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Modeling the effect of intermolecular force on the size-dependent pull-in behavior of beam-type NEMS using modified couple stress theory †

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Abstract

Experimental observations reveal that the physical response of nanostructures is size-dependent. Herein, modified couple stress theory has been used to study the effect of intermolecular van der Waals force on the size dependent pull-in of nanobridges and nanocantilevers. Three approaches including using differential transformation method, applying numerical method and developing a simple lumped parameter model have been employed to solve the governing equation of the systems. The pull-in parameters i.e. critical tip deflection and instability voltage of the nanostructures have been determined. Effect of the van der Waals attraction and the size dependency and the importance of coupling between them on the pull-in performance have been discussed.

Keywords: Pull-in instability; van der Waals (vdW) force; Modified couple stress theory (MCST); Nanocantilever; Nanobridge

1. Introduction

Emerging revolution of nanotechnology gives the opportunity to develop advanced ultra-small systems. Recently, micro/nano-electromechanical systems (MEMS/NEMS) have found enormous engineering applications [1-7]. A beam-type MEMS/NEMS constructed from two conductive electrodes, which one (beam) is movable and the other one is fix (ground). When electrostatic force exceeds the elastic resistance of the beam, the pull-in instability occurs and the beam suddenly adheres to the ground. Instability of MEMS in micro-scales has been investigated by previous researchers neglecting nano-scale phenomena. However, in sub-micron the nano-scale phenomena should be considered in instability models.

The first issue is the presence of van der Waals (vdW) force in nano-scale distances. This attraction can significantly influence the NEMS performance when the initial gap between the components of nanostructure is typically below several ten nanometers. In this case, the attraction between two surfaces is proportional to the inverse cube of the separation [8, 9]. In recent years, various approaches such as finite element methods [8, 9] and developing lumped models [10, 11] are utilized to investigate the effect of vdW force on nano-systems.

Another nano-scale phenomenon is size dependent behavior of structures that cannot be modeled via classic theories. However, by applying non-classic continuum theories i.e. non-local elasticity [12], couple stress theory [13], strain gradient theory [14], modified couple stress theory (MCST) [15], etc. This size effect can be attributed to material length scale parameters. In MCST, the length scale parameters are reduced to only one constant [16-20]. Some studies have been conducted on modeling the size-dependent response of MEMS using MCST [16, 21-29]. Rokni et al. [21] and Baghani et al. [22] used MCST to investigate the size dependent pull-in behavior of micro-beams. Noghrehabadi et al. [23] studied the pull-in of nano-beams in liquid media. Zhang et al. [24] investigated the size-dependent behavior of electrostatic viscoelastic beams. Kong [25] and Yin et al. [26] established size-dependent pull-in models for electrostatic microactuators. However, few researchers have investigated the interaction between vdW force and size-effect in NEMS [16, 28, 29].

Herein, effect of vdW force on the size-dependent pull-in of nanobridges and nanocantilevers is investigated using MCST. Different approaches e.g. differential transformation method (DTM), developing lumped model and numerical solution are applied to solve the governing equation. The concept of DTM was presented by Zhou [30]. This method determines the solution in the form of polynomials. The concept of DTM is derived from Taylor series expansion, but it does not evaluate the derivatives symbolically. The equation and boundary conditions are transformed into a system of algebraic equations.

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2. Theoretical model

2.1 Fundamental of modified couple stress theory

At nano-scale, the gradient deformations vary sharply, hence the stresses and strains depend on the shrinking length scale of the structures [31]. To model these gradient effects, a higher order strain gradient theory was introduced with three length scale parameters relevant to dilatation, deviatoric and symmetric rotation gradients [31]. These three parameters can be combined into one measurable parameter under the assumptions of MCST [31]. As the characteristic length of the deformation field becomes significantly larger than the material length scale parameter, strain gradients effect becomes negligible because the strain terms are much larger than their scaled gradient terms [32]. In this case, the results obtain via MCST is the same as that of classic theory. Based on the MCST, the strain energy density is written as [15]:

$$\bar{u} = \frac{1}{2}(\sigma : \varepsilon + m : \chi) \tag{1}$$

where the stress tensor σ, strain tensor ε, deviator part of the couple stress tensor m and symmetric curvature tensor χ are defined by the following:

$$\sigma = \lambda tr(\varepsilon) I + 2\mu\varepsilon \tag{2a}$$

$$\varepsilon = \frac{1}{2}[(\nabla u) + (\nabla u)^{\mathrm{T}}] \tag{2b}$$

$$m = 2l^2 \mu\chi \tag{2c}$$

$$\chi = \frac{1}{4}[(\nabla.\nabla \times \theta) + (\nabla.\nabla \times \theta)^{\mathrm{T}}] \tag{2d}$$

where u is the displacement vector. In above relations λ, μ, l are Lame constant, shear modulus and material length scale parameter, respectively [33].

2.2 Constitutive equation of beam-type NEMS

Figs. 1(a) and (b) show schematic representation of nanocantilever and nanobridge, respectively. Herein, the structures with a beam length of L, wide of B, thickness of H and initial gap of g are considered. The material parameters are elastic modulus E, and Poisson's ratio ν. The total strain energy for a deformed Euler-Bernoulli beam is the summation of strain energy stored in the beam due to axial forces (U_a) and bending (U_b). Considering MCST, the energy stored in the beam due to axial forces is [26]:

$$U_a = \frac{1}{2}\int_0^L F_a (\frac{dw}{dX})^2 dX \tag{3}$$

where w and F_a are the beam displacement in direction of Z axis and the applied axial force associated with the mid-plane stretching (for nanobridge). Base on MCST, bending strain energy is obtained as [26]:

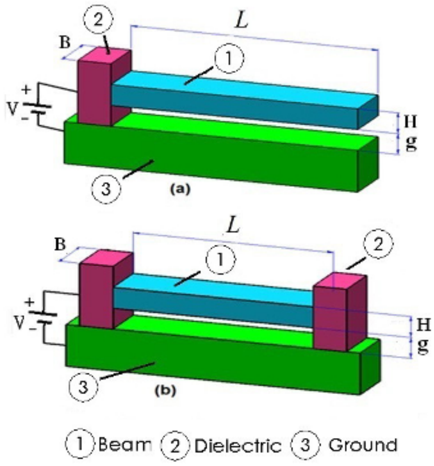


Fig. 1. Representations of (a) nanocantilever; (b) nanobridge.

$$U_b = \frac{1}{2}\int_0^L (EI + \mu A l^2)(\frac{d^2 w}{dX^2})^2 dX \tag{4}$$

where A, I and μ are beam cross section area, 2st cross section moment of area and shear modulus, respectively. Considering the distribution of electrostatic and vdW forces per unit length of the beam (f_{elec} & f_{vdW}), the work by external forces (W_e) can be obtained as:

$$W_e = \int_0^L [f_{elec} + f_{vdW}] dX \ . \tag{5}$$

Now, utilizing Hamilton principle i.e. $\delta(U_a + U_b - W_e) = 0$, in which δ indicates variations symbol, the differential equation of lateral deflection of the system can be derived as:

$$\left(EI + \mu A l^2\right)\frac{d^4 w}{dX^4} - (F_a + F_r)\frac{d^2 w}{dX^2} = f_{elec} + f_{vdW} \tag{6}$$

and the boundary conditions read

$$\left(EI + \mu A l^2\right)\frac{d^3 w}{dX^3} - F_a \frac{dw}{dX} = 0 \ \ or \ \ w\big|_{X=0,L} = 0 \tag{7a}$$

$$\frac{d^2 w}{dX^2} = 0 \ \ or \ \ \frac{dw}{dX}\bigg|_{X=0,L} = 0 \ . \tag{7b}$$

When nanobridge is in tension, the stretching, i.e. the axial force induced due to immovable clamped ends, will occur that can be expressed as [26]

$$F_a = \frac{EA}{2L}\int_0^L \left(\frac{dw}{dX}\right)^2 dX \ . \tag{8}$$

The electrostatic attraction in Eq. (5) is written as the following [17]

$$f_{elec} = \frac{\varepsilon_0 B V^2}{2(g - w(X))^2}\left(1 + 0.65\frac{g - w(X)}{B}\right) \tag{9}$$

where ε_0 is permittivity of vacuum and V is the applied voltage.

