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An equivalent plate model for corrugated-core sandwich panels †

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Abstract

This paper suggests an equivalent plate model to analyze the mechanical behavior of corrugated-core sandwich panels under tensile and bending loads. A homogenization-based theory based on the equivalent energy method is used to obtain the stiffness matrices of corrugated cores of a sandwich panel. Equivalent continuum layers with orthotropic elastic constants corresponding to the membrane and bending stiffness terms of the corrugated layers are determined by using classical lamination theory (CLT). The importance of this work is that an equivalent plate model for corrugated-core sandwich panels can be easily obtained by combining the equivalent energy method and the CLT. The proposed equivalent model is verified by numerical simulations of sandwich panels with sinusoidal and trapezoidal corrugated cores.

Keywords: Sandwich panels; Corrugated plates; Homogenization-based model; Equivalent energy method; Classical lamination theory

1. Introduction

The design of corrugated plates and shells is a major issue in civil, marine and aerospace engineering to enhance the strength and stiffness of structures. A sandwich panel constructed with corrugated laminates can provide a much better strength and stiffness of structures. Numerical simulation of these complex-shaped structures requires a great deal of human effort in preparing analysis models with a fine mesh. An equivalent model for corrugated laminates and corrugated-core sandwich panels is highly required to simplify analysis models of their complex configurations. However, it is not easy to obtain an equivalent model representing the mechanical behavior of corrugated-core sandwich panels under varied situation loads. To solve these problems, effective methods are being studied constantly to develop a simplified model for corrugated laminates.

Homogenization-based modeling techniques for corrugated plates and laminating materials with different mechanical properties have attracted considerable attention. Seydel [1] studied the shear strength of corrugated panels by using a simple formula for an orthotropic plate with equivalent stiffness properties when the direction of the corrugation is constant and the wavelength of the curve is very short as compared with the side length of the plate. For the triangular corrugation, Wang and Chung [2] described truss-core plate models and compared the accuracy with finite element (FE) simulations. Valdevit et al. [3] studied possible combinations of different layers of triangular corrugations. Briassoulis [4] and McFarland [5] presented the equivalent bending stiffness for corrugated plates with sine-wave and rectangular corrugations. Kress and Winker [6-8] derived the exact numerical equation of an equivalent orthotropic plate for circular corrugations. They also studied the effect of geometry changes by using FE analysis. Xia et al. [9] suggested an equivalent plate modeling method that allows for the geometry of arbitrary corrugations to be analyzed easily and accurately.

Some approaches based on homogenization-based theory, determining the equivalent stiffness properties of corrugated-core sandwich panels, have been proposed. Luo et al. [10] developed numerical expressions for the bending stiffness of a sandwich panel which has a flexural plate between two flat plates. Carlsson et al. [11, 12] presented the homogenized properties such as transverse shear stiffness and bending stiffness of the corrugated board by an analytical method. Aboura et al. [13], Talbi et al. [14] and Buannic et al. [15] studied homogenization model based on laminate theory for sandwich panels, and compared numerical results with experimental data. Biancolini [16] used an FE numerical approach for evaluating the stiffness parameters.

In this paper, we propose a simple approach for evaluating equivalent plate models of corrugated-core sandwich panels. First, the stiffness matrices of corrugated cores of a sandwich panel are obtained by using the homogenization-based modeling method based on the equivalent energy method [9], which

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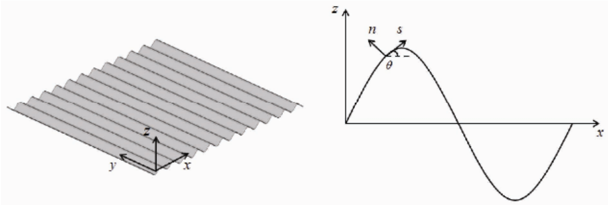


Fig. 1. Definition of the coordinate systems.

is very efficient for arbitrary corrugations. Second, the membrane and bending stiffness terms of the corrugated layers are converted into equivalent continuum layers with orthotropic elastic constants. We present an efficient method to determine the thickness and the orthotropic elastic constants of the equivalent continuum layers by matching the stiffness terms of the corrugated layers with the reduced stiffness components in the classical lamination theory (CLT). Finally, an equivalent plate to account for the mechanical behavior of corrugated-core sandwich panels can be easily determined by integrating mechanical properties and geometric parameters through the thicknesses of the equivalent continuum layers. The effectiveness of the present method is verified by comparing the results of FE analyses of corrugated-core sandwich panels and the equivalent plate models.

