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On the buckling behavior of piezoelectric nanobeams: An exact solution †

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Abstract

In this paper, thermoelectric-mechanical buckling behavior of the piezoelectric nanobeams is investigated based on the nonlocal theory and Euler-Bernoulli beam theory. The electric potential is assumed linear through the thickness of the nanobeam and the governing equations are derived by Hamilton's principle. The governing equations are solved analytically for different boundary conditions. The effects of the nonlocal parameter, temperature change, and external electric voltage on the critical buckling load of the piezoelectric nanobeams are discussed in detail. This study should be useful for the design of piezoelectric nanodevices.

Keywords: Analytical solution; Critical buckling load; Nonlocal elasticity theory; Piezoelectric nanobeam

1. Introduction

Due to the superior properties of piezoelectric materials, they have very applications in smart structures or systems. They experience mechanical deformations when placed in an electric field, and become electrically polarized under mechanical loads and therefore, make them appropriate for a variety of electromechanical systems used for vibration and noise control [1-3], energy harvesting [4], cooling devices [5], telecommunication, and sensor networks [6]. With the development of the material technology, piezoelectric materials have been employed in micro/nano structures such as, micro/nano electromechanical systems (MEMS/NEMS) [7], nanoresonators [8], chemical sensors [9] and biosensors [10]. Because of the demands in applications and high sensibility of MEMS/NEMS to external excitations, obtaining the mechanical properties of these nanoscale devices has attracted a lot of attention. Due to wide applications of nanobeams in engineering, such as nanowires, nanoprobes, atomic force microscope (AFM), and nanosensors, they have attracted a lot of consideration [11-13]. Hence, many investigations have been carried out about buckling, vibration, noise control and bending of nanobeams [14-18]. However, direct employ of classical continuum theory in nanostructures leads to wrong results in predicting their mechanical behavior because the classical theory cannot capture the size effects. Since the experimental methods at the nanoscale are too time-consuming, so many researchers in nanotechnology use modified continuum models based on the concept of nonlocal elasticity for modeling the nanostructures. Such theories contain information about the forces between atoms, and the internal length scale is introduced into the constitutive equations as a material parameter. Between these theories, Eringen's theory, due to its simplicity and high accuracy, has found wide application in studying the behavior of nanostructures. Studies show that the results obtained from this theory are in good agreement with the results obtained by the method of molecular dynamics [19-22]. Therefore, in many studies about the nanostructures such as nanobeam, nanoplate and nanoshell, researchers use this theory to discuss about the buckling [23], linear and nonlinear vibration [24-26] of the nanostructures. H. Ali-Akbari et al [27] investigated the instability of large diameter single-walled carbon nanotubes (SWCNTs) conveying fluid based on the molecular mechanics. I. Lee and J. Lee [28] reported measurement of random uncertainties in resonant characteristics (resonance frequency and quality factor) of microelectromechanical system (MEMS) resonators by employing different methods to extract resonant characteristics of four different MEMS resonators which are either clamped-free or clamped-clamped beams. H. Moeenfard and M.T. Ahmadian [29] studied the static pull-in and equilibrium behavior in electrostatically actuated torsional micromirrors. Perturbation method, the method of straight forward expansion is utilized to find the pull-in angle of the mirror. Wang and Song [30] converted nanoscale mechanical energy into Electrical energy by means of piezoelectric zinc oxide nanowire (NW) arrays. Wang et al. [31] developed a nanowire nanogenerator that is driven by an ultrasonic wave to produce continuous direct-current output. Su et al. [32] converted mechanical energy into electric energy,

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which has been demonstrated in GaN nanorods. In these applications, the mechanical deformation, induced by the deflection of cantilevered NW with a probe tip or ultrasonic wave, produces an electric response that is sensed through the probe tip. Yan and Jiang [33] analyzed the electromechanical coupling (EMC) behavior of piezoelectric nanowires (NWs) with the surface effect. In addition, they studied the influence of surface effects, including residual surface stress, surface elasticity and surface piezoelectricity on the vibrational and buckling behaviors of piezoelectric nanobeams by using the Euler–Bernoulli beam theory [34]. Ke and Wang [35] investigated the linear and Ke et al. [36] investigated the nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory and Timoshenko beam theory by using the differential quadrature (DQ) method. Because nanobeams are important structures widely used in the piezoelectric nanodevices, the buckling analyses of the piezoelectric nanobeams are of primary importance in the design of the piezoelectric nanodevices such as nanoresonators, field effect transistors and sensors. According to the best of authors' knowledge, there is no exact solution for the buckling analyses of the piezoelectric nanobeams based on the nonlocal theory. Therefore, in this article we have tried to provide an exact solution for the buckling analyses of piezoelectric nanobeams for various boundary conditions.

