int_0^1 - a_{10} \frac{\partial^2}{\partial t^2} d\phi$
 $T(x, y, t) = \sum_{n=1}^4 H_{3n} R_n \exp(-k_n y + \omega t + i \alpha x).$

(11)
 2 $a_5 \Omega \frac{\partial}{\partial t} |\psi_1 + [\nabla^2 - a_5 \frac{\partial^2}{\partial t^2} + a_5 \Omega^2] \psi_2 = 0,$ (10)
 $\psi_1 + [\nabla^2 - a_7 - a_8 \frac{\partial}{\partial t} - a_{10} \frac{\partial^2}{\partial t^2}] \phi$
 $\psi_1 + [\nabla^2 - a_7 - a_8 \frac{\partial}{\partial t} - a_{10} \frac{\partial^2}{\partial t^2}] \phi$
 $\psi_2 + [\nabla^2 - a_8 \frac{\partial}{\partial t} - a_{10} \frac{\partial^2}{\partial t^2}] \$ u *or*
 $-\alpha_6 \nabla^2 w_1 + [\nabla^2 - a_7 - a_6 \frac{\partial}{\partial t} - a_{10} \frac{\partial^2}{\partial t^2}] \n\phi$ $T(x, y, t) = \sum_{n=1}^4 H_{4n} R_n \exp(-k_n y + \omega t + i a x),$ (24)
 $-\alpha_1 \frac{\partial^2}{\partial t^2} \nabla^2 w_1 - a_{11} \frac{\partial \phi}{\partial t} + (\epsilon_1 + \epsilon_2 \frac{\partial}{\partial t}) \nabla^2 \theta$ (11) $\sigma_{xx}(x, y, t) = \sum_{n=1}$ Composed in terms of the normal modes as the following
 $[w_1, w_2, \phi, \theta, T, \sigma_{ij}](x, y, t) = [w_1^*, w_2^*, \phi^*, \theta^*, T^*, \sigma_{ij}^*](y)$
 $\qquad \text{exp}(\omega t + i a x).$ $\qquad \qquad (13)$

Using Eq. (13) then Eqs. (9)-(12) take the form and the mode of the $-\varepsilon_1 \frac{\partial^2}{\partial t^2} \nabla^2 \psi_1 - a_{11} \frac{\partial \phi}{\partial t} + (\varepsilon_3 + \varepsilon_2 \frac{\partial}{\partial t}) \nabla^2 \theta$ $\sigma_{xy}(x, y, t) = \sum_{n=1}^4 H_{6n} R_n \exp(-k_n y + \omega t + i a x)$. (26)

The solution of the considered physical quantities can be $\frac{\varepsilon_1}{\sigma_2} H_{6n} R_n \exp(-k_n y +$ $\frac{\partial^2}{\partial t^2} (1 - a_{12} \nabla^2) \neq 0.$ (12) $\sigma_{xy}(x, y, t) = \sum_{r=1}^4 H_{8n} R_n \exp(-k_n y)$

The solution of the considered physical quantities can be Since $R_n(n = 1, 2, 3, 4)$ being soor

normposed in terms of the normal modes as the $-\frac{\partial^2}{\partial t^2}(1-a_{12}\nabla^2)\theta = 0.$ (12) $\sigma_{xy}(x,y,t) = \sum_{i=1}^{\infty}H_{8n}R_n \exp(-k_ny + \omega t)$

solution of the considered physical quantities can be Since $R_n(n=1,2,3,4)$ being some coosed in terms of the normal modes as the following $-\frac{\partial^2}{\partial t^2}(1-a_{12}\nabla^2)\theta = 0.$ (12) $\sigma_{xy}(x,y,t) = \sum_{n=1}^{\infty} H_{8n}R_n \exp(-k_n y + k_0 z)$

The solution of the considered physical quantities can be

composed in terms of the normal modes as the following H_{4n} can be obtained ution of the considered physical quantities can be