2. Equivalent modeling for corrugated plates

In this study, we consider a plate with periodic corrugations in one direction as shown in Fig. 1. The global coordinate system (*x, y, z*) is defined by the corrugation direction, and the local coordinate system that changes along the corrugated surface are indicated in this figure. The local coordinate system (*s, n*) is defined by the tangent direction to the corrugated surface in the *xz* plane, and the normal direction to the corrugated surface in the *xz* plane, respectively. Hence, the position ***r*** of a point on the corrugated surface can be written as

$$\mathbf{r}(s,y) = x(s)\mathbf{i} + y\mathbf{j} + z(s)\mathbf{k}, \tag{1}$$

where ***i, j*** and ***k*** are the unit vectors in *x, y* and *z* directions, respectively.

The macroscopic mechanical behavior of the corrugated plate can be represented by a Kirchhoff orthotropic plate ignoring the coupling stiffness matrix, and the corresponding constitutive equation can be written as

$$\begin{Bmatrix} \bar{N}_x \\ \bar{N}_y \\ \bar{N}_{xy} \\ \bar{M}_x \\ \bar{M}_y \\ \bar{M}_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & 0 & 0 & 0 & 0 \\ \bar{A}_{12} & \bar{A}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{A}_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{D}_{11} & \bar{D}_{12} & 0 \\ 0 & 0 & 0 & \bar{D}_{12} & \bar{D}_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{D}_{66} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_{xy} \\ \bar{\kappa}_x \\ \bar{\kappa}_y \\ \bar{\kappa}_{xy} \end{Bmatrix} \tag{2}$$

where $\bar{\varepsilon}_x$, $\bar{\varepsilon}_y$, $\bar{\varepsilon}_{xy}$, $\bar{\kappa}_x$, $\bar{\kappa}_y$ and $\bar{\kappa}_{xy}$ denote the effective strain and curvature components, and $\bar{N}_x$, $\bar{N}_y$, $\bar{N}_{xy}$, $\bar{M}_x$, $\bar{M}_y$ and $\bar{M}_{xy}$ denote the effective force and moment components with respect to the middle plane of the corrugated plate.

The constitutive equation in the local coordinate system is expressed as follows:

$$\begin{Bmatrix} N_s \\ N_y \\ N_{sy} \\ M_s \\ M_y \\ M_{sy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_s \\ \varepsilon_y \\ \varepsilon_{sy} \\ \kappa_s \\ \kappa_y \\ \kappa_{sy} \end{Bmatrix}. \tag{3}$$

The strain energy over a unit cell whose length corresponding to one period of the corrugation is calculated as

$$U = \frac{1}{2}\iint N^T S N ds dy, \tag{4}$$

where $N = \left[N_s, N_y, N_{sy}, M_s, M_y, M_{sy}\right]^T$ and S indicates the inverse matrix for the stiffness matrix in Eq. (3). The stiffness terms in Eq. (2) can be related to the constitutive equation in Eq. (3) by using the equivalent energy method. The strain energy U of the corrugated plate should be equal to the strain energy $\bar{U}$ of the equivalent orthotropic plate such that

$$U = \bar{U} = \frac{1}{2}(2b)(2c)\begin{Bmatrix} \bar{\varepsilon} \\ \bar{\kappa} \end{Bmatrix}^T \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{D} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon} \\ \bar{\kappa} \end{Bmatrix}, \tag{5}$$

where 2c is the period of the corrugation and b is the width of the unit cell. In order to obtain the stiffness terms in Eq. (2) as a function of the corrugation geometry, different boundary conditions are applied to the corrugated plate and the equivalent orthotropic plate, respectively. Xia et al. [9] presented a homogenization method for a basic unit cell by using the six strain boundary conditions. In this method, the stiffness terms of the equivalent plate are estimated by applying in-plane and out-of-plane deformations. Detailed procedures for the derivation of the equivalent orthotropic plate from Eq. (5) are described well in Ref. [9]. The stiffness terms are listed in Table 1 in which parameters I_1 and I_2 are dependent on the corrugation shape. Specifically, the parameters in the stiffness terms for sine-wave and trapezoidal corrugations are given in Table 2.

3. Equivalent modeling for corrugated-core sandwich panels

In this study, we consider sandwich panels constructed with corrugated layers between flat plates as shown in Fig. 2. The corrugated laminates are glued together with layers of flat

Table 1. Stiffness terms of corrugated plates.

