

MHD mixed convection slip flow in a vertical parallel plate microchannel heated at asymmetric and uniform heat flux†

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Abstract

Developing steady laminar flow and mixed convection heat transfer of a Newtonian conducting fluid in an open-ended vertical parallel plate microchannel under the effect of a uniform magnetic field are numerically studied. The effects of the modified mixed convection parameter, $\frac{Gr}{1}$ $\frac{G}{Re}$, the Hartmann number, M, the Knudsen number, Kn, and the heat flux ratio, r_q , on the velocity and temperature profile

are investigated. It is revealed that the velocity profile is strongly influenced by magnetic field. In fact, with an increase in the Hartmann number the velocity decreases for both Kn = 0 and 0.1 and for all mixed convection parameter values. The effect of magnetic force on the velocity profile is stronger, with respect to the temperature profile. In addition, with an increase in M, the slip velocity increases on both hot and cold walls for $r_q = 0$ and $r_q = 1$. It is observed that the friction factor coefficient has significant increases with an increase in the Hartmann number.

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Keywords: MHD; Mixed convection; Slip flow; Microchannel

1. Introduction

In recent decades, there has been given growing importance to micro flow due to its new applications in microfluidic system devices and Micro-electro-mechanical systems (MEMS), such as biodetection, biotechnology, chemical reactors, electronic cooling medical, etc [1-5]. MHD (Magnetohydrodynamic) mixed convection slip flow has many applications such as microelectrochemicall cell transport, micro heat exchanging, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions, and microchip cooling [6, 7]. In the vast majority of these applications, it is necessary to control fluid motion, stir, and separate fluids. In these cases, magnetohydrodynamic (MHD) offers a low-priced, adjustable and flexible means of performing these functions [8, 9]. As reported in Refs. [10, 11], the continuum approach may be valid in microchannel flow depends on the Knudsen number.

It is defined as the ratio of the fluid mean free path to the characteristic length scale of the physical domain. The traditional continuum hypothesis is valid, albeit with modified boundary conditions. As long as $(Kn \le 10^{-3})$, the flow is assumed to be a continuum flow, while for large value of Knudsen (Kn≥10), the flow is called a free-molecular flow. However, the range $(10^{-3} \le Kn \le 10^{-1})$, is the near continuum region and is known as the slip flow regime. In this work, the the slip flow regime is assumed to predict the fluid flow and thermal behavior of the fluid. A large part of the literature focuses on the forced convection gas flow in microchannels and microtubes as given in Refs. [12-14], but a few works has been done on the mixed convection and natural convection in microdevices. Earlier work on mixed convection was performed by Avci and Aydin [15]. They analytically studied fully developed laminar mixed convection in a vertical parallel plate microchannel with isothermal boundary conditions. They also developed their research to a channel with uniform heat flux boundary conditions [16]. The problem of natural convection slip-flow in an open-ended vertical parallel plate channel with asymmetric wall temperature was investigated by Chen and Weng [17]. It was found that the effects of rarefaction and fluid-wall interaction lead to an increase in volume flow rate and to a decrease in heat transfer rate. Ibrahim and Hady investigated MHD mixed convection flow over a horizontal plate [18]. The problem of fully developed mixed convection flow of a micropolor fluid between two vertical parallel plates with asymmetrical temperature at boundary conditions has been studied by Gorla et al. [20]. A numerical study of the fluid flow characteristics and heat transfer behavior in

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the developing region of a microchannel with isothermal boundary conditions at wall was performed by Biswall et al. [21]. Reddapa et al studied MHD fully developed laminar and steady mixed convection slip flow in a vertical parallel plate microchannel with asymmetrical heat flux at the channel walls [22]. Their study was carried out for a constant pressure gradient along the channel. The problem of fully developed MHD mixed convection of a conducting Newtonian fluid in a vertical parallel plate microchannel with asymmetrical wall temperature was studied analytically by Krishna et al. [23]. They shown that with an increase in the Hartman number, the velocity increases near the cold wall, but it is in opposite condition near the hot wall. Recently Niazmand and Rahimi studied mixed convection gaseous slip flow in an open-ended parallel plate microchannel heated at uniform heat flux, but with different heat flux ratios [24]. It is seen that increasing $\frac{Gr}{Re}$, leads to an increase in heat transfer and friction factor.

