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Mean-value first-order saddlepoint approximation based collaborative optimization for multidisciplinary problems under aleatory uncertainty †

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Abstract

Reliability-based multidisciplinary design optimization (RBMDO) has received increasing attention in engineering design for achieving high reliability and safety in complex and coupling systems (e.g., multidisciplinary systems). Mean-value first-order saddlepoint approximation (MVFOSA) is introduced in this paper and is combined with the collaborative optimization (CO) method for reliability analysis under aleatory uncertainty in RBMDO. Similar to the mean-value first-order second moment (MVFOSM) method, MVFOSA approximated the performance function with the first-order Taylor expansion at the mean values of random variables. MVFOSA uses saddlepoint approximation rather than the first two moments of the random variables to estimate the probability density and cumulative distribution functions. MVFOSA-based CO (MVFOSA-CO) is also formulated and proposed. Two examples are provided to show the accuracy and efficiency of the MVFOSA-CO method.

Keywords: Reliability-based multidisciplinary design optimization; Mean-value first-order saddlepoint approximation; Aleatory uncertainty; Probability density function; Cumulative distribution function

1. Introduction

Multidisciplinary design optimization (MDO) is traditionally formulated as a deterministic problem that assumes the nonexistence of uncertainty. However, the real design and manufacturing processes are often affected by unavoidable uncertainties that cause system performance and output variations [1]. These variations may cause design solutions to be infeasible or unreliable. For high reliability and safety in MDO problems, reliability-based multidisciplinary design optimization (RBMDO) is one research focus. Sues et al. [2] created response surface models at the system level to replace the computationally expensive simulation models and relieve RBMDO of its reliability analysis and computation burden. Sues and Cesare [3] proposed an RBMDO framework where reliability analysis was decoupled from the optimization loop. Reliability is initially computed before the first execution of the optimization loop and then updated iteratively after. During this process, approximate reliability constraints are used. In Ref [4], a multi-stage parallel implementation of probabilistic design optimization integrated the existing reliability analysis method into MDO frameworks. The concurrent subsystem optimization was widely used to search for the most probable point (MPP) [5-7]; the collaborative reliability analysis method was also broadly used [8, 9]. In Ref. [10], a sequential optimization and reliability assessment (SORA) method for RBMDO was proposed. The deterministic formulation of MDO in SORA was constructed by using the MPP from the previous iteration. Following each optimization loop, reliability analysis was conducted at the optimal solution of the deterministic MDO to check the probability constraint feasibilities. Huang et al. [11] proposed an enhanced SORA (ESORA) method to further improve computational efficiency for reliability analysis. ESORA addresses the constant and varying variances of random design inputs. Zhang et al. [12] introduced probability and possibility analyses into RBMDO and proposed an RBMDO with discrete and continuous variables of various uncertainties to simultaneously analyze aleatory and epistemic uncertainties. Xiao et al. [13] used interval variables to consider epistemic uncertainty and proposed the unified uncertainty analysis method to estimate the failure probabilities of complex systems with both epistemic and aleatory uncertainties.

A number of works have been conducted on RBMDO but some issues still require further exploration. In some cases, the performance function is expensive to evaluate but the mean-value first-order Second Moment (MVFOSM) method is feasible. MVFOSM is highly efficient and easy to use. However,

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it has obvious accuracy deficiencies [14-16]. MVFOSM uses only the first two moments of random variables instead of the complete distribution information and assumes that the response is normally distributed. The mean-value first-order saddlepoint approximation (MVFOSA) is used for aleatory uncertainty analysis in the mentioned situations to improve analysis accuracy while retaining high efficiency. Compared with MVFOSM, MVFOSA is relatively accurate because it utilizes complete distribution information [15]. Instead of simply using the first two moments of random variables, MVFOSA estimates the probability density function (PDF) and cumulative distribution function (CDF) of the performance function by saddlepoint approximation. The MVFOSA method is combined with the collaborative optimization (CO) method in the present paper to solve the RBMDO problem. CO is a bi-level MDO method for large scale and distributed-analysis engineering design problems. CO contains optimization problems at both system and discipline levels. The system-level optimization problem optimizes the system objective and the coordinate consistency between the coupling disciplines, whereas the discipline-level optimization problem minimizes the discrepancy between the design variables and their targets [17].

The rest of the paper is organized as follows. Section 2 introduces the RBMDO problem. Commonly used reliability analysis methods, including simulation and approximation, are also briefly reviewed. Sec. 3 provides the fundamental analysis of MVFOSA is provided. Sec. 4 explains the proposed method, namely, the MVFOSA-based CO (MVFOSA-CO), including the strategy, procedure, and formulas. Sec. 5 portrays two examples to demonstrate the accuracy and efficiency of the proposed method. Finally, Sec. 6 concludes.

2. RBMDO problems and simulation or approximation methods for reliability analysis

RBMDO is formulated as follows:

$$\begin{aligned} &\min f\left(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\mu}_{\mathbf{X}_s},\boldsymbol{\mu}_{\mathbf{Y}}\right) \\ &\text{s.t. } P\left[g_i\left(\mathbf{d},\mathbf{X},\mathbf{X}_s,\mathbf{Y}\right)\le 0\right]\ge\left[R_i\right]=1-\left[p_f\right] \\ &\mathbf{d}^L\le\mathbf{d}\le\mathbf{d}^U, \mathbf{X}^L\le\boldsymbol{\mu}_{\mathbf{X}}\le\mathbf{X}^U, \mathbf{X}_s^L\le\boldsymbol{\mu}_{\mathbf{X}_s}\le\mathbf{X}_s^U, \\ &\mathbf{Y}^L\le\boldsymbol{\mu}_{\mathbf{Y}}\le\mathbf{Y}^U, \mathbf{X}_{\mathrm{DV}}=\left\{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\mu}_{\mathbf{X}_s},\boldsymbol{\mu}_{\mathbf{Y}}\right\},\ i=1,2,\dots,n \end{aligned} \quad , \tag{1}$$

where $f(\bullet)$ denotes the system objective function; $g_i(\bullet)<0$ is the safe or successful region; $g_i(\bullet)>0$ is the failure region; $g_i(\bullet)=0$ is the limit state surface, which is the boundary between success and failure; $\left[R_i\right]$ is the required reliability of the probabilistic constraints $P\left[g_i(\bullet)\le 0\right]$; $\left[p_f\right]$ is the allowable probability of failure; $\mathbf{X}_{\mathrm{DV}}$ denotes the design variables; $\mathbf{d}$ is the deterministic design variables; $\mathbf{X}$ denotes the random discipline design variables; $\mathbf{X}_s$ denotes the random sharing design variables; $\mathbf{Y}$ denotes the random coupling design variables, $\mathbf{Y}=\left\{\mathbf{Y}_{\bullet i},\mathbf{Y}_{i\bullet}\right\}$, $i=1,2,\dots,n$; $\mathbf{Y}_{\bullet i}$ is the input coupling variables to the i th discipline; $\mathbf{Y}_{i\bullet}$ is the output coupling variables from the i th discipline; $\boldsymbol{\mu}$ denotes the mean value of random variables and random parameters; superscripts L and U denote the lower and upper bounds, respectively; n is the total number of disciplines.