Stiffness term	Expression
$\bar{A}_{11}$	$\dfrac{2c}{\dfrac{I_1}{A_{11}}+\dfrac{I_2}{D_{11}}}$
$\bar{A}_{12}$	$\dfrac{A_{12}}{A_{11}}\bar{A}_{11}$
$\bar{A}_{22}$	$\dfrac{A_{12}}{A_{11}}\bar{A}_{12}+\dfrac{l}{c}\dfrac{A_{11}A_{22}-A_{12}^2}{A_{11}}$
$\bar{A}_{66}$	$\dfrac{c}{l}A_{66}$
$\bar{D}_{11}$	$\dfrac{c}{l}D_{11}$
$\bar{D}_{12}$	$\dfrac{D_{12}}{D_{11}}\bar{D}_{11}$
$\bar{D}_{22}$	$\dfrac{1}{2c}(I_2A_{22}+I_1D_{22})$
$\bar{D}_{66}$	$\dfrac{c}{l}D_{66}$

Table 2. Parameters in the stiffness terms for sin-wave and trapezoidal corrugations.

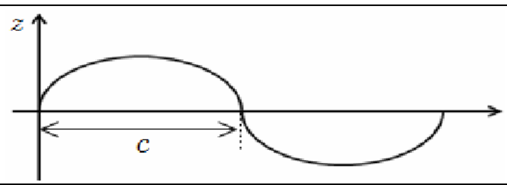
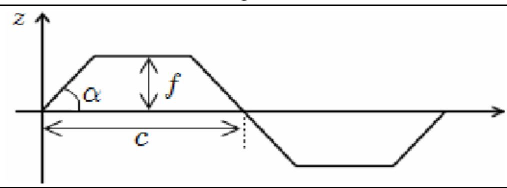
Sin-wave

$I_1=\int_0^{2l}\left(\dfrac{dx}{ds}\right)^2 ds$ $I_2=\int_0^{2l} z^2 ds$
Trapezoid

$I_1=\dfrac{4f\cos\alpha}{3\sin\alpha}+2c-\dfrac{4f}{\tan\alpha}$ $I_2=\dfrac{4f^3}{3\sin\alpha}+2f^2\left(c-\dfrac{2f}{\tan\alpha}\right)$

plate. The corrugated laminates are symmetric with respect to the middle plane of the sandwich panel, and the corrugation directions of the stacking layers are mutually orthogonal with each other. In general, the membrane stiffness and the bending stiffness along the corrugation direction are significantly different from those along the orthogonal direction of the corrugation direction. However, the corrugated laminates with mutually perpendicular corrugation directions show much higher membrane stiffness and bending stiffness in both directions. In particular, it is possible to obtain a symmetric mechanical behavior with respect to the middle plane of the corrugated-core sandwich panel when the corrugated laminates are symmetric with respect to the middle plate.

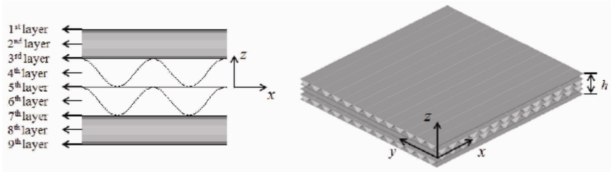


Fig. 2. Geometry of the corrugated-core sandwich panel with four-corrugated layers.

The layer number of the corrugated-core sandwich panel is assigned sequentially from the top plate to the bottom plate, as indicated in Fig. 2 where represents the entire thickness of the corrugated-core sandwich panel. As shown in Fig. 2, the even layers of the sandwich panel (2, 4, 6, 8 layers) are the corrugated layers, and the odd layers are the flat plates (1, 3, 5, 7, 9 layers) joining the corrugated layers. For convenience, we describe the present method for the sandwich panel composed of four-corrugated layers shown in Fig. 2. The stiffness matrices of the individual corrugated layers can be easily evaluated by using the equivalent energy method mentioned described in the previous section. As a result, the effective forces and moments in each corrugated layer can be expressed by the effective strains and curvatures:

$$\begin{Bmatrix}\mathbf{N}_{(k)}\\ \mathbf{M}_{(k)}\end{Bmatrix}=\begin{bmatrix}\bar{A}_{ij} & 0\\ 0 & \bar{D}_{ij}\end{bmatrix}_{(k)}\begin{Bmatrix}\bar{\boldsymbol{\varepsilon}}_{(k)}\\ \bar{\boldsymbol{\kappa}}_{(k)}\end{Bmatrix},\quad k=2,4,6,8, \tag{6}$$

where k is the sequential layer number in the corrugated-core sandwich panel.

