

Efficiency improvement of the highly accelerated life testing system by using multiple hammers[†]

Yeong-Shu Chen* and Le Hong Chuong

Department of Mechanical Engineering, Yuan Ze University, Chungli, Taiwan

(Manuscript Received March 5, 2014; Revised July 17, 2014; Accepted August 19, 2014) --

Abstract

Highly accelerated life testing (HALT) is a critical method that combines thermal loadings and a series of vibration step stresses in testing the reliability of electronic products. HALT typically utilizes multiple hammers as the driving force of vibration excitations. In this study, we investigated an existing HALT system and the improved design of the hammer system. We also examined the responses of each table to a variety of effects, such as different combinations of hammers and variations in the impact force angles from the hammers to the table. The investigation begins with a theoretical analysis of the forces of impact exerted by the hammers on the table of the HALT system. Thereafter, ANSYS Workbench software is used to build the simulation model that will verify the accuracy of the theoretical results. Finally, the accelerations of the table in an actual HALT system are measured and analyzed for comparison with the foregoing results.

<u> La componenta de la compo</u>

Keywords: HALT; Hammer; Highly accelerated life testing; Impact; Shock response spectrum

1. Introduction

Highly accelerated life testing (HALT) is extensively utilized in environmental stress screening to ensure the reliability of electronic systems. HALT is an effective tool employed during the product development period by triggering failure modes induced by potential design weaknesses. These weaknesses can be diagnosed and corrected before the product is launched for mass production. Identifying the weakest points in a system or product enables design improvements to increase the overall reliability of the product. Failure at the weakest points in a system leads to system breakdown in the field several months or years after the release of the product.

During the HALT processes for general products, various stress levels, such as those caused by temperature, vibration, pressure, humidity, and radiation, alone or combined, are tested to assess the product life cycle. However, the HALT machine generally used for testing the current consumer electronic products typically includes only different levels of "temperature and vibration" stresses. Basically, the objectives of the HALT system are to determine multiple failure modes and root causes, determine functional operating limits, determine functional destruct limits, and focus on thermal as well as vibration stresses (separately at first and then combined).

E-mail address: yschen@saturn.yzu.edu.tw

Under this type of test, the operating limit is defined as the stress level prior to where a failure is identified, but the functionality of the unit under test is recovered when the stress is reduced. By contrast, the destruct limit indicates the stress level where a failure is identified and the functionality of the unit under test is not recovered when the stress is reduced. HALT was developed specifically for solid-state electronics, but can be applied to any product with the correct equipment. The purpose of the HALT system is most effectively met by testing at the lowest possible subassembly. Fig. 1 shows a typical HALT machine with its test chamber and driving hammers mounted under the table.

Vibration step stress test is one of the commonly used HALT test procedures. The goal in using this method is to identify the vibration-related failures by mechanical fatigue testing of the device such that its weakest portions will fail quickly. Fig. 2 shows a typical step acceleration input to the HALT machine. The vibration step increment is typically 5 Grms (the root-mean-square acceleration in terms of gravity). The dwell time at each level is at least 10 min. A full functional test is conducted at the end of the dwell. Thereafter, the vibration stress increases again and the dwell is repeated. Generally, once the operating limit is determined, vibration step stressing continues beyond the operating limit to the destruct limit.

Vibration exertions in the HALT system originate from four basic sources, namely, (1) electromagnetic shaker, (2) hydrau-

^{*}Corresponding author. Tel.: +886 3 4638800, Ext.2473, Fax.: +886 3 4558013

[†] Recommended by Editor Yeon June Kang

[©] KSME & Springer 2014

Fig. 1. (a) Typical HALT machine; (b) test chamber; (c) driving hammer.

Fig. 2. Example of a vibration step stress test.

lic shaker, (3) multiple pneumatic hammers, and (4) electromagnetic shaker combined with a skewed fixture. The first two sources are typically utilized in products that undergo a uniaxial vibration testing. Multiaxial testing is generally conducted for the last two sources in the reliability enhancement testing. Although most HALT systems utilize multiple pneumatic hammers to drive the table, this method has many disadvantages, such as poor table uniformity, uncontrollable frequency spectrum, small amount of low-frequency energy, and minimal displacement. The test loading condition is significantly different from the actual conditions in which the products operate [1-3]. Liu reported that, among these four sources of vibration, only the electromagnetic shaker combined with a skewed fixture (such as that developed by the author) can produce triaxial vibrations simultaneously [4, 5]. Subsequently, Li et al. [6] developed a more concrete design of the skewed fixture.

Knowing how a table moves under the impact of these hammers can help researchers make improved choices when testing products. In a real-world testing conducted by the authors by controlling the number of acting hammers, they observed that using more hammers does not necessarily generate more displacement of the table. Moreover, using more hammers and manipulating different combinations of them can make the test unit move freely (under the condition of no fixture) with specific patterns on the table. With the development in the miniaturization of electromagnetic shakers into electromagnetic hammers (also called electrodynamic or ED hammer), ED hammers can now be used in the HALT system. In addition to being more efficient than pneumatic hammers, ED hammers can control the individual impact frequency and force, as well as produce shocks at low frequencies [7].

Installing hammers on a testing table can result in various motion combinations. This study aims to analyze the motion of a table when subjected to the impact of various hammer combinations. Based on the results, the performance of the HALT system can be optimized.

2. HALT system model

A typical HALT system consists of a table with driving hammers beneath it. Four hammers, numbered I, II, III, and IV, are installed in the system analyzed in this study. The response of the table is of interest because it can reflect many different excitation patterns on the test specimen. Therefore, this study examines the response of the table to forces induced by the use of either an individual hammer or multiple hammers simultaneously.

2.1 Theoretical model

The HALT system considered in this study consists of a flat table mounted on a set of four springs and four hammers mounted under the table at different orientations, as shown in Fig. 3. The force and frequency of the impact from the hammers are triggered by a controller system.

The first part of the study focuses on the response of the table (i.e., displacement, acceleration, and force) to the various types of impact by the hammers (e.g., the impact of only one hammer or of multiple hammers simultaneously). The initial theoretical force analyses indicate that the forces of impact from the hammers are redistributed equivalently at each of the springs. This finding is the fundamental measure in handling hammer forces throughout the study.

Fig. 4 shows a coordinate system (*OXYZ*) with origin *O* located at the center of the table. The *OXY* plane coincides with the plane of the table. The springs are equidistant from one another in a circle (*O*, *R*) (i.e., the circle centered at *O* with radius *R*). Similarly, the four hammers are attached alternately to the springs and lie on a circle (O, r) . The hammers are installed at an angle α inclined to the plane of the table such that the forces exerted on the table are also inclined at angle *α* to the *OXY* plane. beso f impact by the hammers (e.g., the impact of only one
mmer or of multiple hammers simultaneously). The initial
orientical force analyses indicate that the forces of impact
orient the hammers are redistributed equival separated into the naments is e.g., ute impact or only one
hammer or of multiple hammers simultaneously). The initial
theoretical force analyses indicate that the forces of impact
from the hammers are redistributed equiva

2.2 Action of a single hammer

We assumed that Hammer I exerts force $H(t)$ instantaneously on the table, as indicated in Fig. 5(a). This force can be separated into three component forces, that is, H_X, H_Y, H_Z ,

Fig. 3. Construction of an ED hammer system with its table.

Fig. 4. Coordinate systems defined in the hammer system.

along three coordinate directions, expressed as follows:

$$
\vec{H}(t) = \vec{H}_X(t) + \vec{H}_Y(t) + \vec{H}_Z(t)
$$

The magnitudes of each of the force components are expressed as follows:

$$
H_Z(t) = H(t)\sin(\alpha),
$$

\n
$$
H_X(t) = H(t)\cos(\alpha)\cos(\beta),
$$

\n
$$
H_Y(t) = H(t)\cos(\alpha)\sin(\beta),
$$

r to the plane of the table (i.e., plane *OXY*) applied by each of the hammers and β wing angle between $H'(t)$ (i.e., the projection of rce $H(t)$ onto the *OXY* plane; the magnitude of $|\vec{H}'(t)| = H(t)\cos(\alpha)$ and the *X*-axis.

H the *H* sin As the *H* is the *H* the *H* the *H* the *H* the *H* is the *H* the *H* the *H* is Example the strainer of the band continue of the band continue of the force $\vec{H}(t) = \vec{H}_X(t) + \vec{H}_Y(t) + \vec{H}_Z(t)$

and three coordinate direct of the hammer with the forces acting at the springs on the table to simplify the analysis of the displacement of the table. The force component typically generates two types of motion on the table. One type is a vertical motion along the *Z*-direction. The other type is a rotational motion around the *X*-axis. Following the principle of the equilibrium of rigid bodies [8-10], this force can be replaced by the four equivalent forces on the $\frac{1}{1-\frac{1}{2}}$ Spring the contrast density the space of the space o *Z EH*(*t*)cos(*a*)cos(*B*),
 Z EH(*t*)cos(*a*)sin(*B*),

the incline angle of force $\overrightarrow{H}(t)$ to the plane of the

plane OXY applied by each of the hammers and β

plane OXY applied by each of the hammers and springs, $\left(\overline{S^iM^X}\right)_z$, $i=1$ to 4, that represent the numbers of springs at the four locations and their directions as shown in Fig. 1. The contract of the form of the set of the s *z* is of force $\vec{H}(t)$ to the plane of the

lied by each of the hammers and β
 $\begin{cases}\n\frac{1}{2} \left(\frac{S}{I}T \right)_Y = -\mu_Y - \mu_Y \rho_Y\end{cases}$
 $\begin{cases}\n\frac{1}{2} \left(\frac{S}{I}T \right)_Y = \mu_Y - \mu_Y \rho_Y\end{cases}$

cen $\vec{H}'(t)$ (i.e., the projection of $\lim_{x \to 0} \lim_{x \to 0} \frac{\text{arccos}(x)}{x}$ is the force along the *Z*-

axis on Spring 1 (i.e., S1), which generates a moment (resulting from force \vec{H}_Z) with respect to the *X*-axis (i.e., M^X). Thus,