We use $\mathbf{X}_{\mathrm{R}}$ to denote the set of random variables $\mathbf{X}_{\mathrm{R}}=\left\{\mathbf{X},\mathbf{X}_s,\mathbf{Y}\right\}$ and express the function relationship between performance function G and the design variables as $G=g\left(\mathbf{d},\mathbf{X}_{\mathrm{R}}\right)$. The probability of the probabilistic constraints $P\left[g_i(\bullet)\le 0\right]$ can be described as the CDF of $g_i(\bullet)$ and is theoretically calculated by Eq. (2).

$$F_G=P\left[g_i\left(\mathbf{d},\mathbf{X}_{\mathrm{R}}\right)\le 0\right]=\int_{g_i\left(\mathbf{d},\mathbf{X}_{\mathrm{R}}\right)\le 0} f_{\mathbf{X}_{\mathrm{R}}}\left(\mathbf{X}_{\mathrm{R}}\right)\mathrm{d}\mathbf{X}_{\mathrm{R}}\ , \tag{2}$$

where $f_{\mathbf{X}_{\mathrm{R}}}\left(\mathbf{x}_{\mathrm{R}}\right)$ is the joint PDF of $\mathbf{X}_{\mathrm{R}}$. However, arriving at the analytical solution by using Eq. (2) is difficult because of the high dimensionality of random variables and the nonlinear integration boundary $G=g\left(\mathbf{d},\mathbf{X}_{\mathrm{R}}\right)$ [18]. Therefore, simulation or approximation methods are widely used.

Simulation or approximation methods have three types: (1) sampling-based methods, (2) moment matching methods, and (3) MPP-based methods [15]. Sampling-based methods are easy to apply and can provide accurate probability estimations with sufficient simulations [19-22]. However, sampling-based methods are inefficient for many engineering problems with high reliability and computationally expensive performance functions [16]. Moment-matching methods ease computational difficulties by approximating the distribution of performance functions and by fitting the first few moments [23-26]. MVFOSM is one of the commonly used moment matching methods. It uses the first two statistical moments and employs first-order Taylor expansion at the mean values of random variables. The moment matching method is highly efficient. However, its accuracy is generally lower than that of sampling-based methods [15]. MPP-based methods have a good balance between efficiency and accuracy. The first-order reliability method (FORM) and second-order reliability method (SORM) [27, 28] approximate the performance function with Taylor expansion at the MPP. However, the MPP location is also an optimization problem and needs more function evaluations than MVFOSM, thus making FORM and SORM computationally expensive for complex and coupling systems [15]. The original random variables in $\mathbf{X}$ -space have to be transformed into standard normal variables for FORM and SORM [29], thus increasing the nonlinearity of the performance function [30, 31]. First-order saddlepoint approximation (FOSA) is proposed for reliability analysis to avoid random variable transformation [31]. FOSA linearizes the performance function in the original random $\mathbf{X}$ -space without any random variable transformation. The expansion point, that is, the most likelihood point (MLP), has the highest probability density on the limit state $g\left(\mathbf{X}_{\mathrm{R}}\right)=0$. However, finding the MLP in FOSA also requires an optimization process.

Table 1. CGFs of some common distributions.

Distribution	PDF	CGF
Normal	$f(x)=\left(1/\sqrt{2\pi}\sigma\right)\exp\left[(x-\mu)^2/2\sigma^2\right]$	$K(t)=\mu t+\frac{1}{2}\sigma^2t^2$
Uniform	$f(x)=1/(b-a)$	$K(t)=\ln\left(e^{bt}-e^{at}\right)-\ln(b-a)-\ln(t)$
Exponential	$f(x)=\alpha\exp(-\alpha x)$	$K(t)=-\ln(1-t/\alpha)$
Gumbel	$f(x)=\left(1/\sigma\right)\exp\left[(x-\mu)/\sigma\right]$ $\exp\left\{-\exp\left[(x-\mu)/\sigma\right]\right\}$	$K(t)=\mu t+\ln\Gamma(1-\sigma t)$
Gamma	$f(x)=\beta^\alpha/\Gamma(\alpha)x^{\alpha-1}e^{-\beta x}$	$K(t)=\alpha\left[\ln\beta-n(\beta-t)\right]$
χ^2	$f(x)=\left[1/\Gamma(n/2)2^{n/2}\right]x^{n/2-1}e\left(-\frac{1}{2}x\right)$	$K(t)=-\frac{1}{2}n\ln(1-2t)$

3. Mean-value first-order saddlepoint approximation

MVFOSA was introduced in reliability analysis by Huang and Du [15] not only for its similar efficiency and robustness to MVFOSM but also for its relatively high accuracy. MVFOSA uses saddlepoint approximation to evaluate the CDF and PDF of the performance function. MVFOSM has been used in reliability sensitivity analysis [32] and structural reliability analysis [33].

In MVFOSA, performance function G is linearized in the original random space by using first-order Taylor expansion. The expansion point is at values of deterministic variables $\mathbf{d}_i^*$ and mean values of random variables $\mu_{\mathbf{X}_{Ri}}$. The first-order approximation function $G\approx\hat{G}=\hat{g}(\mathbf{d},\mathbf{X}_R)$ can be expressed as follows:

$$\hat{G}=\hat{g}(\mathbf{d},\mathbf{X}_R)=g\left(\mathbf{d}^*,\boldsymbol{\mu}_{\mathbf{X}_R}\right)+\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{d}_i}\right|_{\mathbf{d}_i}\left(\mathbf{d}_i-\mathbf{d}_i^*\right)+\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}\left(\mathbf{X}_{Ri}-\mu_{\mathbf{X}_{Ri}}\right). \tag{3}$$

The cumulant generating function (CGF) of $\mathbf{X}_R$ was denoted as $K_{\mathbf{X}_R}(t)$. Table 1 lists the CGFs of some commonly used distributions [31]. Two useful properties of CGF are given as follows [15]:

Property I. If $\mathbf{X}_R=(X_{R1},X_{R2},\dots,X_{Rn})$ are independent random variables and their corresponding CGFs are $K_{X_{Ri}}(t)$ $(i=1,2,\dots,n)$, then the CGF of $Y=\sum_{i=1}^n X_{Ri}$ is $K_Y(t)=\sum_{i=1}^n K_{X_{Ri}}(t)$.