The distances from the middle plane of the corrugated-core sandwich panel to the middle planes of individual corrugated layers should be considered in the evaluation of equivalent stiffness matrix of the corrugated-core sandwich panel. It is well known that the classical lamination theory (CLT) is an efficient approach to estimate the mechanical behavior of laminated layers by integrating mechanical properties and geometric parameters through the thicknesses of continuum layers. However, the equivalent plates in Eq. (6) cannot be applied directly to the CLT because mechanical properties and geometric parameters of laminated continuum layers are not specified in the stiffness terms. To use the CLT, in other way, we assume that the $\left[\bar{A}_{ij}\right]$ and $\left[\bar{D}_{ij}\right]$ in Eq. (6) can be written by

$$\left[\bar{A}_{ij}\right]=\left[\bar{Q}_{ij}\right]t^*, \tag{7}$$

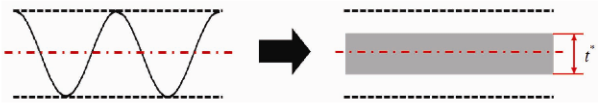


Fig. 3. An equivalent continuum layer of a corrugated layer.

$$\left[\bar{D}_{ij}\right]=\left[\bar{Q}_{ij}\right]\frac{\left(t^*\right)^3}{12}, \tag{8}$$

where t^* indicates the thickness of an equivalent continuum layer of the corrugated layer, and $\left[\bar{Q}_{ij}\right]$ are the so-called reduced stiffness components for a plane stress state. Note that Eqs. (7) and (8) correspond to the membrane and bending stiffness terms of the continuum laminated layers in the CLT, respectively. The reduced stiffness components $\left[\bar{Q}_{ij}\right]$ are expressed by

$$\begin{aligned}&\bar{Q}_{11}=\frac{\bar{E}_1}{1-\bar{\nu}_{12}\bar{\nu}_{21}}, \qquad \bar{Q}_{22}=\frac{\bar{E}_2}{1-\bar{\nu}_{12}\bar{\nu}_{21}}\\ &\bar{Q}_{12}=\bar{\nu}_{12}\bar{Q}_{22}=\bar{\nu}_{21}\bar{Q}_{11}, \qquad \bar{Q}_{66}=\bar{G}_{12}\end{aligned} \tag{9}$$

with

$$\frac{\bar{\nu}_{12}}{\bar{E}_1}=\frac{\bar{\nu}_{21}}{\bar{E}_2}, \tag{10}$$

where $\bar{E}_1, \bar{E}_2, \bar{\nu}_{12}$ and $\bar{\nu}_{21}$ are the orthotropic elastic constants of the equivalent continuum layer. The orthotropic elastic constants in Eq. (9) with the reciprocal relation in Eq. (10) can be evaluated by using Eqs. (7) and (8), respectively, when the thickness of the equivalent continuum layer is given. However, the orthotropic elastic constants evaluated by using Eq. (7) are different from those by using Eq. (8) when the thickness of the equivalent continuum layer is taken arbitrarily. In this study, the thickness of the equivalent continuum layer is determined to minimize the differences between the elastic properties for the membrane deformation in Eq. (7) and those for the bending deformation in Eq. (8). As a result, the equivalent continuum layers have the orthotropic elastic constants and the thickness so as to represent the mechanical behavior for both membrane stiffness and bending stiffness of the corrugated layers. Fig. 3 illustrates an equivalent continuum layer with the thickness t^* for a sine-wave corrugated layer.

Since the corrugated layers or the corrugated cores of a sandwich panel are modeled by the equivalent continuum layers as illustrated in Fig. 4, the equivalent stiffness matrix of the corrugated-core sandwich panel can then be estimated by integrating mechanical properties and geometric parameters through the equivalent continuum layers. Although empty spaces exist between the equivalent continuum layers and the flat plates, the equivalent continuum layers converted from the corrugated layers by using the CLT can be assumed to be deformed in accordance with the effective strains and curvatures with respect to the middle plane of the corrugated-core sandwich panel. Hence, the equivalent stiffness terms of the corrugated-core sandwich panel can be expressed as

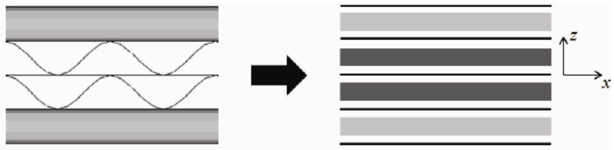


Fig. 4. Equivalent continuum layers of corrugated layers or corrugated cores of a sandwich panel.

