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# Identification of pseudo-natural frequencies in a beam-moving mass system with periodic passages [†]

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**Abstract**

The response of a linear time-invariant (LTI) system to harmonic input generates a harmonic output with constant frequency but varying magnitude and phase. Many structural dynamic systems have been modeled as linear time-varying periodic (LTP) systems. Previous studies have reported that the response of an LTP system to an exponential input establishes an infinite number of frequencies. These studies have presented a new, exponentially modulated periodic signal space and a corresponding harmonic transfer function as useful tools in the operational modal analysis of LTP systems. In consideration of this new approach, this study mainly identifies the frequencies of a typical LTP system, such as a beam that is subject to the intermittent passage of moving masses. Upon obtaining the harmonic transfer function for the beam-moving mass system, conventional frequency domain methods for LTI systems are used to derive the frequency characteristics of the LTP system from the system response. These methods include the peak-picking method. As expected in an LTP system, an infinite number of pseudo-natural frequencies resonate in the beam-moving mass system.

*Keywords*: Beam-moving mass system; Harmonic transfer function; Linear time-periodic systems; Modulated periodic signal space; Operational modal analysis

## 1. Introduction

According to Ewins [1], a primary assumption for the applicability of experimental modal analysis is that the structure must remain time invariant, that is, the parameters to be determined should be constant. Identification techniques for linear time-invariant (LTI) systems have been developed by many researchers, including Andersen and Brincker [2], Fu and Hua [3], Brincker et al. [4], and Peeters and Ventura [5]. However, identification and modeling techniques for systems with time-varying parameters remain under investigation and development. The time-varying characteristics of an in-operation system correspond to valuable information for machine monitoring and diagnosis. Thus, Mathelin and Lozanob [6] and Bonato et al. [7] explored the benefits of determining the time-varying behavior of in-operation systems. These researchers established an effective modeling technique to extract the exact specifications of these systems.

Solving problems under ambient excitations is inherently difficult when traditional modal analysis methods are used; however, these methods can be regarded as a base for generating techniques to identify the traits of systems with time-varying characteristics. Numerous techniques have been proposed for the identification of linear time-varying (LTV) systems in either time or time-frequency domain, including those developed by Liu [8], Bellizzi et al. [9], Xu et al. [10], and Zhang et al. [11]. Liu and Kujath [12] adopted a new method that considers the time history of a linear time varying-periodic (LTP) system in time domain to determine some of the modal parameters of systems. These researchers employed a subspace-based algorithm that uses a multitude of force responses to identify the successive discrete transition matrices of LTP systems. This method is among of the few developed for LTP systems in time domain. Allen [13] presented a frequency-domain approach for intuitively characterizing an LTP system according to the methodology, as well as algorithms based on LTI systems. In fact, this method develops pre- and post-processing techniques to modify measured data and to extract pseudo frequency using conventional LTI methods. Han et al. [14] theoretically analyzed the natural frequencies of a spur—gear—pair system as a time-variant system on the basis of Floquet theory. The influences of the periodically time-varying parameters of mesh stiffness, including sideband frequency and contact ratio, on natural system frequencies in the stable and unstable regions were illustrated in detail. In 2010, Allen et al. [15] applied Wereley's [16] signal space definition to determine the modal parameters of a wind turbine

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blade. This work utilized the concept of harmonic transfer function to develop a system identification method for LTP systems that relies only on output. The proposed method can be considered an extension of the operational modal analysis (OMA) approach to LTP systems.

Antonacci et al. [17] investigated vibration-based parametric identification techniques in the field of structural control and health monitoring for a three-dimensional structure in a beam-moving mass system. These researchers compared several LTI identification methods. Bellino et al. [18] conducted an experimental dynamic analysis of nonlinear beams under moving loads and applied the results to an actual problem. In addition, Dyniewicz et al. [19], Ariaei et al. [20], and Bilello et al. [21] studied realistic problems related to beam—moving mass systems. Different approaches have been established based on the availability of a mathematical model to investigate this system.