Property II. If X_R is a random variable and its CGF is $K_{X_R}(t)$, then the CGF of $Y=aX_R+b$ (a and b are constants) is $K_Y(t)=K_{X_R}(at)+bt$.

For example, if X_R follows χ^2 distribution with CGF $K_{X_R}(t)=-(1/2)n\ln(1-2t)$, then the CGF of Y is $K_Y(t)=-(1/2)n\ln(1-2at)+bt$.

On the basis of the above two properties, the CGF of $\hat{G}$ is given by Eq. (4).

The saddlepoint t_s can be determined by solving Eq. (5) [15, 34], where $K'_G(t)$ is the first-order derivative of CGF. Once both the CGF of $\hat{G}$ and saddlepoint t_s are obtained, saddlepoint approximation can be applied for PDF and CDF estimations.

$$K_{\hat{G}}(t)=\left(g\left(\mathbf{d}^*,\boldsymbol{\mu}_{\mathbf{X}_R}\right)+\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{d}_i}\right|_{\mathbf{d}_i}\left(\mathbf{d}_i-\mathbf{d}_i^*\right)-\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}\mu_{\mathbf{X}_{Ri}}\right)t+\sum_{i=1}^n K_{\mathbf{X}_{Ri}}\left(\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}t\right), \tag{4}$$

$$K'_{\hat{G}}(t)=\left(g\left(\mathbf{d}^*,\boldsymbol{\mu}_{\mathbf{X}_R}\right)+\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{d}_i}\right|_{\mathbf{d}_i}\left(\mathbf{d}_i-\mathbf{d}_i^*\right)-\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}\mu_{\mathbf{X}_{Ri}}\right)+\sum_{i=1}^n\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}K'_{\mathbf{X}_{Ri}}\left(\left.\frac{\partial G}{\partial\mathbf{X}_{Ri}}\right|_{\mu_{\mathbf{X}_{Ri}}}t\right)=0. \tag{5}$$

A simple formula for computing the PDF of $\hat{G}$ is given as follows [15, 35]:

$$f_{\hat{G}}\approx\left[\frac{1}{2\pi K''_{\hat{G}}(t_s)}\right]^{1/2}e^{\left[K_{\hat{G}}(t_s)\right]}, \tag{6}$$

where $K''_{\hat{G}}(\bullet)$ is the second-order derivative of CGF. Two concise formulas are proposed for calculating the CDF of $\hat{G}$ [36, 37]:

$$F_{\hat{G}}=P\left[\hat{g}(\mathbf{X}_R)\le 0\right]\approx\Phi(w)+\phi(w)\left(\frac{1}{w}-\frac{1}{v}\right). \tag{7}$$

or

$$F_{\hat{G}} = P\left[\hat{g}\left(\mathbf{X}_{\mathrm{R}}\right) \le 0\right] \approx \Phi\left(w + \frac{1}{w}\log\frac{v}{w}\right), \tag{8}$$

where $\Phi(\bullet)$ and $\phi(\bullet)$ are the CDF and PDF of the standard normal distribution, respectively.

$$w = \mathrm{sign}\left(t_s\right)\left\{2\left[-K_{\hat{G}}\left(t_s\right)\right]\right\}^{1/2}, \tag{9}$$

and

$$v = t_s\left[K''_{\hat{G}}\left(t_s\right)\right]^{1/2}, \tag{10}$$

where

$$\mathrm{sign}\left(t_s\right) = \begin{cases} 1, & \text{if } t_s > 0 \\ 0, & \text{if } t_s = 0 \\ -1, & \text{if } t_s < 0 \end{cases}.$$

MVFOSA uses full distribution information. Thus, MVFOSA is generally more accurate than MVFOSM [15]. MVFOSA requires only a process of finding one saddlepoint without any integration or optimization. Thus, MVFOSA is employed and combined with CO in this paper to solve RBMDO problems under the aleatory uncertainty.

4. Collaborative optimization under aleatory uncertainty

4.1 SORA method

CO was introduced and developed to maintain multidisciplinary engineering characteristics [38-42]. CO decomposes the design problem at the system and discipline levels. At the discipline level, local constraints are satisfied while the discrepancies between the design variables and their target values are minimized. At the system level, target values are determined for design variables and the system objective is optimized.

CO has many advantages [43]. First, multidisciplinary feasibility can be maintained by using compatibility constraints. Moreover, CO enjoys discipline autonomy. Thus, discipline analysis is easy and can be processed in parallel. Finally, CO can keep disciplinary feasibility at the optimization process. In engineering applications, CO was employed for the design of launch vehicles [40, 44] and aircraft configurations [42]. CO has also been widely used in decision-making [45] and conceptual design [46].

4.2 MVFOSA-CO

The MVFOSA-CO procedure is presented as follows:

Step 1: Set the initial values for system-level design variables as $\mathbf{d}_i^{\mathrm{sys},(0)}$, $\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{sys},(0)}$, $\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(0)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(0)}$, and $\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(0)}$ and the initial values for discipline-level design variables as $\mathbf{d}_i^{\mathrm{dis},(0)}$, $\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{dis},(0)}$, $\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{dis},(0)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{dis},(0)}$, and $\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{dis},(0)}$, respectively. Superscripts sys and dis denote system level and discipline-level design variables, respectively. Subscript i denotes discipline i. At $k=1$, k denotes the k th cycle.

