Springer

KSME

Journal of Mechanical Science and Technology 29 (1) (2015) 79~83
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-014-1212-1

The prediction of dynamic fatigue life of multi-axial loaded system †

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(Manuscript Received March 31, 2014; Revised July 31, 2014; Accepted August 31, 2014)

Abstract

The purpose of this paper is to compare with estimation of equivalent fatigue load in time domain and frequency domain and estimate the fatigue life of structure with multi-axial vibration loading. The fatigue analysis with two methods is implemented with various signals like random, sinusoidal signals. Also, an equivalent fatigue life estimated by rainflow cycle counting in time domain is compared with results estimated with probability density function of each signal in frequency domain. In case of frequency domain, equivalent fatigue life can be estimated through Dirlik's method with probability density function. The work proposed in this paper compared the fatigue damage accumulated under uni-axial loading to that induced by multi-axial loading. The comparison was performed for a simple cantilever beam exposed to vibrations of several directions. For verification of estimation performance of fatigue life, results are compared to those of FEM analysis (ANSYS).

Keywords: Equivalent fatigue life (EFL); Multi-axial loading; Power spectral density (PSD); Probability density function (PDF)

1. Introduction

Lifetime prediction and reliability assessment of a mechanical system under severe conditions, where the dynamic load shows invariably irregular and random, is an important problem. In real field, most mechanical systems are excited by repeated dynamic loads, and performing numerical analysis to predict fatigue life helps to shorten cost and time for development. Traditionally, fatigue life has been analyzed in time domain, in which all input loading and output stress are time signals and its equivalent fatigue life is calculated by rainflow counting [1]. This approach needs the stress time history at every point, so it has a practical problem if time history is too long or the structure is complex to analyze. Compared to typical static and quasi-static time domain approach, the frequency domain approach is efficient and less trivial. In the last few years the fatigue life has been totally formulated in frequency domain by Dirlik's formula [2, 3] and Bendat theory [4]: the former is the most theoretical result, the latter is the best practical, and similar results to the estimates in time domain. Dirlik's method is empirical closed form, which can obtain the probability density function (PDF) of rainflow ranges from the power spectral density (PSD). The frequency domain method can yield many advantages: (i) a systematic understanding of system behavior due to modal parameters, (ii) a more computationally efficient fatigue analysis procedure, and (iii) an effective method extended to a structure under multi-axial loading.

Moreover, these fatigue analysis methods have been implemented uni-axially. In a practical mechanical system, stresses are not a uni-axial but bi-axial or in general multi-axial. Recent studies have shown the differences between simultaneous multi-axial and sequentially applied uni-axial loading [5, 6]. Bonte [7] provides a formula to evaluate phase differences to determine the equivalent spectral density under multi-axial loadings. Aykan [8] compared the fatigue damage accumulated under uni-axial loading to that induced by multi-axial loading and the summed damage values obtained from uni-axial loading are lower than those of simultaneous three-axis loading. Ragan [9] compares the equivalent fatigue life estimated about various signals such as sinusoidal, random and compound signal with the conventional time domain method and Dirlik's method.

Our purpose was to compare with estimation of equivalent fatigue load in time domain and frequency domain, and estimate the fatigue life of a structure with multi-axial vibration loading. The comparison was performed for a simple cantilever beam, which is exposed to vibrations of single direction

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† This paper was presented at the FEOFS 2013, Jeju, Korea, June 9-13, 2013. Recommended by Guest Editor Jung-Il Song

and simultaneous multiple directions. For verification of estimation performance of fatigue life, results are compared to those of FEM analysis.