Beam-moving mass systems are time-varying systems that can be modeled as LTP systems on the basis of certain assumptions. To the best of the authors' knowledge, no specific research on identifying the modal parameters of beam-moving mass systems, or LTV/LTP systems, can be extracted from the few reports on modal analysis and on the derivation of modal parameters of time-varying systems

In the current paper, a modal analysis of the LTP systems presented in Refs. [15, 16] is presented in Sec. 2. The pseudo-natural frequencies of a beam with intermittent moving mass passage are then identified using the aforementioned approach, as described in Sec. 3. Owing to the time-varying nature of beam-moving mass systems, mode shapes cannot be defined in LTP systems unlike in LTI systems. Therefore, a method is established for the OMA of a beam-moving mass system to obtain pseudo-natural frequencies. The development of this method is the main objective and contribution of the present study. Finally, the paper is concluded in Sec. 4.

## 2. Modal analysis of a linear time-periodic system

In this section, the basic features that characterize LTP systems are first explained. Then, the reason why traditional modal analysis methods cannot be used in the modal analysis of a LTP system is rationalized. Next, a suitable method for identifying the pseudo-natural frequencies of LTP systems is introduced. As explained in the following section, the method is implemented on a beam-moving mass system as an example of the capability of LTP systems to evaluate the performance of the proposed algorithm.

### *2.1 Response of linear time-periodic systems to harmonic inputs*

Dynamic equations for a LTP system in the state space model are normally given by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \\ y(t) &= C(t)x(t) + D(t)u(t), \end{aligned} \tag{1}$$

where all of the coefficient matrices are periodic, that is,

$$\begin{aligned} A(t+T) = A(t),\ B(t+T) = B(t), \\ C(t+T) = C(t),\ D(t+t) = D(t). \end{aligned} \tag{2}$$

$\omega_p = 2\pi / T$ denotes the system frequency.

In the absence of $D(t)$, the response of the system to a general input $u(t)$, is given by

$$\begin{aligned} x(t) &= \Phi(t,0)x_0 + \int_0^t \Phi(t,\tau)B(\tau)u(\tau)d\tau, \\ y(t) &= C(t)\Phi(t,0)x_0 + \int_0^t C(t)\Phi(t,\tau)B(\tau)u(\tau)d\tau. \end{aligned} \tag{3}$$

The first part is considered the homogeneous response, whereas the second is regarded as force response. The transition matrix is denoted by $\Phi(t,\tau)$ in Eq. (3).

Given a single harmonic input $u(t) = u_0 e^{j\omega_0 t}$, $x(t)$ can be written as [16]:

$$\begin{aligned} x(t) &= \Phi(t,0)\{x_0 - \sum_{m=-\infty}^{\infty} (s_m I - Q)^{-1} \bar{B}_m u_0\} \\ &\quad + P(t) \sum_{m=-\infty}^{\infty} (s_m I - Q)^{-1} \bar{B}_m u_0 e^{s_m t}. \end{aligned} \tag{4}$$

where $\bar{B}_m$ and $P(t)$ are derived from the Fourier expansion of $B(t)$; $s_m = \omega_0 + jm\omega_p$; and $Q$ is a constant matrix defined in Ref. [16]. The first term in total response vanishes at $t \to \infty$ if the system is strictly stable. The steady-state response is then given by

$$x_{ss}(t) = P(t) \sum_{m=-\infty}^{\infty} (s_m I - Q)^{-1} \bar{B}_m u_0 e^{s_m t}. \tag{5}$$

Although the test signal considered is a single harmonic signal, a steady-state response generally generates an infinite number of harmonics. These harmonics differ from one another in multiples of the fundamental frequency $\omega_n$ (pumping frequency) [14]. The steady-state output response is given by

$$y_{ss}(t) = C(t)x_{ss} = C(t)P(t) \sum_{m=-\infty}^{\infty} \left[ (s_m I - Q)^{-1} \bar{B}_m u_0 \right] e^{s_m t}. \tag{6}$$

Note that in general, $y(t+T) \neq y(t)$.