After this short report of open literature on MHD mixed convection in microchannels, according to the author' s knowledge, it seems the problem of MHD developing mixed convection in a vertical microchannel with uniform and asymmetric heat flux has not yet been investigated. As the first study on this topic, this research focuses on the effect of magnetic field and rarefaction of gas on both the developing and fully developed hydrodynamical and thermal behavior of mixed convection heat transfer.

2. Problem and formulation

The sketch of channel geometry and its coordinates x and y are shown in Fig. 1. The height of the plate, L, is assumed to be ten times larger than its width, b, in order to guarantee that fully developed conditions in the downstream of the channel will occur. The aim of this research is to analyze MHD mixed convection of a gas, with $Pr = 0.71$, in a vertical parallel plate microchannel with walls having uniform, but different heat flux, under the influence of a uniform transverse magnetic field, B_0 . It is assumed that the magnetic field and also the magnetic Reynolds number are very small, and therefore, the induced magnetic field due to motion of the electrically conducting fluid is negligible. Hall effect and joule heating are also negligible. The governing equations of mass, momentum, and energy using the usual busssinesq approximation, ignoring viscose dissipation (it is reasonable because the flow speed and the prandtle number are small) and with temperature independent thermo physical properties, for two dimensional, steady, laminar and incompressible flow are as follows..

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\tag{1}
$$

$$
\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{-\partial p}{\partial x} + \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + \rho g \beta \left(T - T_0 \right) - \sigma B_0^2 u \tag{2}
$$

Fig. 1. Channel geometry.

$$
\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \frac{-\partial p}{\partial y} + \frac{\partial}{\partial x} \left(\mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial y} \right) \tag{3}
$$

$$
\rho c p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) \tag{4}
$$

where μ is the dynamic viscosity, p is the pressure, B_0 is the uniform Magnetic field strength, σ is the electrical conductivity of the fluid, cp is the specific heat, K is the thermal conductivity, T is the temperature and t is the time. Based on gas Kinetic theory, due to the rarefaction effect, slip flow and temperature jump will occur near the channel walls in the slip flow regime .

Applying the first order model , the slip velocity is defined in Ref. [23] :

$$
u_s = \frac{-2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y}\big|_{wall} \,. \tag{5}
$$

The jump temperature at the wall is defined in Ref. [23].

$$
T_{gas} - T_{wall} = -\frac{2 - \sigma_T}{\sigma_T} \cdot \frac{2\gamma}{\gamma + 1} \cdot \frac{\lambda}{\rho r} \frac{\partial T}{\partial y} \big|_{wall} \ . \tag{6}
$$

Here, u_s is the slip velocity, λ is the molecular mean free path, and σ_V and σ_T are the tangential momentum and thermal accommodation coefficients, respectively, which are obtained experimentally. γ is the specific heat ratio, and T_{gas} is the gas temperature. σ_V and σ_T are dependent on the gas type and surface material, but in this study, for simplicity, they are supposed as unity [21]. The governing equations are solved by applying the subsequent boundary conditions:

Inlet:

 $v = 0$, $u = u_{in}$, $T = T_{in}$, where u_{in} and T_{in} are the con-

Table 1. Grid study results.