For the flat plates $(k=1, 3, 5, 7, 9)$,

$$\left[\hat{A}^*_{ij}\right]=\left[Q_{ij}\right]_k\left(z^t_k-z^b_k\right), \tag{11}$$

$$\left[\hat{D}^*_{ij}\right]=\frac{1}{3}\left[Q_{ij}\right]_k\left(\left(z^t_k\right)^3-\left(z^b_k\right)^3\right). \tag{12}$$

For the corrugated layers $(k=2, 4, 6, 8)$,

$$\left[\bar{A}^*_{ij}\right]=\left[\bar{Q}_{ij}\right]_k\left(z^{*t}_k-z^{*b}_k\right), \tag{13}$$

$$\left[\bar{D}^*_{ij}\right]=\frac{1}{3}\left[\bar{Q}_{ij}\right]_k\left(\left(z^{*t}_k\right)^3-\left(z^{*b}_k\right)^3\right), \tag{14}$$

where $\left[Q_{ij}\right]_k$, z^t_k and z^b_k in Eqs. (11) and (12) are the reduced stiffness components and *z* coordinates on the top and bottom surfaces of the flat plates, respectively, and $\left[\bar{Q}_{ij}\right]_k$, z^{*t}_k and z^{*b}_k in Eqs. (13) and (14) are the reduced stiffness components determined from Eqs. (7) and (8) and *z* coordinates on the top and bottom surfaces of the equivalent continuum layers, respectively.

Consequently, the constitutive equation of the equivalent plate model of the corrugated-core sandwich panel can be expressed by

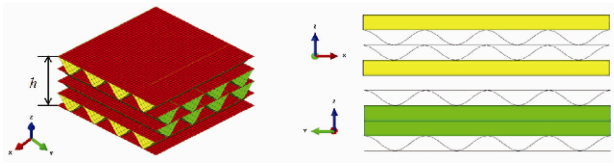
$$\begin{Bmatrix}\tilde{N}^*_x\\ \tilde{N}^*_y\\ \tilde{N}^*_{xy}\\ \tilde{M}^*_x\\ \tilde{M}^*_y\\ \tilde{M}^*_{xy}\end{Bmatrix}=\begin{bmatrix}\tilde{A}^*_{11} & \tilde{A}^*_{12} & 0 & 0 & 0 & 0\\ \tilde{A}^*_{12} & \tilde{A}^*_{22} & 0 & 0 & 0 & 0\\ 0 & 0 & \tilde{A}^*_{66} & 0 & 0 & 0\\ 0 & 0 & 0 & \tilde{D}^*_{11} & \tilde{D}^*_{12} & 0\\ 0 & 0 & 0 & \tilde{D}^*_{12} & \tilde{D}^*_{22} & 0\\ 0 & 0 & 0 & 0 & 0 & \tilde{D}^*_{66}\end{bmatrix}\begin{Bmatrix}\tilde{\varepsilon}^*_x\\ \tilde{\varepsilon}^*_y\\ \tilde{\varepsilon}^*_{xy}\\ \tilde{\kappa}^*_x\\ \tilde{\kappa}^*_y\\ \tilde{\kappa}^*_{xy}\end{Bmatrix}, \tag{15}$$

with

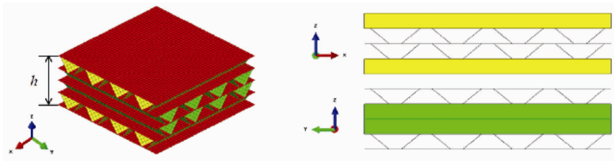
$$\left[\tilde{A}^*_{ij}\right]=\left[\hat{A}^*_{ij}\right]+\left[\bar{A}^*_{ij}\right], \tag{16}$$

$$\left[\tilde{D}^*_{ij}\right]=\left[\hat{D}^*_{ij}\right]+\left[\bar{D}^*_{ij}\right], \tag{17}$$

where $\tilde{\varepsilon}_x$, $\tilde{\varepsilon}_y$, $\tilde{\varepsilon}_{xy}$, $\tilde{\kappa}_x$, $\tilde{\kappa}_y$ and $\tilde{\kappa}_{xy}$ denote the effective



(a)



(b)

Fig. 5. Representative parts of finite element models and cross-sectional views of corrugated-core sandwich panels: (a) sine-wave corrugation; (b) trapezoidal corrugation.

strain and curvature components, and $\tilde{N}_x$, $\tilde{N}_y$, $\tilde{N}_{xy}$, $\tilde{M}_x$, $\tilde{M}_y$ and $\tilde{M}_{xy}$ denote the effective force and moment components with respect to the middle plane of the corrugated-core sandwich panel.

