

Numerical simulation of droplet evaporation between two circular plates†

Hangjin Ban and Gihun Son^{*}

Department of Mechanical Engineering, Sogang University, 35 Baekboem-ro, Mapo-gu, Seoul, 121-742, Korea

(Manuscript Received December 15, 2014; Revised February 11, 2015; Accepted February 17, 2015) --

Abstract

Numerical simulation is performed for droplet evaporation between two circular plates. The flow and thermal characteristics of the droplet evaporation are numerically investigated by solving the conservation equations of mass, momentum, energy and mass fraction in the liquid and gas phases. The liquid-gas interface is tracked by a sharp-interface level-set method which is modified to include the effects of evaporation at the liquid-gas interface and contact angle hysteresis at the liquid-gas-solid contact line. An analytical model to predict the droplet evaporation is also developed by simplifying the mass and vapor fraction equations in the gas phase. The numerical results demonstrate that the 1-D analytical prediction is not applicable to the high rate evaporation process. The effects of plate gap and receding contact angle on the droplet evaporation are also quantified.

,我们就会在这里的时候,我们就会在这里的时候,我们就会在这里的时候,我们就会在这里的时候,我们就会在这里的时候,我们就会在这里的时候,我们就会在这里的时候,我们

Keywords: Contact angle hysteresis; Droplet evaporation; Level-set method; Mass transfer

1. Introduction

Droplet evaporation on a solid surface has received significant attention as an efficient fabrication process for various microstructures [1-4]. Despite extensive experimental and theoretical studies of evaporation process [4-6], its general predictive model has not yet been developed due to the complexity of the interfacial flow including heat transfer, mass transfer, phase change and contact line dynamics.

Recently, as a more simplified and easily controllable process, the evaporation of a liquid droplet in the confined geometry between two circular transparent plates was investigated by Clément and Leng [7]. They obtained 2-dimensional (or 1 dimensional axisymmetric in the ideal case) images of the temporal evolution of an evaporating droplet in confined geometry and compared theoretical prediction based on the 1 dimensional diffusion equation of vapor number density with measured droplet contact area. Subsequently, the confined geometry was employed by Leng [8] for the evaporation of a hard-sphere colloidal suspension. Although the evaporation process is expected to be well defined and easily controllable in the confined geometry, it is not clear that the characteristics of flow, heat and mass transfer are negligible across the film thickness normal to the plates considering no-slip and contact angle condition on the plates.

Numerical simulations for general analysis of droplet

E-mail address: gihun@sogang.ac.kr

evaporation were performed in several studies using a finite element method (FEM) [10, 11] and a body-fitted movinggrid method [12]. The Lagrangian methods generally are not easy to handle breaking of the interface. Such difficulties can be overcome with Eulerian interface tracking methods such as the volume-of-fluid (VOF) method and a level-set (LS) method.

Briones et al. [13] computed droplet impact and evaporation using the explicit VOF method in the commercial CFD code FLUENT. However, the coupled interface conditions for the vapor concentration, temperature and evaporation flux are not easy to implement in the VOF method.

Tangui et al. [14] developed the LS method for computation of droplet evaporation combining with the ghost fluid approach to accurately implement the interface conditions. Subsequently, Son [15, 16] developed the LS method for droplet impact and evaporation on a solid surface by employing an iterative calculation procedure for the coupled interface conditions for the temperature and vapor fraction.

In this study, the LS method is further developed for simulations of droplet evaporation between two circular plates and validation test of its theoretical prediction. The effects of plate gap and receding contact angle on the droplet evaporation are investigated.

2. Numerical analysis

The present numerical approach is based on the sharpinterface LS formulation developed by Son [15, 16] for droplet evaporation on a solid surface. The LS method is extended

^{*}Corresponding author. Tel.: +82 2 705 8641, Fax.: +82 2 712 0799

[†] Recommended by Associate Editor Hyoung-gwon Choi

[©] KSME & Springer 2015

Fig. 1. Configuration for analysis of liquid droplet evaporation between two circular plates.

for droplet evaporation between two circular plates, as depicted in Fig. 1. The liquid-gas interface is tracked by the LS function ϕ , which is defined as a signed distance from the interface. The negative sign is chosen for the gas phase and the positive sign for the liquid phase. We assume here that the gas is an ideal mixture of air and vapor, while the liquid is a pure substance and the interface temperature is below the boiling temperature. Fig. 1. The liquid-gas interface is tracked by the LS and gas phases with a source term for the ϕ , which is defined as a signed distance from the porization is re-formulated as the energ ϕ , which is defined as a sig Fig. 1. The liquid-gas interface is tracked by the LS and gas phases w
 ϕ , which is defined as a signed distance from the porization is re-form the regainties ign is chosen for the gas phase and phase with the set ign Here the solution *v* is eventual as a sign using using the meaning production ψ , when is eventual as the energy equation is the state of the gap base with the state transition is meaning equation. The interface is i acce. The negative sign is chosen for the gas phase and phase with the saturation temperature at the sign for the liquid phase. We assume here that the Dirichlet boundary condition. The interface sign for the liquid is a

2.1 Governing equation

The conservation equations of mass, momentum, energy, and the vapor mass fraction *Y* for the liquid-gas region are expressed as follows [15, 16]:

$$
\nabla \cdot \mathbf{u} = v_{lg} \dot{m} \mathbf{n} \cdot \nabla H \tag{1}
$$

We use a
substate and the interface temperature is below the other
changing the numerical results.
\nThe conservation equations of mass, momentum, energy,
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\nwhere the interface velocity **U** is expressed as follows [15, 16]:
\n
$$
\nabla \cdot \mathbf{u} = v_{1g} \sin \mathbf{n} \cdot \nabla H
$$
\n
$$
\frac{\partial (\mathbf{u} \cdot \mathbf{u}_f \cdot \nabla \mathbf{u}_f) = -[\nabla p + (\sigma \mathbf{x} - v_{1g} \dot{m}^2) \nabla H] \quad \text{where the interface velocity U is expressed\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\n
$$
\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0
$$
\nwhere the interface velocity **U** is expressed
\n
$$
\rho_f c_f \frac{\partial T_f}{\partial t} + \mathbf{u}_f \cdot \nabla T_f) = \nabla \cdot k_f \nabla T_f
$$
\nif $\phi \neq 0$;\n
$$
T_f = T_{int} \text{ if } \phi = 0
$$
\n
$$
\rho_g \frac{\partial \phi}{\partial t} + \mathbf{u}_g \cdot \nabla Y = \nabla \cdot \rho_g \frac{\partial \nabla Y}{\partial t} \text{ if } \phi < 0
$$
\n
$$
Y = Y_{int}
$$
\nif $\phi = 0$ \n
$$
\frac{\partial \phi}{\partial \tau} = S(\phi)(1 - |\nabla \phi|)
$$
\n
$$
\rho_g \frac{\partial Y}{\partial t} + \mathbf{u}_g \cdot \nabla Y = \nabla \cdot \rho_g \frac{\partial \nabla Y}{\partial t} \text{ if } \phi < 0
$$
\n
$$
Y = Y
$$
$$

$$
\rho_f c_f \left(\frac{\partial T_f}{\partial t} + \mathbf{u}_f \cdot \nabla T_f \right) = \nabla \cdot k_f \nabla T_f \quad \text{if } \phi \neq 0; \tag{3}
$$
\n
$$
T_f = T_{int} \quad \text{if } \phi = 0 \qquad \qquad \frac{\partial \phi}{\partial t} = S(\phi)(1 - \phi) \quad \text{if } \phi = 0; \tag{3}
$$

+
$$
\nabla \cdot \hat{\mu}(\nabla \mathbf{u} - v_{1g} \sin \nabla H + (\nabla \mathbf{u}_f)^T
$$
] (2) $\sin \mathbf{n} / \rho_f$. The LS function is reintializ
\ntion from the interface by obtaining a s
\nthe equation.
\n $\rho_f c_f(\frac{\partial T_f}{\partial t} + \mathbf{u}_f \cdot \nabla T_f) = \nabla \cdot k_f \nabla T_f$ if $\phi \neq 0$;
\n $T_f = T_{int}$ if $\phi = 0$
\n $\rho_g(\frac{\partial Y}{\partial t} + \mathbf{u}_g \cdot \nabla Y) = \nabla \cdot \rho_g \hat{D} \nabla Y$ if $\phi < 0$;
\n $Y = Y_{int}$ if $\phi = 0$
\n $\mathbf{H} = 1$ if $\phi > 0$; $H = 0$ if $\phi \leq 0$
\n $\mathbf{H} = 1$ if $\phi > 0$; $H = 0$ if $\phi \leq 0$
\n $\mathbf{H} = \nabla \phi / |\nabla \phi|$; $\kappa = \nabla \cdot n$
\n $\mathbf{u}_f = \mathbf{u} + v_{fg} \sin(1 - H)$; $\mathbf{u}_g = \mathbf{u} + v_{fg} \sin(1 - H)$ (6) Hineimabolic boundary condition during the
\n $\hat{\rho} = \rho_g (1 - F) + \rho_f F$; $\hat{\mu}^{-1} = \mu_g^{-1} (1 - F) + \mu_f^{-1} F$ (8)
\n $\hat{\mu}^{-1} = \mathbf{h}_g^{-1} (1 - F) + \mu_f^{-1} F$ (9) 2.3 Boundary conditions
\n $\hat{\mu}^{-1} = \frac{M_v \rho_{v,sat}(T_{int})}{M_{tot}} = \frac{M_v \rho_{v,sat}(T_{int})}{M_v \rho_{v,sat}(T_{int}) + M_a [\rho_{\infty} - \rho_{v,sat}(T_{int})]$ (11) At the bottom and top walls ($y = 0$, y)
\n $\dot{m} = \mathbf{n} \cdot \rho_g D_g$

where

$$
H = 1 \quad \text{if} \quad \phi > 0; \quad H = 0 \quad \text{if} \quad \phi \le 0 \tag{5}
$$

$$
\mathbf{n} = \nabla \phi / |\nabla \phi|; \qquad \kappa = \nabla \cdot n \tag{6}
$$

$$
\mathbf{u}_l = \mathbf{u} + v_{lg} \dot{m} \mathbf{n} (1 - H); \qquad \mathbf{u}_g = \mathbf{u} + v_{lg} \dot{m} \mathbf{n} H
$$
 (

$$
\hat{\rho} = \rho_g (1 - F) + \rho_l F; \qquad \hat{\mu}^{-1} = \mu_g^{-1} (1 - F) + \mu_l^{-1} F \qquad (1 - F) = \mu_l^{-1} F
$$

$$
\hat{k}^{-1} = k_g^{-1}(1 - F) + k_l^{-1}F\tag{9}
$$

$$
\hat{D}^{-1} = D_g^{-1}(1 - F) \tag{10}
$$

$$
Y_{int} = \frac{M_{\nu} p_{\nu, sat}(T_{int})}{M_{\nu} p_{\nu, sat}(T_{int}) + M_a [p_{\infty} - p_{\nu, sat}(T_{int})]}
$$
(11)

here
\n
$$
H = 1 \text{ if } \phi > 0; \quad H = 0 \text{ if } \phi \le 0
$$
\n
$$
\mathbf{u}_l = \mathbf{u} + v_{lg} \dot{m} \mathbf{n} (1 - H); \quad \mathbf{u}_g = \mathbf{u} + v_{lg} \dot{m} \mathbf{n} H \qquad (5) \quad \text{Simples that a}
$$
\n
$$
\hat{\rho} = \rho_g (1 - F) + \rho_l F; \quad \hat{\mu}^{-1} = \mu_g^{-1} (1 - F) + \mu_l^{-1} F \qquad (8)
$$
\n
$$
\hat{k}^{-1} = k_g^{-1} (1 - F) + k_l^{-1} F \qquad (9) \qquad 2.3 \text{ Boundary } c
$$
\n
$$
D^{-1} = D_g^{-1} (1 - F) \qquad (10) \qquad \text{The boundary } c
$$
\n
$$
Y_{int} = \frac{M_v p_{v, sat}(T_{int})}{M_v p_{v, sat}(T_{int}) + M_a [p_{\infty} - p_{v, sat}(T_{int})]} \qquad (11) \qquad \text{At the bottom}
$$
\n
$$
\dot{m} = \mathbf{n} \cdot \rho_g D_g \nabla Y / (1 - Y_{int}) = \frac{\mathbf{n} \cdot (k_l \nabla T_l - k_g \nabla T_g)}{h_{lg}}. \qquad (12) \qquad \mathbf{u} = 0, \quad T =
$$
\nHere, the subscript *f* denotes the liquid phase (*l*) for $\phi > 0$ At the symm