2. Equivalent fatigue load estimation

2.1 Conventional fatigue load estimation in the time domain

Miner popularized the linear damage rule first proposed by Palmgren in 1924. In general, this rule, variously called Miner's rule, states that the damage fraction, D, is

$$D = \frac{n}{N}, \tag{1}$$

where N is the number of cycle to failure of constant stress level S and n is the number of cycles accumulated. The damage fraction is a number between zero and unity, failure is reached when D = 1. If the damage induced in any single cycle is proportional to the stress range amplitude raised to the m-th power, where m is a material parameter, Wohler's equation may be expressed as

$$NS^m = K, \tag{2}$$

$$D = \frac{NS^m}{K}, \tag{3}$$

where K is a material parameter proportional to the number of cycles a material can withstand before failure. It is sometimes useful to define the fatigue damage in terms of an equivalent fatigue load, which is the constant amplitude stress that would cause an equivalent amount of damage as the variable amplitude stress time series. Using the following definition,

$$EFL = \left(\sum_{i=1}^{N} \frac{S_i^m}{N} \right)^{1/m}. \tag{4}$$

And it is possible to describe Eq. (4) in the same pattern as Eq. (3):

$$D = \frac{N\left(EFL\right)^m}{K}. \tag{5}$$

In this simulation, the material parameter, m, is taken as 5 for steel structures.

2.2 Fatigue load estimation in the frequency domain [10]

Vibration-induced fatigue life estimation is a frequency domain approach used when the input loading or the stress history obtained from the structure is a vibratory load and can be specified using statistical information like a random process. The random time processes are usually described in frequency domain by the power spectral density (PSD) function. Given a measurement for a duration T of the random time process $y(t)$, its PSD, $G_{yy}(f)$, is:

$$G_{xx}(f) = \lim_{T\to\infty} \frac{2}{T} E\left[\left| X(f,T) \right|^2 \right]. \tag{6}$$

The transfer function relates between the response to the excitation both for random and deterministic signals. That is, the response of the system will be the input multiplied by the transfer function.

$$G_{yy}(f) = H(f)H^*(f)G_{xx}(f), \tag{7}$$

where '*' means the complex conjugate. Then when system is excited by multiple partially correlated inputs, the PSD of the response becomes

$$G_{yy}(f) = \sum_{i}^{q} \sum_{j}^{q} H_i(f)H_j(f)G_{ij}(f). \tag{8}$$

The probability density function (PDF) of stress ranges is evaluated based on the spectral moments, which are derived from the stress PSD. The nth spectral moment of the PSD is expressed in Eq. (9).

$$m_n = \int_0^\infty f^n G_{yy}(f)df = \sum_{k=1}^{L} f_k^n G_{yy,k}(f_k)\delta f. \tag{9}$$

Dirlik proposed a method to estimate the probability density function of stress ranges that is intended to be applicable to both wide-band and narrow-band processes. The formula for Dirlik's stress range PDF is

$$p(S) = \frac{\frac{D_1}{Q} e^{-Z/Q} + \frac{D_2 Z}{R^2} e^{-(Z^2/2R^2)} + D_2 Z e^{-Z^2/2}}{2\sqrt{n_0}}, \tag{10}$$

where an normalized stress range (Z), a regularity factor (γ), and mean frequency (x_m) is defined as follows:

$$Z = \frac{S}{2\sqrt{n_0}},$$

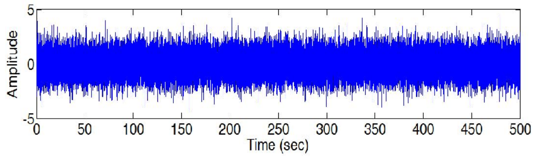
$$\gamma = \frac{n_2}{\sqrt{n_0 n_4}},$$

$$x_n = \frac{n_1}{n_0}\sqrt{\frac{n_2}{n_4}}.$$

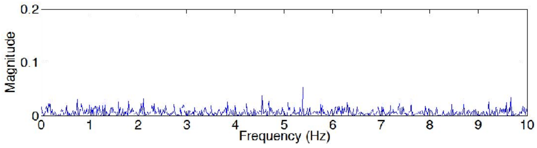
Finally the fatigue life in the frequency domain can be acquired with the PDF, p(S).

$$EFL = \left(E\left[S^m \right] \right)^{1/m}, \tag{11}$$

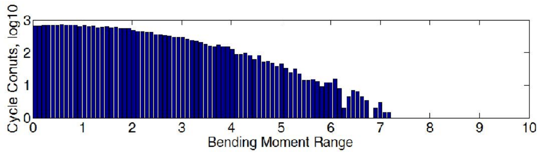
where $E\left[S^m\right] = \int_0^\infty S^m p(S)dS$.