Step 2: Solve the system-level optimization problem (Eq. (11)). Compatibility constraints J_i should be less than or equal to ε, which is a small positive number. Eq. (11) aims to optimize the system objective and obtain the solutions of $\mathbf{d}_i^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(k)}$, and $\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(k)}$. Thereafter, send these variables to discipline i at the discipline level as design parameters.

$$\begin{aligned}
&\min\ f\left(\mathbf{d}_i^{\mathrm{sys},(k)}, \boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{sys},(k)}, \boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(k)}, \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(k)}, \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(k)}\right) \\
&\text{s.t. } J_i = \left(\mathbf{d}_i^{\mathrm{sys},(k)} - \mathbf{d}_i^{\mathrm{dis},(k-1)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{dis},(k-1)}\right)^2 \\
&\qquad + \left(\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{dis},(k-1)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{dis},(k-1)}\right)^2 \\
&\qquad + \left(\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{dis},(k-1)}\right)^2 \le \varepsilon,\ i = 1, 2, \ldots, n
\end{aligned} \tag{11}$$

Step 3: Solve the discipline-level optimization problems (Eq. (12)), where discipline optimization problem tasks minimize J_i, which denotes the compatibility constraints at the system level. In the discipline optimization process, system-level design variables are treated as design parameters, and discipline optimization and reliability analysis are nested. Reliability analysis consists of three steps: (1) linearize the performance function $G \approx \hat{G} = \hat{g}\left(\mathbf{d}, \mathbf{X}_{\mathrm{R}}\right)$ (Eq. (3)) at the values of deterministic variables $\mathbf{d}^{\mathrm{sys},(k)}$ and the mean values of discipline-level random variables $\boldsymbol{\mu}_{\mathbf{X}_{\mathrm{R}i}}^{\mathrm{sys},(k)}$ by using first-order Taylor expansion; (2) calculate the CGF of the performance function $\hat{G}$ (Eq. (4)); (3) estimate the CDF and PDF of the performance function $\hat{G}$ (Eqs. (6)-(8)).

Step 4: Check the convergence. Send the values of $\mathbf{d}_i^{\mathrm{dis},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{dis},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{dis},(k)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{dis},(k)}$, and $\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{dis},(k)}$ to the system level and calculate compatibility constraints. If all compatibility constraints satisfy $J_i \le \varepsilon, i = 1, 2, \ldots, n$ and the value of the system objective function is stable, go to Step 5; otherwise, set $k = k+1$ and go to Step 2.

Step 5: Stop the optimization process. Export final solutions $\mathbf{d}_i^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_i}^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(k)}$, $\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(k)}$, and f.

The flowchart of MVFOSA-CO is shown in Fig. 1.

$$\begin{aligned}
&\min\ J_i = \left(\mathbf{d}_i^{\mathrm{sys},(k)} - \mathbf{d}_i^{\mathrm{dis},(k)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}i}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{X}i}^{\mathrm{dis},(k)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{dis},(k)}\right)^2 \\
&\qquad + \left(\boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{Y}_{\bullet i}}^{\mathrm{dis},(k)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{sys},(k)} - \boldsymbol{\mu}_{\mathbf{Y}_{i\bullet}}^{\mathrm{dis},(k)}\right)^2 \\
&\text{s.t. } P\left[\hat{g}_i\left(\mathbf{d}_i^{\mathrm{dis},(k)}, \mathbf{X}_i^{\mathrm{dis},(k)}, \mathbf{X}_s^{\mathrm{dis},(k)}, \mathbf{Y}_{\bullet i}^{\mathrm{dis},(k)}, \mathbf{Y}_{i\bullet}^{\mathrm{dis},(k)}\right) \le 0\right] \ge \left[R_i\right], \\
&\qquad \mathbf{d}^L \le \mathbf{d}_i^{\mathrm{dis},(k)} \le \mathbf{d}^U,\ \mathbf{X}^L \le \boldsymbol{\mu}_{\mathbf{X}i}^{\mathrm{dis},(k)} \le \mathbf{X}^U,\ \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s}^{\mathrm{dis},(k)} \le \mathbf{X}_s^U \\
&\qquad \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}}^{\mathrm{dis},(k)} \le \mathbf{Y}^U,\ \mathbf{X}_{\mathrm{DV}} = \left\{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}}\right\},\ i = 1, 2, \ldots, n
\end{aligned} \tag{12}$$

5. Examples

Two examples are used to show the accuracy and efficiency of the proposed method. MVFOSA-CO, MVFOSM-based CO

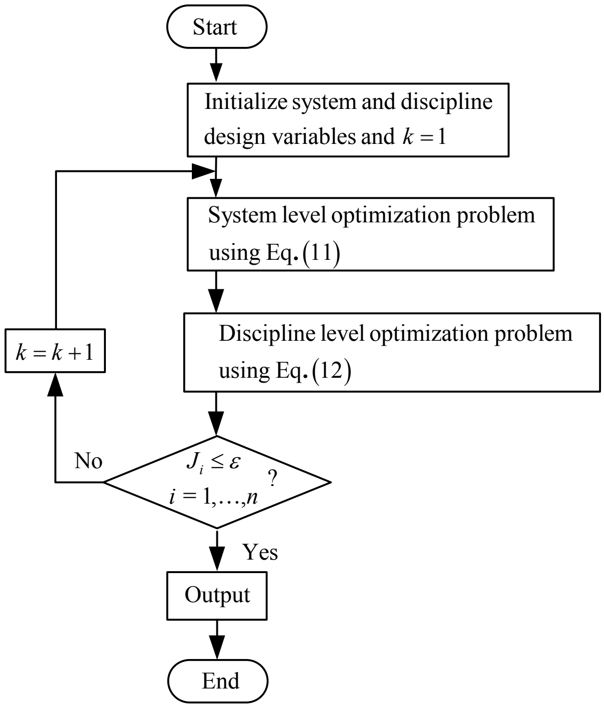

Fig. 1. Flowchart of MVFOSA-CO.

(MVFOSM-CO), and MCS-based CO (MCS-CO) are compared. The results obtained by MCS-CO are used as reference for accuracy comparison.

5.1 Mathematical example

The mathematical problem is provided as a simple test problem. The integrated framework of the design optimization problem is given as follows:

$$
\begin{aligned}
&\text{Find} \quad x_1, x_2, x_3, y_{12}, y_{21} \\
&\min \; f = (y_{12}-1)^2 + x_1^2 + x_2^2 + (y_{21}-2)^2 + x_3^2 \\
&\text{s.t.} \; g_1 = x_1 x_2^2 + y_{12} - 1 \le 0, \; g_2 = x_3^2 + y_{12}^2 + y_{21} - 5 \le 0, \\
&\quad y_{12} = x_1 - x_2 + 2y_{21}, \quad y_{21} = x_3 - y_{12}, \\
&\quad -5 \le x_1 \le 0, \quad 0 \le x_2 \le 1, \quad 0 \le x_3 \le 5, \\
&\quad 0 \le y_{12} \le 10, \quad 0 \le y_{21} \le 10
\end{aligned}
\tag{13}
$$

where f is the system objective, and x_1, x_2, x_3, y_{12} and y_{21} are the design variables.