l given the mathemation for analysis of liquid droplet evaporation be multaneously solved using a Newton-Raphes

in for their stable calculation [15]. Note

droplet evaporation between two circular plates, as de-

tion ernota as a signed distance from the potrzainon is re-ionitation as the energy equation

sign is chosen for the gas phase and phase with the staturation temperature at the interface temperature is below the boil-

liquid **Example 12**
 Example 12 as a signed distance from the portzation is re-formulated as the ener-

in chosen for the gas phase and phase with the saturation temperature

phase. We assume here that the Dirichlet boundary condition. The inter-

etemp is denoted as a signed disk
and is denoted in the portzation is tellocated as the elergy endicative sign is chosen for the gas phase and phase with the stutration temperature at the liquid phase. We assume here that the D regative sign is chosen for the gas phase and

phase with the saturation temperature at the interface

in for the liquid phase. We assume here that the Dirichlet boundary condition. The interface temperature is

a obtaine and the gas phase (*g*) for $\phi \le 0$, and *H* is the discontinuous step function. The velocity $\mathbf{u}_f(\mathbf{u}_f)$ or \mathbf{u}_g) for each phase is extrapolated into the entire domain (or a narrow band near the interface) by using the velocity ψ and *H* is the discontinuous
step function. The velocity $\mathbf{u}_f(\mathbf{u}_l \text{ or } \mathbf{u}_g)$ for each phase is ex-
trapolated into the entire domain (or a narrow band near the
interface) by *le and Technology 29 (6) (2015) 2401-2407*

and the gas phase (g) for $\phi \le 0$, and *H* is the discontinuous

step function. The velocity $\mathbf{u}_f(\mathbf{u}_i \text{ or } \mathbf{u}_g)$ for each phase is ex-

rapolated into the entire dom v_{lg} *m***n** $\cdot \nabla H$. The effective properties, ($\hat{\rho}$, $\hat{\mu}$, and *D*), are evaluated from a fraction function *F*, which is described in Ref. [17]. In Eqs. (11) and (12), Y_{int} , T_{int} , and \dot{m} are simultaneously solved using a Newton-Raphson iterative algorithm for their stable calculation [15]. Note that the formulation for energy conservation, given by Eq. (3), is different from that in Refs. [15, 16]. The energy equation for the liquid and gas phases with a source term for the latent heat of vaporization is re-formulated as the energy equation for each phase with the saturation temperature at the interface as a Dirichlet boundary condition. The interface temperature is obtained from Eq. (12) with the latent heat of vaporization. This formulation is much more efficient to implement without changing the numerical results. Fig. in Leg., T_m i, $r_{\rm int}$, $r_{\rm int}$, and $m_{\rm th}$ and the current transform iterative algo-
Itaneously solved using a Newton-Raphson iterative algo-
m for their stable calculation [15]. Note that the formula-
1 for e tion for energy conservation, given by Eq. (3), is different
from that in Refs. [15, 16]. The energy equation for the liquid
and gas phases with a source term for the latent heat of va-
porization is re-formulated as the

2.2 Level-set equations

In the LS formulation, the interface is described as $\phi = 0$.
The zero level set of ϕ is advanced as

$$
\frac{\partial \phi}{\partial t} + \mathbf{U} \cdot \nabla \phi = 0 \tag{13}
$$

The main of the Higuid is a obtained from Eq. (12) with the latent head
acc temperature is below the boil-
This formulation is much more efficient to in
changing the numerical results.
2.2 Level-set equations
ns of mass ideal mixture of air and vapor, while the liquid is a

deal mixture of air and vapor, while the liquid is a

obtained from Eq. (12) with the latent heat of vapor

marce and the interface temperature is below the boil-

Th **EXECUTE AN INSTERTANT CONSULTER (SURVER)**
 EXECUTE AN INSTERTANT CONSULTS:
 EXECUTE AN INSTERTANT CONSULTS:
 EXECUTE AN INSTERTANT CONSULTS:
 Y For the liquid-gas region are The zero level set of ϕ is advanced 2.2 Level-set equations

tion *Y* for the liquid-gas region are
 $y = \frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$
 $y + (\sigma x - v_{1g} \dot{m}^2) \nabla H$
 $\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$
 $\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$
 $\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$
 $\frac{\partial \phi}{\partial t} + U$ Solution is the matrice end of the metric of expectations

one operator of the liquid-gas region are
 $\frac{\partial \phi}{\partial t$ **Example 2.2 Level-set equations**

example the interface is described to vapor mass fraction Y for the liquid-gas region are
 $\therefore v_{ig} \sin \cdot \nabla H$
 $= v_{ig} \sin \cdot \nabla H$
 $= v_{ig} \sin \cdot \nabla H$
 $+ \mathbf{u}_f \cdot \nabla \mathbf{u}_f = -(\nabla p + (\alpha x - v_{ig} \$ (1) $\frac{\partial u}{\partial t} + v \cdot \nabla \psi = 0$