ded in terms of the normal modes as the following
 $\theta, \theta, T, \sigma_y[(x, y, t) = [w_1^{'}, w_2^{'}, \phi^{\dagger}, \theta^{'}, \sigma^{'}, \tau^{'}, \sigma_y^{{\dagger}}](y)$
 $\exp(\omega t + i \alpha x)$.
 $[W_1^{'}, W_2^{'}, \phi^{'}, T^{'}, \sigma_y^{{\dagger}}$ 1 15 [D] [D] 0. *¹⁴ ¹⁶* - - - + - = *b a b b* ^y ^f ^q (17) Consider the boundary conditions to determine the coeffi- $\theta(x, y, t) = \sum_{n=1}^{n} H_{3n} R_n \exp(-\kappa_n y + \omega t + i \alpha x),$ (25)
 $T(x, y, t) = \sum_{n=1}^{4} H_{3n} R_n \exp(-\kappa_n y + \omega t + i \alpha x).$ (24)
 $\sigma_{xx}(x, y, t) = \sum_{n=1}^{4} H_{5n} R_n \exp(-\kappa_n y + \omega t + i \alpha x),$ (25)
 $\sigma_{yy}(x, y, t) = \sum_{n=1}^{4} H_{6n} R_n \exp(-\kappa_n y + \omega t + i \alpha x).$ (26)
 $\sigma_{$ to avoid the unbounded solutions at infinity. Then the non dimensional boundary conditions at the surface of the material are given at $y = 0$ as follows: $\sigma_{xy}(x, y, t) = \sum_{n=1}^{n} H_{8n} R_n \exp(-k_n y + \omega t + i \alpha x).$ (27)

Since $R_n(n = 1, 2, 3, 4)$ being some coefficients and H_{1n} to
 \int_{4n}^{∞} can be obtained from elimination the functions between

s. (14)-(17).
 Boundary conditi

(1) The mechanical boundary conditions are

(i) The normal stress condition (mechanically stressed by constant force), so that

$$
\sigma_{yy} = -p_l \exp(\omega t + i \, a \, x),\tag{28}
$$

Where p_l is the magnitude of the applied force in the halfspace.

(ii) The tangential stress condition (stress free), then

$$
\sigma_{xy} = 0,\tag{29}
$$

(2) The condition of the voids (the volume fraction field is constant in y-direction). This implies that

$$
\frac{\partial \phi}{\partial y} = 0. \tag{30}
$$

(3) The thermal condition (the half-space subjected to ther mal shock applied to the boundary). This leads to

$$
T = p_2 \exp(\omega t + i \, a \, x). \tag{31}
$$

Substituting the expressions of the considered quantities in the above boundary conditions, to obtain the parameters. After applying the inverse of matrix method, one can get the values

(3) The thermal condition (the half-space subjected to the-
\nul shock applied to the boundary). This leads to
\n
$$
T = p_2 \exp(\omega t + i \alpha x)
$$
.
\n(31) Fig. 2. The distribution of *u* again:
\n p_2 is applied constant temperature to the boundary.
\nSubstituting the expressions of the considered quantities in
\nthe four constants $R_n (n = 1, 2, 3, 4)$.
\n(h)
\n $\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} = \begin{pmatrix} H_{61} & H_{62} & H_{63} & H_{64} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ -k_1 H_{21} & -k_2 H_{22} & -k_3 H_{23} & -k_4 H_{24} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ p_2 \end{pmatrix}$.
\n(32) Fig. 3. The distribution of *T* again
\nwith dots and dashed with d
\nHence obtain the expressions for the physical quantities of
\nthe behavior of the physical
\nin 2D in the behavior of the physical
\nin 2D in the operator of the physical
\nin 2D in the operator of the physical

the plate surface.

4. Numerical results and discussion

In order to illustrate the obtained theoretical results in the preceding section, following Dhaliwal and Singh in Ref. [22] both types II and III of (G-N) theory during $\Omega = 0.1$ rad / s, the Magnesium crystal-like thermoelastic material with voids $\alpha^* = 0.00051/K$ and $d = 10^{-15}$ fo the Magnesium crystal-like thermoelastic material with voids was chosen for purposes of numerical evaluations. All the units of parameters used in the calculation are given in SI units.

with dots and dashed with dots represented by the behavior of the physical quantities of the photon using the surface.
\n**Numerical results and discussion**
\n**Numerical results and the calculation are given in SI units.**
\n**in** 2D in the context of both types II and III of (G-N) theory during *Q* = 0.1 rad / *s*,
\n**Nagnesium crystal-like thermodastic material with voids**
\n
$$
\alpha^2 = 0.00051/K
$$
 and $d = 10^{-15}$ for *g* = 9.8 *m*/*s*², 0. Figs.
\n**Nagnesium crystals in the calculation are given in SI units.**
\n**in** 10-13 depict the distribution of the physical quantities
\nits of parameters used in the calculation are given in SI units.
\n**in** 10-13 depict the distribution of the physical quantities
\n $\alpha^2 = 0.01 \text{ rad} / s$, $g = 9.8 \text{ m/s}^2$, $0. \text{ Figs.}$
\n**in** 20-1 rad / *s*, $g = 9.8 \text{ m/s}^2$, $0. \text{ Figs.}$
\n**in** 20-1 rad / *s*, $g = 9.8 \text{ m/s}^2$, $0. \text{ Figs.}$
\n**in** 20-1 rad / *s*, $g = 9.8 \text{ m/s}^2$, $0. \text{ Figs.}$
\n**in** 20-1 rad / *s*, $g = 9.8 \text{ m/s}^2$, $0. \text{ Figs.}$
\n**in** 20-1 rad / *s*, $g = 9.8 \text{ m/s}^2$, $0. \text{$