The problem is modified into a MDO problem including two disciplines given in Fig. 2. In the modified problem, coupling variables y_{12} and y_{21} affect each discipline. The distribution information of all random design variables is given in Table 2. We assume two distribution types: the normal and Gumbel distribution. The formulations of the MVFOSA-CO optimization problem including two disciplines are provided in Eqs. (14)-(16).

The MVFOSA-CO approach analysis for the mathematical example is provided in Fig. 3.

Table 2. Distribution information of random variables in the mathematical problem.

Variables	Mean	Standard deviation	Distribution 1	Distribution 2
x_1	μ_{x_1}	$0.001\mu_{x_1}$	Normal	Gumbel
x_2	μ_{x_2}	$0.001\mu_{x_2}$	Normal	Gumbel
x_3	μ_{x_3}	$0.001\mu_{x_3}$	Normal	Gumbel
y_{12}	$\mu_{y_{12}}$	$0.001\mu_{y_{12}}$	Normal	Gumbel
y_{21}	$\mu_{y_{21}}$	$0.001\mu_{y_{21}}$	Normal	Gumbel

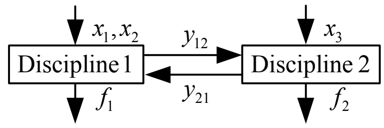

Fig. 2. MDO problem of the mathematical example.

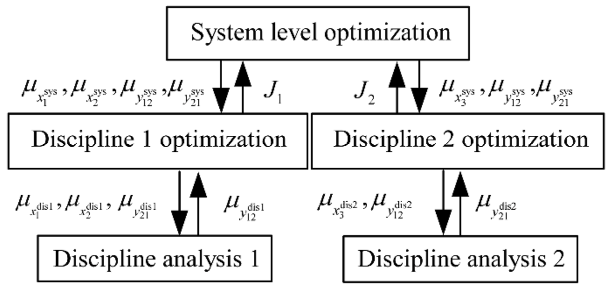

Fig. 3. MVFOSA-CO approach analysis for the mathematical example.

(1) System optimization problem

$$
\begin{aligned}
&\text{Find} \quad \mu_{x_1^{sys}}, \mu_{x_2^{sys}}, \mu_{x_3^{sys}}, \mu_{y_{12}^{sys}}, \mu_{y_{21}^{sys}} \\
&\min \; f = \left(\mu_{y_{12}^{sys}} - 1\right)^2 + \left(\mu_{x_1^{sys}}\right)^2 + \left(\mu_{x_2^{sys}}\right)^2 \\
&\qquad + \left(\mu_{y_{21}^{sys}} - 2\right)^2 + \left(\mu_{x_3^{sys}}\right)^2 \\
&\text{s.t.} \quad J_1 \le 0.01, \quad J_2 \le 0.01
\end{aligned}
\tag{14}
$$

(2) Discipline 1 optimization problem

$$
\begin{aligned}
&\text{Find} \quad \mu_{x_1^{dis1}}, \mu_{x_2^{dis1}}, \mu_{y_{12}^{dis1}}, \mu_{y_{21}^{dis1}} \\
&\min \; J_1 = \left(\mu_{x_1^{sys}} - \mu_{x_1^{dis1}}\right)^2 + \left(\mu_{x_2^{sys}} - \mu_{x_2^{dis1}}\right)^2 \\
&\qquad + \left(\mu_{y_{12}^{sys}} - \mu_{y_{12}^{dis1}}\right)^2 + \left(\mu_{y_{21}^{sys}} - \mu_{y_{21}^{dis1}}\right)^2 \\
&\text{s.t.} \quad P_1\left[x_1^{dis1}\left(x_2^{dis1}\right)^2 + y_{12}^{dis1} - 1 \le 0\right] \ge 0.96, \\
&-5 \le \mu_{x_1^{dis1}} \le 0, \quad 0 \le \mu_{x_2^{dis1}} \le 1, \quad 0 \le \mu_{y_{21}^{dis1}} \le 10, \\
&0 \le \mu_{y_{12}^{dis1}} \le 10, \quad \mu_{y_{12}^{dis1}} = y_{12}\left(\mu_{x_1^{dis1}}, \mu_{x_2^{dis1}}, \mu_{y_{21}^{dis1}}\right)
\end{aligned}
\tag{15}
$$

Table 3. Optimization results of the mathematical example (Case 1: Normal distribution).

	$\mu_{x_1^{sys}}$	$\mu_{x_2^{sys}}$	$\mu_{x_3^{sys}}$	$\mu_{y_{12}^{sys}}$	$\mu_{y_{21}^{sys}}$	f
MVFOSA-CO	−0.3177	0.3142	0.9056	0.4133	0.5251	3.6312
MVFOSM-CO	−0.3177	0.3142	0.9056	0.4133	0.5251	3.6312
MCS-CO	−0.3177	0.3142	0.9057	0.4134	0.5252	3.6314

Table 4. Reliability of the probabilistic constraints and calculation efficiency of the mathematical example (Case 1: Normal distribution).

	R_1	R_2	n_1	n_2	n_s	Calculation time
MVFOSM-CO	0.9739	0.9864	5924	4444	50	5 min 50 s
MVFOSA-CO	0.9739	0.9864	5924	4444	50	5 min 50 s
MCS-CO	0.9831	0.9907	5930	4408	50	-

Table 5. Optimization results of the mathematical example (Case 2: Gumbel distribution).

	$\mu_{x_1^{sys}}$	$\mu_{x_2^{sys}}$	$\mu_{x_3^{sys}}$	$\mu_{y_{12}^{sys}}$	$\mu_{y_{21}^{sys}}$	f
MVFOSM-CO	−0.3106	0.3051	0.9113	0.4231	0.5098	3.6609
MVFOSA-CO	−0.3177	0.3142	0.9057	0.4133	0.5251	3.6313
MCS-CO	−0.3177	0.3142	0.9056	0.4134	0.5252	3.6315

Table 6. Reliability of probabilistic constraints and calculation efficiency of the mathematical example (Case 2: Gumbel distribution).