cience and Technology 28 (12) (2014) 4815-4831 4817
\naxis on Spring 1 (i.e., S1), which generates a moment (resulting from force
$$
\vec{H}_Z
$$
) with respect to the X-axis (i.e., M^X). Thus,
\n
$$
\sqrt{\left(\frac{S^4M^X}{\epsilon^2}\right)_Z + \left(\frac{S^3M^X}{\epsilon^2}\right)_Z + \left(\frac{S^3M^X}{\epsilon^2}\right)_Z + \left(\frac{S^4M^X}{\epsilon^2}\right)_Z - \frac{1}{27}
$$
 (1)
\n
$$
\frac{4}{\epsilon^2} \left(\frac{S^3M^X}{\epsilon^2}\right)_Z a = \vec{H}_Z r
$$
\nwhere *r* is the distance of the force component \vec{H}_Z of the
\nharmonic to the center of table *O* and *a* is the distance of the
\nth spring from the X-axis. Given that the springs are equilis-
\ntant to one another in circle (*O*, *R*), as shown in Fig. 5(a),
\n $a = R(\sin 45^\circ) = \left(\frac{\sqrt{2}}{2}\right) R$. Solving Eq. (1) results in
\n
$$
\begin{cases}\n\left(\frac{S^1M^X}{\epsilon^2}\right)_Z = \left(\frac{S^2M^X}{\epsilon^2}\right)_Z = \frac{\sqrt{2r} + R}{4R} H_Z \uparrow \\
\left(\frac{S^3M^X}{\epsilon^2}\right)_Z = \left(\frac{S^2M^X}{\epsilon^2}\right)_Z = \frac{\sqrt{2r} + R}{4R} H_Z \downarrow\n\end{cases}
$$
\nwhere † indicates the forces pointing to the positive direction
\nalong the Z-axis, whereas † indicates the forces pointing to the
\nnegative direction along the Z-axis.
\nSimilarly, in Fig. 5(a), force component \vec{H}_Y can move the
\nthe data be the S-axis and can be replaced by the four transla-
\ntional of these leads to the original by the form two
\nfrom the Y-axis and can be replaced by the form two
\nthe Y-axis. For example, as shown in Fig. 5(b), $\left(\frac{S^2T}{\epsilon}\right)_Y$, is the
\nforce on Spring 2 (i.e., S2) that can generate a translational
\nmovence (i.e., *T*) along the Y-axis, expressed as follows:
\n
$$
\begin{cases}\n\frac{4}{
$$

where *r* is the distance of the force component \vec{H}_Z of the hammer to the center of table *O* and *a* is the distance of the *i*th spring from the *X*-axis. Given that the springs are equidistant to one another in circle (O, R) , as shown in Fig. 5(a),

$$
a = R(\sin 45^\circ) = \left(\frac{\sqrt{2}}{2}\right)R
$$
. Solving Eq. (1) results in

$$
\begin{cases}\n\left(S^1 M^X\right)_Z = \left(S^2 M^X\right)_Z = \frac{\sqrt{2r} + R}{4R} H_Z \uparrow \\
\left(S^3 M^X\right)_Z = \left(S^4 M^X\right)_Z = \frac{\sqrt{2r} - R}{4R} H_Z \downarrow\n\end{cases}
$$
\n(2)

where ↑ indicates the forces pointing to the positive direction along the *Z*-axis, whereas ↓ indicates the forces pointing to the negative direction along the *Z*-axis.

 $a = R(\sin 45^\circ) = \left(\sqrt[3]{2}/2\right)R$. Solving Eq.

and Solving Eq. (sin 45°) = $\left(\sqrt{2}/2\right)R$. Solving Eq.

and Solving The Solving Theorem is the time system.

edirections, expressed as follows:
 $\sqrt{r(t) + H_Z(t)}$.
 $\sqrt{r(t) + H_Z(t)}$.
 gauve direction along the z-axis.
Similarly, in Fig. 5(a), force component \vec{H}_Y can move the table along the *Y*-axis and can be replaced by the four translational forces located at each of the springs in the direction of force on Spring 2 (i.e., S2) that can generate a translational movement (i.e., *T*) along the *Y*-axis, expressed as follows: indicates the forces pointing to the positive direction
 Z-axis, whereas \downarrow indicates the forces pointing to the

direction along the *Z*-axis.
 Ny, in Fig. 5(a), force component \vec{H}_Y can move the

ig the *Y*- $(S^1M^X)_z = (S^2M^X)_z = \frac{\sqrt{2r} + R}{4R}H_z \uparrow$
 $(S^3M^X)_z = (S^4M^X)_z = \frac{\sqrt{2r} - R}{4R}H_z \downarrow$ (2)
 $(\frac{S^3M^X}{2})_z = (S^4M^X)_z = \frac{\sqrt{2r} - R}{4R}H_z \downarrow$ (2)
 $\neq \uparrow$ indicates the forces pointing to the positive direction

g the Z-axis, w M^X $Z = (S^2 M^X)$ $Z = \frac{\sqrt{2r + R}}{4R} H_Z \uparrow$ (2)
 M^X $Z = (S^4 M^X)$ $Z = \frac{\sqrt{2r - R}}{4R} H_Z \downarrow$ (2)

indicates the forces pointing to the positive direction
 Z -axis, whereas \downarrow indicates the forces pointing to the

direct $\left[\left(S^1 M^X\right)_Z = \left(S^2 M^X\right)_Z = \frac{\sqrt{2r} + R}{4R} H_Z \uparrow\right]$
 $\left[\left(S^3 M^X\right)_Z = \left(S^4 M^X\right)_Z = \frac{\sqrt{2r} - R}{4R} H_Z \downarrow\right]$

ere \uparrow indicates the forces pointing to the positive direction

mg the Z-axis, whereas \downarrow indicates the for $\begin{cases}\n\int (S^1 M^X)_Z = (S^2 M^X)_Z = \frac{S^2 T + K_1}{4R} H_Z \uparrow$
 $\left| (S^3 M^X)_Z = (S^4 M^X)_Z = \frac{\sqrt{2r - R}}{4R} H_Z \downarrow \right|$

cre \uparrow indicates the forces pointing to the positive direction

mg the Z-axis, whereas \downarrow indicates the forces poi one anomer in enter (0, n), as shown in rig. 5(a),

in 45°) = $(\sqrt{2}/2)R$. Solving Eq. (1) results in
 M^X)_Z = (S^2M^X) _Z = $\frac{\sqrt{2r} + R}{4R}H_Z$ [†]
 M^X)_Z = (S^4M^X) _Z = $\frac{\sqrt{2r} - R}{4R}H_Z$ (2)

indicates the forc ere r is the distance of the force component \vec{H}_z of the
mmer to the center of table O and a is the distance of the *i*-
since to the *X*-axis. Given that the spring are quidis-
t to one another in circle (O, H) , as s **Solution** the *S*-axis, whereas 4 indicates the forces pointing to the direction along the *Z*-axis.

Finalized in the *Z*-axis and can be replaced by the four translated and the calong the *Y*-axis and can be replaced b

$$
\begin{cases} \sum_{i=1}^{4} \left(\overrightarrow{S^i T} \right)_Y = \vec{H}_Y = \vec{H}_{OY} \\ \sum_{i=1}^{4} \left(\overrightarrow{S^i T} \right)_Y a = \vec{M}_{OZ} = 0 \end{cases}
$$
 (3)

 r_{r} Therefore,

$$
(S1T)Y = (S2T)Y = (S3T)Y = (S4T)Y = \frac{H_Y}{4} \uparrow,
$$
 (4)

where ↑ indicates the forces pointing to the positive direction along the *Y*-axis, whereas ↓ indicates the forces pointing to the negative direction along the *Y*-axis.

ganve direction along the *1*-axis.
Force component H_X generates two types of motion on the table. One type is a linear motion along the *X*-direction. The other type is a rotational motion in the *Z*-direction. The motion along the *X*-direction can be replaced by the four translational forces located at the four springs, as shown in Fig. 5(b), and expressed as follows:

18
\n18
\n18
\n18
\n17. -S. Chen and L. H. Chung / Journal of Mechanical Science and Technology 28 (12) (2014) 4813
\n
$$
\begin{cases}\n\frac{4}{r^2} \left(\overrightarrow{S}^T \right)_X = \overrightarrow{H}_X = \overrightarrow{H}_{OX} \\
\frac{4}{r^2} \left(\overrightarrow{S}^T \right)_X a = 0\n\end{cases}
$$
\n(5)

Eq. (5) produces the following equation:

$$
\left(S^{1}T\right)_{X} = \left(S^{2}T\right)_{X} = \left(S^{3}T\right)_{X} = \left(S^{4}T\right)_{X}
$$
\n
$$
= \frac{\sqrt{2}H_{X}r}{4R} \to,
$$
\n(6)

where \rightarrow shows the forces pointing to the positive direction along the *X*-axis, whereas \leftarrow indicates the forces pointing to the negative direction along the X-axis.

The moment component can rotate the table in the *Z*-axis and can be replaced by the four tangential forces that lie on circle (O, R) and at each of the springs, as indicated in Fig. 5(b) and expressed as follows:

$$
\sum_{i=1}^{4} \left(\overrightarrow{S^i M^2} \right) R = \vec{H}_X r = \vec{M}_{OZ}.
$$
 (7)

Eq. (7) results in the following equation:

$$
(S1MZ) = (S2MZ) = (S3MZ) = (S4MZ) = \frac{HXr}{4R}.
$$
 (8)

The force of the spring that generates the moment, $\left(S^{i}M^{2}\right)$, can be separated into two component forces on the *OXY* plane along the *X*- and *Y*-axes. Fig. 5(b) indicates that

$$
\left(\overline{S^iM^Z}\right)_X + \left(\overline{S^iM^Z}\right)_Y = \left(\overline{S^iM^Z}\right)
$$
\n(9)

or

The force of the spring that generates the moment,
\n
$$
\frac{\sqrt{3}i_N}{3}
$$
, can be separated into two component forces on the
\n XY plane along the *X*- and *Y*-axes. Fig. 5(b) indicates that
\n $(S^i M^Z)_{x} + (S^i M^Z)_{y} = (S^i M^Z)$ (9)
\nthat ger
\n $(S^i M^Z)_{x} = (S^i M^Z)_{y} = \frac{H_x r}{4R} \sin(45^\circ)$
\n $= \frac{\sqrt{2}H_x r}{8R}$ (10)
\n W (e.g., Ha

The foregoing calculations for the analysis of the effects of the force from a single hammer on the table indicate that this hammer force with its equivalent components can be replaced by a series of forces (such as those causing translation and rotation) along each of the axes of the coordinate system rotation) along each of the axes of the coordinate system $(QXYZ)$. S^i_X is the force on spring *i* along the *X*-direction, $QXYZ$). S'_{x} is the force on spring *i* along the *X*-direction, and S'_{z} is the force on spring *i* along the *Y*-direction, and S'_{z} site directions, except for the is the force on spring *i* along the *Z*-direction, calculated as follows:

Fig. 5. Equivalent force system on the springs.

