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# Vibration analysis of viscoelastic tapered micro-rod based on strain gradient theory resting on visco-pasternak foundation using DQM †

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**Abstract**

In the present work, based on strain gradient theory, the free vibration analysis of tapered viscoelastic micro-rod resting on visco-Pasternak foundation is investigated. The material properties of micro-rod are assumed the visco-elastic and modeled as the Kelvin-Voigt. Using Hamilton's principle and energy method, the governing equation of motion of viscoelastic micro-rods is derived, then this obtained equation using the differential quadrature method (DQM) for different boundary conditions is solved. In this study, the effects of various parameters such as the structural damping coefficient, Winkler and Pasternak foundation modulli, damping coefficient of the elastic medium and material length scale parameters on the non-dimensional natural frequencies of viscoelastic micro-rod are investigated. The results show that with an increase in the Winkler and Pasternak coefficients, the natural frequency increases as well as the obtained non-dimensional natural frequencies by MCST and SGT decrease by increasing the material length to radius ratio. It can be seen that the non-dimensional frequency for SGT is higher than that of the other theories. It is shown that the non-dimensional frequencies increase by increasing the damping coefficient for all theories. Moreover, at the specified value of damping coefficient of the elastic medium, the variation of non-dimensional natural frequency is approximately smooth.

*Keywords*: DQM; Strain gradient theory; Vibration analysis; Viscoelastic tapered micro-rod; Visco-Pasternak foundation

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## 1. Introduction

Recently, extensive researches using many various theories on the mechanical behavior of materials at the micro and nano scale are studied. In most of these analyzes, the material properties are considered as isotropic, composite, or functionally graded (FG) but visco-elastic materials are used in a number of various applications such as a buffer under heavy loads to prevent or reduce the damage that can be caused by machine vibration and by reducing shock damages that are generated by working machinery parts. Lei et al. [1] investigated the dynamic behavior of nonlocal viscoelastic damped nanobeams. They employed the Kelvin-Voigt viscoelastic model and Timoshenko beam theory to establish the governing equations of motion, also using transfer function methods (TFM), the natural frequencies and frequency response functions (FRF) are calculated for nanobeams with various boundary conditions. They showed that the real part of the natural frequencies decreases generally with an increase in the values of the nonlocal parameter. Pouresmaeeli et al. [2] studied the vibration characteristics of a viscoelastic nanoplate using the nonlocal plate theory by including the effect of viscoelastic foundation. They solved this equation using Navier's type solution for simply supported nanoplate. Their results can be seen that the frequency decreases significantly with increasing the structural damping coefficient as well as the damping coefficient of foundation. Akgöz and Civalek [3] presented the longitudinal free vibration analysis of axially FG microbars based on strain gradient elasticity theory. In their study, Rayleigh-Ritz method is utilized to obtain an approximate solution to this problem for clamped-clamped (C-C) and clamped-free (C-F) boundary conditions. They obtained that the natural frequencies evaluated by the newly developed non-classical model are always greater than those obtained by the classical model in all cases. Kahrobaiyan et al. [4] illustrated the longitudinal behavior of microbar using strain gradient theory. They used the Hamilton's principle in order to derive the governing equation of equilibrium. Their result showed that a good agreement between the strain gradient finite element and analytical results is observed. Simsek and Reddy [5] investigated the static, bending, and free vibration of FG microbeams based on the modified couple stress theory (MCST) and various higher order beam theories. They showed the efficacy of the material length scale parameter, different material compositions, and shear deformation on the bending and free vibration behavior