A direct method is difficult to determine on the basis of this equation. The traditional definition of transfer function cannot simply be extended to LTP systems. Therefore, a different test input signal, namely, the exponentially modulated periodic (EMP) signal, has been defined to render an LTP system analogous to an LTI system in terms of frequency response.

**Definition**: An EMP signal is expressed as a complex Fourier series of an LTP system with main frequency $\omega_p$ modulated by the exponential signal $e^{j\omega_0 t}$:

$$u(t) = e^{j\omega_0 t}\sum_{n=-\infty}^{\infty} u_n e^{jn\omega_p t} = \sum_{n=-\infty}^{\infty} u_n e^{s_n t}; \quad \text{with} \quad s_n = j(\omega_0 + n\omega_p). \tag{7}$$

This signal structure is used to define the compatible transfer function for LTP systems.

### *2.2 Linear time-periodic system transfer function*

In this section, a modal transition function is introduced through Eqs. (6) and (7). This function can be converted to a modal form.

Once Eq. (6) has been expressed as its Fourier series expansion and some algebraic operations have been performed, the output in the frequency domain can be expressed as follows

$$\begin{aligned} & y(\omega) = G(\omega)u(\omega) \\ & \text{where } G(\omega) = \sum_{r=1}^{N}\sum_{n=-\infty}^{\infty}\frac{1}{j\omega - s_n}\bar{C}_{r,n}\bar{B}_{r,n} \\ & \text{and } \quad \bar{C}_{r,n} = \left[\cdots\bar{C}^T_{r,n-1}\bar{C}^T_{r,n}\bar{C}^T_{r,n+1}\cdots\right], \\ & \qquad \bar{B}_{r,n} = \left[\cdots\bar{B}^T_{r,n-1}\bar{B}^T_{r,n}\bar{B}^T_{r,n+1}\cdots\right]. \end{aligned} \tag{8}$$

Definitions and details of the variables are given in Ref. [15].

Eq. (8) takes exactly the same mathematical form as the expression for the frequency response function (FRF) matrix of an LTI system. Thus, the same traditional algorithms can be applied to identify the frequency parameters of an LTP system.

### *2.3 Operational modal analysis of an ltp system*

In practice, input is unknown in the industry and engineers should determine the modal parameters of a system from the output signal alone. Furthermore, OMA methods based on the power spectral density (PSD) of signals are used.

Given a system with transfer function $G(\omega)$, the PSD of the output signal is defined as follows:

$$S_{yy}(\omega) = \frac{1}{M}\sum_{k=1}^{M} y_k(\omega)y_k(\omega)^H. \tag{9}$$

This variable can also be calculated from the PSD of the input signal with

$$S_{yy}(\omega) = G(\omega)S_{uu}(\omega)G(\omega)^H \tag{10}$$

where $(\ )^H$ denotes the Hermitian or complex conjugate transpose.

When $G(\omega)$ in Eq. (10) is replaced with its modal representation in Eq. (8), the following formula is obtained

$$\begin{aligned} & S_{yy}(\omega) = \sum_{r=1}^{N}\sum_{n=-\infty}^{\infty}\sum_{l=1}^{N}\sum_{k=-\infty}^{\infty}\frac{\bar{C}_{r,n}W(\omega)_{r,n,l,k}\bar{C}^H_{l,k}}{(j\omega - s_r)(j\omega - s_l)} \\ & \text{where } W(\omega)_{r,n,l,k} = \bar{B}_{r,n}S_{uu}(\omega)\bar{B}^H_{l,k}. \end{aligned} \tag{11}$$

This finding shows that the auto spectrum of the output can be approximated by the sum of modal contributions if $W(\omega)_{r,n,l,k}$ is reasonably flat. For LTI systems, input spectrum $S_{uu}(\omega)$ should be flat. This flatness is also required in LTP systems, and the condition may be more stringent in this case.