4. Numerical examples

The present method is validated by comparing FE results for corrugated-core sandwich panels and equivalent plate models. A commercial code ABAQUS/standard is used to perform the FE analyses. A linear elastic material with Young's modulus $E = 210\,\text{GPa}$ and Poisson's ratio $\nu = 0.3$ is used for the FE simulations of the corrugated-core sandwich panels. The equivalent stiffness terms in Eq. (15) are directly used for the equivalent plate models in ABAQUS/standard. Sandwich panels with sine-wave and trapezoidal corrugations are considered in this study. Representative parts of FE models using 4-node shell elements (S4R) are shown in Fig. 5. Horizontal and vertical lengths of the corrugated-core sandwich panels are 500 *mm*. and the average size of elements is 1 *mm*. The total thickness is varied by $h = 8, 10, 12, 14, 16\,\text{mm}$ to investigate the effect of the thickness on the validity of the present method. The distances between the corrugation crests are $2.0, 2.5, 3, 3.5, 4.0\,\text{mm}$ for $h = 8, 10, 12, 14, 16\,\text{mm}$, respectively.

The thicknesses of the equivalent continuum layers of the corrugated layers or the corrugated cores with sine-wave and trapezoidal corrugations are determined to minimize the differences between orthotropic elastic constants evaluated by using Eq. (7) and those by using Eq. (8). The thickness of the equivalent continuum layers depends on the corrugation height and the corrugation shape. The orthotropic elastic constants evaluated by using Eqs. (7) and (8), which represent the membrane and the bending deformations of the corrugated layers, respectively, are plotted in Fig. 6 when the total thickness of the corrugated-core sandwich panel with the sine-wave corrugation is 16 *mm*. As indicated in these figures, the thicknesses in Eqs. (7) and (8) to give the same orthotropic elastic

Table 3. The thickness t^* and orthotropic elastic constants of the equivalent continuum layers for sin-wave and trapezoidal corrugations.

Continuum thickness & elastic constants		Total thickness h (*mm*)				
		8	10	12	14	16
t^* (*mm*)	Sin-wave corrugation	1.67	2.09	2.53	2.96	3.38
	Trapezoidal corrugation	1.73	2.16	2.59	3.02	3.45
$\bar{E}_1$ (*MPa*)	Sin-wave corrugation	1.65 x10^5	1.28 x 10^5	1.10 x 10^5	9.42 x 10^4	8.24 x 10^4
	Trapezoidal corrugation	1.37 x 10^5	1.15 x 10^5	1.01 x 10^5	9.21 x 10^4	8.53 x 10^4
$\bar{E}_2$ (*MPa*)	Sin-wave corrugation	6.03 x 10^4	3.16 x 10^4	1.24 x 10^4	1.06 x 10^4	9.29 x 10^3
	Trapezoidal corrugation	5.73 x 10^3	2.77 x 10^3	1.50 x 10^3	8.95 x 10^2	5.65 x 10^2
$\bar{\nu}_{12}$	Sin-wave corrugation	0.32	0.32	0.33	0.34	0.34
	Trapezoidal corrugation	0.31	0.32	0.33	0.33	0.34

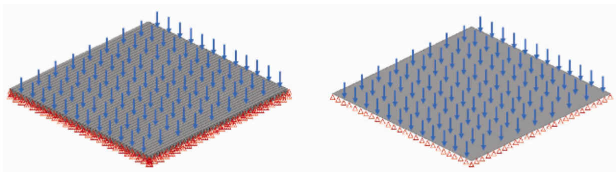


Fig. 6. Loads and boundary constraints for corrugated-core sandwich panels and equivalent plates.

constants $\bar{E}_1$ and $\bar{E}_2$, respectively, are almost similar. In other words, the orthotropic elastic constants evaluated by using Eq. (7) are almost similar to those by using Eq. (8) for a chosen thickness of the equivalent continuum layer. In our numerical examples, the thickness and the orthotropic elastic constants of the equivalent continuum layers of the corrugated layers in the sandwich panels with $h = 8, 10, 12, 14\,\text{mm}$ are similarly determined by using Eqs. (7) and (8). It means that the membrane and bending stiffness terms resulted from the equivalent energy method can be converted into the equivalent continuum layers with the orthotropic elastic constants. The thicknesses and the orthotropic elastic constants of the equivalent continuum layers for the sandwich panels with sine-wave and trapezoidal corrugations are listed in Table 3.