(7)
\n(7)
\n(8)
\nFig. 5. Equivalent force system on the springs.
\n
$$
\frac{H_X r}{4R}
$$
\n(8)
\n
$$
\int \frac{\overline{s}^i x}{s^i y} = \left(\overline{s^i}\right)_x + \left(\overline{s^i}\right)_x^2
$$
\n(9)
\n
$$
\frac{1}{s^i y} = \left(\overline{s^i}\right)_x + \left(\overline{s^i}\right)_x^2
$$
\n(11)
\n
$$
\frac{1}{s^i y} = \left(\overline{s^i}\right)_x + \left(\overline{s^i}\right)_x^2
$$
\n(12)

X $\left(\frac{S^2 M^2}{N}\right)_x = \left(\frac{S^2 M^2}{N}\right)_x = \left(\frac{S^2 M^2}{N}\right)_x = \frac{H_x r}{8R}$
 X $\left(\frac{S^2 M^2}{N}\right)_x = \left(\frac{S^2 M^2}{N}\right)_x = \left(\frac{S^2 M^2}{N}\right)_x = \left(\frac{S^2 M^2}{N}\right)_x$
 X plane along the *X*- and *Y*-axes. Fig. 5(b) indicates that
 X $R \frac{\sin(\pi s)}{100}$ values of these forces are provided in Table 1. **Example 19** and the spring equation:

Fig. 5. Equivalent force system on the springs.

Ing that generates the moment,
 $\frac{1}{S'}$ ing that generates the moment,
 $\frac{1}{S'}$ ing that generates the moment of the moment of th = = ° ⁼ As shown in Eq. (11), the force of Spring 1 along the *X* direction, \overline{S}^1_X , is the result of two forces, namely, a force \rightarrow , and another force that $\overline{}$. The corresponding

The aforementioned analyses are summarized as follows:

(1) When a table is impacted by the other single hammers (e.g., Hammer II, III, or IV), the motion of the table can be analyzed in a manner similar to the analysis of Hammer I, except that the geometric relationship between the hammer and springs must be considered. For example, the force components on Spring 1 resulting from the impact of Hammer III are exactly similar in magnitude to those on Spring 3 resulting from the impact of Hammer I. However, they point in opposite directions, except for the forces in the *Z*-direction, as shown in Fig. 6(a). Moreover, the force components on Spring 1 resulting from the impact of Hammer II are exactly similar

	Y.-S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815~4831				4819	
Axis	Table 1. The total force of each spring along the three axes.					
Spring	\boldsymbol{X}	\boldsymbol{Y}			\boldsymbol{Z}	
$\mathbf{1}$	$\frac{2R + \sqrt{2}r}{8R} H \cos(\alpha) \cos(\beta)$	$\frac{2R\sin(\beta)+\sqrt{2}r\cos(\beta)}{H\cos(\alpha)}$ 8R			$\frac{\sqrt{2}r+R}{4R}H\sin(\alpha)$	
$\overline{2}$	$\frac{2R + \sqrt{2}r}{8R} H \cos(\alpha) \cos(\beta)$	$\frac{2R\sin(\beta)-\sqrt{2}r\cos(\beta)}{8R}H\cos(\alpha)$			$\frac{\sqrt{2}r+R}{4R}H\sin(\alpha)$	
$\overline{3}$	$\frac{2R-\sqrt{2}r}{8R}H\cos(\alpha)\cos(\beta)$	$\sqrt{2R\sin(\beta) - \sqrt{2}r\cos(\beta)}$ H cos(a) $\overline{8R}$			$\frac{\sqrt{2}r - R}{4R} H \sin(\alpha)$	
$\overline{4}$	$\frac{2R-\sqrt{2}r}{8R}H\cos(\alpha)\cos(\beta)$	$\sqrt{2R\sin(\beta) + \sqrt{2}r\cos(\beta)}$ H cos(α) $\overline{8R}$			$\frac{\sqrt{2}r - R}{4R} H \sin(\alpha)$	
			using Hammer I for the case with fixed angles of $\alpha = 45^{\circ}$ and $\beta = 0^{\circ}$.		Table 2. Total forces on each of the springs along the three axes in	
		Axis Spring	\boldsymbol{X}	Y	Z	
		1	$\frac{\sqrt{2}R+r}{8R}H\rightarrow$	$\frac{r}{8R}H\nearrow$	$\frac{2r+\sqrt{2}R}{8R}H\uparrow$	
		2	$\frac{\sqrt{2}R+r}{8R}H\rightarrow$	$\frac{r}{8R} H \swarrow$	$\frac{2r+\sqrt{2}R}{8R}H\uparrow$	
	$(\mathbb{S}^1_\mathcal{M})_\mathcal{T}$ $(S^{\perp}\!T)_{\chi}$ $\mathscr{F}_{\mathbb{Z}_2}$	$\overline{3}$	$\frac{\sqrt{2}R-r}{H\rightarrow}$	$\frac{r}{\sqrt{2}}H\sqrt{2}$	$\frac{2r-\sqrt{2}R}{\sqrt{2}R}H\downarrow$	

Table 1. The total force of each spring along the three axes.

(a) Impact by Hammer III

(b) Impact by Hammer II

Fig. 6. Force components at Spring 1 caused by the impact of a single hammer.

to those on Spring 4 resulting from the impact of Hammer I. However, they point in opposite directions, except for the forces in the *Z*-direction, as shown in Fig. 6(b).

(2) Given a change in the direction of the force of the impact (i.e., variation in angles, namely, incline angle *α* and swing angle β) resulting from the mounting of the hammers to the table, all force components on the spring also change.

(3) The appropriate hammer location, contact angle of the

Table 2. Total forces on each of the springs along the three axes in using Hammer I for the case with fixed angles of $\alpha = 45^{\circ}$ and $\beta = 0^{\circ}$.

$\frac{\sqrt{2}r\cos(\beta)}{H\cos(\alpha)}$			$\frac{\sqrt{2}r+R}{4R}H\sin(\alpha)$
$\sqrt{2}r\cos(\beta)$ H cos(α) R			$\frac{\sqrt{2}r - R}{4R} H \sin(\alpha)$
$\frac{\sqrt{2}r\cos(\beta)}{H\cos(\alpha)}$ \overline{R}			$\frac{\sqrt{2}r - R}{4R} H \sin(\alpha)$
	using Hammer I for the case with fixed angles of $\alpha = 45^{\circ}$ and $\beta = 0^{\circ}$.		Table 2. Total forces on each of the springs along the three axes in
Axis Spring	\boldsymbol{X}	Y	Ζ
$\mathbf{1}$	$\frac{\sqrt{2}R+r}{8R}H\rightarrow$	$\frac{r}{8R}H\!\nearrow\!$	$\frac{2r+\sqrt{2}R}{8R}H\uparrow$
2	$\frac{\sqrt{2}R+r}{8R}H\rightarrow$	$\frac{r}{8R} H \swarrow$	$\frac{2r+\sqrt{2}R}{8R}H\uparrow$
\mathfrak{Z}	$\frac{\sqrt{2}R-r}{8R}H\rightarrow$	$\frac{r}{8R} H \swarrow$	$\frac{2r-\sqrt{2}R}{8R}H\downarrow$
$\overline{4}$	$\frac{\sqrt{2}R-r}{H}$ 8R	$\frac{r}{8R} H \nearrow$	$\frac{2r-\sqrt{2}R}{8R}H\downarrow$
point to.	using new substitutions of symbols by letting $k = \frac{R}{r} = const \ge 1, \quad D_1 = \frac{r}{8R}H,$		are usually fixed by the manufacturer. Changing their posi- tions or contact angle (generally, $\alpha = 45^{\circ}$) is impossible. The projected force of the impact of each hammer on the OXY plane is tangential to circle (O, r) ($\beta = 0^{\circ}$). We consider a type of fixed (special) case using Hammer I with $\alpha = 45^{\circ}$ and $\beta =$ 0°. By substituting these numbers into Table 1, the component forces are rewritten similarly, as shown in Table 2. The ar- rowheads for each of the forces indicate the directions they Understanding the effects and relationships of these pa- rameters is difficult because of the many parameters included in this table. Thus, all of these parameters were simplified (12)
and	$E_1 = \frac{\sqrt{2}R}{8R}H = (k\sqrt{2})D_1 \Leftrightarrow E_1 > D_1$		

rces are rewritten similarly, as shown in Table 2. The ar-
wheads for each of the forces indicate the directions they
int to.
Understanding the effects and relationships of these pa-
meters is difficult because of the many parameters included
this table. Thus, all of these parameters were simplified
ing new substitutions of symbols by letting

$$
k = \frac{R}{r} = const \ge 1, \quad D_1 = \frac{r}{8R}H,
$$

$$
k = \frac{\sqrt{2}R}{r}H = (k\sqrt{2})D_1 \Leftrightarrow E_1 > D_1
$$
(12)

$$
E_1 = \frac{\sqrt{2}R}{8R}H = (k\sqrt{2})D_1 \Leftrightarrow E_1 > D_1
$$

4820				Y.-S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815~4831
	Table 3. Spring forces in each of the three axes in using Hammer I, as represented by the shortened parameters.			The components of the deflection of a spring along the three directions can be calculated using the equations in Table 1 and
Axis Spring	\boldsymbol{X}	Y	Ζ	Eq. (11) . However, the combined effects of the hammers on deflections along all of the three axes should also be consid-
	$bD_1 \rightarrow$	$D_1 \nearrow$	dD_1 \uparrow	ered. It is supposed that the bottom of the spring is fixed on a
\overline{c}	$bD_1 \rightarrow$	$D_1 \swarrow$	dD_1 \uparrow	plane, where it attaches to a frame structure to support the
3	$cD_1 \rightarrow$	$D_1 \swarrow$	$eD_1 \downarrow$	entire system. As indicated in Fig. 4, a coordinate system $(O2 -$ $X'-Y'-Z'$) is set on this fixed plane, with $O'Z'$ pointing in the
4	$cD_1 \rightarrow$	$D_1 \nearrow$	$eD_1 \downarrow$	same direction as OZ ($O'Z' \equiv OZ$) and $O'X'$ being parallel to
	$b = k\sqrt{2} + 1$, $c = k\sqrt{2} - 1$, $d = k\sqrt{2} + 2$, $e=2-k\sqrt{2}$ $\Leftrightarrow d>b>c>e\geq 0$.		(13)	$OX (O'X' N' X)$ and $O'Y'' N' Y$. A lateral movement as well as a rotational motion around the Y' and X' axes occur because force components $S^i{}_x$ and $S^i{}_y$ in Fig. 5 act on top of the spring. Meanwhile, the table to which the springs are attached has the same motion as the deflections of the spring. Thus, the final deflection of the table along the Z' -axis can be deter-
			The formulas in Table 2 are shown again in Table 3 with	mined. Similarly, the force component $S^i z$ can cause simul-
			these substitutions. Thus, the advantage of using these new symbols can be addressed. For example, after checking the	taneous deflections along the Z' - and Y' -axes. To better describe the effects of the springs, the physical
			forces at Springs 1 and 2 under the action of a single hammer	principle for combining multiple springs [11-15] can be util-