	R_1	R_2	n_1	n_2	n_s	Calculation time
MVFOSM-CO	0.9837	0.9891	6195	4532	51	5 min 10 s
MVFOSA-CO	0.9659	0.9704	5888	4423	50	5 min 48 s
MCS-CO	0.9662	09758	5918	4408	50	-

(3) Discipline 2 optimization problem

$$\begin{aligned}
&\text{Find } \mu_{x_3^{dis2}}, \mu_{y_{12}^{dis2}}, \mu_{y_{21}^{dis2}} \\
&\min\ J_2 = \left(\mu_{x_3^{sys}} - \mu_{x_3^{dis2}}\right)^2 + \left(\mu_{y_{12}^{sys}} - \mu_{y_{12}^{dis2}}\right)^2 \\
&\qquad + \left(\mu_{y_{21}^{sys}} - \mu_{y_{21}^{dis2}}\right)^2 \\
&\text{s.t. } P_2\left[\left(x_3^{dis2}\right)^2 + \left(y_{12}^{dis2}\right)^2 + y_{21}^{dis2} - 5 \le 0\right] \ge 0.96, \\
&\quad 0 \le \mu_{x_3^{dis2}} \le 5,\ \ 0 \le \mu_{y_{12}^{dis2}} \le 10, \\
&\quad 0 \le \mu_{y_{21}^{dis2}} \le 10,\ \ \mu_{y_{21}^{dis2}} = y_{21}\left(\mu_{x_3^{dis2}}, \mu_{y_{12}^{dis2}}\right).
\end{aligned} \tag{16}$$

The target reliability of each probabilistic constraint is 0.96. System optimization is solved by using sequential quadratic programming (SQP), and the discipline optimization problems are solved by using a genetic algorithm (GA). The results from different methods are given in Tables 3-6. A sufficiently large number of simulations (10^6) is used; thus, the MCS-CO results are considered accurate references. R_1 and R_2 are the reliabilities of probabilistic constraints 1 and 2, respectively; n_s is the number of iterations in system optimization problem; n_1 and n_2 are the numbers of iterations in discipline optimization 1 and 2, respectively.

In Tables 3 and 4, for the special case where all random variables are normally distributed, MVFOSA-CO and MVFOSM-CO produce similar results and have the same values for system and discipline analyses because MVFORM is a special case of MVFOSA [15]. In Tables 5 and 6, MVFOSA-CO generates more accurate results than MVFOSM-CO. However, both methods have almost the same efficiency because MVFOSA-CO uses the complete distribution information of random variables and has good performance in the tail regions.

5.2 Speed reducer design

The second problem is derived from NASA MDO evaluation examples [47]. This problem represents the design of a speed reducer and is posed as an artificial multidisciplinary problem comprising three subsystems: subsystem 1 (Bearing group 1 and Shaft 1), subsystem 2 (Bearing group 2 and Shaft 2), and subsystem 3 (Gear 1 and Gear 2) (Fig. 4).

The problem has three sharing design variables. We assume that aleatory uncertainty is associated with some input variables. All random variables are described by Gumbel distribution. The details of the design variables are given in Table 7.

The system objective f is to minimize the speed reducer volume. The system objective and constraints are as follows:

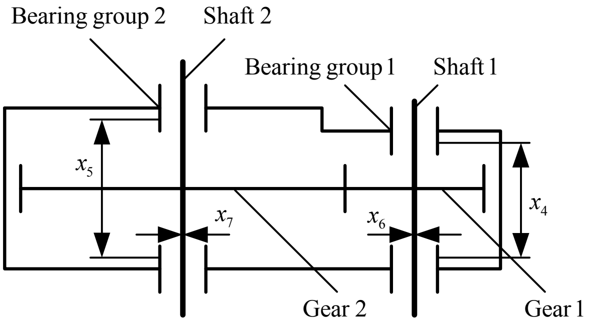

Fig. 4. Speed reducer design.

$$\min f = 0.7854x_1x_2^2\left(3.333x_3^2 + 14.933x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.477\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4x_6^2 + x_5x_7^2\right).$$

s.t. $g_1 = 27/\left(x_1x_2^2x_3\right) - 1 \le 0$: Upper bound on the bending stress of the gear tooth.

$g_2 = 397.5/(x_1x_2^2x_3^2) - 1 \le 0$: Upper bound on the contact stress of the gear tooth.

$g_3 = 1.93x_4^3/(x_2x_3x_6^4) - 1 \le 0$, $g_4 = 1.93x_5^3/(x_2x_3x_7^4) - 1 \le 0$: Upper bounds on the transverse deflection of the shaft.

$g_5 = A_1/B_1 - 1100 \le 0$, $g_6 = A_2/B_2 - 850 \le 0$: Upper bounds on the stresses of the shaft,

$$A_1 = \left[\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9\times10^6\right]^{0.5},$$

$$A_2 = \left[\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5\times10^6\right]^{0.5}, \quad B_1 = 0.1x_6^3, \quad B_2 = 0.1x_7^3.$$

$g_7 = x_2x_3 - 40 \le 0$, $g_8 = x_1/x_2 - 12 \le 0$,

$g_9 = -x_1/x_2 + 5 \le 0$: Dimensional restrictions based on space.

$g_{10} \sim g_{23}$: Dimensional restrictions of design variables (Table 7).

$g_{24} = (1.5x_6 + 1.9)/x_4 - 1 \le 0$, $g_{25} = (1.1x_7 + 1.9)/x_5 - 1 \le 0$: Design condition for the shaft based on experience.

The MVFOSA-CO analysis for the speed reducer MDO problem is shown in Fig. 5.

MVFOSA-CO optimization problem formulations, including three disciplines are provided in Eqs. (17)-(20).

(1) System optimization problem

$$\begin{aligned}
&\text{Find } x_1^{sys}, x_2^{sys}, x_3^{sys}, \mu_{x_4^{sys}}, \mu_{x_5^{sys}}, \mu_{x_6^{sys}}, \mu_{x_7^{sys}} \\
&\min f = 0.7854x_1^{sys}\left(x_2^{sys}\right)^2\left(3.333\left(x_3^{sys}\right)^2 + 14.933x_3^{sys} - 43.0934\right) \\
&\quad - 1.508x_1^{sys}\left(\left(\mu_{x_6^{sys}}\right)^2 + \left(\mu_{x_7^{sys}}\right)^2\right) + 7.477\left(\left(\mu_{x_6^{sys}}\right)^3 + \left(\mu_{x_7^{sys}}\right)^3\right) \\
&\quad + 0.7854\left(\mu_{x_4^{sys}}\left(\mu_{x_6^{sys}}\right)^2 + \mu_{x_5^{sys}}\left(\mu_{x_7^{sys}}\right)^2\right) \\
&\text{s.t. } J_1 < \varepsilon,\ J_2 < \varepsilon,\ J_3 < \varepsilon
\end{aligned} \tag{17}$$