2. Nonlocal theory for piezoelectric materials

The layout of a problem is shown in Fig. 1. Consider a straight uniform nanobeam with the length L and rectangular cross-section with width b, and thickness h. Based on the theory of nonlocal piezoelectricity, the stress tensor and the electric displacement at a reference point depend not only on the strain components and electric-field components at same position but also on all other points of the body [20]. The nonlocal constitutive behavior for the piezoelectric material can be given as follows:

$$\sigma_{ij} = \int_V \alpha\left(\left|x'-x\right|,\tau\right)\left[C_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x') - \lambda_{ij}\Delta T\right]dx' \quad (1)$$

$$D_i = \int_V \alpha\left(\left|x'-x\right|,\tau\right)\left[e_{ikl}\varepsilon_{kl}(x') + \Xi_{ik}E_k(x') + p_i\Delta T\right]dx' \quad (2)$$

$$\sigma_{ij,j} = \rho\ddot{u}_i,\ D_{i,i} = 0, \quad (3)$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ E_i = -\Phi_{,i} \quad (4)$$

where σ_{ij}, ε_{ij}, D_i, E_i and u_i are the stress, strain, electric displacement, electric field and displacement components, respectively; C_{ijkl}, e_{kij}, Ξ_{ik}, λ_{ij}, p_i and ρ are the fourth-order elasticity tensor, piezoelectric constants, dielectric constants, thermal moduli, pyroelectric constants and mass density, respectively; ΔT is the temperature change and Φ is the electric potential. The nonlocal attenuation function $\alpha(|x'-x|,\tau)$ represents nonlocal modulus. $|x_0 - x|$ being the distance (in Euclidean norm). $\tau = e_0a/l$ is defined as the scale coefficient that

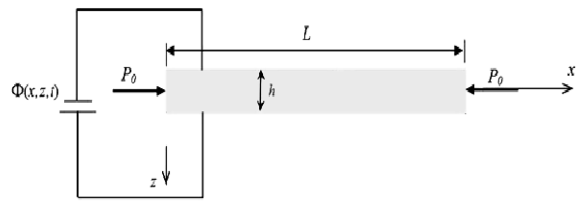

Fig. 1. Piezoelectric nanobeam under thermo-electro-mechanical loadings.

incorporates the small scale factor, where e_0 is a material constant determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics. a and l are the internal (e.g. lattice parameter, granular size) and external characteristic lengths (e.g. crack length, wavelength) of the nanostructures, respectively. According to Eringen [20, 21], the constitutive Eqs. (1) and (2) can be converted to the equivalent differential constitutive equations form as

$$\sigma_{ij} - (e_0a)^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T, \quad (5)$$

$$D_i - (e_0a)^2\nabla^2 D_{ii} = e_{ikl}\varepsilon_{kl} + \Xi_{ik}E_k + p_i\Delta T, \quad (6)$$

where ∇^2 is the Laplace operator.

3. The governing equations

Based on the Euler-Bernoulli beam theory (EBT), the axial displacement, u, and the transverse displacement of any point of the beam, w, are expressed as:

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w(x,t)}{\partial x} \quad (7)$$

$$w(x,z,t) = w_0(x,t) \quad (8)$$

where u_0 and w_0 are the displacement of the midplane of the beam at the point $(x,0)$ (i.e., $z = 0$) and t is the time. The axial strain ε_{xx} of Euler–Bernoulli beam theory is expressed as:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}. \quad (9)$$