Grid resolution	Nu
20×20	5.4245
50×50	5.3928
100×60	5.3374
150×70	5.3421
Niazmand and Rahimi [23]	5.3376

stant and uniform velocity and ambient air temperature at the channel inlet

Outlet:

$$
\frac{\partial u}{\partial y} = 0 , \frac{\partial v}{\partial y} = 0 , \frac{\partial T}{\partial y} = 0 .
$$
 (7)

At right wall:

$$
v = 0
$$

$$
u = \frac{2 - \sigma_v}{\sigma_v} D_h K n \frac{\partial u}{\partial y} \Big|_{y=0}, \quad \frac{\partial T}{\partial y} \Big|_{y=0} = -q \Big|_{\mathcal{A}}^v. \tag{8}
$$

At left wall:

 $v = 0$

$$
u = -\frac{2 - \sigma_v}{\sigma_v} D_h K n \frac{\partial u}{\partial y}|_{y=H} , \frac{\partial T}{\partial y}|_{y=H} = \frac{q_2^{\prime \prime}}{K} .
$$
 (9)

3. Numeric model and solution

The governing equations were solved by using a finite volume method. The QUICK scheme is taken into account for convection terms, while for diffusion terms, central differencing is used. Pressure velocity coupling is made with the SIMPLE-C algorithm. To verify grid independency, comprehensive computations have been carried out for various node numbers in terms of average Nusselt number, and friction factor coefficient number for $\frac{Gr}{Re} = 50$, Kn = 0.1, pr = 0.71.

The results of this analysis are presented in Table 1. It is observed that there is a very small difference between the quantities in the grids with 100×60 nodes and 150×70 nodes. Consequently, a rectangular grid system with 100×60 nodes is adopted for computational domain. In order to validate the numerical code, the fully developed velocity and temperature profiles compare with those analytical solution given by Avci and Aydin [16]. As exhibit in Figs. 2(a) and (b) the results are in good accord with the corresponding values of Avci and Aydin.

4. Results and discussions

To demonstrate the results of this study, the following nondimensional variables are used. The dimensionless temperature ($\overline{\theta}$) is selected so that, it will become invariant in the fully developed region as reported in Ref. [23].

Fig. 2. Comparison of the fully developed non-dimension velocity and non-dimension temperature profiles with those of Avci and Aydin for Kn = 0 and 0.06, rq = 1, $\frac{Gr}{Re}$ = 50.

$$
X = \frac{x}{L}, Y = \frac{y}{H}, U = \frac{u}{u_{in}}, V = \frac{vRe}{v_{in}}, Re = \frac{\rho UH}{\mu},
$$

\n
$$
Gr = \frac{\rho^2 g \beta q_1 H^4}{k^2}
$$

\n
$$
\overline{\theta} \text{ or } \theta^* = \theta - \theta_w = \frac{T - T_{s,c}}{q_1 H} - \frac{T_w - T_{s,c}}{q_1 H}, T_m = \frac{\int PUT dA}{\int PU dA},
$$

\n
$$
h = \frac{q}{T_w - T_m}, Nu = \frac{hH}{K}, rq = \frac{q_1}{q_2}, M = \left(\frac{\sigma B_0^2 H^2}{\mu}\right)^{1/2}
$$

\n
$$
Kn = \frac{\lambda}{H}.
$$
\n(10)

4.1 Fully developed velocity profiles

In order to investigate the influence of Hartmann number,

Fig. 3. Fully developed dimensionless velocity profiles for different M, Kn and rq.

M, on dimensionless fully developed velocity profiles, Fig. 3 is plotted, for rq = 0 and rq = 1, at $\frac{Gr}{Re} = 10$, and $\frac{Gr}{Re} = 200$, with $M = 0$, $M = 2$, $M = 5$, for $Kn = 0$ and $kn = 0.1$ respectively. It is noted that slip velocity increases at the channel walls with an increase in the Knudsen number. In addition it is revealed that with increasing mixed convection parameter Gr Re , buoyancy force increases, and hence the velocity gradi-

ent increases near the hot wall. This effect is more significant in larger Knudsen numbers as shown in Figs. 3(g) and (h). In fact, when the mixed convection parameter is low, the forced convection mechanism dominates, so that there is no difference between the velocity profiles for $rq = 0$ and $rq = 1$, but with an increase in $\frac{Gr}{Re}$, buoyancy effect control heat transfer mechanism. Also an increase in M, leads to a decrease in velocity, with an increase in the velocity gradient near the microchannel walls, for both $rq = 0$ and 1 in all mixed convection parameter values.