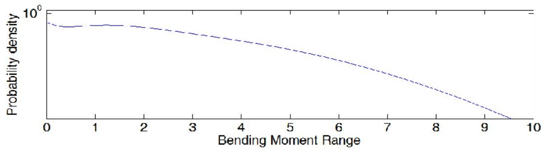
(a) Time signal histories



(b) Power spectral density (PSD)



(c) Stress range histogram



(d) Probability density function (PDF)

Fig. 1. Stress histogram and spectral density of sample time signal histories.

2.3 Numerical simulation for random signal

To compare Dirlik's method to the conventional time domain method, the equivalent fatigue life is estimated with Gaussian white noise: $\mu = 0, \sigma = 1$.

Figs. 1(a) and (b) show time signal histories of white noise and its power spectral density. The stress histogram of signal has a similar pattern with the probability density function in Figs. 1(c) and (d), and the equivalent fatigue load in the frequency domain overestimates those estimated with stress histogram in the time domain by 5.5%.

3. Simulations for estimating fatigue life

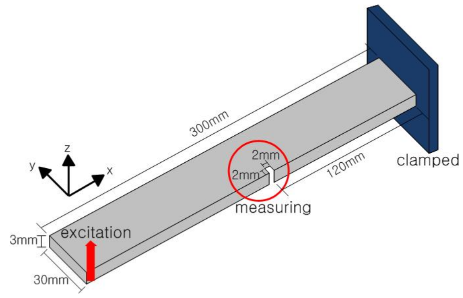
To compare the performance for estimating fatigue life, consider the simple notched cantilever beam like Fig. 2. The material properties of a typical specimen are listed in Table 2. The steel sample model has a notch of 2 mm size at one side to facilitate the stress concentration and fatigue damage under repeated dynamic loading. The material constants of Eq. (5) are determined by S-N curve. The material constant for sample steel structure is $m = 5$ and $K = 4.06\times10^{88}$, and the allowable endurance limit, is 250 MPa reached after $N = 1.28\times10^{6}$ [10]. The natural frequency is identified by the experimental modal analysis (EMA) and the first two natural frequencies are 27.3 Hz, 171.25 Hz. 6700 hexahedral elements have been used in the finite element discretization. The natural frequencies have become converged completely to certain values beyond this element number. One end is clamped and the other end is subjected to a vibration excitation of the first natural frequency. The vibration excitation is applied along the Z-axis direction in uni-axial loading case like Fig. 2(a). In case of multi-axial loading of Fig. 2(b), the excitation that comprised of Z-axis direction and Y-axis direction loading is applied simultaneously [6].

Table 1. Results of estimated equivalent fatigue load with two methods.

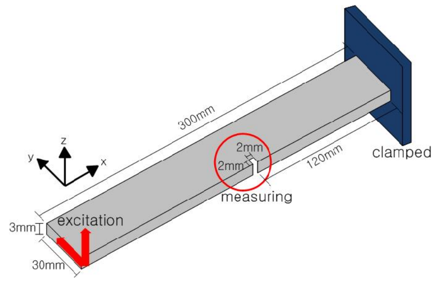
	Time domain	Frequency domain	Difference (%)
EFL	4.019	4.253	5.5

Table 2. Material properties of cantilever.

Young's modulus (GPa)	200
Poisson`s ratio	0.3
Density (kg/m^3)	7850



(a) Uni-axial loading



(b) Multi-axial loading

Fig. 2. Analysis model of cantilever.

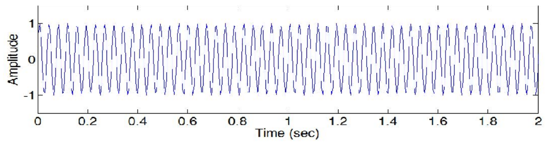
or instance, Eq. (1) is used to calculate a response surface as follows:

3.1 Fatigue life for uni-axial loading

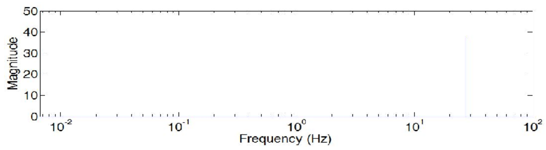
The uni-axial vibration of the first natural frequency (27.3 Hz) is applied at the end of beam in transverse direction. The fatigue life is predicted by the conventional time domain method, Dirlik's method, and ANSYS fatigue and the numerical results of comparison between three methods are presented in Table 2. The location of the most severe damage accumulation is a side notch area in transverse direction, so Fig. 3

Table 3. Fatigue life of uni-axial loading.