Terms with minimal denominators dominate the summation Eq. (14). If the sidebands of mode $r$ do not overlap with those of mode $l$, then this minimum is attained when $r = l$ and $n = k$. Consequently $S_{yy}(\omega)$ is expressed as

$$S_{yy}(\omega) = \sum_{r=1}^{N}\sum_{n=-\infty}^{\infty}\frac{\bar{C}_{r,n}W(\omega)_{r,n,r,n}\bar{C}^H_{r,n}}{[j\omega - (s_r)]^2}. \tag{12}$$

Thus, the auto spectrum can be approximated in the vicinity of dominant mode shapes by considering one contribution at a time, as in LTI cases.

The FRF matrix for LTI systems takes a well-known form that consists of peaks near the natural frequencies of each mode. A similar relationship can be derived for LTP systems. Therefore, the proposed output-only system identification method can be summarized briefly as follows:

(1) Response $y(t)$ of an LTP system is recorded to a broadband input.

(2) Output vector $y(\omega)$ is generated by applying the transform $y(\omega)=FFT[y(t)]$.

(3) The auto spectrum of $y(\omega)$ is computed by averaging multiple blocks of time history, as expressed in Eq. (9).

(4) LTI output-only identification method is employed to identify the frequency characteristics from the auto spectrum. A simple, applicable identification method for this purpose is the peak-picking method.

To reiterate, a relevant mode shape concept does not exist in LTP systems owing to the time-varying nature of these systems. Therefore, pseudo-natural and sideband frequencies can be identified through the procedure described above.

## 3. Pseudo-natural frequency identification of a beam-moving mass system

Preventive maintenance may be implemented and the structural safety of bridges guaranteed using health-monitoring techniques. Such techniques can generate valuable information for the detailed inspection, repair, and rehabilitation of bridges. Current structural identification methods based on ambient vibration data are essential solutions for the online monitoring of such structures. In many cases, a bridge can be represented by a beam-moving mass. Thus, the process of identifying the pseudo-natural frequencies of an LTV system given periodic passages of mass is explained in this section.

A simple model for the system is shown in Fig. 1.

The equation for beam motion was developed by Ghorbani

Table 1. Numerical values for simulation of system.

| $m$ (kg/m) | $L$ (m) | $M$ (kg) | $EI$ (N*m$^2$) | $V$ (m/s) |
|---|---|---|---|---|
| 20 | 30 | 250 | 60 × 106 | 5 |

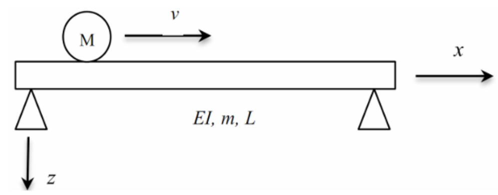


Fig. 1. Schematic of a beam with moving mass.

and Keshmiri [21] in consideration of all of the inertia effects of moving mass and the discretization of the transverse motion of the beam with beam eigenfunctions.

$$\tilde{M}(t)\ddot{X}+\tilde{C}(t)\dot{X}+\tilde{K}(t)X=\tilde{f}(t)\,, \tag{13}$$

where

$$\begin{aligned}&\tilde{M}_{ij}(t)=\delta_{ij}+\frac{M}{m}\phi_i(vt)\phi_j(vt),\ \tilde{C}_{ij}(t)=\frac{2vM}{m}\phi_i(vt)\phi_j'(vt),\\&K_{ij}(t)=\omega_i^2\delta_{ij}+\frac{Mv^2}{m}\phi_i(vt)\phi_j''(vt),\ f_i(t)=\frac{Mg}{m}\phi_i(vt).\end{aligned} \tag{14}$$

$\phi_i(x)$ is the normalized $i$-th mode shape of the beam, and $\int_0^L \phi_i\phi_j dx = \delta_{ij}$.

Assuming the existence of a periodic passage for the moving mass, the system becomes an LTP system; thus,

$$\begin{aligned}&\tilde{M}(t)=\tilde{M}(t+T),\ \ \tilde{C}(t)=\tilde{C}(t+T),\\&\tilde{K}(t)=\tilde{K}(t+T),\ \ \tilde{f}(t)=\tilde{f}(t+T),\end{aligned} \tag{15}$$

where $T=L/v$.

The following values are applied as system parameters for simulation purposes.