In the analysis of both macro scale and nanoscale piezoelectric beams, the electric variables must also satisfy the Maxwell's equations. In this article, we use Wang [37] electric potential model to describe the electric potential distribution across the thickness of piezoelectric nanobeam. The electric potential at any point of the piezoelectric nanobeam is assumed as

$$\Phi(x,z,t) = -\cos(\beta z)\phi(x,t) + \frac{2zV_0}{h}e^{i\Omega t} \quad (10)$$

where $\beta = \pi/h$, z is measured from the mid-plane of the nanobeam in the transverse direction; $\phi(x,t)$ is the electric potential in the x direction; V_0 is the external electric voltage and Ω is the natural frequency of the piezoelectric nanobeam. According to Eqs. (4) and (10), the electric fields is given as

$$E_x = -\frac{\partial \phi}{\partial x} = \cos(\beta z)\frac{\partial \phi}{\partial x}, \tag{11}$$

$$E_z = -\frac{\partial \phi}{\partial z} = -\beta \sin(\beta z)\phi - \frac{2V_0}{h}\mathrm{e}^{i\Omega t}. \tag{12}$$

The strain energy of the Euler-Bernoulli piezoelectric nanobeam is

$$\Pi_s = \frac{1}{2}\int_0^L \int_{-h/2}^{h/2} \left(\sigma_{xx}\varepsilon_{xx} - D_x E_x - D_z E_z\right) dz dx. \tag{13}$$

The kinetic energy Π_k is expressed as

$$\Pi_k = \frac{1}{2}\int_0^L \left[m_0 \left(\frac{\partial w_0}{\partial t}\right)^2 \right] dx \tag{14}$$

where $m_0=\rho h$. It is assumed that width b is unit. The work done by the external force is

$$\Pi_E = \frac{1}{2}\int_0^L \left[N_p \left(\frac{\partial w_0}{\partial x}\right)^2 \right] dx \tag{15}$$

where N_P is the normal force induced by the axial force P_0.

For a beam type structure, both thickness and width are much smaller than its length. Therefore, for beams with the transverse motion in the x–z plane, the nonlocal constitutive relations Eqs. (5) and (6) can be approximated to one-dimensional form as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = c_{11}\varepsilon_{xx} - e_{31}E_z - \lambda_1 \Delta T, \tag{16}$$

$$D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = \Xi_{11} E_x, \tag{17}$$

$$D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31}\varepsilon_{xx} + \Xi_{33} E_z + p_1 \Delta T. \tag{18}$$

The governing equations and the boundary conditions will be derived by using Hamilton's principal. Now Considering the Hamilton's principle

$$\int_0^t (\delta \Pi_s + \delta \Pi_k - \delta \Pi_E) dt = 0. \tag{19}$$

The governing equations of piezoelectric nanobeams based on Euler–Bernoulli beam theory can be rewritten as

$$\frac{\partial N_x}{\partial x} = m_0 \left(1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u_0}{\partial t^2} \tag{20}$$

$$F_{31}\frac{\partial^2 w_0}{\partial x^2} - X_{11}\frac{\partial^2 \phi}{\partial x^2} - X_{33}\phi = 0 \tag{21}$$

$$\begin{aligned} & D\frac{\partial^4 w_0}{\partial x^4} + F_{31}\frac{\partial^2 \phi}{\partial x^2} + \\ & (N_p + N_T + N_E)\frac{\partial^2}{\partial x^2}\left(w_0 - (e_0 a)^2 \frac{\partial^2 w_0}{\partial x^2} \right) = \\ & m_0 \left(1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 w_0}{\partial t^2} \end{aligned} \tag{22}$$

where N_T and N_E are the normal force induced by the temperature change ΔT and external electric voltage V_0, which may be written as

$$N_T = -\lambda_1 h \Delta T \text{ and } N_E = 2e_{31}V_0. \tag{23}$$

and

$$\begin{aligned} & D = \int_{-h/2}^{h/2} -c_{11} z^2 dz, F_{31} = \int_{-h/2}^{h/2} e_{31}\beta \sin(\beta z) z dz, \\ & X_{11} = \int_{-h/2}^{h/2} \Xi_{11}\cos^2(\beta z) dz, X_{33} = \int_{-h/2}^{h/2} \Xi_{33}\left[\beta \sin(\beta z)\right]^2 dz. \end{aligned} \tag{24}$$

Note that for the homogeneous piezoelectric nanobeam Eq. (20) is decoupled with Eqs. (21) and (22). Therefore, in the following analysis, we only consider the governing Eqs. (21) and (22) to describe the buckling behavior of the piezoelectric nanobeams [38, 39].