(a) $\frac{\partial v}{\partial t} + u_y \cdot \nabla u_y = -[\nabla p + (\sigma x - v_{yz} \dot{m})\nabla H + (\nabla u_y)^T]$
 $+ \nabla \cdot \hat{\mu}[\nabla u - v_{yz} \dot{m} \nabla H + (\nabla u_y)^T]$
 $\frac{\partial v}{\partial t} + v_y \cdot \hat{H}[\nabla u - v_y \dot{m} \nabla H + (\nabla u_y)^T]$
 $\frac{\partial v}{\partial t} + v_z \cdot \nabla T = \nabla \$ $\hat{\rho}_1 \frac{\partial Y}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f \cdot \mathbf{l} = 0$
 $\hat{\rho}_2 \frac{\partial Y}{\partial t} + \mathbf{u}_f \cdot \nabla T_f \cdot \hat{H} \nabla H + (\nabla \mathbf{u}_f \cdot \nabla T_f) = \nabla \cdot k_f \nabla T_f$ **if** $\phi = 0$ **ii** $\frac{\partial \phi}{\partial \tau_0} = S(\phi)(1 - |\nabla \phi|)$ **ii** $\frac{\partial \phi}{\partial \tau_0} = S(\phi)(1 - |\nabla \phi|)$ \dot{m} *n* / ρ_f . The LS function is reinitialized to a distance function from the interface by obtaining a steady-state solution of the equation. ined from Eq. (12) with the latent heat of vaporization.

formulation is much more efficient to implement without

ging the numerical results.
 Level-set equations

the LS formulation, the interface is described as $\phi =$ om Eq. (12) with the latent heat of vaporization.

lation is much more efficient to implement without

e numerical results.
 et equations

formulation, the interface is described as $\phi = 0$.

level set of ϕ is advanc ained from Eq. (12) with the latent heat of vaporization.

s formulation is much more efficient to implement without

neging the numerical results.

Level-set equations

n the LS formulation, the interface is described as Example 11 is expressed as $U = u +$

ELS function is reinitialized to a distance func-

interface by obtaining a steady-state solution of
 $(1-|\nabla \phi|)$

if $|\phi| \le h/2$

otherwise. (15)

(15)

(id spacing and the formulation Level-set equations
 $Level-set equations$
 $Level-set equations$

In the LS formulation, the interface is described as $\phi = 0$.

The zero level set of ϕ is advanced as
 $\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$ (13)

ere the interface velocity U is expressed as Level-set equations

In the LS formulation, the interface is described as $\phi = 0$.

The zero level set of ϕ is advanced as
 $\frac{\partial \phi}{\partial t} + U \cdot \nabla \phi = 0$ (13)

ore the interface velocity U is expressed as $U = u +$
 $\frac{1}{\rho$

$$
\frac{\partial \phi}{\partial \tau_0} = S(\phi)(1 - |\nabla \phi|) \tag{14}
$$

$$
S(\phi) = 0 \qquad \text{if } |\phi| \le h/2
$$

$$
= \frac{\phi}{\sqrt{\phi^2 + h^2}} \qquad \text{otherwise.} \tag{15}
$$

+**u**_f · ∇F_f = ∇ · $k_f \nabla T_f$ if $\phi = 0$

(3) $T_f = T_{int}$ if $\phi = 0$

(3) $\frac{\partial \phi}{\partial \tau_0} = S(\phi)(1 - |\nabla \phi|)$

(4) $S(\phi) = 0$ if $|\phi| \le h/2$
 $Y = Y_{int}$ if $\phi = 0$

(4) $S(\phi) = 0$ if $|\phi| \le h/2$
 $Y = Y_{int}$ if $\phi = 0$

(5) He (3)
 $T_f = T_{in}$ if $\phi = 0$
 $T_f = T_{in}$ if $\phi = 0$
 $T = T_{in}$ if $\phi = 0$

(5) Simples that a near-zero level set rather *M_{pP_{sed}* (T_n) *M*_{*N*} $\frac{F}{dt}$ **a** $T_y = T_{int}$ if $\phi = 0$
 $\frac{\partial \phi}{\partial t_0} = S(\phi)(1-|\nabla \phi|)$
 *M*_p $\frac{\partial f}{\partial t} + \mathbf{u}_g \cdot \nabla Y$ $= \nabla \cdot \rho_g \hat{D} \nabla Y$ if $\phi < 0$;
 $Y = Y_{int}$ if $\phi = 0$
 A $\frac{\partial \phi}{\partial t^2 + h^2}$ otherwi} (3)
 $T_f = T_{int}$ if $\phi = 0$
 $Y = Y_{int}$ if $\phi = 0$
 $Y = Y_{int}$ if $\phi = 0$
 $H = 0$ if $\phi \le 0$
 $H = 0$ if $\phi \le 0$
 $H = 0$ if $\phi \le 0$
 $K = \nabla \cdot n$

(5) Here *h* is grid spacing and the formulation
 $K = \nabla \cdot n$

(5) Simplies tha (a) $V = \nabla \cdot \rho_g \hat{D} \nabla Y$ if $\phi < 0$;
 $Y = Y_{int}$ if $\phi = 0$
 $Y = Y_{int}$ if $\phi = 0$
 $Y = Y_{int}$ if $\phi = 0$
 $\frac{\partial}{\partial \phi^2 + h^2}$ otherwise.
 $\therefore H = 0$ if $\phi \le 0$
 $\therefore H = 0$ if $\phi \le 0$

(5) Here *h* is grid spacing and the for *k* $\vec{r} = 0$
 $\vec{r} = 0$
 L_{int} If φ = 0

if φ = 0

if φ = 0

if φ = 0

(4)
 $S(\phi) = 0$ if $|\phi| \le h/2$

if φ = 0
 $= \frac{\phi}{\sqrt{\phi^2 + h^2}}$ otherwise.
 $\phi \le 0$

(5) Here *h* is grid spacing and the formulation of sign fit
 $\phi \le 0$

(5) Simplies tha Here *h* is grid spacing and the formulation of sign function *S* implies that a near-zero level set rather than ϕ is used as the immobile boundary condition during the reinitialization procedure. $\frac{\partial \phi}{\partial \tau_0} = S(\phi)(1 - |\nabla \phi|)$ (14)
 $S(\phi) = 0$ if $|\phi| \le h/2$
 $= \frac{\phi}{\sqrt{\phi^2 + h^2}}$ otherwise. (15)