The system is numerically simulated, and system response $X(t)$ is applied in the analysis.

### 3.1 Single-DOF system

First, the beam is simulated as a single degree of freedom (DOF) system. Fig. 2 shows the PSD plot of the beam displacement response obtained using Eq. (9) in the absence of any moving mass. The beam is excited by a white signal, and beam response is measured. The first natural frequency of the beam is approximately 3 Hz.

Fig. 3 depicts the PSD plot for the displacement of the same system (1-DOF system) in the presence of a periodic moving mass with period $L/v$. The mass motion introduces a new natural frequency called pseudo-natural frequency at roughly 2.6 Hz. This frequency is in fact the deviated value of the first natural frequency of the beam (3.0 Hz) and introduces side-

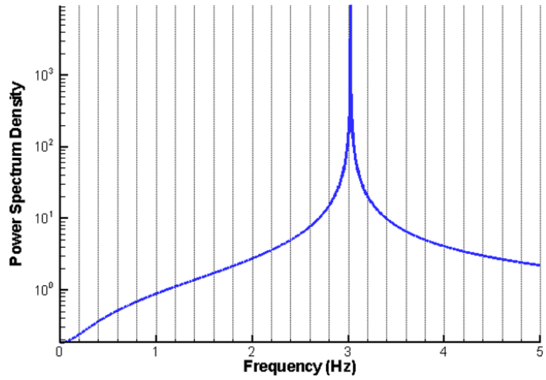


Fig. 2. PSD plot of a simply supported beam modeled by its first eigenfunction.

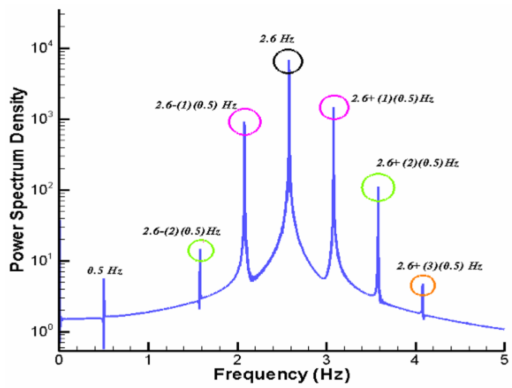


Fig. 3. PSD density of the 1-DOF beam-moving mass system.

band frequencies into the PSD plot of beam response. These sideband frequencies distinguish the concepts of modal analysis for LTI and LTP systems.

Two sets of pseudo-natural frequencies can be identified using the peak-picking method displayed in Fig. 3. The first set is recognized to fit the relation

$$s_n=\omega_1\pm n\omega_p\,,\quad n=0,1,....\,, \tag{16}$$

where $\omega_1=2.6\,\text{Hz}$ is the modified first natural frequency and $\omega_p=2\pi v/L$. The second set is the frequency of the forcing function $\tilde{f}(t)$ and its integer multiples, that is,

$$\tilde{s}_n=n\omega_p\,,\quad n=1,2,.... \tag{17}$$

The first group is in fact the contribution of the unforced vibration of the system, whereas the second group corresponds to the harmonically forced vibration of the LTP system. The amplitudes of the second group of frequencies drop significantly over time, as illustrated in Fig. 3. The peaks at 1.6, 2.1, 2.6, 3.1, 3.6, and 4.1 Hz belong to the first group, whereas that at 0.5 Hz belongs to the second group. The other frequencies of the groups are not clearly visible in the figure because of their low energy or amplitude.

Table 2. Modified and sideband frequencies.

| Number of frequency | Modified main frequency (Hz) | Sideband frequency (Hz) |
|---|---|---|
| First frequency | 2.6 | 4.1 |
| | | 3.6 |
| | | 3.1 |
| | | 2.1 |
| | | 1.6 |
| Second frequency | 10.7 | 12.7 |
| | | 12.2 |
| | | 11.7 |
| | | 11.2 |
| | | 10.2 |
| | | 9.7 |
| | | 9.2 |

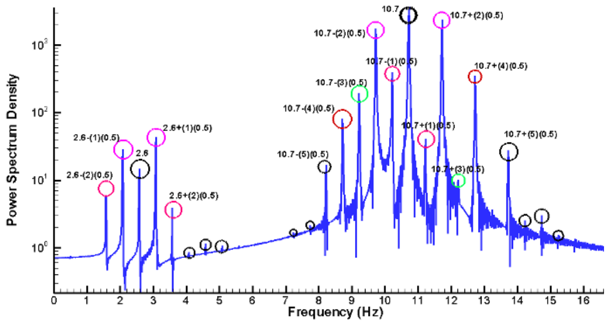


Fig. 4. PSD of the two-DOF beam-moving mass system.