4.2 Fully developed temperature profiles

Fig. 4 illustrates the effects of Knudsen number, Kn, mixed convection parameter, $\frac{Gr}{Re}$, and Hartmann number, M, for rq

Fig. 4. Fully developed non-dimension temperature for different M, Kn and rq.

 $= 0$ and rq $= 1$, on the non-dimensional fully developed temperature profile. It is observed that an increase in M, leads to an increase in the magnitude of dimensionless temperature for both $rq = 0$ and $rq = 1$, but variations in temperature are greater for $rq = 1$. It is important to note that the temperature profiles are less affected than the velocity profiles by the magnetic field. Also, it is shown that temperature jump increases at the channel walls with an increase in the Knudsen number. In addition in contrast to the velocity profile which is significantly affected by mixed convection parameter, $\frac{Gr}{Re}$, the dimensionless temperature shows insignificant changes with mixed convection parameter.

4.3 Average and local Nusselt number

Fig. 5 shows the variation of local Nusselt number, Nu, for different Hartmann numbers, M, Kundsen number, Kn and rq $= 0$ along the microchannel walls. Increasing the Hartmann number, M leads to an increase in average Nusselt number for both values of heat flux ratio. It is due to the rising at temperature gradient at the channel wall in the presence of magnetic field. Fig. 6 exhibits the influence of mixed convection parameter, $\frac{Gr}{1}$ $\frac{G}{Re}$, on the average Nusselt number at different valuse of Hartman number for $Kn = 0.05$ and $rq = 1$. It can be seen that increasing buoyancy force, slightly increases the heat

Fig. 5. Variation of local Nusselt number for different M, Kn, and rq.

Fig. 6. Variations of the average Nusselt number with mixed convection parameter, $\frac{Gr}{Re}$ for Kn = 0.05 and rq = 1.

transfer coefficient and therefore, the average Nusselt number. The effect of heat flux ratio, rq, on the average Nusselt number is shown in Fig. 6. It is observed that with increasing, rq, the average Nusselt number increases linearly even though magnetic a field exists. The effect of Harman number, M, on the thermal entrance length, $X_{eth} = \frac{x_{eth}}{L}$, is presented in Ta-

Fig. 7. Variations of the average Nusselt number with heat flux ratio for Kn = 0.1 and $\frac{Gr}{Re}$ = 200.

ble 2. It is revealed that the Presence of magnetic field has no important effect on the thermal entrance length. This trend is same for all values of Knudsen number.

4.4 Friction factor coefficient

The local friction factor coefficient at the channel wall as

Table 2. Variations of dimensionless thermal entrance length for different Hartmann and Knudsen number values, $rq = 1$, $Gr / Re = 10$.

Hartman number	$M = 0$	$M = 2$	$M = 5$
$Kn = 0$	0.2624	0.2623	0.2622
$Kn = 0.05$	0.3384	0.3383	0.3395
$Kn = 0.1$	0.3443	0.3444	0.3443

Table 3. Variations of friction factor coefficient for different values of Hartman number and Knudsen number, $rq = 1$, $Gr / Re = 10$.

reported in Ref. [23] is defined as:

$$
\Gamma = \frac{D_h}{u_{in}} \left(\frac{\partial u}{\partial y} \right) wall \tag{11}
$$

Table 3 presents the local friction factor coefficient, Γ , at the inlet section of the channel walls for various values of Hartmann number. It can be seen that with an increase in the Hartmann number, the friction factor increases greatly at the inlet of the channel walls for $Kn = 0$, 0.05 and 0.1. In fact under the effect of the magnetic field the magnetic force leads to an increase in the velocity gradients near the channel walls, and therefore the friction factor increases significantly. This effect is reduced by increasing the Knudsen number. In fact, at higher Kn, slip velocity increases and hence, velocity gradient decreases near the channel walls.