In the present study, the exact solutions for the buckling of piezoelectric nanobeams for different boundary conditions are obtained. It is assumed that the electric potential is zero at ends of the nanobeam. Setting $\partial / \partial t = 0$ and solving Eqs. (21) and (22) for ϕ gives

$$\begin{aligned} \phi = & \frac{X_{11}D}{X_{33}F_{31}}\frac{\partial^4 w}{\partial x^4} + \frac{F_{31}}{X_{33}}\frac{\partial^2 w_0}{\partial x^2} \\ & + \frac{(N_p + N_E + N_T)X_{11}}{X_{33}F_{31}}\frac{\partial^2}{\partial x^2}\left(w_0 - \mu \frac{\partial^2 w_0}{\partial x^2}\right). \end{aligned} \tag{25}$$

Applying the operator $\partial^2/\partial x^2$ to the Eq. (22) and substitute obtained equation into Eq. (21) gives a decoupled fourth-order ordinary differential equation as follow:

$$\frac{d^4 W}{dx^4} + \eta \frac{d^2 W}{dx^2} + NW = 0 \tag{26}$$

where $\eta = \dfrac{D + F_{31}^2 / X_{33} + NX_{11} / F_{31}X_{33} - \mu N}{X_{11}D / X_{33} - \mu N X_{11} / F_{31}X_{33}}$.

The general solution of Eq. (26) can be written as

$$W(x) = C_1 \sinh(\lambda_1 x) + C_2 \cosh(\lambda_1 x) + C_3 \sinh(\lambda_2 x) + C_4 \cosh(\lambda_2 x) \tag{27}$$

where $\lambda_1 = \left(\frac{-\eta + \sqrt{\eta^2 - 4N}}{2} \right)^{1/2}, \lambda_2 = \left(\frac{-\eta - \sqrt{\eta^2 - 4N}}{2} \right)^{1/2}$

and C_k (k = 1,...4) are four unknown constants to be determined from four boundary conditions at each ends of nanobeam, $x = 0, L$.

By imposing the four boundary conditions at $x = 0$ and L a system of four homogeneous linear algebraic equations is obtained. Setting the determinant of the coefficient matrix equal to zero, we obtain an algebraic equation for the determination of critical buckling loads of the piezoelectric nanobeam.

4. Numerical results

In this section, we will present the analytical study on the thermo-electro-mechanical buckling and vibration of piezoelectric nanobeam developed in the previous sections. Assume that the nanobeam is made of PZT-4 with the material properties listed in Table 1 [35, 36]. The effects of the dimensionless nonlocal parameter μ, temperature change ΔT, external electric voltage V_0 and slender ratio L/h on the critical buckling load of the piezoelectric nanobeams are discussed in detail. The following parameters are used in computing the numerical values: L =10000 (nm) and h = 500 (nm). Since there are no theoretical, experimental and molecular dynamic results for the critical buckling load of the nonlocal piezoelectric nanobeams in the existing literature to compare our model against it, therefore, if we neglect the piezoelectric effect, the present model can be directly reduced to the nonlocal elastic nanobeam model. Reddy [16] has been analytically investigating the critical buckling load of an elastic nanobeam based on the nonlocal theory in detail. Table 2 gives the Nondimensional critical buckling load ($\bar{N} = -NL^2 / C_{11} I$) of an S-S nonlocal Euler-/Bernoulli nanobeam. The parameters used in this example are [16]: L = 10, E = 30×10^6, ρ = 1, ν = 0.3. The obtained results show that our results are in good agreement with the analytical results obtained by Reddy [16]. The Nondimensional buckling loads for clamped-clamped (C-C), simply supported-simply supported (S-S), clamped-simply supported (C-S), and clamped-free (C-F) boundary conditions are tabulated in Table 3. Based on these results, it can be seen that the critical buckling loads related to all boundary conditions are reduced with growing the value of nonlocal parameter, which demonstrates this fact that with incorporating the nanoscale size-effects, the stiffness of nanobeam decreases. Also, it is observed that for lower L/h, nonlocality is more important and this effect is lost for higher L/h. Table 4 shows the effect of the external electric voltage V_0 on the Nondimensional critical buckling load of the piezoelectric nanobeam with ΔT = 0ºC and different values of nonlocal parameter. It is seen that the critical buckling loads of the piezoelectric nanobeam are not sensitive to the external electric voltage change. With the increase of the external voltage, the critical buckling loads of the piezoelectric nanobeam increase slightly, which illustrates a larger change in the external voltage needs to cause more reduction in the stiffness of nanobeams and hence leads to lower critical buckling load of the piezoelectric nanobeams. It is seen the positive voltage increases the critical buckling load of piezoelectric nanobeams, whereas the negative voltage has the opposite effect. That is because axial tensile and compressive forces are generated in the nanobeam by the applied positive and negative voltages, respectively. Fig. 3 shows the effect of slenderness ratio on Nondimensional critical buckling loads of