Table 3. Spring forces in each of the three axes in using Hammer I, as represented by the shortened parameters.

$$
b = k\sqrt{2} + 1, \quad c = k\sqrt{2} - 1, \quad d = k\sqrt{2} + 2,
$$

\n
$$
e = 2 - k\sqrt{2} \quad \Leftrightarrow d > b > c > e \ge 0.
$$
 (13)

The formulas in Table 2 are shown again in Table 3 with these substitutions. Thus, the advantage of using these new symbols can be addressed. For example, after checking the forces at Springs 1 and 2 under the action of a single hammer (Hammer I), we observed that the force magnitudes in the *X* direction are smaller than those in the *Z*-direction (given that *dD*₁ \rightarrow $D_1 \rightarrow$ $D_1 \rightarrow$ merely a factor of the difference between *b* and *d* in the *X*- and *Z*-directions, respectively. Similarly, after checking the column under the *X*-direction, we observed that the forces in Springs 1 and 2 are exactly similar. The same situation was observed for the forces in Springs 3 and 4. However, the forces in Springs 3 and 4 have smaller values compared with $b = k\sqrt{2} + 1$, $c = k\sqrt{2} - 1$, $d = k\sqrt{2} + 2$, to the components S'_{xx} and S'_{yy} in Fig. 5 act out to $e - 2 - k\sqrt{2} \implies d \ge b \ge c \ge c \ge 0$.

The formulas in Table 2 are shown again in Table 3 with mind deflection of the table forces under column *X* pointed to the same direction.

2.3 Analysis of table displacement

The motion of the table when impacted by hammers can be analyzed by calculating the displacement at each of the four springs caused by the force components discussed previously.

Fig. 5 shows that, with regard to the motion of the table when impacted by Hammer I, the force of the impact can be replaced with the resulting force components at all four springs. Therefore, the motion of the table can be determined by calculating the displacement of the four springs acted upon by these force components. The analysis in the preceding section indicated that the spring not only was compressed along the *Z*-direction but was also bent in the *X*- and *Y*-directions. S^i_X , S^i_Y , and S^i_Z are the spring forces expressed in Eq. (11). n under the *N*-direction, we observed that the foreces in
 $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8 \leq x_9 \$ observed to the lowes in Springs 3 and 4. However, the $\binom{3}{k}$ is the matrix of the conjugated by a spring 3 and 4 how smaller values compared with $\binom{3}{k}$ is the matrix of the capitalisation based in Springs, 3 and uses in spring, a matrix was matrix values can be exactly in the state in the space of the space of the stiffness matrix of the space of the stiffness matrix of the space of the stiffness of the space of the deflection of

The deflection of the spring can be calculated using the following equation:

$$
\{S\} = \{k_s\} \{\delta\},\tag{14}
$$

the spring along the three axes.

Y.-S. Chen and L. H. Chiong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815-4831

by the shortened parameters.

by the shortened parameters.

 EQ. (11). However, the components of the deflection 20. $\frac{dx}{dx}$ *b b* $\frac{dx}{dx}$ *cD* \rightarrow *b* $\frac{dx}{dx}$ *cD* \rightarrow *k k* advantage *k* become and *k k* become and *k* the deflection of a spring directions can be calculated using the equation of a spring directions c *F*. S. Chen and *L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815-4831*

de 3. Spring forces in each of the three axes in using Hammer 1, as
 $\frac{x}{\sqrt{1-x^2}}$ are components of the deflection F.S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815-4831

3. Spring forces in each of the three axes in using Hammer I, as
 $\frac{ax}{1}$
 $\frac{bx}{1}$
 $\frac{bx}{2}$
 $\frac{bx}{2}$
 $\frac{dx}{2}$ x. S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815–4831

2. 3. Spring forces in each of the three axes in using Hammer 1, as
 $\frac{1}{x}$ The components of the deflection of a sp 2 $bD_1 \rightarrow D_1 \swarrow$ *dD*₁ \uparrow plane, where it attaches to a frame structure to support the entire system. As indicated in Fig. 4, a coordinate system (*O*′- $\begin{array}{ccc} cD_1 \rightarrow & D_1 \swarrow & eD_1 \downarrow & \text{if } X \rightarrow Y \rightarrow Z' \text{ is set on this fixed plane, with } O'Z' \text{ pointing in the } \end{array}$ 4 $cD_1 \rightarrow$ $D_1 \nearrow$ *eD*₁ \downarrow same direction as *OZ* (*O'Z'* = *OZ*) and *O'X'* being parallel to *OX* (*O*′*X*′//*OX*) and *O*′*Y*′//*OY*. A lateral movement as well as a rotational motion around the Y' and X' axes occur because rotational motion around the Y and X' axes occur because
force components $S^i{}_X$ and $S^i{}_Y$ in Fig. 5 act on top of the spring. Meanwhile, the table to which the springs are attached has the same motion as the deflections of the spring. Thus, the final deflection of the table along the *Z*′-axis can be deterfinal deflection of the table along the Z -axis can be deter-
mined. Similarly, the force component $S^{i}z$ can cause simultaneous deflections along the *Z*′- and *Y*′-axes. ctions can be calculated using the equations in Table 1 and

(11). However, the combined effects of the hammers on

ections along all of the three axes should also be consid-

is supposed that the bottom of the spring is (11). However, the combined effects of the hammers on

citions along all of the three axes should also be consid-

is supposed that the bottom of the spring is fixed on a

is supposed that the bottom of the spring is fixe *x* and *Technology* 28 (12) (2014) 4815-4831

the components of the deflection of a spring along the three tectoms can be calculated using the equations in Table 1 and

(11). However, the combined effects of the harmners direction as *OZ* (*OTZ*' = *OZ*) and *OX*^{*N*} being parallel to
 OX^{*N*}/*OX* and *OY*^{*N*}/*OYX* a lateral movement as well as a

conal motion argund the *Y* and *X*^{*Y*} are soccur because

components *S^tx* an *Y*-Z') is set on this fixed plane, with *O*'Z' pointing in the
 Y-Z') is set on this fixed plane, with *O'Z'* pointing in the

ne direction as *OZ* (*O'Z'* = *OZ*) and *O'X'* being parallel to
 $((O'X')/OX)$ and $O'Y'//OY$ $\langle (O/XI/OX)$ and $O'YI/OY$. A lateral movement as well as a
tational motion around the Y' and X' axes occur because
the ce components $S'x$ and $S'v$ in Fig. 5 act on top of the
ting. Meanwhile, the table to which the s ational motion around the \underline{Y}^n and X^n axes occur because
ce components $S^i x$ and $S^i y$ in Fig. 5 act on top of the
ing. Meanwhile, the table to which the springs are attached
the same motion as the deflections o plane, where it attaches to a frame structure to support the entire system (O^x -
kF- P^x - Z) is set on this fixed plane, with OZ pointing is the matric system (O^x - O^x - P^x - Z) is set on this fixed plane, wi *tions: ead Technology* 28 (23) (2014) 4815-4831

The components of the deflection of a spring along the three

directions can be calculated using the culturiost in Table 1 and

Eq. (11). However, the combined effects of

To better describe the effects of the springs, the physical principle for combining multiple springs [11-15] can be utilized to transform the four springs, each with various individual deflections, into one spring located at the center of the table. The stiffness of this transformed single spring along the three axes can be calculated by the following equation:

$$
\begin{Bmatrix}\n(k_t)_{X} \\
(k_t)_{Y} \\
(k_t)_{Z}\n\end{Bmatrix} = \n\left\{\nf\begin{bmatrix}\nk_x^i \\
k_y^i \\
k_z^i\n\end{bmatrix}\right\}\n\quad \text{or} \quad\n\left\{k_t\right\} = \n\left\{f\left(k^i\right)\right\},
$$
\n(15)

ness with components in the *X*-, *Y*-, and *Z*-axes designated as of the stiffness matrix for each of the original four springs in the three axes (i.e., k_X^i , k_Y^i , and k_Z^i), where *i* represents the number of springs $(i = 1-4)$.

As shown in Fig. 7, for the force along the *X*′-axis, force As shown in Fig. 7, for the force along the X⁻-axis, force component $S^i{}_X$ (*i* = 1-4) can be replaced by \overrightarrow{F}_X at the component $S'_{X}(i = 1-4)$ can be replaced by F'_{X} at the center of the table. Similarly, force component S'_{Y} can be center of the table. Similarly, force component S_Y can be replaced by \overline{F}_Y at the center of the table. For example, for teplaced by F_Y at the center of the table. For example, for the motion of a table impacted by \vec{F}_X , the lateral deflection δ_{X_1} along the *X*[']-axis can be calculated as follows:

$$
\delta_{X1} = \frac{F_X}{(k_t)_X}.\tag{16}
$$

to transform the four springs, each with various individ-
effections, into one spring located at the center of the
The stiffness of this transformed single spring along the
axes can be calculated by the following equation If the deflection is small, then the change in the length of the spring can be ignored. This lateral deflection causes the longitudinal axis of the spring to rotate at an angle of $\gamma = \arcsin\left(\frac{\delta_{X_1}}{\ell}\right)$, where *L* is the length of the spring that $\langle i_k \rangle$, $\langle k_i \rangle$, and $\langle k_i \rangle$, respectively. $\{f(k')\}$ is a function
the stiffness matrix for each of the original four springs in
the exerce, $\langle k_i, k_j \rangle$, and k'_2 , where *i* represents the
three axes (i.e., k'_r , a unchanged (Fig. 7(c)). Similarly, the displacement of the table along the *Z*-axis, that is, δ_{Z_1} , at spring locations 2 and 3 (position A in Fig. 7(c)) can also be calculated when the table is

(c) Rotation on top of the transformed spring

Fig. 7. Transformation of multiple springs into a single spring.

impacted by \vec{F}_X . Impact force \vec{F}_X only causes the rotation i of the table along the *Y*-axis, as shown in Fig. 7.