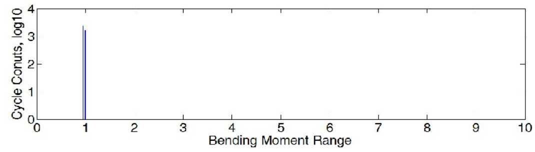
Method	Life (cycle)	Difference (%)
Time domain	11,214	12.1
Dirlik	13,915	9.0
ANSYS fatigue	12,765	-



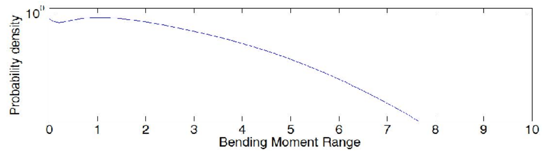
(a) Load history



(b) Power spectral density (PSD)



(c) Cycle counting (cycle log10)



(d) Probability density function (PDF)

Fig. 3. Comparison of time and frequency domain methods for predicting fatigue life.

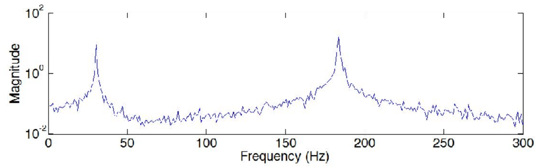
shows time load history, its power spectral density, the rainflow-counted stress histogram, and Dirlik's stress range probability density function.

The equivalent fatigue life (EFL) by the conventional time domain method in Eq. (5) and Dirlik's method in Eq. (13) are compared to that by ANSYS Fatigue. Dirlik's equivalent fatigue life estimate is greater than that of ANSYS fatigue about 9.0% and in general overestimates that of time domain.

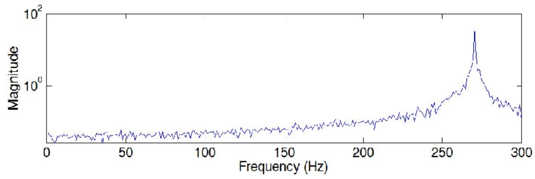
3.2 Fatigue life for multi-axial loading

The practical structure will be multi-axial and simply bi-axial. This study proposed a systematic procedure to calculate an equivalent stress from multi-axial stresses. The proposed method for multi-axial loading includes the following steps as follows:

(i) The dynamic loading for input uses the random excitation, and it can be described by $G_{ii}(f)$, the power spectral density function.

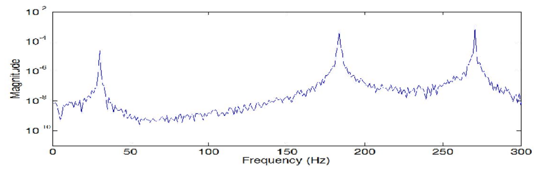


(a) Z-axis (H_i)

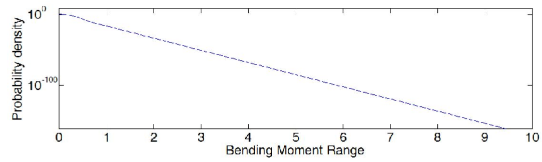


(b) Y-axis (H_j)

Fig. 4. Frequency response function.



(a) Power spectral density



(b) Probability density function

Fig. 5. Data in case of bi-axial loading.

(ii) The transfer functions $H(f)$ that relate each random excitation to the response stress can be obtained by FE model. In this study, the random input is two channels in the Z-axis and Y-axis, and the output stress is one channel in the Z-axis.

(iii) The response stress PSD can be obtained from the excitation PSD using a transfer functions like as Eq. (8).

(iv) The probability density function (PDF) $G_o(f)$ of the stress ranges can be obtained from spectral moments and the empirical formula to model random signals.