These numerical values used to obtain the distribution of the real part of the displacement u the temperature T , the stress σ_{xx} and the change in the volume fraction field ϕ with the distance y for (G-N) theory of types II and III. In each graph the solid and dashed lines represent the solution in the context of the (G-N) theory of type II and the solid

Fig. 3. The distribution of T against y while $\Omega = 0.1$ rad / s, 0.

Hence obtain the expressions for the physical quantities of (G-N) theory of type III. Figs. 2-5 represent the change in The $\begin{pmatrix} R_0 & H_{12} & H_{63} & H_{64} & H_{65} & H_{64} & H_{66} & H_{67} & H_{68} & H_{69} & H_{60} \ R_4 \end{pmatrix}$ = $k_1H_{21} - k_2H_{22} - k_3H_{23} - k_4H_{24}$ = $k_4H_{41} - H_{42} - H_{43} - H_{44}$ = $H_{41} - H_{42} - H_{44}$ = $H_{41} - H_{42} - H_{43}$ = $H_{41} - H_{42} - H_{44}$ $\begin{vmatrix} R_1 & M_1 & M_2 & M_3 & M_4 \\ R_5 & R_6 & R_7 & M_2 & R_5 \end{vmatrix} = \begin{vmatrix} R_1 & M_1 & M_2 & M_3 & M_4 \\ R_1 & M_2 & R_3 & M_4 & M_4 \end{vmatrix} = \begin{vmatrix} 0 & 0.32 \\ 0 & P_1 & 0.33 \\ 0 & P_2 & 0.3 \end{vmatrix}$ with dots and dashed with dots represent the eolution using white Q= with dots and dashed with dots represent the solution

1.2.4 with dots and dashed with dots represent the solution

the behavior of the physical quantities of

the behavior of the physical quantities against distant

in 2 hoe obtain the expressions for the physical quantities of

the babavior of type III. Figs. 2.5 represent the ch

the baby of the physical quantities signist

in 2D in the context of both types II and III o

unerical resul sions for the physical quantities of (G-N) theory of type III. Figs. 2-5 represent the behavior of the physical quantities aga in 2D in the context of both types II amplies and theoretical results in the physical quantiti plate surface.

the behavior of the physical quantities against distance y

in 2D in the context of both types II and III of (G-N)

In order to illustrate the obtained theoretical results in the

tor $\Omega = 0.1 rad/s$, $0.$ Fig **and discussion**
 $\frac{1}{2}$ where one through the context of bour lybes II and III or (G-N)
 $\frac{1}{2}$ context of the physical quantities against distance y in 2D in the context of the

particular case of the physical qua Fig. 2. The distribution of u against y while $\Omega = 0.1$ rad / s, 0.
 $\begin{bmatrix}\n\frac{601}{16} & \frac{601}{16} & \frac{601}{16} \\
\frac{601}{16} & \frac{601}{16} & \frac{600}{16} \\
\frac{601}{16} & \frac{600}{16} & \frac{600}{16} \\
\frac{601}{16} & \frac{601}{16} & \frac{600}{16} \\
\frac{601}{16} & \$ Fig. 3. The distribution of T against y while $\Omega = 0.1 rad/s$, 0.