Thus, calculating the deflections of general spring positions through the center locations of the springs designated as A and B is easy, as indicated in Fig. 7(c) and the following equation:

$$
\begin{cases}\n\delta_{Z1} = \left[R - L \tan(\gamma)\right] \sin(\gamma) \\
\delta_{Z2} = \left[R + L \tan(\gamma)\right] \sin(\gamma)\n\end{cases}
$$
\n(17)

 δ_{z1}
 $\frac{1}{\sqrt{z_{z1}}}$
 $\frac{1}{\sqrt{z_{z2}}}$
 $\frac{1}{\sqrt{z_{z1}}}$
 $\frac{1}{\sqrt{z_{z2}}}$
 $\frac{1}{\$ In the case that the motion of the table is impacted by Hammer I, the deflections along the *Z*-axis for Springs 2 and 3 (Location A) move upward and the deflections of Springs 1 and 4 (Location B) move downward.

Similarly, when considering the table being impacted by Similarly, when considering the table being impacted by $\frac{1}{f}$ translational force \vec{F}_y , the deflections along the *Z*-axis of $\frac{1}{g}$ Springs 1 and 2 move upward and the deflections of Springs 3 springs 1 and 2 move upward and the deflections of springs 3 systems and 4 move downward. Force component \vec{H}_Z along the Z- their position direction resulting from the impact of Hammer I generate a bending moment around the *X*-axis. The deflections of the four springs along the *Y*-axis are equal to one another.

By combining all of the deflection components along the directions of the three axes for the case of Hammer I in exerting \uparrow force $\vec{H}(t)$ on a table, we can summarize the deflections of the table at the four springs as follows:

- Deflections along the X-direction: $\delta_{1X} = \delta_{2X} > \delta_{3X} = \delta_{4X}$; R
- ¹ Deflections along the *Y*-direction: $\partial_{1Y} = \partial_{4Y} > \partial_{2Y} = \partial_{3Y}$;
 $\sum_{Y} S_Y$
	-

nce and Technology 28 (12) (2014) 4815-4831

• Deflections along the X-direction: $\delta_{1X} = \delta_{2X} > \delta_{3X} = \delta_{4X}$;

• Deflections along the Y-direction: $\delta_{1X} = \delta_{2X} > \delta_{3X} = \delta_{4X}$;

• Deflections along the Z-direction: ferent locations, it results in different force components along k $\frac{\cancel{x}}{\cancel{x}}$ $\frac{x}{\cancel{x}}$ the three axes at a specific position on the table. When comparing the forces in the last column of Table 3 for the spring forces in the *Z*-axis, we observed that Springs 1 and 2 have exactly the same values. The same observation was obtained for Springs 3 and 4. However, the deflections at each of the sience and Technology 28 (12) (2014) 4815-4831
 Band 11 4821
 Conflections along the *Z***-direction:** $\delta_{1X} = \delta_{2X} > \delta_{3X} = \delta_{4X}$;
 Conflections along the *Z***-direction:** $\delta_{1Z} > \delta_{1Z} > \delta_{3Z} > \delta_{4Z}$;
 Conflection $\overrightarrow{O_{2Z}} > O_{1Z} > O_{3Z} > O_{4Z}$, as previously mentioned in the summa-
rized conclusions of the deflections of the table. Moreover, Fig. $7(c)$ shows that the deflection value at the center of the table (Location "O") is the smallest compared with those on the outer edges of the locations of the four springs. \int dif- \overrightarrow{O} These analyses show that, when a single hammer acts at difs at dif- 11 $\overline{}$ $\delta_{27} > \delta_{17} > \delta_{37} > \delta_{47}$, as previously mentioned in the summa- $R \longrightarrow R_{X} \longrightarrow R_{X}$ B

The entire table plane moves either in a translational or rotational manner because the table is sufficiently rigid. The ac- R_{rel} celerations at each point on the table depend on the corresponding deflections of that point because acceleration is the δ_{X} $\frac{A}{1}$ $\frac{\delta_{\text{X}}}{2}$ B_| $\frac{}{B}$ B| $\frac{}{B}$ $\frac{}{$

2.4 Force impact and spring stiffness z2

 $\frac{1}{2}$ $\frac{1}{2}$ For $\frac{1}{2}$ and $\frac{1}{2$ $\frac{d}{dx}$ $\frac{d}{dx}$ **EVALUATE:**

The second derivative of displacement with
 $\frac{1}{16}$ and $\frac{1}{16}$ and **EVALUATE:** By
 R AND **EVALUATE:** By
 R AND **EVALUATE:** By
 R AND **EVALUATE:** By

(c) Rotation on top of the transformed spring

(c) Rotation on top of the transformed spring

magnitude and direction of multiple spr eclerations at each point on the table

sponding deflections of that point because
 $\delta_{21} = \frac{1}{2}R - L \tan(r)\sin(r)$
 $\delta_{22} = \frac{1}{2}R + L \tan(r)\sin(r)$

the case that the motion of the table is impacted by \vec{F}_x . Impact force $\vec{F$ $\frac{2\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{2}}$
 $\frac{2.4\text{ Force impact and springs stiffness}}{2\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{2-\frac{1}{2}}\sqrt{$ In the HALT systems, the force of the impact from each hammer creates an instantaneous shock on the table. The magnitude and direction of this shock are fully dependent on the force of the impact. The force of this impact results from the motion of the piston inside the hammer housing. As the piston moves up and down in the cylinder, the direction of the impact changes accordingly. This change also causes a change in the direction of the shock pulse on the table.

The stiffness of the spring also affects the motion of the table. As shown in Eq. (16), spring stiffness and the motion of the table are inversely proportional to each other.

3. Table responses when multiple hammers are combined

When the table is impacted by the simultaneous forces of multiple hammers, the total resultant force at a specific spring is not simply the sum of the forces of each individual hammer. The direction of the corresponding forces that act on the same spring must be considered.

impacted by F_x . Impact force F_x only causes the rotation impact changes accordingly illusion the eyling cost

of the table along the P-axis, as shown in Fig. 7.

Thus, calculating the deficed cost of general sping pos Theoretically, the locations and incline angles of the hammers mounted under the table can be changed to obtain the greatest impact. However, in reality, the hammers in HALT systems are usually fixed by the manufacturer. Thus, changing their positions or contact angle is impossible. Fig. 8 shows a typical installation of four equidistant hammers whose top surfaces come in contact with the table by lying on a circular path centered at *O* with radius *r*. In fact, these hammers were installed beneath the table with a contact surface of 45° relative to the table, with the orientation of the hammer pointing at the tangential direction of the circle $(0, r)$. Thus, the projected force of the impact of each hammer on the *OXY* plane (e.g.,

 $H1_X$) is tangential to circle (O, r) , whereas the other projection of impact force points vertically away from the table surface along the *Z*-axis. Then, we consider several cases with the same settings of the hammers, except that the number of hammers used is changed.

The general case in an actual system is that all four hammers simultaneously act with a magnitude similar to their 4822

If A_x) is tangential to circle (O, r), whereas the other projection of Mechanical Science and Technology 28 (12) (2014) 4815-4831

If A_x) is tangential to circle (O, r), whereas the other projection of impact fo impact forces (i.e., $|\overrightarrow{H1}| = |\overrightarrow{H2}| = |\overrightarrow{H3}| = |\overrightarrow{H4}| = |\overrightarrow{H}|$) (Table 4). The magnitudes of the component forces in each of the three axes (i.e., X, Y, and Z) are similar in all spring positions. However, the directions they point to are different. For example, as indicated in column X, the values are all equal to *4D,* and the forces of Springs 1 and 2 point to the right, whereas those of Springs 3 and 4 point to the left. This observation is explained by Eq. (12), where the simplified parameter for a 3522

Single hammer is designated as *D*₁ . The *R* is the context of the state of the *R* respective of a regular term is designated as *D* respective points verifiedly away from the thirb sum-

since an any single as *H1*_x) is tangential to circle (*O*, *r*), whereas the other projections we have the same force components at all the spring positions when four ham-
there along the Z-axis Then, we consider several cases with the
face 222 *Y. S. Chen and L. H. Chaong /Jeannal of Mechanical Science and Technology 28 (12) (2014) 4815–4531

(T₁x) is tangential to circle (<i>Q*, *r*), whereas the other projections of implies are components at all the sprin $H1_X$) is tangential to circle (O, r) , whereas the other projection of impact force points vertically away from the table surface along the Z-axis. Then, we consider several cases with the measure such according of the ha $|\overrightarrow{H4}| = |\overrightarrow{H}|$), which provides the total hammer force equal to same settings of the hammers, except that the number of

Integrances used is changed.

The general case in an actual system is that all four ham-

The general case in an actual system is that all four ham-

Impact forces for the Y-axis except for the similarity of both the values and directions of all spring forces on the Z-axis. All forces shown in Table 4 result in the translation of the table along the Z-axis and the rotation around the Z-axis. The HALT test performed

in this situation excites the specimen mostly in the Z direction while simultaneously producing minor rotations. In the case with only three hammers (e.g., Hammers I, III, and IV), the magnitudes of the force components along the Yand Z-axes at Springs 1 and 4 are larger than those in the previous case with four hammers (Table 5). In other words, the use of all four hammers does not always generate larger forces. The table moves along the Y- and Z-axes and simultaneously rotates around the Z-axis in this case. Accordingly, the speci-

The ratios among all force components in the overall exerting force on the table.

men is subjected to loadings in these two directions.