(2) Optimization problem for discipline 1

$$\begin{aligned}
&\text{Find } x_1^{dis1}, x_2^{dis1}, x_3^{dis1}, \mu_{x_4^{dis1}}, \mu_{x_6^{dis1}} \\
&\min J_1 = \left(x_1^{sys} - x_1^{dis1}\right)^2 + \left(x_2^{sys} - x_2^{dis1}\right)^2 + \left(x_3^{sys} - x_3^{dis1}\right)^2 \\
&\quad + \left(\mu_{x_4^{sys}} - \mu_{x_4^{dis1}}\right)^2 + \left(\mu_{x_6^{sys}} - \mu_{x_6^{dis1}}\right)^2 \\
&\text{s.t. } P_1\left[g_3\begin{pmatrix} x_2^{dis1}, x_3^{dis1} \\ x_4^{dis1}, x_6^{dis1} \end{pmatrix} \le 0\right] \ge 0.95,\ P_2\left[g_5\begin{pmatrix} x_2^{dis1}, x_3^{dis1} \\ x_4^{dis1}, x_6^{dis1} \end{pmatrix} \le 0\right] \ge 0.95, \\
&\quad P_3\left[g_{24}\left(x_4^{dis1}, x_6^{dis1}\right) \le 0\right] \ge 0.95, \\
&\quad g_1\left(x_1^{dis1}, x_2^{dis1}, x_3^{dis1}\right) \le 0,\ g_2\left(x_1^{dis1}, x_2^{dis1}, x_3^{dis1}\right) \le 0,\ g_7\left(x_2^{dis1}, x_3^{dis1}\right) \le 0, \\
&\quad g_8\left(x_1^{dis1}, x_2^{dis1}\right) \le 0,\ g_9\left(x_1^{dis1}, x_2^{dis1}\right) \le 0
\end{aligned}. \tag{18}$$

(3) Optimization problem for discipline 2

$$\begin{aligned}
&\text{Find } x_1^{dis2}, x_2^{dis2}, x_3^{dis2} \\
&\min J_2 = \left(x_1^{sys} - x_1^{dis2}\right)^2 + \left(x_2^{sys} - x_2^{dis2}\right)^2 + \left(x_3^{sys} - x_3^{dis2}\right)^2 \\
&\text{s.t. } g_1\left(x_1^{dis2}, x_2^{dis2}, x_3^{dis2}\right) \le 0,\ g_2\left(x_1^{dis2}, x_2^{dis2}, x_3^{dis2}\right) \le 0, \\
&\quad g_7\left(x_2^{dis2}, x_3^{dis2}\right) \le 0,\ g_8\left(x_1^{dis2}, x_2^{dis2}\right) \le 0,\ g_9\left(x_1^{dis2}, x_2^{dis2}\right) \le 0
\end{aligned}. \tag{19}$$

(4) Optimization problem for discipline 3

$$\begin{aligned}
&\text{Find } x_1^{dis3}, x_2^{dis3}, x_3^{dis3}, \mu_{x_5^{dis3}}, \mu_{x_7^{dis3}} \\
&\min J_3 = \left(x_1^{sys} - x_1^{dis3}\right)^2 + \left(x_2^{sys} - x_2^{dis3}\right)^2 + \left(x_3^{sys} - x_3^{dis3}\right)^2 \\
&\quad + \left(\mu_{x_5^{sys}} - \mu_{x_5^{dis3}}\right)^2 + \left(\mu_{x_7^{sys}} - \mu_{x_7^{dis3}}\right)^2 \\
&\text{s.t. } P_4\left[g_4\begin{pmatrix} x_2^{dis3}, x_3^{dis3} \\ x_5^{dis3}, x_7^{dis3} \end{pmatrix} \le 0\right] \ge 0.95,\ P_5\left[g_6\begin{pmatrix} x_2^{dis3}, x_3^{dis3} \\ x_5^{dis3}, x_7^{dis3} \end{pmatrix} \le 0\right] \ge 0.95, \\
&\quad P_6\left[g_{25}\left(x_5^{dis3}, x_7^{dis3}\right) \le 0\right] \ge 0.95, \\
&\quad g_1\left(x_1^{dis3}, x_2^{dis3}, x_3^{dis3}\right) \le 0,\ g_2\left(x_1^{dis3}, x_2^{dis3}, x_3^{dis3}\right) \le 0,\ g_7\left(x_2^{dis3}, x_3^{dis3}\right) \le 0, \\
&\quad g_8\left(x_1^{dis3}, x_2^{dis3}\right) \le 0,\ g_9\left(x_1^{dis3}, x_2^{dis3}\right) \le 0
\end{aligned}. \tag{20}$$

The required reliability for each probability constraint is 0.95, and the compatibility constraint accuracy ε is 0.001. System optimization is solved by using SQP, and discipline optimization problems are solved by using GA. The accuracy and efficiency of MVFOSA-CO are compared with MVFOSM-CO and MCS-CO in Tables 8 and 9. $R_i\left(i = 1 \sim 6\right)$ denotes the reliabilities of probabilistic constraints. The objective function optimization histories are shown in Fig. 6 by using three different methods. The methods have similar optimization histories, as well as similar number of iterations n_s in the system optimization problem and iterations n_1, n_2,

Table 7. Details of the design variables in the speed reducer design problem.

Variables	Description	Mean	Standard deviation	Distribution	Lower bound	Upper bound
x_1	Gear face width	-	-	-	2.6	3.6
x_2	Teeth module	-	-	-	0.3	1.0
x_3	Number of teeth of pinion	-	-	-	17	28
x_4	Distance between bearings 1	μ_{x_4}	$0.001\mu_{x_4}$	Gumbel	7.3	8.3
x_5	Distance between bearings 2	μ_{x_5}	$0.001\mu_{x_5}$	Gumbel	7.3	8.3
x_6	Diameter of shaft 1	μ_{x_6}	$0.001\mu_{x_6}$	Gumbel	2.9	3.9
x_7	Diameter of shaft 2	μ_{x_7}	$0.001\mu_{x_7}$	Gumbel	5	5.5

Table 8. Optimization results of the reducer design.

	x_1^{sys}	x_2^{sys}	x_3^{sys}	$\mu_{x_4^{sys}}$	$\mu_{x_5^{sys}}$	$\mu_{x_6^{sys}}$	$\mu_{x_7^{sys}}$	f
MVFOSM-CO	3.4273	0.6504	18	7.3003	7.6884	3.3208	5.2642	2888.5820
MVFOSA-CO	3.4247	0.6457	18	7.3005	7.6865	3.3231	5.2635	2866.0874
MCS-CO	3.4236	0.6442	18	7.3005	7.6856	3.3235	5.2633	2859.1180

Table 9. Reliability of probabilistic constraints and calculation efficiency of the reducer design.