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of FG micro beams. They established that in the FG microbeams model, the size-dependence effect is more significant. Reddy [6] reformulated bending, buckling, and vibration analysis of nanobeam based on various beam theories. He showed that the inclusion of the nonlocal effect increases the value of deflection and decreases the critical buckling load and natural frequencies. In the other work, he [7] studied the microstructure-dependent couple stress theories of FG beams. Murmu and Pradhan [8] carried out the buckling analysis of a single-walled carbon nanotube (SWCNT) embedded in an elastic medium based on nonlocal elasticity and Timoshenko beam theories using differential quadrature method (DQM). They showed that the critical buckling loads of SWCNT are strongly dependent on the nonlocal small-scale coefficients and on the stiffness of the surrounding medium. They showed that the influence of small-scale coefficients diminishes with higher aspect ratios for both Winkler- and Pasternak-type models. Lim et al. [9] studied the free torsional vibration of nanotubes based on nonlocal stress theory. Their results indicate that natural frequency of nanotubes increases with increasing of the nonlocal parameter. Furthermore, they derived the critical speed of axially moving nanorods/nanotubes and it is concluded that this critical speed is significantly influenced by the nonlocal parameter. Firouz-Abadi et al. [10] carried out free vibration analysis of nanocones using a nonlocal continuum model. They used a novel approach for derivation of the governing equations of motion and the Galerkin technique is used to obtain the natural frequencies of vibrations. Their results showed that the fundamental natural frequencies decrease with the increase in the apex angle. Also, it is observed from their results that for lower values of the small-scale parameter, the dependency of the natural frequencies to the small-scale parameter decreases. Using nonlocal elasticity theory, Murmu and Adhikari [11] investigated the longitudinal vibration analysis of double-nanorod systems. Their obtained results can be useful to study the axially vibrating complex multiple-nanobeam system in nano-opto-mechanical system. Danesh et al. [12] illustrated the axial vibration analysis of a tapered nanorod based on nonlocal elasticity theory using DQM. They showed that the nonlocal effect plays an important role in the axial vibration of nanorods. Also, the nonlocal frequencies are always smaller than their local counterparts. Aydogdu [13] presented the axial vibration analysis of nanorods (Carbon nanotubes) embedded in an elastic medium using nonlocal elasticity theory. Their results showed that the axial vibration frequencies of embedded nanorods are highly over estimated by the classical continuum rod model which ignores the effect of small length scale. Huang [14] studied the nonlocal effects of longitudinal vibration nanorod with internal long-range interactions. Their results showed that the nanorod becomes stiffer due to the internal long-range interactions. Heireche et al. [15] presented the nonlocal elasticity effect on vibration characteristics of protein microtubules. They offered a simple and effective new approach to study vibration characteristics of microtubules. Zhang and Fu [16] presented pull-in analysis of electrically actuated viscoelastic microbeams based on a MCST. They employed the Galerkin method to solve the equation. Their results showed that the instantaneous pull in voltage, durable pull-in voltage and pull-in delay time predicted by this newly developed model is larger than that predicted by the classical beam model. Akgöz and Civalek [17] investigated the free vibration analysis of axially FG tapered Euler-Bernoulli microbeams based on the MCST. They employed the Rayleigh-Ritz solution method to obtain an approximate solution for the free transverse vibration problem and also observed that dimensionless natural frequencies predicted by classical theory (CT) are always smaller than those obtained by MCST. Ghorbanpour Arani et al. [18] studied the nonlocal vibration analysis of coupled system of double-layered graphene sheets (CS-DLGS) embedded on visco-Pasternak foundation for various boundary conditions using DQM. Their results indicated that the frequency ratio of the CS-DLGSs is more than the single-layered graphene sheet (SLGS). Mohammadimehr and Rahmati [19] investigated the Small scale effect on electro-thermo-mechanical vibration analysis of single-walled boron nitride nanorods (SWBNNRs) under electric excitation. They taken into account the effects of the small scale, aspect ratio, and C-C and C-F boundary conditions on the natural frequency. Their results indicated that the axial displacement of SWBNNRs increases with an increase in the temperature change and also, for the piezoelectric coefficient, it is the same. Mohammadimehr et al. [20] carried out torsional buckling of double-walled carbon nanotubes (DWCNT) on Winkler and Pasternak foundations using nonlocal elasticity theory. It is shown from their results that the nonlocal critical buckling load is lower than the local critical buckling load. Rahmati and Mohammadimehr [21] presented vibration analysis of non-uniform and non-homogeneous boron nitride nanorods embedded in an elastic medium under combined loadings using DQM. Kahrobaiyan et al. [22] investigated the strain gradient beam element using the finite element method (FEM). They indicated that there is a good agreement between the experimental and the strain gradient based on FEM results while the difference between the experimental and the classical FEM results is significant. Kong et al. [23] carried out static and dynamic analysis of micro beams based on strain gradient theory (SGT) and also found that the beam deflections decrease and natural frequencies increase remarkably when the thickness of the beam becomes comparable to the material length scale parameter. K. A. Lazopoulos and A. K. Lazopoulos [24] studied the bending and buckling analysis of thin strain gradient elastic beams. Mohammadimehr et al. [25] considered the buckling analysis of DWCNT embedded in an elastic medium under axial compression using non-local Timoshenko beam theory. Their results showed that the critical buckling load can be overestimated by the local beam model if the small-scale effect is overlooked for long nanotubes are compared their results with those obtained using molecular mechanics. Ghorbanpour Arani et al. [26] illustrated the