Table 6 shows the force value at the spring locations when four hammers are used (Fig. 9). All hammer forces in the figure point to the same direction at angles $\alpha = 45^{\circ}$ and $\beta = 0^{\circ}$. The spring force value ratio with different acting directions along the X- and Z-axes is $X:Z = 1:1$.

In summary, the ratio of spring forces along the three axes (i.e., X:Y:Z) varies when the hammer angles (i.e., incline angle α and swing angle β) in the design of a HALT system is altered. This variation identifies the most efficient position for specimen attachment on a HALT system table.

The preceding cases yield some interesting findings. For example, all forces on the X-direction in each spring location cancel out one another (i.e., Springs 1 and 2 have force $4D \rightarrow$ and Springs 3 and 4 have force $4D\leftarrow$). The similar cancellations are also observed in the spring forces on the Y-direction (Table 4). The table still rotates even though all forces are cancelled because of the existing resulting moment. By con-

Table 4. Force components at all the spring positions when four hammers are used.

Y.-S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815~4831				
o circle (O, r) , whereas the other projec-				Table 4. Force components at all the spring positions when four ham-
points vertically away from the table sur-	mers are used.			
Then, we consider several cases with the	Axis	X	Y	Z
	Spring			
		$4D \rightarrow$	$4D\lambda$	$4E \uparrow$
	2	$4D \rightarrow$	$4D\swarrow$	$4E \uparrow$
	3	$4D \leftarrow$	$4D\swarrow$	4 $E \uparrow$
e hammers, except that the number of nged. in an actual system is that all four ham- act with a magnitude similar to their $\overrightarrow{H1}$ = $\overrightarrow{H2}$ = $\overrightarrow{H3}$ = $\overrightarrow{H4}$ = $\overrightarrow{H1}$) (Table 4).	$\overline{4}$	$4D \leftarrow$	$4D\lambda$	$4E \uparrow$
he component forces in each of the three				
d Z) are similar in all spring positions. ons they point to are different. For exam-		Ζ	H ₃	

Fig. 8. Forces of impact on the table from each of the four hammers.

Fig. 9. Forces of impact on the table from each of the four hammers with the same direction.

trast, the forces in the spring locations in Table 6 indicate that the resulting force is maximized and minimized along certain directions by changing the mounting angles of the hammers. Correspondingly, the forces along the X- and Z-directions are the maximum, whereas the forces along the Y-direction are the minimum (i.e., zero).

4. Simulation of a HALT system

The preceding sections have presented the theoretical analysis for the mechanics of a HALT system. The current section describes the commercially available ANSYS Workbench software used to create a HALT model, which evaluates the theoretical model.

The model comprises a flat table mounted on a set of four springs. All springs are fixed on the bottom of the table (Fig. 4). The geometric dimensions of the table and the springs are as follows:

Table 5. Force components at all the spring positions when three hammers (i.e., I, III, and IV) are used.

Axis Spring			
	$3D \rightarrow$	$3D + E$	$3E+2D$ \uparrow
	$3D \rightarrow$	$3D - E \swarrow$	$3E-2D$ ^{\uparrow}
	$3D \leftarrow$	$3D - E \swarrow$	$3E-2D$ ^{\uparrow}
	$3D \leftarrow$	$3D+E^2$	$3E+2D$ ^{\uparrow}

Table 6. Forces at each spring location in the case indicated in Fig. 9.

• Table: diameter = 500 mm and thickness = 100 mm.

• Springs: outside diameter (Do) = 50 mm, wire diameter $(dw) = 4$ mm, number of active coils $(n) = 10$, and total height $= 100$ mm.

The hammers and springs are equally spaced on a circle centered at O with a diameter of 200 mm.

The deflections at the four spring positions are respectively shown in Figs. 10(b)-(d) with regard to the assumption that the table is impacted by a single hammer acting at location I. This hammer has exactly the same mounting angles (i.e., $\alpha = 45^{\circ}$ and $\beta = 0^{\circ}$) as those used in the current HALT system (Sec. 2). If we suppose that the table and hammers are rigid, then the comparison of the large spring deflections with the corresponding results for the deflections at all springs along the three axes is summarized as follows: and Technology 28 (12) (2014) 4815-4831

and Technology 28 (12) (2014) 4815-4831

in Tigs. 10(b)-(d) with regard to the assumption that the

is impacted by a single hammer acting at location I. This

ner has exactly the s *nce and Technology 28 (12) (2014) 4815-4831*

4823

The deflections at the four spring positions are respectively

by own in Figs. 10(b)-(d) with regard to the assumption that the

le is impacted by a single hammer actin and Technology 28 (12) (2014) 4815-4831

4823

e deflections at the four spring positions are respectively

in in Figs. 10(b)-(d) with regard to the assumption that the

is impacted by a single hammer acting at location I The deflections at the four spring positions are respectively
own in Figs. 10(b)-(d) with regard to the assumption that the
le is impacted by a single hammer acting at location I. This
mmer has exactly the same mounting a e elerections at the bour spring positions are respectively
in in Figs. 10(b)-(d) with regard to the assumption that the
is impacted by a single hammer acting at location 1. This
err has exactly the same mounting angles (13

14

13

14

13

14

15. Force components at all the spring positions when three ham-

15. Force components at all the spring positions when three ham-

13*D* \rightarrow

13*D* 23

23

24. France and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815-4831

35. Force components at all the spring positions when three ham-

23. The deflections at the four spring positio 23

33

33*D* + 3*D* + 3*D + 5²* 3*E +2D* + 3*B*

33*D* + 3*D + 3<i>D + 3E +2D* + 3*B*

343

32 + 3*D + 3D + 5² 3<i>E +2D* + 3*D* + 3*D + 52* 3*E +2D* + 3*D* + 5*C Y.-S. Chen and L. H. Chuong / Journal of Mechanical Science and Technology 28 (12) (2014) 4815-4831*

43

5. Force components at all the spring positions when three ham-

..e., I, III, and IV) are used.

Axis
 $\begin{array}{rcl}\$

- Among the plots in Fig. 10, the most noteworthy case is that of the deflections along the X-direction. Fig. 10(c) are positive.
- $4E \rightarrow 0$ $4E \uparrow$ tive, whereas those of Springs 2 and 3 are negative, as shown in Fig. 10(d). • For the deflections along the Y-direction, $\delta_{1Y} = \delta_{4Y} >$ $\delta_{2Y} = \delta_{3Y}$. The deflections of Springs 1 and 4 are posi
	- upward, whereas those at Springs 3 and 4 point downward as shown in Fig. 10(b). • For the deflections in the Z-direction, $\delta_{2z} > \delta_{1z} >$

All spring deflections are completely consistent with the explanations from the previous section. Thus, the theoretical model accurately depicts the table response to the hammer impact.

Fig. 10. Deflections of the table impacted by $\overrightarrow{H}(t)$.

Fig. 11. Accelerations along the Z-axis at Spring 1 when acted upon by each single hammer.

Fig. 12. Table accelerations along the Z-axis at Spring 1 when acted upon by Hammers II and III.

4.1 Response of the table to different impacts (i.e., a single hammer at various locations, multiple hammers, and a change in the hammer angle)

The theoretical analysis shows that the table responds differently at a fixed point when a single hammer acts at different locations. This result is derived by considering four separate cases of a single hammer acting at one of the four hammer positions (i.e., I, II, III, and IV). The HALT system model contains four hammers equidistantly arranged from one another in a circular path around the center of the table (Fig. 4). These hammers come into direct contact with the table. At each moment of impact, the control of the hammer through the piston displacement inputs is identical. The differences in the maximum acceleration for all cases at Spring 1 along the Z-axis are shown in Fig. 11. For example, as shown in Figs. 11(a) and (d), the maximum accelerations are 14.1 and 27.4 G for Hammers I and IV, respectively.

Fig. 12 indicates that the maximum acceleration at Spring 1 is 23.6 G when Hammers II and III impacted the table simultaneously. However, this acceleration does not exceed that of Hammer IV (i.e., 27.4 G) impacting the table, as shown in Fig. $11(d)$.

The theoretical analysis data also show that the force components on the spring change when the impacting force direction is varied (i.e., angle α or β). These angles depend on the mounting angle of the hammer to the table. Correspondingly, the table motion is changed. The acceleration along the three axes of the table at Spring 1 when Hammer I acts on the table with a swing angle of $\beta = -90^{\circ}$ is shown in Fig. 13(a). Correspondingly, the projected force of the hammer on the O-XYZ plane is along the negative Y-axis. Therefore, the overall resulting table motion is mainly along the Z- and Y-axes. However, a small oscillating motion occurs along the X-axis because the force applied to the table is asymmetric. Thus, changing the swing angle *β* of Hammer I also changes the table movements. For example, the maximum acceleration (i.e., 29.3 G) for Hammer I with angle $\beta = -90^{\circ}$ along the Zaxis becomes 14.1 G with angle $\beta = 0^{\circ}$ (Fig. 11(a)).

4.2 Hammer stroke and spring stiffness effects on the table response

When the piston moves up and down inside the hammer housing, its movement causes the direction of the acceleration shock pulses to change accordingly. The two following strokes are observed when a table is impacted by a single hammer acting at location I from time period $t = 0-0.14$ s and at the start of the piston movement:

Single-impact stroke: the piston impacts only the anvil at the front end of the hammer. This type of stroke also occurs when a gap exists between the front end of the hammer and the table (i.e., the bottom of the hammer is installed at the frame but its front end does not make contact with the table).

Fig. 13. Model responses to hammer I with angle $\beta = -90^\circ$.

(a) Accelerations of the table along the Z-axis for the single-impact stroke

(b) Acceleration of the table along the Z-axis for the dual-impact stroke

Fig. 14. Effects of piston stroke on the table accelerations.

Dual-impact stroke: the piston impacts both ends of the cylinder.

Figs. 14(a) and (b) shows the table acceleration along the Zaxis at Spring 1. The two plots indicate that the piston hitting the front end of the hammer generates only a unidirectional impact (Fig. 14(a)), whereas the piston hitting both ends of the hammer can result in shock pulse accelerations in both positive and negative directions (Fig. 14(b)).

(c) Acceleration along the Y-axis (d) Acceleration along the Z-axis

The magnitude and shape of the time history shock pulse also change when the spring stiffness changes. The table acceleration in response to two different values of spring stiffness is shown in Fig. 15. Fig. 15(b) shows double the spring stiffness of Fig. 15(a). The figures show that a stiffer spring has earlier peak acceleration and smaller magnitude than a softer spring.