Uniform pressure $p = 0.2\,\text{MPa}$ is applied on the top surfaces of the corrugated-core sandwich panels and the equivalent plates with boundary constraints on all displacement and rotation at the sides of FE models, as illustrated in Fig. 7. The stiffness matrix of the corrugated-core sandwich panels can be obtained by using the CLT for the equivalent continuum layers and the flat plates. The vertical displacements of the corrugated-core sandwich panels and the equivalent plates are plotted in Figs. 8 and 9 when the total thicknesses are $8\,\text{mm}$ and $16\,\text{mm}$, respectively. Note that the sandwich panels with sine-wave corrugations show higher stiffnesses than the sandwich

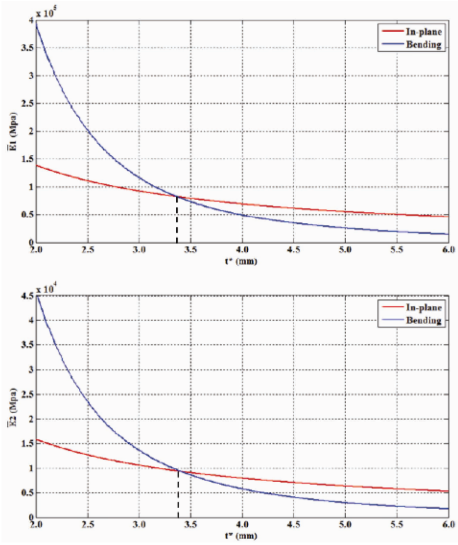


Fig. 7. Orthotropic elastic constants $\overline{E}_1$ and $\overline{E}_2$ with varying the thickness t^* of the equivalent continuum layer when the total thickness of the corrugated-core sandwich panels with sine-wave corrugation is $h = 16$ mm.

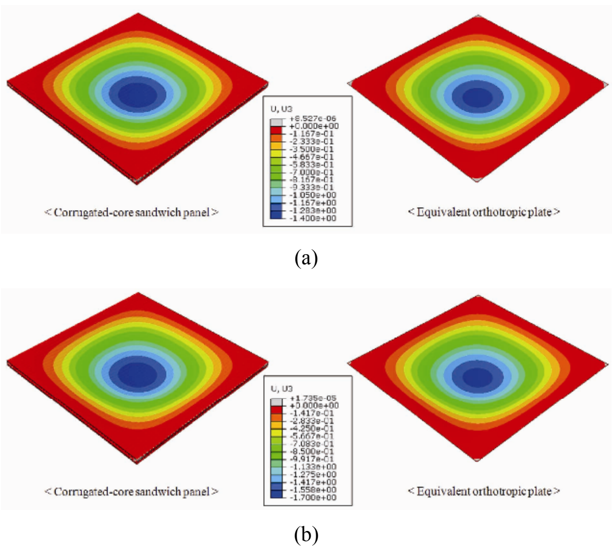


Fig. 8. Displacement contours of finite element results for the corrugated-core sandwich panels with the thickness $h = 8$ mm and the equivalent plates: (a) sine-wave corrugation; (b) trapezoidal corrugation.

panels with trapezoidal corrugations. We obtained similar results for the corrugated-core sandwich panels with $h = 10, 12, 14$ mm. The vertical displacement contours indicate that the equivalent plate models can be used effectively instead of the complex-shaped models of corrugated-sandwich panels. The relative displacement errors of vertical displacements at the center of the corrugated-core sandwich panels $h = 8, 10, 12, 14, 16$ mm and their corresponding equivalent plates are plotted in Fig. 10. The relative displacement errors for the trapezoidal corrugations are relatively higher than those for the sine-wave corrugations. As the total thickness

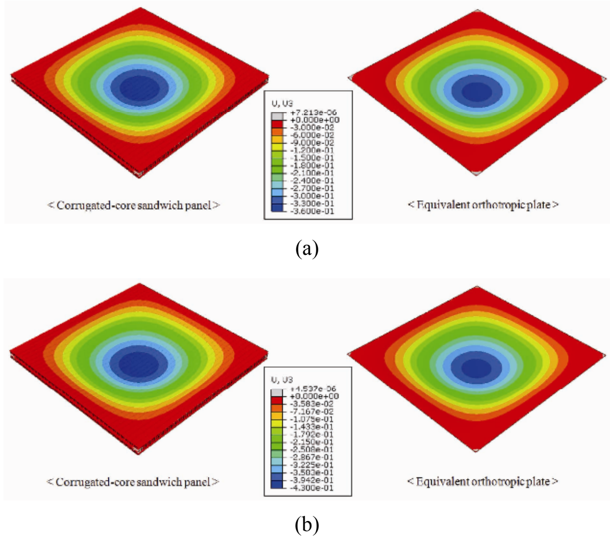


Fig. 9. Vertical displacement contours of finite element results for the corrugated-core sandwich panels with the thickness $h = 16$ mm and the equivalent plates: (a) sine-wave corrugation; (b) trapezoidal corrugation.