Here *h* is grid spacing and the formulation of sign function

implies that a near-zero level set rather than ϕ is use *T* if $|\phi| \le h/2$
 T otherwise. (15)
 T otherwise. (15)
 T and the formulation of sign function

that a near-zero level set rather than ϕ is used as
 T if ϕ is used as
 T and *T* conditions
 T and *T* is *y y* $\frac{f}{f_0} = S(\phi)(1 - |\nabla \phi|)$ (14)
 $f_{\overline{f_0}} = S(\phi)(1 - |\nabla \phi|)$ if $|\phi| \le h/2$
 $= \frac{\phi}{\sqrt{\phi^2 + h^2}}$ otherwise. (15)

are h is grid spacing and the formulation of sign function

memobile boundary conditions

memobile boundary $S(\phi) = 0$ if $|\phi| \le h/2$ (15)
 $= \frac{\phi}{\sqrt{\phi^2 + h^2}}$ otherwise. (15)

Here *h* is grid spacing and the formulation of sign function

mplies that a near-zero level set rather than ϕ is used as

immobile boundary condition

2.3 Boundary conditions

The boundary conditions used in this study are as follows (Refer to Fig. 1).

$$
\mathbf{u} = 0, \quad T = T_w, \quad \frac{\partial Y}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = \mp \cos \theta \,. \tag{16}
$$

At the symmetry boundary ($r = 0$)

$$
\mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial r} = \frac{\partial T}{\partial r} = \frac{\partial Y}{\partial r} = \frac{\partial \phi}{\partial r} = 0 \tag{17}
$$

$$
\frac{\partial \mathbf{u}}{\partial r} = \frac{\partial T}{\partial r} = \frac{\partial \phi}{\partial r} = 0, \quad p = 0, \quad Y = Y_0.
$$
 (18)

3. Results and discussion

The computations of droplet evaporation between two circular plates are conducted using the water and air properties at 1atm:

$$
\rho_l = 997 kg / m^3, \qquad \rho_g = 1.18 kg / m^3
$$
\n
$$
\mu_l = 8.91 \times 10^{-4} Pa \cdot s, \qquad \mu_g = 1.85 \times 10^{-5} Pa \cdot s
$$
\n
$$
c_l = 4.18 \times 10^3 J / kg \cdot K, \qquad c_g = 1.01 \times 10^3 J / kg \cdot K
$$
\n
$$
k_l = 0.595 W / mK, \qquad k_g = 2.55 \times 10^{-2} W / mK
$$
\n
$$
D_g = 2.6 \times 10^{-5} m^2 / s, \qquad \sigma = 7.2 \times 10^{-2} N / m.
$$
\nThe saturated-vapor pressure is evaluated as a function of
\nnperature from the steam table [19].\n
$$
p_{v, sat} = \exp\left(9.487 - \frac{3.893 \times 10^3}{T_{int} + 230.47}\right), \qquad (19) \qquad \frac{\text{g}}{\text{g}}
$$
\nhere $p_{v, sat}$ is given in *MPa* and T_{int} in °C. We vary para-
\ntrically the receding contact angle θ_c and the gap G of

The saturated-vapor pressure is evaluated as a function of temperature from the steam table [19].

$$
p_{v,sat} = \exp\left(9.487 - \frac{3.893 \times 10^3}{T_{int} + 230.47}\right),\tag{19}
$$

 \degree C. We vary parametrically the receding contact angle θ_r and the gap *G* of two circular plates while keeping the advancing contact angle $G = 32 \mu m$ during the late period of droplet evaporation. $\theta_a = 90^\circ$, the wall temperature as $T_w = 90^\circ C$ and the initial \degree C, the vapor mass fraction at the right

with $\theta_r = 80^\circ$, G = 32 μ m and four different grid s $= 4 \mu m$, $h = 2 \mu m$, $h = 1 \mu m$, and $h = 0.5 \mu m$. The temporal variations of liquid droplet contact radius are plotted in Fig. 2.

The droplet contact radius decreases with evaporation. A small peak is observed during the late period of droplet evaporation when the liquid droplet between two plates is split into two single droplets. The split and elongated droplets are quickly deformed to have a spherical-cap shape due to the surface tension minimizing the interface area, as depicted in Fig. 3. This results in the sharp increase of droplet contact radius. Thereafter, the droplet contact radius decreases again.

The evaporation time for $h = 4 \mu m$, $h = 2 \mu m$, $h = 1 \mu m$, and $h = 0.5 \mu m$ is $\tau_e = 0.209 s$, $\tau_e = 0.184 s$, $\tau_e = 0.172 s$, and $\tau_e = 0.166 s$, respectively. The relative difference of the computed results between successive mesh sizes is observed to be small as the mesh size decreases. Therefore, most of computations in this study are done with $h = 1 \mu m$ to save computing time without losing the accuracy of the numerical results.

Fig. 2. Effect of mesh size on the temporal variation of the liquid droplet-plate contact radius for $\theta_r = 80^\circ$ and $G = 32 \mu m$.