5. Experimental results

This study collects the test data using a real HALT system designed with ED hammers. The current system is unique because it uses ED hammers to produce the input shock pulses to the table instead of the typical pneumatic hammers used by other HALT systems. The hammers are equidistantly arranged from one another around a circular path at the center of the table. Furthermore, the ED hammers together with the controller system were attached in direct contact to the table (Fig. 4).

All hammers are maintained at the same frequency and input power during the test. The table accelerations are measured using two accelerometers (i.e., Acc. 1 and Acc. 2). These accelerations occur on the Z-axis close to Spring 1 and the center of the table.

The time data of the accelerations on the table are analyzed to understand the excitation produced by the HALT machine. The following response data are measured from the locations on the table of a real multi-hammer HALT machine. Two useful methods are further employed to analyze the maximum accelerations in cases using a single hammer and combined multi-hammers (i.e., combinations of Hammers I and II; Hammers I, II, and IV; and Hammers I, II, III, and IV).

Fig. 15. Effects of spring stiffness on the accelerations of the table along the Z-axis.

Henderson [16, 17] provided the two best methods to analyze a machine with six degrees of freedom (6-DOF). The first method is shock response spectrum (SRS) based on the highkurtosis seismic vibration analysis. Biot [18] explored the SRS, while Smithson [19] proposed SRS for 6-DOF machines. The second method uses rainflow analysis to produce a plot known as peak probability distribution function (PPDF) [20].

The SRS is a frequency domain data adopting the time domain peak accelerations and the corresponding natural frequencies of a series of test objects to verify a shock excitation. For example, military standards MIL-STD-810 C, D, and E and other related products prefer the SRS to specify environmental excitation for laboratory simulation. These specifications range from product response to transient and seismic shock. Accordingly, the SRS "fingerprints" such excitations in terms of their stress-related time domain properties [18]. The HALT machine excitations in the current study dynamically occur over periods of seconds. Thus, the SRS is a valuable analysis tool to describe peak amplitudes versus frequency.

The PPDF only measures peak accelerations instead of measuring both the accelerations and instantaneous acceleration history. Although various forms of rainflow analysis exist, the resulting PPDF is a plot of the number of peak occurrences with narrow amplitude ranges. In case of vibration or shock, the PPDF counts the number of acceleration peaks with sigma amplitudes that fall within the defined range bins. A relationship exists between the measured root mean square (RMS) level and the PPDF plot for the current acceleration responses. The RMS value of a signal is equal to a standard deviation

Table 7. Maximum accelerations measured at Spring 1 and at the center of the table when impacted by multiple hammers.

Number of		Test	Maximum acceleration value (G)			
hammers	Hammer		number Acc. 1 (Spring 1)	Acc. 2 (Center of the table)		
	I	$\mathbf{1}$	13.6	2.2		
$\,1$	П	\overline{c}	9.3	3.0		
	Ш	3	10.9	3.1		
	IV	$\overline{4}$	25.7	2.7		
	I and II	5	13.6	3.8		
	I and III	6	15.1	3.7		
$\overline{2}$	I and IV	7	32.4	2.9		
	II and III	8	12.3	3.3		
	II and IV	9	25.0	4.1		
	III and IV	10	30.8	3.6		
	I, II, and III	11	14.8	4.3		
3	I, II, and IV	12	38.2	5.5		
	II, III, and IV	13	32.7	4.2		
	I, III, and IV	14	37.2	4.4		
$\overline{4}$	I, II, III, and IV	15	32.1	5.2		
30						
25 20 Acceleration(G) 15 $10\,$ 5 0				t(s)		
-5^{0}	0.2 $\overline{4}$	0.6	1.0 $_{\rm b.8}$ 1.2	1.4 1.6		

Fig. 16. Time history of accelerations both at Spring 1 measured by Acc. 1 and the center of the table measured by Acc. 2 on a real HALT machine acted upon by Hammer IV.

 $Acc.1 - Acc.2$

-15 -10 -5

with a zero mean. The standard deviation is usually represented by a sigma.

Table 7 shows the maximum table accelerations measured near Spring 1 and at the center of the table for the following cases: (1) a single hammer (i.e., Hammers I and II), (2) a combination of two hammers (i.e., Hammers I and II and Hammers I and III), (3) three hammers acting simultaneously (i.e., Hammers I, II, and III), and (4) all four hammers acting simultaneously (i.e., Hammers I, II, III, and IV). The measurement results indicate a significant difference between the acceleration values at the center of the table and at Spring 1. For example, differences are observed between the acceleration values (i.e., 13.6, 9.3, 10.9, and 25.7 G) at Spring 1 when the table is impacted by each of the four hammers. However, only small acceleration variations (i.e., 2.2, 3.0, 3.1, and 2.7 G) occur at the center of the table. Significant differences are also observed between the accelerations at the center of the table and at Spring 1 for all cases. This difference is shown by

taking the time domain signals at the two locations using a continuous 1.6 s measurement (Fig. 16). The case where Hammer IV is directly mounted on the table (Fig. 4) is shown in Fig. 16. The piston inside the hammer immediately rebounds with small accelerations after hitting the anvil at the front end of the hammer. Thus, the magnitudes for each of the instantaneous accelerations are larger on the upper half than those on the lower half (Fig. 16). Furthermore, the directions of the acceleration shock pulses on the table vary from positive to negative (i.e., from approximately +25 G to -10 G).

Using a combination of multiple hammers does not necessarily result in greater impact power. For example, using only three hammers (i.e., I, II, and IV) in Test 14 generates a maximum acceleration of 38.2 G at Spring 1. This acceleration is even higher than that generated by using all four hammers (i.e., I, II, III, and IV; 32.1 G; Table 2).

The PPDF plots with RMS values are shown in Figs. 17(a)- (g). These plots typically originate from the 1.6 s time domain test records on the HALT machine table (Fig. 16). The differences on the RMS values and the number of peaks per second are observed between the cases. Based on the PPDF method, the acceleration effectiveness differs because of the fatigue occurrence on the test specimen for each of the cases using different hammers. The case illustrated in Fig. 17(f) is the most effective because both the highest number of peaks per second and the highest RMS values are achieved. Therefore, using three hammers (i.e., I, II, and IV) provides the best efficiency in the HALT machine.

An SRS plot from the HALT machine table for all foregoing testing cases is shown in Fig. 17(h). Accordingly, the response spectrum magnitudes in all cases are extremely small and without significant difference from each other at low frequencies. The peak accelerations dramatically increase when the repetitive frequency of the hammer increases up to 10 Hz in all cases except those using single hammers (i.e., III and IV). Furthermore, the peaks are observed at around 100 Hz in all cases. These peaks possibly denote the natural frequency of the table. Furthermore, Fig. 17(h) shows that the modal responses of the table are different, that is, they depend not only on the number of hammers in operation but also on the acting position of the hammer.

Another experiment is conducted to examine the effects of changing the swing angle (*β*) of a hammer. Correspondingly, a cantilever beam that serves as a typical test specimen is attached on top of the table at the Hammer IV position by orienting along the X-axis (Fig. 4). The swing angle *β* of driving Hammer I varies in two different cases with angles $\beta = 0^{\circ}$ and −90°. The original hammer, which points at the positive Xaxis, has a swing angle of $\beta = 0^{\circ}$. $\beta = -90^{\circ}$ denotes that the hammer rotates clockwise along its fixed center point with a swing angle of 90° with respect to the positive X-axis (Fig. 13). The table was impacted by a single hammer with the same input setup as the hammer in the two preceding cases. The time history of accelerations at the two measurement positions (i.e., center of the table and beam) is shown in Fig.

19. The accelerations at the center of the table are nearly unchanged under the two different swing angles. By contrast, apparent differences are observed on the beam responses. These differences include peaks at 60 G and 30 G for $\beta = 0^{\circ}$ and $\beta = -90^{\circ}$, respectively. The PPDF with RMS for these two cases is also illustrated in Fig. 19. The higher peak number, which focuses on the smaller sigma numbers in the case $\beta = 0^{\circ}$, denotes the test effectiveness.

6. Summary and conclusions

The table responses to various hammer design combinations in a HALT system determine the effects on the tested products under different impact types. This study examines the problem by initiating a theoretical model that analyzes the impact of a hammer with equivalent spring forces at four table locations, where the springs are mounted. These spring forces are the exerting forces in a subsequent simulation. The piston generates the corresponding impact forces when it moves up and down inside the cylindrical hammer housing. The effects of the impact forces and the spring stiffness on the table motion are also examined theoretically. These theoretical calculations are then verified using a finite element simulation analysis of the HALT system. The satisfactory consistency between the simulation and analytical results reveals that the currently proposed theoretical analysis is correct.

To measure the table accelerations at two specified locations (i.e., near Spring 1 and at the center of the table), the real HALT system in the experiment is impacted by either a single hammer or multiple hammers. The resulting outcomes at these two locations indicate that the theoretical results agree well with the experimental results. Both the PPDF and SRS methods are used to analyze the time domain history of the experimental results. These methods are further utilized to determine how these results could help the HALT system user. For example, Fig. 17(h) provides an overall view of the SRS responses for each of the different combinations of hammers. The PPDF and SRS methods provide the users a better understanding of the appropriate hammers to use in replicating a desired SRS.

Furthermore, the presented methods are proven suitable for solving the HALT system responses. One of the most noteworthy results is the occurrence of the strongest table response accelerations when three hammers are used. Moreover, the experimental data on the SRS plot show the excitation performance in the frequency domain for each of the hammer design combinations. Using three hammers leads to the best frequency domain characteristics, that is, the accelerations are strongest over a wide frequency range. In addition, using a combination of multiple hammers does not necessarily result in higher impact strength. Therefore, one can test products more effectively by choosing a logical combination of multiple hammers. However, users are unable to switch between hammer combinations in the current impact table designs. Thus, the manufacturer should provide design flexibility by

Fig. 17. PPDF and SRS plots for the table responses at Spring 1 for each case using different hammers.

Fig. 18. Time history of accelerations at the center of the table and on a beam acted upon by hammer I with different swing angles.

Fig. 19. PPDF from the beam data in Fig. 18.

allowing the users to optionally turn on and off some of the redundant hammers. Waste of resources is avoided by selecting the optimum hammer excitations. Furthermore, the impact forces are better controlled by choosing the appropriate hammer combination.