### *3.2 Two-DOF system*

Fig. 4 shows a similar PSD plot for the displacement response of the beam-moving mass LTP system simulated by two eigenfunctions. Once the second DOF of the beam response has been measured and the peak-picking method applied, the peaks at 2.6 and 10.7 Hz are recognized as the first and second natural frequencies of the beam that were modified by time-varying effect. The other peaks presented in the figure are the pumping (Sideband) frequencies, and the values are listed in Table 2.

### *3.3 Five-DOF system*

Fig. 5 shows the beam displacement response with five DOFs along with the peaks and sideband frequencies. When peak-picking method is used, 23.6, 41.6, and 68.0 Hz are identified as the third, fourth, and fifth pseudo-natural frequencies. The visible sideband frequencies of these main frequencies are near the main frequencies.

Nonetheless, the differentiation of main frequencies from sideband frequencies in the PSD plot is also a concern for the authors, as is determining the original (unmodified) natural frequencies of the beam in the above analysis. These issues are currently being investigated and are expected to be highlighted in future works.

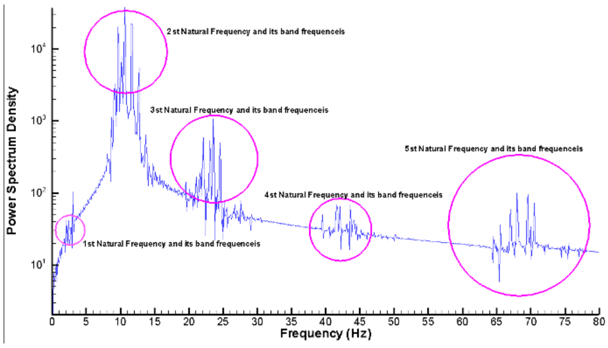


Fig. 5. PSD of the five-DOF beam-moving mass LTP system.

## 4. Conclusions

Pseudo-natural frequencies were identified in a beam-moving mass system constructed as an LTP system in this study. Following a literature review and an introduction of previous works, the mathematical foundation for LTP systems and their response to a simple harmonic input was described briefly. Unlike the response of an LTI system, the response of an LTP system to a simple harmonic input generates an infinite number of frequencies. Thus, the traditional transfer function cannot be employed in an LTP system. Nonetheless, the transfer function can be modified for such systems according to the EMP signal space defined by Werely and Allen. Given this signal space, harmonic transfer function, and the nature of system responses, the pseudo-natural frequencies of a beam-moving mass system were identified for the first time using an OMA procedure that considers the PSD of beam motion. The beam-moving mass system was modeled by single-, two-, and five-DOF systems. In conclusion, natural beam frequencies decrease due to moving mass motion. Furthermore, moving mass introduces a set of band frequencies for each modified or reduced natural frequency. Aside from the reduced natural frequencies and their respective sideband frequencies, another peak was observed in the PSD of the system response. This new peak was related to forcing frequency. In future works, original natural frequencies should be distinguished from modified ones.

## Nomenclature

$\omega_p$ : Pumping frequency
$\Phi(t,\tau)$ : Transition matrix
$\omega_0$ : Input frequency
$s_m$ : Band frequencies
$G(\omega)$ : Transfer function
$S_{yy}(\omega)$ : Power spectral density
$\phi_i(x)$ : Normalized mode shape
$T$ : Moving mass passage period
$\omega_1$ : Time-affected natural frequency
$\tilde{s}_n$ : Forcing frequency and its integer multiples

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