Pasternak effect on the buckling of embedded SWCNT using non-local cylindrical shell theory. They indicated that the Winkler-type spring and Pasternak-type shear constants increase the non-local critical buckling load under general loads. Simsek [27] studied the dynamic analysis of an embedded microbeam carrying a moving microparticle based on MCST. He used the Hamilton's principle to derive the governing equations of motion. His results showed that the material length scale parameter, Poisson's ratio, velocity of the microparticle and elastic medium constant play an important role on the dynamic behavior of the microbeam. Filiz and Aydogdu [28] examined the axial vibration of carbon nanotube (CNT) heterojunctions using the nonlocal elasticity theory. They used the constitutive equations based on the nonlocal elasticity theory and found that the axial vibration frequencies of CNT heterojunctions are highly over estimated by the classical rod model because of ignoring the effect of small length scale. Narendar and Gopalakrishnan [29] presented the axial wave propagation in coupled nanorod system with considering the nonlocal effects. Their analysis showed that the wave characteristics are highly over estimated by the classical rod model; also the nonlocal parameter introduces certain band gap region in axial wave mode, where no wave propagation occurs. Simsek [30] investigated the nonlocal effects on the free longitudinal vibration of axially functionally graded tapered nanorods. He utilized Eringen's nonlocal elasticity theory and Galerkin method to obtain the natural frequencies. He observed that the effect of the nonlocal parameter on the free vibration frequencies rises as the mode number increases. Narendar and Gopalakrishnan [31] carried out the nonlocal effects on ultrasonic wave characteristics of nanorods. They developed the nonlocal Euler-Bernoulli model for nanorods and explicit expressions derived for wave numbers and wave speeds of nanorods. In the other work, they [32] illustrated ultrasonic wave dispersion characteristics of a nanorod. They used strain gradient models to analyze the ultrasonic wave behavior in nanorod. They showed that the fourth order strain gradient model gives the approximate results over the second order strain gradient model for dynamic analysis. Lee [33] studied the free vibration analysis of beams with non-ideal clamped boundary conditions. In his study, it can be seen that the free vibration analysis of the Euler-Bernoulli beam is carried out analytically, and the pseudo spectral method is employed to accommodate the non-ideal boundary conditions in the free vibration analysis of Timoshenko beam. Ghorbanpour Arani et al. [34] presented the nonlinear thermo free vibration and instability of viscose fluid-conveying double-walled carbon nanocones. They simulated the nanocone as a clamped-clamped Euler-Bernoulli's beam embedded in an elastic foundation of the Winkler and Pasternak type. Their results showed that the nonlocal effect on flow field is remarkable on frequency and critical fluid velocity of double-walled carbon nanocones. Also, the nonlinear frequency and critical flow velocity decrease with increasing the nonlocal parameter and cone semi-vertex angle.

In the present work, free vibration analysis of viscoelastic tapered microrod is discussed by using DQM. The tapered microrod is rested on the visco-pasternak foundation. The strain gradient theory is used to derive the equations of motion. Finally the effects of various parameters such as, material length scale, viscoelastic structural damping coefficient, internal damping coefficient, Winkler and Pasternak foundation modulus, are investigated on the natural frequencies of tapered microrod.

## 2. Strain gradient theory

The mechanical analysis of structures in micro and nano scales is more significant; because of the size effects play a role important in these scales. According to the strain gradient theory proposed by Lam et al. [35], the strain energy $U$ in a linear elastic isotropic material is expressed as follows:

$$\Pi_p = \frac{1}{2}\int_{\forall}\left[\sigma_{ij}\varepsilon_{ij} + P_i\gamma_i + \tau_{ijk}\eta_{ijk} + m_{ij}\chi_{ij}\right]d\forall \tag{1}$$

$$\varepsilon_{ij} = \frac{1}{2}\left[u_{i,j} + u_{j,i}\right] \tag{2}$$

$$\gamma_i = \varepsilon_{mm,i} \tag{3}$$

$$\begin{aligned}\eta_{ijk} &= \frac{1}{3}\left[\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}\right] - \frac{1}{15}\delta_{ij}\left[\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right] \\ &- \frac{1}{15}\delta_{jk}\left[\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right] - \frac{1}{15}\delta_{ki}\left[\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right]\end{aligned} \tag{4}$$

$$\chi_{i,j} = \frac{1}{2}\left[\theta_{i,j} + \theta_{j,i}\right] \tag{5}$$

$$\theta_i = \frac{1}{2}\left[\nabla \times u\right]_i \tag{6}$$

where $\varepsilon_{ij}$, $\gamma_i$, $\eta_{ijk}$, $\chi_{ij}$, $\theta_i$ and $\delta_{ij}$ define the strain tensor, dilatation gradient vector, deviatoric stretch gradient tensor, symmetric rotation gradient tensor, rotation vector and the kronecker delta. The strain gradient theory, higher-order stresses tensor can be defined as follows [25]:

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{mm} + 2G\varepsilon_{ij} \tag{7}$$

$$P_i = 2Gl_0^2\gamma_i \tag{8}$$

$$\tau_{ijk} = 2Gl_1^2\eta_{ijk} \tag{9}$$

$$m_{ij} = 2Gl_2^2\chi_{ij} \tag{10}$$

where $l_0, l_1, l_2$ are three material length scale parameters and $G = E/2(1+\nu)$, $\lambda = E\nu/(1+\nu)(1-2\nu)$ denote lame constants. Based on Kelvin-voigt model in viscoelastic structures, elastic and shear modulli defined as $E = E_0(1 + g\partial/\partial t)$, $G = G_0(1 + g\partial/\partial t)$ that $g$ is viscoelastic structural damping coefficient.