Table 1. Material properties of PZT4 [35, 36].

c_{11} (GPa)	e_{31} (Cm^{-2})	Ξ_{11} (CV^{-1} m^{-1})	Ξ_{33} (CV^{-1} m^{-1})	λ_1 (Nm^{-2} K^{-1})	p_1 (Cm^{-2} K^{-1})	ρ (kg m^{-3})
132	-4.1	5.841×10^{-9}	7.124×10^{-9}	4.738×10^5	2.5×10^{-5}	7500

Table 2. Nondimensional critical buckling loads of the simply supported nonlocal elastic nanobeam.

L/h	μ	Reddy [16]	Present
20	0	9.8696	9.8696
	1	8.9830	8.9830
	2	8.2426	8.2426
	3	7.6149	7.6149
	4	7.0761	7.0761
100	0	9.8696	9.8696
	1	8.9830	8.9830
	2	8.2426	8.2426
	3	7.6149	7.6149
	4	7.0761	7.0761

Table 3. Nondimensional critical buckling loads of the piezoelectric nanobeam for different boundary conditions.

L/h	μ	Boundary conditions			
		C-C	S-S	C-S	C-F
100	0	38.3453	9.6957	19.8107	2.4639
	1	29.1639	8.8247	18.0311	2.2426
	2	23.3872	8.0974	16.5450	2.0578
	3	19.8932	7.4808	15.2851	1.9011
	4	17.4905	6.9514	14.2034	1.7665
20	0	38.7815	9.6954	19.8101	2.4638
	1	29.7529	8.8244	18.0305	2.2425
	2	23.4102	8.0971	16.5444	2.0577
	3	22.9099	7.4805	15.2845	1.9010
	4	19.3462	6.9512	14.2030	1.7665

Table 4. The effect of external electric voltage on the Nondimensional critical buckling loads of the piezoelectric nanobeam ($L/h = 20$ and $\Delta T = 0$).

V_0 (v)	Nonlocal parameter μ (*nm*)				
	0	1	2	3	4
-15	9.6953	8.8244	8.0971	7.4805	6.9512
-10	9.6953	8.8244	8.0971	7.4805	6.9512
-5	9.6953	8.8244	8.0971	7.4805	6.9512
0	9.6954	8.8244	8.0971	7.4805	6.9512
5	9.6954	8.8244	8.0971	7.4805	6.9512
10	9.6954	8.8244	8.0971	7.4805	6.9512
15	9.6954	8.8245	8.0971	7.4805	6.9513

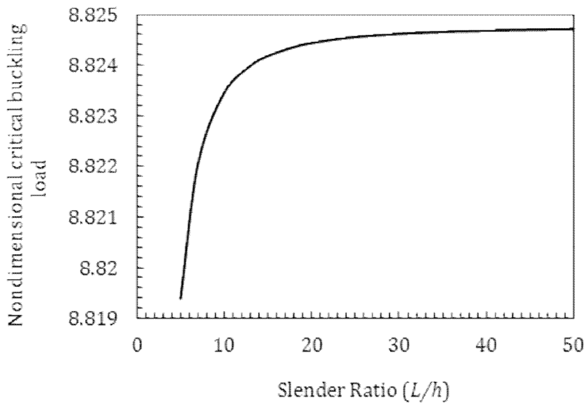

Fig. 2. Effect of slenderness ratio on Nondimensional critical buckling loads of piezoelectric nanobeams for $\mu = 1$, $\Delta T = 0$.