	R_1	R_2	R_3	R_4	R_5	R_6	n_1	n_2	n_3	n_s	Calculation time
MVFOSM-CO	0.9604	0.9678	0.9771	0.9622	0.9707	0.9742	18468	1297	19881	71	23 m 2 s
MVFOSA-CO	0.9568	0.9625	0.9740	0.9587	0.9675	0.9711	23703	1687	25093	93	25 m 45 s
MCS-CO	0.9550	0.9601	0.9749	0.9564	0.9642	0.9727	24292	1364	24937	92	-

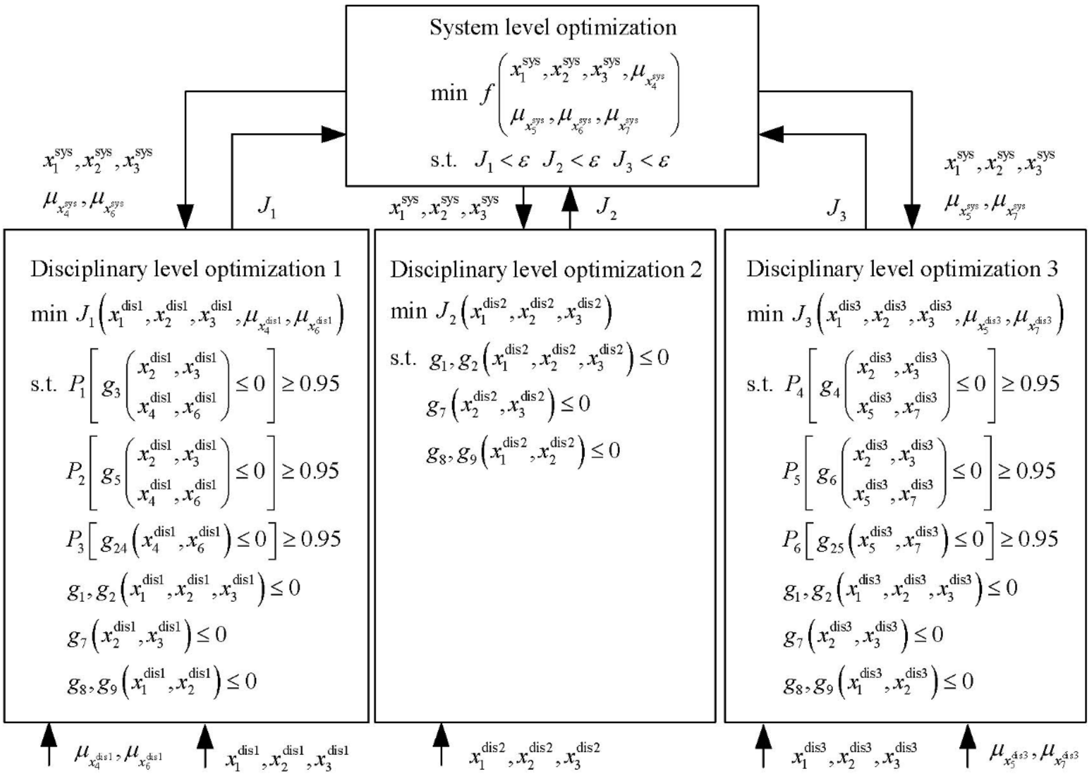

Fig. 5. MVFOSA-CO approach for the speed reducer MDO problem.

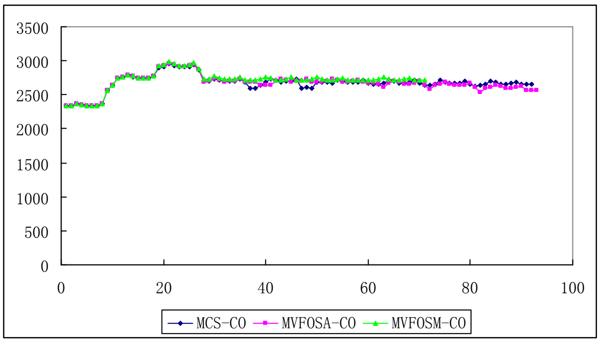

Fig. 6. Optimization histories of three different methods.

and n_3 in discipline optimization problems 1 to 3 (Table 9). This result is attributed to the use of the same MDO method(i.e., the CO method). Furthermore, the optimal solution 2866.0874 obtained by MVFOSA-CO is closer to the reference value 2859.1180 obtained by MCS-CO than the solution 2888.5820 obtained by MVFOSM-CO (Table 8). MVFOSM-CO has the more conservative solution than MVFOSA-CO. MVFOSA-CO uses full distribution information rather than the first two moments of the random variables; thus, MVFOSA-CO performs relatively better in the tail regions. The solution obtained by MVFOSA-CO is more accurate than MVFOSM-CO. Their calculation times are almost the same (Table 9). This example shows that MVFOSA-CO has the same efficiency as MVFOSM-CO but is highly accurate.

6. Conclusions

This work aims to improve reliability analysis accuracy in MDO problem performance functions that are expensive and only respond to traditional MVFOSM methods. MVFOSA-CO is proposed to address RBMDO problems under aleatory uncertainty. The proposed method introduces the first-order Taylor expansion of a performance function at the mean values of random variables and uses saddlepoint approximation to estimate CDF and PDF. MVFOSA-CO has several advantages. First, the bi-level analysis and coordination structure of MVFOSA-CO allows the application of different subspace optimizers among various analysis groups. Different disciplines are easily parallelized and well-suited for conventional discipline organizations. Second, MVFOSA-CO has high accuracy in tail regions while keeping the same efficiency as the MVFOSM. Finally, in MVFOSA-CO, non-normal random variables do not need to be transformed into normal random variables. However, the proposed method is only suitable for RBMDO problems that are under aleatory uncertainty and that have the analytical CGF of the random variable.

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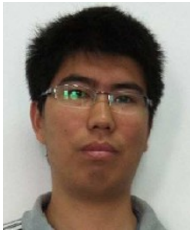

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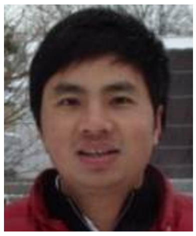

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