## 3. Vibration of viscoelastic tapered microrod

The displacement fields can be expressed for micro-rod as follows:

$$u = U(\mathrm{x,t}) \qquad v = 0 \qquad w = 0. \tag{11}$$

The kinematic equations are written as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial U(\mathrm{x,t})}{\partial x} \qquad \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0. \tag{12}$$

Substituting Eq. (12) into Eq. (3) yields:

$$\gamma_x = \frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \qquad \gamma_y = 0,\ \gamma_z = 0. \tag{13}$$

By using Eq. (12) into Eq. (4), the nonzero component of the kinematic equations are written as follows:

$$\begin{aligned} \eta_{xxx} &= \frac{2}{5}\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \\ \eta_{yyx} = \eta_{xyy} = \eta_{yxy} &= -\frac{1}{5}\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \\ \eta_{zzx} = \eta_{zxz} = \eta_{xzz} &= -\frac{1}{5}\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2}. \end{aligned} \tag{14}$$

With considering the displacement fields and Eqs. (5) and (6), all components of the rotation vector and symmetric rotation gradient tensor are zero.

$$\chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{xy} = \chi_{xz} = \chi_{yz} = 0. \tag{15}$$

By substituting Eqs. (12)-(15) into Eqs. (7)-(10), the higher-order stresses tensor can be rewritten as:

$$\sigma_{xx} = E\frac{\partial U(\mathrm{x,t})}{\partial x} \tag{16}$$

$$P_x = 2Gl_0^2\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \tag{17}$$

$$\begin{aligned} \tau_{xxx} &= \frac{4}{5}Gl_1^2\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \\ \tau_{yyx} = \tau_{yxy} = \tau_{xyy} &= -\frac{2}{5}Gl_1^2\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \\ \tau_{zzx} = \tau_{zxz} = \tau_{xzz} &= -\frac{2}{5}Gl_1^2\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} \end{aligned} \tag{18}$$

$$m_{ij} = 0. \tag{19}$$

Substituting Eqs. (12)-(19) into Eq. (1), the strain energy of the tapered micro-rod can be found as:

$$\Pi_p = \frac{1}{2}\int_0^L\left[A(\mathrm{x})\mathrm{E}\left(\frac{\partial U(\mathrm{x,t})}{\partial x}\right)^2 + DA(\mathrm{x})\left(\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2}\right)^2\right]dx \tag{20}$$

where

$$D = D_0\left[1 + g\frac{\partial}{\partial t}\right], \qquad D_0 = G\left(2l_0^2 + \frac{4}{5}l_1^2\right) \tag{21}$$

$$A(x) = A_0\,\phi(x), \qquad \phi(x) = \left(1 - \frac{3}{4L}x\right). \tag{22}$$

And the kinetic energy is given by [28]:

$$\Pi_k = \frac{1}{2}\rho_0\int_0^L\left[A(\mathrm{x})\left(\frac{\partial U(\mathrm{x,t})}{\partial t}\right)^2\right]dx. \tag{23}$$

The work done by the visco-elastic medium force is considered as [36]:

$$\Pi_w = \frac{1}{2}\int_0^L F(x,t)U(\mathrm{x,t})\,\mathrm{dx} \tag{24}$$

$$F(\mathrm{x,t}) = -\mathrm{k}_w U(\mathrm{x,t}) + \mathrm{k}_G\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} - \mathrm{c}\frac{\partial U(\mathrm{x,t})}{\partial t} \tag{25}$$

where $k_w$, $k_p$ and $c$ are the Winkler, Pasternak and damping modulli. The Hamilton's principle can be expressed as:

$$\int_0^t\left[\delta\Pi_k + \delta\Pi_w - \delta\Pi_p\right]dt = 0. \tag{26}$$

Using Eqs. (20), (23) and (24), and the governing equations of motion for viscoelastic tapered micro-rod based on strain gradient theory are obtained as follows:

$$\begin{aligned} &D_0A_0g\phi(\mathrm{x})\frac{\partial^5 U(\mathrm{x,t})}{\partial x^4\partial t} + D_0A_0\phi(\mathrm{x})\frac{\partial^4 U(\mathrm{x,t})}{\partial x^4} - A_0E_0g\phi(\mathrm{x})\frac{\partial^3 U(\mathrm{x,t})}{\partial x^2\partial t} \\ &-A_0E_0\phi(\mathrm{x})\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} - A_0E_0\frac{\partial\phi(\mathrm{x})}{\partial x}\frac{\partial U(\mathrm{x,t})}{\partial x} - A_0E_0g\frac{\partial\phi(\mathrm{x})}{\partial x}\frac{\partial^2 U(\mathrm{x,t})}{\partial x\partial t} \\ &+D_0A_0\frac{\partial^2\phi(\mathrm{x})}{\partial x^2}\frac{\partial^2 U(\mathrm{x,t})}{\partial x^2} + D_0A_0g\frac{\partial^2\phi(\mathrm{x})}{\partial x^2}\frac{\partial^3 U(\mathrm{x,t})}{\partial x^2\partial t} + k_wU(\mathrm{x,t}) \\ &-\mathrm{k}_G\nabla^2 U(\mathrm{x,t}) + \mathrm{c}\frac{\partial U(\mathrm{x,t})}{\partial t} + \rho_0A_0\phi(\mathrm{x})\frac{\partial^2 U(\mathrm{x,t})}{\partial t^2} = 0. \end{aligned} \tag{27}$$