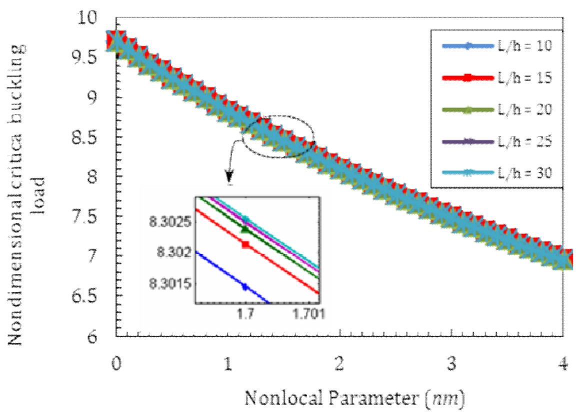

Fig. 3. Effect of slenderness ratio on Nondimensional critical buckling loads of piezoelectric nanobeams for $\mu = 1$, $\Delta T = 0$.

piezoelectric nanobeams for Euler beam model. The difference between the results obtained from the Euler beam model are quite small when $L/h > 20$ but relatively large when $L/h < 20$ and for $L/h > 20$ Nondimensional critical buckling load is stabilized. However, these differences between the results are very little so that it can be neglected. It means that EBT neglects the shear deformation effects and overestimates the

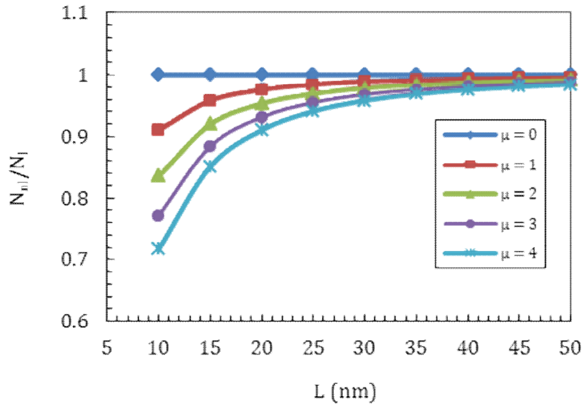

Fig. 4. Variation of the critical buckling load ratio with piezoelectric nanobeams length for different nonlocal parameters, $\Delta T = 0$ and $V = 0$.

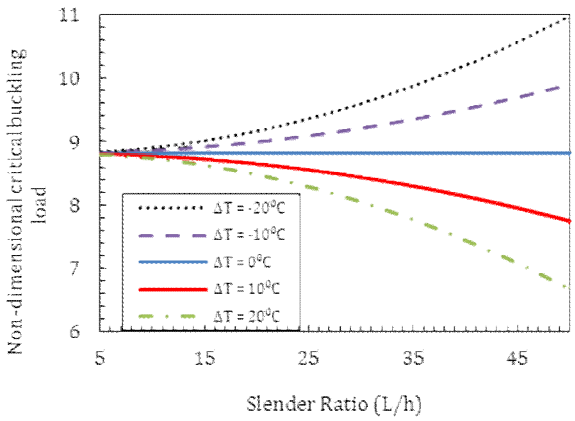

Fig. 5. Effect of slenderness ratio on Nondimensional critical buckling loads of piezoelectric nanobeams for different values of ΔT and $\mu = 1$.

critical buckling load of deep nanobeams. To illustrate the effect of small scale on the buckling load of piezoelectric nanobeam, Nondimensional buckling load versus the variation of nonlocal parameter at different slender ratios are plotted in Fig. 4. It can be observed that for all the slender ratios when the nonlocal parameter increases, the Nondimensional buckling load decreases. Also, it can be seen that there are no differences between the curves for all the slender ratios i.e. EBT cannot consider the shear deformation effects for deep nanobeams.

Nondimensional buckling load versus the variation of nonlocal parameter at different slender ratios are plotted in Fig. 4. It can be observed that for all the slender ratios when the nonlocal parameter increases, the Nondimensional buckling load decreases. In addition, it can be seen that there are no differences between the curves for all the slender ratios i.e. EBT cannot consider the shear deformation effects for deep nanobeams.

In the following discussions, we consider a parameter which defines the relation between local and nonlocal theory in critical buckling load as follow:

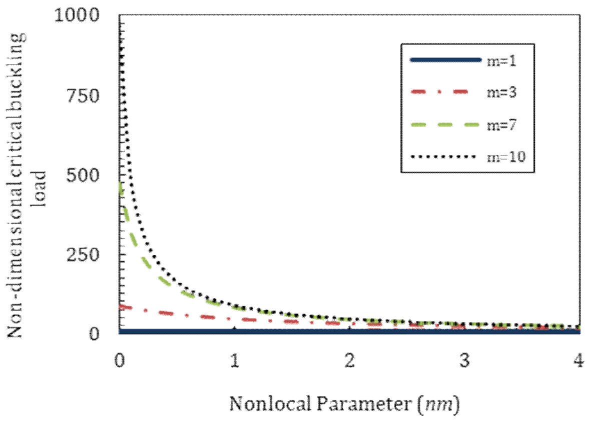

Fig. 6. Effect of nonlocal parameter on the Nondimensional buckling load of piezoelectric nanobeam for different mode number (L/h = 20).