Fig. 3. The distribution of T against y while $\Omega = 0.1 rad/s$, 0.

with dots and dashed with dots represent the solution using

in 2D in the context of both types physical quantities against distance *y* in 2D in the context of **Example 12**
 Example 1 Fig. 3. The distribution of *T* against y while $\Omega = 0.1$ *rad* / *s*, 0.

with dots and dashed with dots represent the solution using

(G-N) theory of type III. Figs. 2-5 represent the change in

the behavior of the phys 10-13 depict the distribution of the physical quantities against distance *y* in 2D in the context of both types of (G-N) Fig. 3. The distribution of *T* against y while $\Omega = 0.1 rad/s, 0$.

with dots and dashed with dots represent the solution using

(G-N) theory of type III. Figs. 2-5 represent the change in

the behavior of the physical quant Fig. 3. The distribution of *T* against *y* while $\Omega = 0.1 rad/s$, 0.

with dots and dashed with dots represent the solution using

(G-N) theory of type III. Figs. 2-5 represent the change in

the behavior of the physical qua tribution of the physical quantities against distance *y* in 2D in Fig. 3. The distribution of T against y while $\Omega = 0.1 rad/s, 0$.
with dots and dashed with dots represent the solution using
the behavior of type III. Figs. 2-5 represent the change in
the behavior of the physical quantities Fig. 3. The distribution of *T* against *y* while $\Omega = 0.1 rad/s, 0$.

with dots and dashed with dots represent the solution using

G-N) theory of type III. Figs. 2-5 represent the change in

he behavior of the physical quant wind oos and assued wind oos represent the solution using
(G-N) theory of type III. Figs. 2-5 represent the change in
the behavior of the physical quantities against distance y
in 2D in the context of both types II and II GEN) mory or type III. rigs. 2-3 represent the cnange in
the behavior of the physical quantities against distance y
in 2D in the context of both types II and III of (G-N)
theory during $g = 9.8$ m/s^2 , $\alpha^* = 0.00051/K$ an one one
and or the physical quantities against distance y
in a 2D in the context of both types II and III of $(G-N)$
theory during $g = 9.8 m/s^2$, $\alpha^* = 0.00051/K$ and $d = 10^{-15}$
for $\Omega = 0.1 rad/s$, 0. Figs. 6-9 show the beha *LD* in the context of both types 1 and 10 or G-N)
theory during $g = 9.8$ *m*/s², $\alpha^2 = 0.00051/K$ and $d = 10^{-15}$
for $\Omega = 0.1 rad/s$, 0. Figs. 6-9 show the behavior of the
physical quantities against distance *y* in 2D in mory during $g = 9.8$ *m*/s², $\alpha = 0.000051/K$ and $a = 10$
tor $\Omega = 0.1$ rad/s, 0. Figs. 6-9 show the behavior of the
physical quantities against distance y in 2D in the context of
both types II and III of (G-N) theory du 16 $1 rad / s$, 0. Figs. 6-9 show the behavior of the
physical quantities against distance y in 2D in the context of
both types II and III of (G-N) theory during $\Omega = 0.1 rad / s$,
 $\alpha^* = 0.00051/K$ and $d = 10^{-15}$ for $g = 9.8 m / s^2$

Fig. 2 shows the distribution of the displacement component *u*; it noticed that in the case of (G-N) of type II the

Fig. 3 explains that the distribution of the temperature in creasing with the increase of the rotation for in both types of (G-N) theory as increasing the rotation for $y > 0$.

Fig. 4 depicts that the distribution of the stress σ_{xx} increasing for both types II and III of (G-N) theory with the increasing of the rotation for $y > 0$.

Fig. 5 expresses that the distribution of ϕ increasing for both types II and III of (G-N) theory with the increase of the rotation for $y > 0$. It explained that the rotation has an effec-

tive role in the distribution of all physical quantities of the problem for the both types II and III of (G-N) theory since the distribution of these quantities varying (increasing or decreasing) with the increase of the rotation value while other physical operators (the gravity, the temperature dependent and the two temperature effect) in the problem are present.

Fig. 6 shows that the distribution of the displacement *u* (G-N) theory, but the distribution of u in the case of type III gravity value. problem for the form types if and it of $(C-N)$ theory since the volume ratection field ϕ decreased in $0 \le y \le 2$ then ing) with the increase of the rotation value while other physi-

ing with the increase of the rotatio

type II and increasing for type III of (G-N) theory for $y > 0$ 6 $\le y \le 10$ and 14 $\le y \le 20$ for both types II and III of (Gwith the increase of the gravity value.