## 4. DQM technique

In this research, the governing equation of motion for viscoelastic tapered micro-rod is obtained by using DQM. According to this method, the derivatives of a function at the point ($x_i$) can be expressed in terms of the value of function in throughout domain as [37-39]:

$$\left.\frac{df}{dx}\right|_{x=x_i} = \sum_{j=1}^{N} A_{ij}^{(1)}y_j \quad , \ i = 1,2,...,N \tag{28}$$

where $A^{(m)}$ and $N$ denote the weighting coefficients associated with the $m^{th}$ order derivative and the number of grid points in the $x$-direction. These coefficients for the first-order derivatives are given by:

$$A_{ij}^{(1)}=\frac{\prod\limits_{\substack{k=1\\k\neq j}}^{N}(x_i-x_k)}{\prod\limits_{\substack{k=1\\k\neq j}}^{N}(x_j-x_k)}\sum_{\substack{m=1\\m\neq j}}^{N}\frac{1}{x_i-x_m} \tag{29}$$

$$A_{ij}^{(1)}=\begin{cases}\dfrac{\prod\limits_{\substack{m=1\\m\neq i,j}}^{N}(x_i-x_m)}{\prod\limits_{\substack{m=1\\m\neq j}}^{N}(x_j-x_m)}, & (i,j=1,2,3,...,N;i\neq j)\\ \sum\limits_{\substack{m=1\\m\neq i}}^{N}\dfrac{1}{(x_i-x_m)}, & (i=j=1,2,3,...,N).\end{cases} \tag{30}$$

Also, the weighting coefficients of higher-order derivatives can be considered as follows:

$$A_{ij}^{(r)}=\begin{cases}r\left(A^{(r-1)}{}_{ii}A^{(1)}{}_{ij}-\dfrac{A^{(r-1)}{}_{ij}}{x_i-x_j}\right), & (i,j=1,2,3,...,N;i\neq j)\\ -\sum\limits_{\substack{m=1\\m\neq i}}^{N}A^{(r)}{}_{im}, & (i=j=1,2,3,...,N)\end{cases} \tag{31}$$

$$A^{(2)}=A^{(1)}A^{(1)} \qquad A^{(r)}=A^{(1)}A^{(r-1)}. \tag{32}$$

A well-accepted set of the grid points is given by the Gauss-Lobatto-Chebyshev points for interval [0, L]:

$$\bar{x}_i=\frac{x_i}{L}=\frac{1}{2}\left\{1-\cos[\frac{(i-1)\pi}{(N-1)}]\right\},\ (i=1,2,3,...,N). \tag{33}$$

The governing equation of motion for viscoelastic tapered micro-rod based on strain gradient theory resting on visco-pasternak foundation using DQM can be obtained as:

$$\begin{aligned}&D_0A_0g\phi(x_i)\sum_{j=1}^{N}A_{ij}^{(4)}\dot{u}_j+D_0A_0\phi(x_i)\sum_{j=1}^{N}A_{ij}^{(4)}u_j\\&-E_0A_0g\phi(x_i)\sum_{j=1}^{N}A_{ij}^{(2)}\dot{u}_j-E_0A_0\phi(x_i)\sum_{j=1}^{N}A_{ij}^{(2)}u_j\\&-E_0A_0\frac{\partial\phi(x_i)}{\partial x}\sum_{j=1}^{N}A_{ij}^{(1)}u_j-E_0A_0g\frac{\partial\phi(x_i)}{\partial x}\sum_{j=1}^{N}A_{ij}^{(1)}\dot{u}_j\\&+D_0A_0\frac{\partial^2\phi(x_i)}{\partial x^2}\sum_{j=1}^{N}A_{ij}^{(2)}u_j+D_0A_0g\frac{\partial^2\phi(x_i)}{\partial x^2}\sum_{j=1}^{N}A_{ij}^{(2)}\dot{u}_j\\&+k_wu(x_i,t)-k_p\nabla^2u(x_i,t)+C_0\dot{u}(x_i,t)+\rho A_0\phi(x_i)\ddot{u}(x_i,t)=0.\end{aligned} \tag{34}$$