Fig. 3. Evolution of the liquid-gas interface for $\theta_r = 80^\circ$ and

3.1 Comparison with theoretical prediction

Computations are carried out to validate the assumptions used by Clément and Leng [7] for the theoretical analysis for liquid droplet evaporation between two circular plates. Assuming that the vapor fraction field is 1-dimensional axisymmetric, the vapor fraction equation is expressed as g. 3. Evolution of the liquid-gas interface for $\theta_r = 80^\circ$ and
 $= 32 \mu m$ during the late period of droplet evaporation.
 I Comparison with theoretical prediction

Computations are carried out to validate the assumpt *l* by Clément and Leng [7] for the theoretical analysis for
 d droplet evaporation between two circular plates. As-

ing that the vapor fraction field is 1-dimensional axisym-

ic, the vapor fraction equation is expres 3. Evolution of the liquid-gas interface for $\theta_r = 80^\circ$ and
 $\frac{32 \mu m \text{ during the late period of droplet evaporation.}}{22 \mu m \text{ during the late period of droplet evaporation.}}$
 Comparison with theoretical prediction

show Clément and Leng [7] for the theoretical analysis for

d droplet e ²⁰ ⁻¹⁰ ⁰ ¹⁰ ²⁰ ⁻²⁰ ⁻¹⁰ ⁰ ¹⁰ ²⁰ ⁻²⁰ ⁻¹⁰ ⁰ ⁰ ²⁰

⁻²*gmm*) ¹⁰ ²² *2m* during the laquid-gas interface for $\theta_r = 80^\circ$ and

²³ *Lmm* during the late period of droplet evaporation.
 Compa ons are carried out to validate the assumptions
nent and Leng [7] for the theoretical analysis for
exaporation between two circular plates. As-
the vapor fraction field is 1-dimensional axisym-
apor fraction equation is e Computations are carried out to validate the assumptions

dd y Clément and Leng [7] for the theoretical analysis for

uid droplet evaporation between two circular plates. As-

ming that the vapor fraction field is 1-dimen mputations are carried out to validate the assumptions
y Clément and Leng [7] for the theoretical analysis for
droplet evaporation between two circular plates. As-
g that the vapor fraction field is 1-dimensional axisym-
 R and Leng [7] for the theoretical analysis for
droplet evaporation between two circular plates. As-
droplet evaporation between two circular plates. As-
g that the vapor fraction field is 1-dimensional axisym-
t, the v tion the term of the theoretical analysis for

diment and Leng [7] for the theoretical analysis for

blet evaporation between two circular plates. As-

the vapor fraction field is 1-dimensional axisym-

vapor fraction equ

$$
\frac{\partial Y}{\partial t} + u \frac{\partial Y}{\partial r} = D_g \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial Y}{\partial r} \quad \text{if } r > R.
$$
 (20)

$$
\frac{\partial Y}{\partial t} + u \frac{\partial Y}{\partial r} = D_g \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial Y}{\partial r}
$$
 if $r > R$. (20)
The interface velocity is determined as

$$
\frac{\partial R}{\partial t} = \frac{\rho_g D_g}{\rho_l (1 - Y_{int})} \left(\frac{\partial Y}{\partial r} \right)_{r=R}
$$
 (21)
Neglecting the transient and convection terms and assuming
constant interface temperature in Eq. (20), we have

$$
\ln(\frac{R}{R_o}) \frac{\partial R^2}{\partial t} = \frac{2 \rho_g D_g (Y_{int} - Y_o)}{\rho_l (1 - Y_{int})}
$$
 (22)

Neglecting the transient and convection terms and assuming the constant interface temperature in Eq. (20), we have

$$
\ln\left(\frac{R}{R_o}\right)\frac{\partial R^2}{\partial t} = \frac{2\rho_g D_g (Y_{int} - Y_o)}{\rho_l (1 - Y_{int})}.
$$
\n(22)

Fig. 4. Effect of plate gap on the temporal variation of liquid droplet radius with different conditions for $\theta_r = 80^\circ$: (a) $G = 16 \,\mu\text{m}$ and; (b) $G = 64 \,\mu\text{m}$.

Assuming that the interface temperature is constant as

$$
u = -\frac{\rho_l - \rho_g}{\rho_l} \frac{R}{r} \frac{dR}{dt}, \qquad \frac{dR}{dt} = \frac{\rho_g D_g}{\rho_l (1 - Y_{int})} \left(\frac{\partial Y}{\partial r}\right)_{r=R}.
$$
 (23)

The 1-D moving boundary problem can be easily solved using the following coordinate transformation

$$
\xi = r - R \tag{24}
$$

Eq. (20) is transformed as

$$
\frac{\partial Y}{\partial t} + (u - \frac{dR}{dt}) \frac{\partial Y}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} D_g r \frac{\partial Y}{\partial r} \quad \text{if } \xi > 1.
$$
 (25)

and solve.

In Fig. 4, 1-dimensional theoretical predictions are compared with the numerical results for different plate gaps. The 1-dimensional predictions show a good agreement with the numerical results obtained without/with the transient and conassumed. When including the effect of heat transfer, however, the numerical prediction of evaporation time is much longer than the 1-dimensional predictions. The evaporation time increases with the plate gap as $\tau_e \sim G^{0.322}$, which is estimated from the least square fitting from the numerical results.

Fig. 5. Evolution of an evaporating liquid-gas interface for $\theta_r = 80^\circ$ and $G = 32 \mu m$.

Fig. 6. Velocity field for $\theta_r = 80^\circ$ and $G = 32 \,\mu\text{m}$.

3.2 Interfacial motion and the associated flow, temperature and vapor fraction fields

64 μ m.

Summing that the interface temperature is constant as
 $\frac{2}{\lambda}$ **i** $\frac{1}{\lambda}$ **g** $\frac{1}{\lambda}$ and including convection term as $u \ne 0$, we obtain
 $= -\frac{\rho_1 - \rho_g}{\rho_1} \frac{R}{r} \frac{dR}{dt}$, $\frac{dR}{dt} = \frac{\rho_g D_g}{\rho_1 (1$ ing that the interface temperature is constant as

and including convection term as $u \ne 0$, we obtain
 $\frac{2\sqrt{t^2 - t^2}}{t^2}$, $\frac{dR}{dt} = \frac{t^2 \sqrt{t^2}}{t^2}$ (23)

D moving boundary problem can be easily solved us

Fig. 6 **Example 14 Am**
 Example 14 the interface temperature is constant as
 Example 14 the interface temperature is constant as
 t = $\frac{\rho_r - \rho_s}{\rho_l} \frac{R}{r} \frac{dR}{dt}$, $\frac{dR}{dt} = \frac{\rho_s D_s}{\rho_l (1 - Y_m)} \left(\frac{\partial Y}{\partial r}\right)_{r=R}$. (23)
 $+(u-\frac{u\alpha}{l})\frac{\partial}{\partial t}=-\frac{v\alpha}{2}D_g r^{\frac{\alpha}{l}}$ if $\xi>1$. (25) contact line is pinned while the contact angle reaches the re-Examing that the interface temperature is constant as
 T_{m} and including convection term as $u \ne 0$, we obtain
 $u = -\frac{\rho_1 - \rho_g}{\rho_1} \frac{R}{r} \frac{dR}{dt}$, $\frac{dR}{dt} = \frac{\rho_g D_g}{\rho_1(1 - Y_{m})} \left(\frac{\partial Y}{\partial r}\right)_{r=R}$. (23)

Although co The transformed equations are straightforward to discretize into two droplets at $t = 164.5 \text{ ms}$ without any satellite drop-The 1-D moving boundary problem can be easily solved us-

ing the following coordinate transformation

ing the following coordinate transformation
 $\xi = r - R$.