Fig. 4 shows the transfer functions obtained from numerical analysis with FE model. The first is the transfer function between dynamic loading in the Z-axis and the stress response in the Z-axis and the latter is between dynamic loading in the Y-axis and the stress response in the Z-axis.

Fig. 5(a) is the stress response of maximum stress point nearby side notch under simultaneously bi-axial loading and Fig. 5(b) is the PDF of stress ranges obtained with Dirlik's formula.

Table 4 is the equivalent fatigue life under each uni-axial loading and simultaneous bi-axial loading. The maximum stress point for two-axis loading is same nearby side notch,

Table 4. Fatigue life of multi-axial loading.

	Z-axis	Y-axis	Bi-axial
Dirlik	29,654	15,273	12,284
ANSYS	37,677	13,494	11,879
Difference (%)	8.2	7.2	4

and the equivalent fatigue life to each loading is shown in Table 4. The equivalent fatigue life under Y-axis loading is 50% smaller than when it is under Z-axis loading, so it shows that the cantilever beam is significantly weaker to Y-axis loading. The predicted results of the equivalent fatigue life shows very identical to those of ANSYS regardless of direction of dynamic loading. The proposed method to predict the fatigue life of a structure under multi-axial loading has an accurate estimate of EFL that has a difference of only 8.2% to that of ANSYS.

4. Conclusion

This paper compares the systematic procedure to estimate the equivalent fatigue life (EFL) under uni-axial loading state in the time domain with frequency domain method, and yields to estimate the fatigue life of structure under multi-axial vibration loading.

First, the estimation of the fatigue life for typical time signal shows very similar results to each other by only 5.5%. Specially, the equivalent fatigue life under uni-axial loading shows that the frequency domain is more identical value to ANSYS results than the time domain. Therefore, the fatigue life of a structure under the random loading input can be calculated effectively in the frequency domain.

Secondly, the equivalent fatigue life under multi-axial loading is very identical to ANSYS, so these results demonstrate that the proposed method can be an attractive tool to evaluate the fatigue life of a structure under multi-axial random vibration.

Additional studies to compare the fatigue life between sequentially uni-axial loadings and simultaneously multi-axial loading and to evaluate the influence of phase difference between input loadings are necessary in the future.

Acknowledgment

This research was financially supported by the Ministry of Education, Science Technology (MEST) and National Research Foundation of Korea (NRF) through the Human Resource Training Project for Regional Innovation.

References

[1] J. A. Bannantine, J. J. Comer and J. L. Handrock, *Fundamentals of metal fatigue analysis*, Prentice Hall (1989) 198-200.
[2] T. Dirlik, Application of computers to fatigue analysis, *Ph.D. Thesis*, University of Warwick, UK (1985).
[3] N. W. Bishop, Analytical fatigue life assessment of fatigue life assessment of vibration induced fatigue damage, *Proceedings of MSC world Users Conference* (1995).
[4] J. S. Bendat, Probability functions for random responses, *NASA report on contract NASA-5-4590* (1964).
[5] R. M. Frech, R. Handy and H. L. Cooper, A comparison of simultaneous and sequential single-axis durability testing, *Experimental Techniques* (2006) 32-37.
[6] W. Whiteman and M. S. Berman, Fatigue failure results for multi-axial versus uni-axial stress screen vibrating testing, *Shock and vibration* (2002) 319-328.
[7] M. H. A. Bonte, A. Boer and R. Liebregts, Determining the on Mises stress power spectral density for frequency domain fatigue analysis including out-of-phase stress components, *Journal of sound and vibration*, 302 (2007) 379-386.
[8] M. Ayken and M. Celik, Vibration fatigue analysis and multi-axial effect in testing of aerospace structure, *Mechanical System and Signal Processing*, 23 (2009) 897-907.
[9] P. Regan and L. Manuel, Comparing estimates of wind turbine fatigue loads using time-domain and spectral methods, *Wind Engineering*, 3 (2) (2007) 83-99.
[10] J. S. Park, Fatigue life estimation of multi-axial loaded system in frequency domain, *Master's Thesis*, Inje University (2010).

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