Eq. (34) can be converted to eigenvalue problem by considering the following form:

$$u(x,t)=U(x)e^{\omega t}. \tag{35}$$

The first four derivatives in respect to $x$ can be written as follows:

$$A=A^{(1)}\quad B=A^{(2)}\quad C=A^{(3)}\quad D=A^{(4)} \tag{36}$$

$$a_{ij}=\phi|_{x=x_i}\delta_{ij}\quad b_{ij}=\left.\frac{d\phi}{d\zeta}\right|_{x=x_i}\delta_{ij}\quad c_{ij}=\left.\frac{d^2\phi}{d\zeta^2}\right|_{x=x_i}\delta_{ij}. \tag{37}$$

Substituting Eqs. (35) and (36) into Eq. (34) yields:

$$\begin{aligned}&D_0A_0g\omega[a]\cdot[D]\cdot\{U\}+D_0A_0[a]\cdot[D]\cdot\{U\}\\&-E_0A_0g\omega[a]\cdot[B]\cdot\{U\}-E_0A_0[a]\cdot[B]\cdot\{U\}\\&-E_0A_0[b]\cdot[A]\cdot\{U\}-E_0A_0g\omega[b]\cdot[A]\cdot\{U\}\\&+D_0A_0[c]\cdot[B]\cdot\{U\}+D_0A_0g\omega[c]\cdot[B]\cdot\{U\}\\&+k_w\{U\}-k_p\nabla^2\{U\}+C_0\omega\{U\}+\rho A_0\omega^2[a]\cdot\{U\}=0.\end{aligned} \tag{38}$$

The matrix form of viscoelastic tapered micro-rod can be obtained as:

$$\left(\omega^2[M]+\omega[C]+[K]\right)\{u\}=\{0\} \tag{39}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping, and stiffness matrices, respectively that are expressed as:

$$\begin{aligned}&[M]=\rho A_0[a]\\&[C]=D_0A_0g[a]\cdot[D]-E_0A_0g[a]\cdot[B]-E_0A_0g[b]\cdot[A]\\&+D_0A_0g[c]\cdot[B]+C_0\\&[K]=D_0A_0[a]\cdot[D]-E_0A_0[a]\cdot[B]-E_0A_0[b]\cdot[A]\\&+D_0A_0[c]\cdot[B]+k_w-k_p[B].\end{aligned} \tag{40}$$

The various boundary conditions using DQM are given by:
Simply supported:

$$u_i=0\quad\sum_{j=1}^{N}\left[A^{(2)}{}_{ij}u_j\right]=0.$$

Clamped:

$$u_i=0\quad\sum_{j=1}^{N}\left[A^{(1)}{}_{ij}u_j\right]=0. \tag{41}$$

Free:

$$\sum_{j=1}^{N}\left[A^{(3)}{}_{ij}u_j\right]=0,\quad\sum_{j=1}^{N}\left[A^{(2)}{}_{ij}u_j\right]=0.$$

Table 1. The mechanical and geometrical properties of tapered micro-rod.

| | |
|---|---|
| Density (Kg/m^3) | 7300 |
| Poisson's ratio | 0.3 |
| Elastic module (Pa) | 5.76E+07 |
| Maximum radius to length (R/L) | 0.125 |
| Winkler coefficient (N/m$^3$) | 1.00E+06 |
| Pasternak Coefficient (N/m) | 1.00E+07 |
| Damping coefficient of the elastic medium (Kg/m^2/s) | 1.49E+04 |
| Structural damping coefficient (1/s) | 0.0061 |
| CT | $L_1/L = 0$ $L_0/L = 0$ |
| MCST | $L_1/L = 0$ $L_0/L = 0.5$ |
| SGT | $L_1/L = 0.5$ $L_0/L = 0.5$ |

Table 2. The first five non-dimensional natural frequencies of a tapered micro-rod without damping coefficients and three material length scale parameters for C-C and C-F boundary conditions.

| Boundary conditions | Mode | Present work | Simsek [30] | Exact [40] | PT [40] |
|---|---|---|---|---|---|
| C-C | 1 | 3.1416 | 3.1416 | 3.1416 | 3.1416 |
| | 2 | 6.2832 | 6.2832 | 6.2832 | 6.2832 |
| | 3 | 9.4248 | 9.4248 | 9.4248 | 9.4248 |
| | 4 | 12.5654 | 12.5663 | 12.5664 | 12.5664 |
| | 5 | 15.7354 | 15.7080 | 15.7080 | 15.7080 |
| C-F | 1 | 1.5708 | 1.5708 | 1.5708 | 1.5708 |
| | 2 | 4.7124 | 4.7123 | 4.7124 | 4.7124 |
| | 3 | 7.8540 | 7.8539 | 7.8540 | 7.8540 |
| | 4 | 10.9955 | 10.9955 | 10.9956 | 10.9956 |
| | 5 | 14.0560 | 14.1371 | 14.1372 | 14.1372 |

Table 3. The influence of aspect ratios on the non-dimensional first three frequencies based on various theories.