One design parameter (i.e., how the hammers are mounted on the table with regard to incline angle α and swing angle β) is also important to the response of the table in the HALT system. Changing the spring stiffness also changes the magnitude and shape of the time history shock pulses on the table. The corresponding factor investigations help the designers handle the hardware performances of the machine at the design stage (i.e., before the system is manufactured).

In addition, specific impact types result in different responses at different table locations. This observation has been verified in Sec. 2.3, where the impacts of a single hammer (i.e., Hammer I) are analyzed. This hammer excites different responses at each spring location. Nevertheless, the single

hammer has different effects on the table motion at a specific point even at different locations.

One obtains different results (e.g., Tables 3-6) by setting new parameters (i.e., *b*, *c*, *d*, *e*, *E*, and *D*) in the theoretically analyzed formulas (Table 2). The representation of the corresponding force components is simplified at the spring locations along each axis. Thus, recognizing the relationship among these force components and calculating the spring force ratios along all the three axes (i.e., X:Y:Z) are possible. Accordingly, the most efficient test position to mount the specimen on the table of a HALT system becomes apparently achievable. This information is especially beneficial for end users, particularly when deciding on how to mount the test specimen on the table.

Two approaches are used to connect the hammers to a table. One is by directly mounting the hammers under the table, and the other is by leaving a gap between the hammers and the table. The manner by which the hammers are mounted dramatically

changes the shock pulse accelerations on the table. The former results in response pulses with both positive and negative values, whereas the latter results in mostly positive pulses.

Based on all the preceding detailed analyses of the HALT system performances, the future system design can be improved through a user-friendly interface. Such an interface can be created by incorporating the technical information derived from the current study. The interface is then turned into a smart system, which can guide and help the users understand the effects of their chosen settings.

Acknowledgment

The authors express their appreciation for the equipment and funding support provided by King Design Company.

Nomenclature-

a : Distance of the springs to the X- and Y-axes b, c, d, D, e, E, k : Simplified parameters Whereas the latter results in mostly positive pulses.

The axed on all the preceding detailed analyses of the HALT

system performances, the future system design can be im-

by corrected by incorporating the cellmical inf From the propose purses with both positive and negative values,

whereas the latter results in mostly positive pulses.

Merculation, design

Based on all the preceding detailed analyses of the HALT

positive pulses of the \vec{H}_X , \vec{H}_Y , \vec{H}_Z : Component forces along the three axes *k^s* : Spring stiffness { }*^t ^k* : Matrix of the total equivalent spring stiffness *r* : Distance of the hammers to the center of the table *R*_i: Distance of the springs to the center of the table Share The Hender Street of the internal for the effects of their chosen settings.

Acknowledgment

The authors express their appreciation for the equipment

IT D. Lee, J. Zhugh, China

and funding support provided by King **Example 11 Axiao-Yandam System, which can guide and neip the users understand** [6] Li Xiao-Yandam effects of their chosen settings. The authors express their appreciation for the equipment g' the *LE*. The authors expr Treets or the rensen settings.
 ICON The EVENT SURVE SURVENT CONDUCT
 ICON THE SURVENT CONDITER SURVENT CONDUCT
 EXERCURE SURVENT CONDITER SURVENT CONDITION
 EXERCURE SURVENT CONDITION
 EXERCURE SURVENT CONDITION From the current study. The interface is then turned into a

meant system, which can guide and help the users understand [6] Li Xiao-Yang, Jia

he effects of their chosen settings.

Acknowledgment
 $\begin{array}{ll}\n\text{R}^2 & \text{R}^$ $S^1 \times S^1$, S^1 , S^1 z: Component forces on the springs along the three axes **Acknowledgment**

The authors express their appreciation for the equipment

(7) D. Lee, J. Zhuj

changes upport provided by King Design Company.
 Nomenclature-
 17 D. Lee, J. Zhuj

expected the springs to the X- and Y $S^t M^2$): Force of the spring that generates the moment **Acknowledgment**

TD . Lee, J. Zhuge,

The authors express their appreciation for the equipment

ITD . Lee, J. Zhuge,

and funding support provided by King Design Company.

Momenclature

The statues of the new gradient met **dgment**

(17) D. Lee, J. Zhange, Y. C. Clongedu, Chana (2009) 118

(17) D. Lee, J. Zhange, Y. C. Clongent from the equipment

transference of the springs to the X- and Y-axes

eneral phenometric hammer

time of the sprin **Examplement**

Solven the support provided by King Design Company.

IT D. Lee, J. Zhuge, Y. Company support provided by King Design Company.
 IT D. Lee, J. Zhuge, Y. Company from the support provided by King Design Compa For a time surface of the spings of the New Jersey Company
 Nonenclature
 \vec{Y}_L , \vec{U}_L , $\vec{U$ *α*
 α : Usatece or the spinnes of the X- and Y-axes
 b,c, d, D, e, E. k : Simplified parameters
 H_U : Exerting force of hammer
 A is *C*, *P* is *C*, *C* is *C* is a component force show the total equivalent s *δ β* : S, *b*, *B*, *E*, *B*, *C*, *K*, *B*, *C*, *S* is the conduction of the standard in the projection of force *H* is the projection of *B* is the projection of force *H* is the projection of *F* is the projection

- $(S^{i}M^{Z})_{X}$, $(S^{i}M^{Z})_{Y}$: Component forces on the O-XY plane along the X- and Y-axes
- the direction of the X- and Y-axes

Greek symbols

- : Incline angle of force $\vec{H}(t)$ to the plane of the table
- : Swing angle between the projection of force $H(t)$ on the O-XY plane and the X-axis
- δ : Deflection of the spring

Subscripts

i : Spring numbers, *i* = 1 to 4

X, Y, Z : Direction of the forces along the three axes

References

- [1] H. J. Yuan and T. M. Jiang, Characteristics of Omni-Axes Random Vibration Environment and Its Stimulation Effect, *Structure & Environment Engineering*, 33 (2) (2006) 1-5.
- [2] F. F. Wang, Modified highly accelerated life test for aerospace electronics, *Thermo-mechanical Phenomena in Electronic Systems, Proceedings of the Intersociety Conference*, Lunnwood, WA., USA (2002) 940-945.
- [3] G. K. Hobbs, What HALT and HASS can do for your products, *Evaluation Engineering*, 36 (11) (1997) 138-143.
- [4] H. S. Liu, Environmental stress screening equipment: search, evaluation, design, experimentation, *Test Engineering and Management* (1994)10-14.
- [5] H. S. Liu, New approach for production line Environmental Stress Screening, *Test Engineering and Management*, 67 (4) (2005) 18-19.
- [6] Li Xiao-Yang, Jiang Tong-Min and Zhang Wei, A multi-axis vibration fixture based on electromagnetic shaker, *Proceedings of the IEEE 8th Reliability, Maintainability and Safety,* Chengdu, China (2009) 1183-1186.
- [7] D. Lee, J. Zhuge, Y. C. Chuang and Y. S. Chen, Comparing features of the newly developed electrodynamic hammer and the general pneumatic hammer used in environmental stress screening systems, *Proceeding of the Institute of Environmental Sciences and Technology Conference*, Florida, USA (2012) 453-465.
- [8] B. B. Muvdi, A. W. Al-Khafaji and J. W. McNabb, Equilibrium of rigid bodies in three dimensions, *Statics for Engineers*, Springer-Verlag, New York, USA (1997) 228-307.
- [9] H. R. Harrison and T. Nettleton, Force systems and equilibrium, *Principles of Engineering Mechanics*, 2nd ed., Butterworth-Heinemann, London, U.K. (1994) 37-53.
- [10] J. L. Meriam and L. G. Kraige, Force systems, *Engineering Mechanics*, *SI Version: Statics*, 6th ed., John Wiley & Sons Inc., New Jersey, USA (2008) 23-50.
- [11] R. K. Vierck, *Vibration analysis*, 2nd ed., Crowell Company, New York, USA (1979).
- [12] S. Rajasekaran and G. Sankarasubramanian, Fundamental concepts of structures, *Computational Structural Mechanics*, Prentice-Hall, New Delhi, India (2001) 60-62.
- [13] Mario Paz and W. E. Leigh, Undammed single degree-offreedom system, *Structural dynamics Theory and Computation updated with SAP2000*, 5th ed., Kluwer Academic publishers, Massachusetts, USA (2004) 7-8.
- [14] Thomas D. Rossing and Neville H. Letcher, Free and force vibration of simple system, *Principles of Vibration and Sound*, 2nd ed., Springer-Verlag, New York, USA (2004) 15-17.
- [15] S. Graham Kelly, Modeling of SDOF system, *Mechanical Vibrations: Theory and Applications*, SI Edition, 1st ed., Cengage Learning Inc., Connecticut, USA (2012) 62-67.
- [16] G. Henderson, 6DOF machine characteristics and method of measuring effectiveness, *Test Engineering & Management* (1992) 10-12.
- [17] G. Henderson, *Dynamic characteristics of repetitive shock machine*, GHI Systems Incorporated, San Pedro, California, USA (1993).
- [18] M. A Biot, Theory of elastic system vibration response under transient excitation with an application to earth quake resistant building, *Journal of Applied Sciences,* 19 (1933) 260-268.
- [19] Smithson, Stephen A, Shock response spectrum analysis for ESS and STRIFE/HALT measurement*, Proceedings, IES*, San Diego, California, USA (1991).
- [20] N. E. Dowling, Fatigue failure predictions for complicated stress-strain histories, *Journal of Materials*, 7 (1) (1972) 1-17.

Y. S. Chen is an Associate Professor at the Department of Mechanical Engi neering, Yuan-Ze University, Taoyuan, Taiwan. He received his Ph.D. in mechanical engineering from the University of Maryland, College Park. He has worked closely with the industry in areas related to the mechanical reliability assessment

of electronic products, including vibration analysis, structural analysis, packaging design, and material properties investigation.

Le Hong Chuong received his B.S. and M.S. degrees in Mechanical Engineering from the National University of Civil Engineering, Vietnam, in 2000 and 2006, respectively. He is currently pursuing his Ph.D. at the Department of Mechanical Engineering, Yuan-Ze University, Taiwan. His current research

interests include electronic packaging analysis, computeraided engineering analysis, mechanical system design, and automotive engineering.