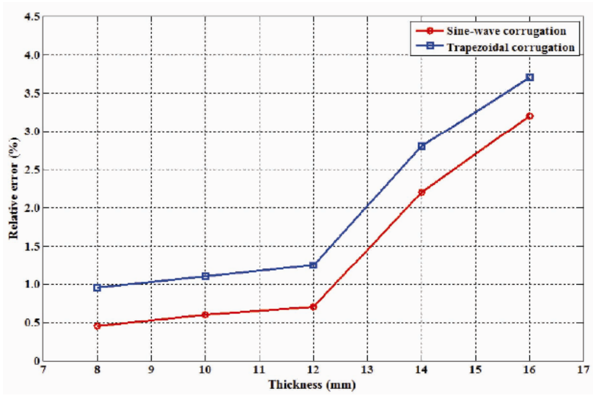


Fig. 10. The relative displacement errors between finite element results of corrugated-core sandwich panels and their equivalent models with varying the total thickness.

increases, the relative displacement errors increase because the ratio between the thickness and the distance between supports in the model increases. We confirm that the accuracy of the equivalent plate model is improved as the ratio between the thickness and support distance of the model decreases.

5. Conclusions

We have proposed equivalent plate models of corrugated-core sandwich panels by using a homogenization-based model for corrugated layers and the CLT. The stiffness matrices of the corrugated layers or the corrugated cores of a sandwich panel can be effectively obtained by using the equivalent energy method. The thickness and the orthotropic elastic properties of the equivalent continuum layers representing the mechanical behavior of the corrugated layers are determined by matching the membrane and bending stiffness terms of the corrugated layers with the reduced stiffness components of the equivalent continuum layers. As a result, the equivalent stiffness matrices of the corrugated-core sandwich panels can be easily obtained by integrating mechanical properties and geometric parameters through the equivalent continuum layers. The present method was verified by comparing FE results for corrugated-core sandwich panels with sine-wave and trapezoidal corrugations and their corresponding equivalent plates. When the total thickness is not large compared to the distance between supports, we confirmed that the present method can provide an easy and efficient way to analyze corrugate-core sandwich panels.

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References

[1] V. E. Seydel, Shear buckling of corrugated plates (in German), *DVL-Jahrbuch* (1931) 233-245.
[2] H. X. Wang and S. W. Chung, Equivalent elastic constants of truss core sandwich plates, *J. of Pressure Vessel Technology*, 133 (4) (2011) 1-6.
[3] L. Valdevit, J. Hutchinson and A. Evans, Structurally optimized sandwich panels with prismatic cores, *International J. of Solid and Structures*, 41 (18-19) (2004) 5105-5124.
[4] D. Briassoulis, Equivalent orthotropic properties of corrugated sheets, *Computers & Structures*, 23 (2) (1986) 129-138.
[5] D. E. McFarland, An investigation of the static stability of corrugated plates loaded in pure shear, *Ph. D. thesis*, University of Kansas, Lawrence, USA (1967).
[6] G. Kress and M. Winkler, Corrugated laminate homogenization model, *Composite Structures*, 92 (3) (2010) 795-810.
[7] M. Winkler and G. Kress, Influence of corrugation geometry on the substitute stiffness matrix of corrugated laminates, *Composite Structures*, 94 (9) (2012) 2827-2833.
[8] G. Kress and M. Winkler, Corrugated laminate analysis: A generalized plane strain problem, *Composite Structures*, 93 (5) (2011) 1493-1504.
[9] Y. Xia, M. I. Friswell and E. I. Flores, Equivalent models of corrugated plates, *International J. of Solid and Structures*, 49 (13) (2012) 1453-1462.
[10] S. Luo, The bending stiffness of corrugated board, *Mechanics of cellulosic materials*, AMD145/MD, 36 (1992) 15-26.
[11] L. A. Carlsson and T. Nordstrand, Evaluation of transverse shear stiffness of structural core sandwich plates, *Composite Structures*, 37 (4) (1997) 145-153.
[12] L. A. Carlsson, T. Nordstrand and B. Westerlind, On the elastic stiffnesses of corrugated core sandwich, *J. of Sandwich Structures and Materials*, 3 (4) (2001) 253-267.
[13] Z. Aboura, N. Talbi, S. Allaoui and M. L. Benzeggagh, Elastic behavior of corrugated cardboard: experiments and modeling, *Composite Structures*, 63 (1) (2004) 53-62.
[14] N. Talbi, A. Batti, R. Ayad and Y. Q. Guo, An analytical homogenization model for finite element modeling of corrugated cardboard, *Composite Structures*, 88 (2) (2009) 280-289.
[15] N. Buannic, P. Cartraud and T. Quesne, Homogenization of corrugated core sandwich panels, *Composite Structures*, 59 (3) (2003) 299-312.
[16] M. E. Biancolini, Evaluation of equivalent stiffness properties of corrugated board, *Composite Structures*, 69 (3) (2005) 322-328.

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