| L/R | m | CT | MCST | SGT |
|---|---|---|---|---|
| 2 | 1 | 5.739602533 | 10.52520902 | 12.60207498 |
| | 2 | 14.55293989 | 101.0917714 | 136.8329423 |
| | 3 | 28.24294621 | 478.1647609 | 659.752744 |
| 4 | 1 | 3.085807326 | 5.538589555 | 6.586412952 |
| | 2 | 7.285328988 | 50.54709996 | 68.41734495 |
| | 3 | 14.18665796 | 239.1604319 | 329.9544915 |
| 6 | 1 | 2.208297581 | 3.881351619 | 4.585170488 |
| | 2 | 4.863514986 | 33.69888821 | 45.61215221 |
| | 3 | 9.50369879 | 159.4924807 | 220.0218553 |
| 8 | 1 | 1.773666015 | 3.056041089 | 3.587224069 |
| | 2 | 3.653248891 | 25.27479158 | 34.20956069 |
| | 3 | 7.16418906 | 119.6586243 | 165.0556235 |
| 10 | 1 | 1.515227347 | 2.563198997 | 2.990372553 |
| | 2 | 2.927699784 | 20.22034117 | 27.36800971 |
| | 3 | 5.762156566 | 95.75840621 | 132.0759535 |
| 20 | 1 | 1 | 1.590851079 | 1.807872892 |
| | 2 | 1.483965239 | 10.11150015 | 13.68493848 |
| | 3 | 2.972432643 | 47.95869365 | 66.11713518 |

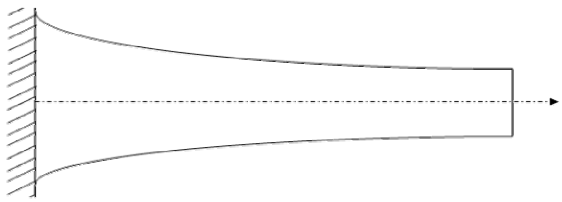

Fig. 1. An schematic of viscoelastic tapered micro-rod.

## 5. The numerical results and discussion

In this article, the free vibration analysis of viscoelastic tapered micro-rod based on strain gradient theory resting on visco-pasternak foundation using DQM is investigated. The mechanical and geometric properties of micro-rod are defined in Table 1.

The results of this research are compared with the obtained results by Simsek [30] and Kiani [40] for the first five non-dimensional natural frequencies of a tapered micro-rod without damping coefficients and three material length scale parameters with considering C-C and C-F boundary conditions in Table 2 that are good agreements between them.

Table 3 shows the influence of aspect ratios on the non-dimensional first three frequencies based on various theories. It can be seen from this Table that the non-dimensional frequencies decrease with an increase in the aspect ratio for a specified *m*. Moreover, the obtained results illustrate that an increase in the value of the aspect ratio gives rise to an decrease in the dimensionless natural frequency for all modes. It is seen that the difference between dimensionless natural frequency obtained by CT, MSCT, and SGT becomes more prominent for higher modes. Applying the size-dependant theory, the stiffness of micro-rod increases, and then the dimensionless natural frequencies evaluated by SGT is higher than the other theories.

Fig. 1 shows an schematic view of viscoelastic tapered micro-rod.

Fig. 2 illustrates the effect of internal damping coefficient ($C_0$) on the non-dimensional frequencies for CT, MSCT, and SGT. It is obvious that the non-dimensional frequencies increase by increasing the damping coefficient for all theories. Furthermore, at the specified value of ($C_0$), the variation of non-dimensional natural frequency is approximately smooth. Also, it is shown from this figure that the non-dimensional frequency for SGT is higher than that of the other theories.

In Fig. 3, the variation of non-dimensional natural frequency with structural damping coefficient of visco-elastic micro-rod for classical and non-classical theories is depicted. It can be seen that the obtained dimensionless frequency by non-classical theories tend to have a greater reduction than with respect to the classical theory for structural damping

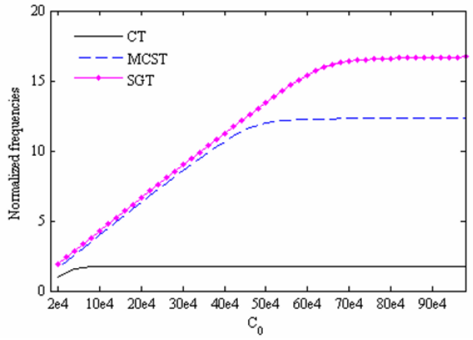


Fig. 2. The non-dimensional natural frequency versus the different damping coefficient on the vibration of the micro-rod for different theories.