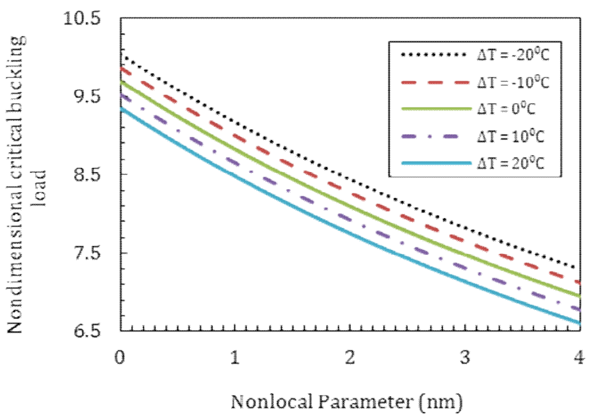

Fig. 7. Effect of nonlocal parameter on the Nondimensional buckling load for different values of ΔT.

$$\frac{N \text{ Calculated by nonlocal theory}}{N \text{ Calculated by local theory}} = \frac{N_{nl}}{N_l}. \tag{28}$$

Critical buckling load ratio versus length of nanobeam, for different nonlocal parameters, μ, has been plotted in the Fig. 5. This figure illustrates that the increase in length of the nanobeam decreases the nonlocal effects and the nonlocal curve converges with the local theory results and this reduction is more significant for short nanobeams. Further, it can be observed that the critical buckling load increases as length of the nanobeam increases. Fig. 6 presents the effect of the temperature change ΔT on the Nondimensional critical buckling loads of the piezoelectric nanobeams with V_0 = 0.0 V and μ = 1. With the increase of the temperature change, the critical buckling loads decrease and the critical buckling loads increase by decreasing the temperature change, for all slender ratios. The reason is that a positive temperature change brings in more reduction in the nanobeam stiffness and hence leads to lower critical buckling loads of the piezoelectric nanobeams and vice versa. It is seen that the temperature change effects are more pronounced for greater slenderness ratio. The effect of mode number of the piezoelectric nanobeam on the variation of Nondimensional buckling load versus nonlocal parameter is illustrated in Fig. 7. As can be seen, Nondimensional buckling load decreases with increasing the nonlocal parameter μ. Also, the small scale effect on the Nondimensional buckling load is more prominent at higher modes. Clearly, the differences between the Nondimensional buckling loads of the piezoelectric nanobeam for different mode numbers are decreased with increasing the nonlocal parameter and the curves converge with the results of mode 1. Fig. 8 shows the effect of the nonlocal parameter on the Nondimensional buckling loads of the piezoelectric nanobeam with V_0 = 0 and for various ΔT. It is observed that the positive temperature change decreases the Nondimensional buckling loads of piezoelectric nanobeams, while the negative temperature change has the opposite effect. Furthermore, it can be seen that all the curves have been decreased with same the slope.

5. Conclusion

In this study, thermoelectric-mechanical buckling behavior of the piezoelectric nanobeams based on the nonlocal and Euler-Bernoulli beam theory is investigated. The Hamilton's principles is used to obtain the governing equations and are solved analytically to obtain the critical buckling loads of piezoelectric nanobeams for various boundary conditions that are clamped-clamped, simply supported-simply supported, clamped-simply supported, and clamped-free. The following conclusions can be derived from the presented study:

- Critical buckling loads decreased as the nonlocal parameter increased for all boundary conditions.
- With the increase of the temperature change, the critical buckling loads decrease and the critical buckling loads increase by decreasing of the temperature change, for all nonlocal parameter.
- It is seen that the positive voltage increases the critical buckling load of piezoelectric nanobeams, whereas the negative voltage has the opposite effect.

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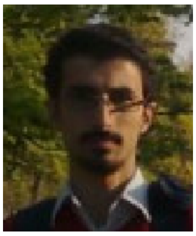

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