(24) 3.7 Interfaced motion and the associated flow, temperatur Fig. 5 shows the evolution of an evaporating liquid-gas interface for $\theta_r = 80^\circ$ and $G = 32 \mu m$. The interface is formed as the liquid volume decreases with evaporation. The ceding contact angle of $\theta_r = 80^\circ$ and then the contact line -120 -90 -60 -30 0 30 60 90 120

t=164.5ms
 $\frac{2}{5}$ 16

begins to -120 -90 -60 -30 0 30 60 90 120

r(μ m)

Fig. 6. Velocity field for $\theta_r = 80^\circ$ and $G = 32 \mu$ m.

3.2 *Interfacial motion and the associated flow, tem* **interference at a finally dependent and the associated flow, temperature and vapor friedly field for** $\theta_t = 80^\circ$ **and** $G = 32 \mu m$ **.**
3.2 **Interfacial motion and the associated flow, temperature and vapor fraction fields**
Fi lets. The velocity, temperature and vapor fraction fields associated with the droplet evaporation are plotted in Figs. 6-8. The liquid phase is almost stationary, whereas the gas velocity is pronounced near the interface. The gas velocity, which is normal to the interface, becomes parallel to the plates away from the interface. The temperature in the liquid-gas phases initially increases due to the heat transfer from the plates and then the temperature field becomes quite uniform except the interface region, where the thermal energy is used for the liquid-vapor phase change. The isothermal pattern near the interface maintains as long as the remaining portion of liquid drop-

Fig. 7. Temperature field for $\theta_r = 80^\circ$ and $G = 32 \mu m$. The interval between temperature contours is 5° C. .

interval between vapor mass fraction contours is 0.05.

Fig. 9. Effect of plate gap on the liquid droplet evaporation for vals between temperature and vapor mass fraction contours are 5° C and 0.05, respectively.

let is not so small, as seen at $t = 100.0$ *ms* of Fig. 7. The temperature is not uniform along the interface, which results in the non-uniform vapor fraction distribution, as depicted in Fig. 8. dimensional except the interface.

3.3 Effect of the gap between two plates

The effect of plate gap *G* on the temperature and vapor fraction field is shown in Fig. 9 while keeping $\theta_r = 80^\circ$. As the gap increases, the temperature field varies in a wider region and the interface temperature become lower in the central Fig. 9. Effect of plate gap on the liquid droplet evaporation for $G = 32 \mu m$: (a) temperature field; (b) vapor mass fraction. The inter-
vals between temperature and vapor mass fraction contours are
 F/g and 0.05, respect uniform with increasing plate gap. As a result, the evaporation

Fig. 10. Effect of plate gap on temporal variation of liquid dropletplate contact radius for $\theta_r = 80^\circ$.

Fig. 11. Effect of receding contact angle on the evolution of an evaporating liquid-gas interface: (a) $\theta_r = 30^\circ$; (b) $\theta_r = 80^\circ$. .

Fig. 12. Effect of receding contact angle on temporal variation of liq-

However, the vapor fraction distribution is almost 1- longer than the 1-dimensional prediction although using the dimensional except the interface small gap of $G = 16 \mu m$. This means that 1-dimensional prerate decreases and the evaporation period increases with the plate gap, as plotted in Fig. 10. Although the gap decreases, the 2-dimensional prediction of evaporation period is much longer than the 1-dimensional prediction although using the diction is not easily applicable to the high-rate evaporation \mathcal{C} .

3.4 Effect of the receding contact angle

Figs. 11 and 12 present the effect of receding contact angle on the temporal variations of an interface and liquid droplet radii. As the receding contact angle decreases, the interface deformation is pronounced as depicted in Fig. 11 and the contact line pinning period increases as seen in Fig. 12. As a result, the evaporation rate increases as the contact angle decreases and the liquid droplet-air contact area increases.

4. Conclusions

Numerical simulations of droplet evaporation in the confined geometry between two circular plates, which is a welldefined and easily controllable evaporation process, were T performed using a level-set method, which was extended to T_f include the effect of evaporation at the liquid-gas interface and T_i dynamic contact angles at the liquid-gas-solid contact line. T_{int} The computations were used to validate the assumptions in the $\qquad u$ 1-dimensional theoretical analysis reported in the literature. \mathbf{u}_f The theoretical prediction showed a good agreement with the numerical results obtained by neglecting the transient and \mathbf{u}_l convection terms and assuming no heat transfer. When including the effects of convection and heat transfer, however, the v_{lg} theoretical prediction of evaporation time was much shorter than the numerical predictions. As the gap decreases, the $2-_{int}$ dimensional numerical prediction is closer to the 1 dimensional theoretical prediction, but the difference is not θ_a negligible even for the small gap. This means that the characteristics of flow, heat and mass transfer are not negligible κ across the film thickness normal to the plates for the case of μ_{g} the high-rate evaporation process. The evaporation rate was μ_l observed to increase as the receding contact angle decreased.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) funded by the Korean government σ (MSIP) (Grant No. 2013R1A2A2A01068333).