Fig. 8 shows that the distribution of the stress σ_{xx} for type II of (G-N) theory is decreasing for $y > 0$. While in the case

increased in 0 2 £ £ *^y* then decreased in 2 7 £ £ *^y* then returns to increasing in 7 20 £ £ *^y* for the case of type II of Contribution of *y* and $\frac{1}{2}$ **y** $\frac{1}{2}$ **y** Five foie in measuremental paramoment of air phase and paramoment the interest of the consequent in the case of the consequent of G-N) theory is decreased in $0 \le y \le 2$ then decreases in the case of type III and introduce Fig. 9 explains that the distribution of the change in the volume fraction field ϕ decreases in the case of type II and increased in 0 2 £ £ *^y* then decreased in 2 10 £ £ *^y* for (G- N) theory of type III with the increase of the gravity value. It observed that the gravity has a great effect on the distribution of all physical quantities in the case of both types II and III of (G-N) theory and the distribution of the physical quantities changing (increasing or decreasing) with the increase of the gravity value while other physical operators (the rotation, the temperature dependent and the two temperature effect) in the problem are available. Fig. 9. The distribution of ϕ against y while $g = 9.8$ m/s^2 , 0.

Fig. 9. The distribution of ϕ against y while $g = 9.8$ m/s^2 , 0.

Fig. 9 explains that the distribution of the change in the volume fraction field Fig. 9. The distribution of ϕ against y while $g = 9.8$ m/s^2 , 0.

Fig. 9 explains that the distribution of the change in the volume fraction field ϕ decreases in the case of type III and increased in $0 \le y \le 2$ the

Fig. 7 determines that the distribution of *T* decreasing for the intervals $0 \le y \le 6$ and $10 \le y \le 14$, but increased in Fig. 10 shows the distribution of the displacement component *u*; it noticed that the distribution of *u* decreased in

> Fig. 11 explains that the distribution of *T* increasing with the increase of the rotation for in both types of (G-N) theory as increasing of α^* value for $y > 0$.

Fig. 12 depicts that the distribution of the stress σ_{rr} in-

creasing for both types II and III of (G-N) theory with the increasing of α^* value for $y > 0$.
Fig. 13 expresses that the distribution of ϕ increasing for

both types II and III of (G-N) theory with the increase of α^* value for $y > 0$. It explained that the temperature dependent properties have a significant role in the distribution of all physical quantities of the problem for the both types II and III of (G-N) theory; as the distribution of the physical quantities

Fig. 16. 3D Curve of σ_{xy} versus the components of distance.

having an alteration (increasing or decreasing) with the increase of the temperature dependent properties while other physical operators (the rotation, the gravity and the two temperature effect) in the problem are attending.

Fig. 14 shows that the distribution of *u* increased in the theory, but the distribution of u in the case of type III de-Fig. 16. 3D Curve of σ_{xy} versus the components of distance.

Fig. 16. 3D Curve of σ_{xy} versus the components of distance.

therease of the temperature dependent properties while other

physical operators (the rotat with the increase of *d*.
Fig. 15 explains that the distribution of the change in the

volume fraction field ϕ increases in the case of types II and III of (G-N) theory with the increase of *d* . It observed that all the curves, continuous and converges to zero, and the two temperature effect has a great effect on the distribution of all physical quantities in the case of both types II and III of (G-N) theory; as the distribution of the physical quantities changing (increasing or decreasing) with the increase of two temperature effect while other physical operators(the rotation, the

gravity and the temperature dependent properties) in the problem exists. 3D curve is representing the complete relation between σ_{xy} against both components of the distance as shown in Fig. 16 where $\Omega = 0.1 \, rad / s$, $\alpha^* = 0.00051 / K$ *M. I. A. Othman and M. I. M. Hilal / Journal of Mechanical Science and Technology 29 (9) (2015) 3739-3746* 3745
gravity and the temperature dependent properties) in the prob-
lence wise representing the complete relation *M. I. A. Othman and M. I. M. Hilal / Journal of Mechanical Science and Technology*

gravity and the temperature dependent properties) in the prob-

lem exists. 3D curve is representing the complete relation [12] M. I. A. fect; under (G-N) theory of type III. This figure is very important to show that the functions are moving in wave propagation.

5. Concluding remarks

(1) The rotation, the gravity and the temperature dependent properties having great role in the distribution of the physical quantities, since these quantities varying with the increase of the physical operators.

(2) The value of all physical quantities converges to zero with an increase in the distance *y* and all functions are continuous.

(3) The deformation of a body depends on the nature of the applied forces as well as the type of boundary conditions.

(4) The two temperature theory has an important effect on many problems in thermoelasticity and on the distribution of the considered physical quantities.