5. Conclusion

Developing steady laminar flow, and mixed convection heat transfer gaseous slip flow in a vertical parallel-plate microchannel, heated by uniform and asymmetric heat flux, under the effect of a transverse uniform magnetic field are investigated numerically. The SIMPEL-C Co-Located scheme is used to solve the governing equations. For validation the numerical method, the results are compared with those of Avci Niazmand and Rahimi and Aydin, and in terms of the Nusselt number, fully developed velocity and temperature distribution. The effects of Hartman number, mixed convection parameter, heat flux ratio and rarefaction of gas are taken into account. The most important findings are as follows:

(1) Presence of the magnetic field has significant effect on the velocity. It causes to an increase on the velocity when Hartmann number increases. Also, at higher mixed convection parameter values, the slip velocity increases on the hot wall and decreases on the cold wall for $rq = 0$. In addition, effect of the magnetic force is more significant on the velocity with increasing $\frac{Gr}{Re}$ and Kn.

(2) Increasing M, leads to an increase on the dimensionless temperature profile. For higher $\frac{Gr}{Re}$ values, and lower heat flux ratio, and greater Kn, magnetic field effect is more significant on the temperature distribution.

(3) It is noted that the Nusselt number variations as a function of heat flux ratio is almost linearly for all Hartmann number values. In addition, there are insignificant variations in the

Nusselt number with an increase in
$$
\frac{Gr}{Re}
$$

(4) There is a great increase in the friction factor coefficient with increasing M.

.

(5) Thermal entrance length has very small variations with an increase in M. This is due to insignificant influence of magnetic field on the temperature profile in the channel entrance region.

: Specific heat at constant pressure, $J/Kg K$

Nomenclature--

 B_0

 \boldsymbol{c}_p

 D_h : Hydraulic diameter, m g : Gravity, m/s^2 Gr : Grashof number, Eq. (10) H : Distance between plates, m K : Thermal conductivity, W/m^2k Kn : Knudsen number, λ/H M : Hartmann number, Eq. (10) Nu : Nusselt number, Eq. (10) p : Pressure, pa

: Magnetic field strength, Tesla

- pr : Prandtl number
- r_q : Heat flux ratio, Eq. (10)
- q : Heat flux, W / mk
- q_1 : Heat flux at right channel wall
- q_1 : Heat flux at left channel wall
- Re : Reynolds number
- Re_m : Magnetic Reynolds number, uH / η

T : Tennerature. K
- : Temperature, K
- T_c : Cold wall temperature
- T_h
 T_m : Hot wall temperature
- : Mean temperature
- T_w : Wall temperature
- u, v : Velocity components in x and y direction, m/s
- U, V : Dimensionless velocities, Eq. (10)
- x, y : Axial and normal coordinates, m
- X, Y : Dimensionless coordinates, Eq. (10)

Greek symbols

- α : Thermal diffusivity, m^2/s
- ^γ : Specific heat ratio
- H : Distance between plates, m
- Γ : Dimensionless local friction factor coefficient
- λ : Molecular mean free path, m
- μ : Dynamic viscosity, pa.s.
- ρ : Density, Kg/m^3
- σ : Electrical conductivity, $1/\Omega m$
- σ_t : Thermal accommodation coefficient
- σ_v : Tangential momentum accommodation coefficient
- v : Kinematic viscosity, m^2/s
- $\overline{\theta}$: Dimensionless temperature, Eq. (10)
- η : Magnetic diffusivity

Subscripts

- s : Slip / jump values
- w : Wall values

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