Nomenclature-

- *cf* : Specific heat of fluid
- *c^g* : Specific heat of gas
- *cl* : Specific heat of liquid
- D_g
 $\stackrel{\sim}{D}$: Diffusion coefficient of gas
- \tilde{D} : Effective diffusion coefficient
- *F* : Fraction function
- *G* : Gap of two circular plates
- *h* : Grid spacing
- *hlg* : Latent heat of vaporization
- *H* : Step function
- *k^f* : Thermal conductivity of fluid
- *k^g* : Thermal conductivity of gas
-
- *k_l* : Thermal conductivity of liquid \hat{k} : Effective thermal conductivity
- \dot{m} : Mass flux across the interface
- *M^a* : Molecular mass of air
- *M^v* : Molecular mass of vapor
- **n** : Unit normal vector
- *p* : Pressure
- *pv,sat* : Vapor saturation pressure p_{∞} : Atmospheric pressure *r, y* : Cylindrical coordinates *R* : Radius of liquid droplet *Ri* : Initial radius of liquid droplet *R^o* : Radius of circular plates *S* : Sign function *t* : Time *T* : Temperature *Tf* : Temperature of fluid *Ti* : Initial temperature *Tint* : Temperature at the liquid-gas interface **u** : Flow velocity vector **u***f* : Flow velocity vector of fluid **u***^g* : Flow velocity vector of gas **u***l* : Flow velocity vector of liquid **U** : Interface velocity vector *vlg* : Difference between the liquid and gas specific volume *Y* : Vapor mass fraction *Yint* : Vapor mass fraction at the interface *Y^o* : Vapor mass fraction at the right boundary θ_a : Advancing contact angle θ_r : Receding contact angle : Interface curvature $\mu_{\rm g}$: Dynamic viscosity of gas μ _l : Dynamic viscosity of liquid $\hat{\mu}$: Effective dynamic viscosity ρ_f : Density of fluid ρ_g : Density of gas ρ_l : Density of liquid
- $\hat{\rho}$: Effective density
- : Surface tension
- τ_0 : Artificial time
- τ_e : Total evaporation time
 ϕ : Distance function from
	- : Distance function from the liquid-gas interface
- ξ : Transformed coordinate

References

- [1] T. Kawase, T. Shimoda, C. Newsome, H. Sirringhaus and R. H. Friend, Inkjet printing of polymer thin film transistors, *Thin Solid Films*, 438 (2003) 279-287.
- [2] H. Koo, M. Chen and P. Pan, LCD-based color filter films fabricated by a pigment-based colorant photo resist inks and printing technology, *Thin Solid Films*, 515 (2006) 896-901.
- [3] J. Berthier, *Microdrops and digital microfuidics*, 1st Ed., William Andrew (2008).
- [4] A. F. Routh, Drying of thin colloidal films, *Reports on Progress in Physics*, 76 (2013) 046603.
- [5] R. G. Picknett and R. Bexon, The evaporation of sessile or pendant drops in still air, *Interface J. of Colloid and Interface Science*, 61 (1977) 336-350.
- [6] H. Y. Erbil, Evaporation of pure liquid sessile and spherical suspended drops: A review, *Adv. Colloid Interface Sci.*, 170 (2012) 67-86.
- [7] F. Clément and J. Leng, Evaporation of liquids and solutions in confined geometry, *Langmuir*, 20 (2004) 6538-6541.
- [8] J. Leng, Drying of a colloidal suspension in confined geometry, *Physical Review*, 82 (2010) 021405.
- [9] L. Daubersies and J. B. Salmon, Evaporation of solutions and colloidal dispersion in confined droplets, *Physical Review E*, 84 (2011) 031406.
- [10] H. Hu and R. G. Larson, Analysis of the microfluid flow in an evaporating sessile droplet, *Langmuir*, 21 (2005) 3963- 3971.
- [11] E. Widjaja and M. T. Harris, Numerical study of vapor phase-diffusion driven sessile drop evaporation, *Computers & Chemical Engineering*, 32 (2008) 2169-2178.
- [12] O. E. Ruiz and W. Z. Black, Evaporation of water droplets placed on a heated horizontal surface, *J. of heat transfer*, 124 (5) (2002) 854-863.
- [13] A. M. Briones, J. S. Ervin, S. A. Putnam, L. W. Byrd and L. Gschwender, Micrometer-sized water droplet impingement dynamics and evaporation on a flat dry surface, *Langmuir*, 26 (2010) 13272-13286.
- [14] S. Tanguy, T. Ménard and A. Berlemont, A level-set method for vaporizing two-phase flows, *J. of Computational Physics*, 211 (2007) 837-853.
- [15] G. Son, A level-set method for analysis of microdroplet evaporation on a heated surface, *J. of Mechanical Science and Technology*, 24 (2010) 991-997.
- [16] G. Son, Numerical simulation of microdroplet impact and evaporation on a solid surface, *J. of Heat Transfer*, 134 (2012) 101502.
- [17] G. Son and V. K. Dhir, A level set method for analysis of

film boiling on an immersed solid surface, *Numerical Heat Transfer, Part B: Fundamentals*, 52 (2007) 153-177.

- [18] J. Fukai, Y. Shiiba, T. Yamamoto, O Miyatake, O. Poulikakos, C. M. Megaridis and Z. Zhao, Wetting effects on the spreading of a liquid droplet colliding with a flat surface: Experiment and modelling, *Physics of Fluids*, 7 (1995) 236-247.
- [19] T. F. Irvine and P. E. Liley, *Steam and gas tables with computer equations*, Academic Press (1984).

Hangjin Ban received a B.S. in Mechanical Engineering from Sogang University in 2014. He is a graduate student of Mechanical Engineering at Sogang University in Seoul, Korea. Ban's research interests are in the area of microfluidics and multiphase dynamics.

Gihun Son received his B.S. and M.S. in Mechanical Engineering from Seoul National University in 1986 and 1988, respectively, and his Ph.D. in Mechanical Engineering from UCLA in 1996. Dr. Son is currently a professor of Mechanical Engineering at Sogang University in Seoul, Korea. His research interests are

in the areas of multiphase dynamics, heat transfer, and power system simulation.