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\n**Appendix**
\n
$$
a_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad a_2 = \frac{b_0 c_1^2}{\omega_1^2 \psi_0 \mu_0}, \quad a_3 = \frac{\beta_0 T_0}{\mu_0},
$$
\n
$$
a_4 = \frac{\rho g c_1^2}{\mu_0 f(T)}, \quad a_5 = \frac{\rho c_1^2}{\mu_0 f(T)}, \quad a_6 = \frac{b_0 \psi_0}{\alpha_0},
$$
\n
$$
a_7 = \frac{\xi_{10} c_1^2}{\alpha_0 \alpha_1^2}, \quad a_8 = \frac{\omega_{10} c_1^2}{\omega_0 \alpha_1^2}, \quad a_9 = \frac{m_0 T_0 \psi_0}{\alpha_0},
$$
\n
$$
a_{10} = \frac{\rho c_1^2 \psi_0}{\alpha_0} \quad a_{11} = \frac{m_0 c_1^2 f(T)}{\rho C_e \psi_0 \alpha_1^3}, \quad a_{12} = \frac{d \omega_1^2}{\rho C_e^2},
$$
\n
$$
a_1 = \frac{\beta_0 f(T)}{\rho C_e}, \quad \varepsilon_2 = \frac{k^* \omega_1^*}{\rho C_e \omega_1^2}, \quad \varepsilon_3 = \frac{k_0 f(T)}{\rho C_e \omega_1^2},
$$
\n<math display="block</p>

3746
\n*M. L. A. Othman and M. L. M. Hild / Journal of Mechanical Science and Technology 29 (9) (2015) 3739-3746
\n
$$
b_2 = a^2 + \frac{a_5(\omega^2 - \Omega^2)}{b_1}, b_3 = \frac{iaa_4 + 2a_5\omega\Omega}{b_1}, b_4 = \frac{a_2}{b_1},
$$
\n**Mohamed I. M. Hild** received his B.Sc.
\n
$$
b_5 = \frac{a_3}{b_1}, b_6 = \frac{a_3a_1}{b_1}, b_7 = iaa_4 + 2a_5\omega\Omega,
$$
\n
$$
b_8 = a^2 - a_5(\omega^2 - \Omega^2), b_9 = a^2 + a_7 + a_8\omega + a_{10}\omega^2,
$$
\n
$$
b_{10} = a_9a_{12}, b_{11} = \varepsilon_3 + \varepsilon_2\omega, b_{12} = a_{12}\omega^2, b_{13} = b_{11} + b_{12},
$$
\n
$$
b_{14} = -\frac{\varepsilon_1\omega^2}{b_{13}}, b_5 = -\frac{a_{11}\omega}{b_{13}}, b_6 = a^2 + \frac{\omega^2}{b_{13}}, D = \frac{d}{b_1},
$$
\n
$$
G_{1n} = (ia - k_n H_{1n}), M_{1n} = -(k_n + ia H_{1n}), n = 1, 2, 3, 4.
$$
\n
$$
H_{5n} = a_{13}(G_{1n} - k_n M_{1n}) + 2ia a_{16}G_{1n} + a_{14}H_{2n} - a_{15}H_{4n},
$$
\n
$$
H_{5n} = a_{16}(-k_n G_{1n} + ia M_{1n})
$$
\n
$$
n = 1, 2, 3, 4.
$$
\n
$$
H_{5n} = a_{16}(-k_n G_{1n} + ia M_{1n})
$$
\n
$$
m = 1, 2, 3, 4.
$$
\n
$$
M_{6n} = a_{16}(-k_n G_{1n} + ia M_{1n})
$$
\n
$$
m = 1, 2, 3, 4.
$$
\n
$$
M_{7n} = a_{15}(\omega -
$$*

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* $+ 2a_5 \omega \Omega$ *,* $b_4 = \frac{a_2}{b_1}$ *,
* $2a_5 \omega \Omega$ *,
* $7 + a_8 \omega + a_{10} \omega^2$ *,
* $a_6 \omega + a_{11} \omega^2$ *,
* $a_7 \omega^2$ *,
* $b_7 \omega^2$ *,
* $c_8 \omega$ *,
* $d_9 \omega$ *,
* d *d* / *Journal of Mechanical Science and Technology 29 (9) (2015) 3739-3746

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 b_{13} , $b_{16} = a^2 + \frac{\omega}{b_{13}}$, $D = \frac{d}{dy}$, stic, thermoelasticity with voids and thermoelasticity with b_{13} . microtemperatures. Have about 9 published papers in the previous fields. Mohamed I. M. Hilal is currently asisstant lecturer of applied mathematics in faculty of scince, Zagazig university, Egypt.