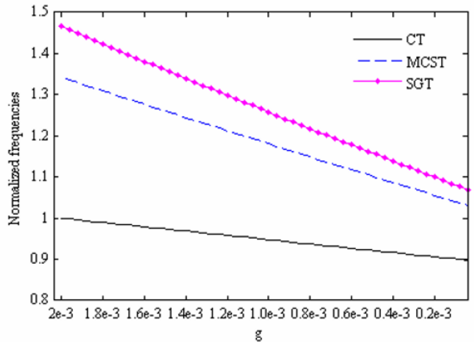


Fig. 3. The non-dimensional natural frequency versus the different structural damping coefficient for different theory.

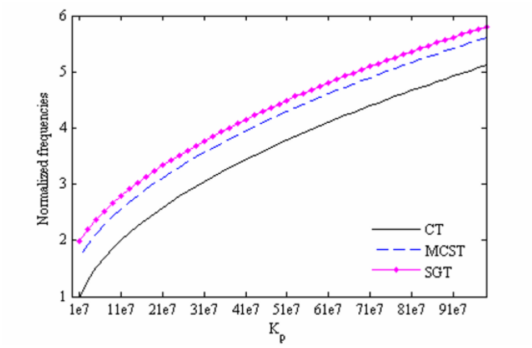


Fig. 4. The non-dimensional natural frequency versus the different Pasternak modulus for different size-dependent effects.

coefficient.

The variation of dimensionless natural frequency versus Pasternak and Winkler foundation modulli for different theories is shown in Figs. 4 and 5, respectively. The results illustrate that by increasing Pasternak modulus, the non-dimensionless frequency increases. This fact is due to stiffer of micro-rod structure.

Fig. 6 indicates the effect of the length to radius on non-dimensional natural frequencies of vicso-elastic micro rods for different modes corresponding to MCST and SGT. It is observed that the obtained non-dimensional natural frequencies by MCST and SGT decrease by increasing in material length

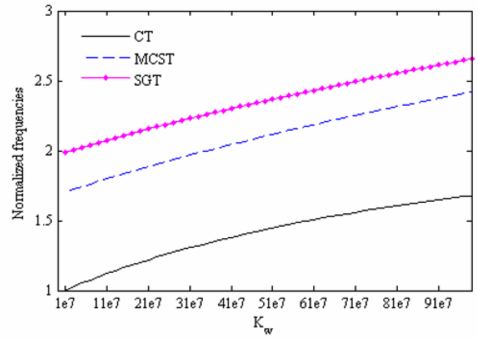


Fig. 5. The non-dimensional natural frequency versus the different Winkler modulus for different size-dependent effects.

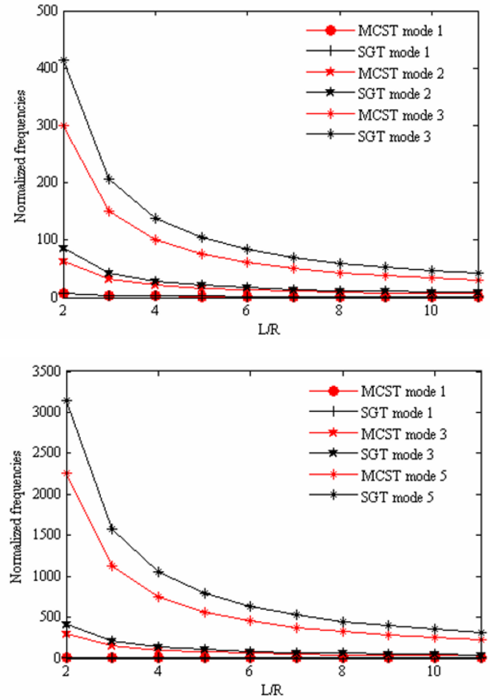


Fig. 6. The non-dimensional natural frequency versus aspect ratios based on various size-dependent effects.

to radius ratio. Additionally, it can be found that for higher modes, the difference between non-dimensional natural frequencies corresponds to MCST and SGT is more significant than lower modes.

## 6. Conclusions

In this work, the vibration analysis of visco-elastic tapered micro-rod resting on visco-pasternak foundation is studied based on strain gradient theory. The effects of visco-Pasternak medium such as Winkler, Pasternak and damping coefficients are taken into account. C-C boundary condition is considered and DQM is utilized to obtain an approximate solution to the vibration problem of strain gradient visco elastic tapered micro rod. The results of this research show that the frequency is significantly affected by the Winkler, Pasternak, structural and internal damping coefficients of viscoelastic micro-rod. It is illustrated that by increasing Pasternak foundation modulus, the dimensionless frequency increases, also the obtained non-dimensional natural frequencies by MCST and SGT decrease by increasing the material length to radius ratio. It can be seen that the obtained dimensionless frequency by non-classical theories tend to have a greater reduction than with respect to the classical theory for structural damping coefficient. Also, it is shown that the non-dimensional frequency for SGT is higher than that of the other theories. It is obvious that the non-dimensional frequencies increase by increasing the damping coefficient for all theories. Furthermore, at the specified value of ($C_0$), the variation of non-dimensional natural frequency is approximately smooth.

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