Considering some idealizations, the vdW attraction in Eq. (5) can be obtained as [29]:

$$f_{vdW} = \frac{\bar{A}B}{6\pi(g - w(X))^3} \tag{10}$$

where Ā is the Hamaker constant.

2.3 Dimensionless equations

By using the substitutions x = X/L and $\bar{w} = w/g$ one can obtain the dimensionless governing equation of the structures, from Eq. (6) as the following:

$$\begin{gathered}(1+\xi)\frac{d^4\bar{w}}{dx^4} - \eta\left[\int_0^1 \left(\frac{d\bar{w}}{dx}\right)^2 dx\right]\frac{d^2\bar{w}}{dx^2} = \\ \frac{\alpha}{(1-\bar{w}(x))^3} + \frac{\beta}{(1-\bar{w}(x))^2} + \frac{\gamma\beta}{1-\bar{w}(x)}\end{gathered}. \tag{11}$$

And the following boundary conditions

$$\bar{w}(0) = \bar{w}'(0) = \bar{w}''(1) = \bar{w}'''(1) = 0 \quad \text{(nanocantilever)} \tag{12}$$

$$\bar{w}(0) = \bar{w}(1) = \bar{w}'(0) = \bar{w}'(1) = 0 \quad \text{(nanobridge)}. \tag{13}$$

In Eq. (11), the dimensionless parameters are identified as

$$\begin{gathered}\alpha = \frac{\bar{A}BL^4}{6\pi EIg^4}, \beta = \frac{\varepsilon_0 BV^2L^4}{2g^3EI}, \gamma = 0.65\frac{g}{B}, \xi = \frac{\mu A}{EI}l^2 \\ \eta = \begin{cases} 0 & \text{nanocantilever} \\ 6\left(\frac{g}{H}\right)^2 & \text{nanobridge}\end{cases}\end{gathered}. \tag{14}$$

Letting $l = 0$, the relations are simplified to those obtained by classic continuum nonlinear geometrically beam theory.

Noted that Eq. (11) is singular for $\bar{w} = 1$. However, this singularity is physically impossible since the instability occurs at lower values of deflection ($\bar{w} < 1$). The integral term and force terms result in high nonlinearity of the equation. There exist no exact solutions for the governing equation, hence, three different approaches are applied to solve the nonlinear differential equation in the next section.

3. Solution methods

3.1 Differential transformation method (DTM)

The basic idea and detail of DTM solution is addressed in Appendix A. Multiply both sides of the Eq. (11) by $(1-\bar{w}(x))^4$ and then apply DTM, the following series solution is provided:

$$\begin{aligned}&\bar{w}(x) = ax^2 + bx^3 + \frac{-8a^3\eta - 18a^2b\eta + 3\beta\gamma + 3\alpha + 3\beta}{72(1+\xi)}x^4 + \\ &\eta b\frac{6a\alpha - 36a^3b\eta - 16a^4\eta - (60a^2 + 81b^2 + 135ab - 6a\beta)(\xi+1)}{900(1+\xi)}x^5 + \\ &\dots\end{aligned} \tag{15}$$

using boundary conditions (Eqs. (12) and (13)). The instability occurs when $d\bar{w}(x=1)/d\beta \to 0$ for nanocantilever and $d\bar{w}(x=0.5)/d\beta \to 0$ for nanobridge. The instability parameters of the system can be determined via the slope of the $\bar{w}$-β graphs by plotting $\bar{w}$ vs. β.

3.2 Iterative numerical method

The governing equations of the structures are numerically solved with an iterative method. By using this approach, Eq. (11) is writing in the following form:

$$\begin{gathered}(1+\xi)\frac{d^4\bar{w}_i}{dx^4} = \frac{\alpha}{(1-\bar{w}_{i-1}(x))^3} + \frac{\beta}{(1-\bar{w}_{i-1}(x))^2} + \\ \frac{\gamma\beta}{1-\bar{w}_{i-1}(x)} + \eta\left[\int_0^1 \left(\frac{d\bar{w}_{i-1}}{dx}\right)^2 dx\right]\frac{d^2\bar{w}_{i-1}}{dx^2}\end{gathered} \tag{16}$$

and the following boundary conditions:

$$\bar{w}_i(0) = \bar{w}_i'(0) = \bar{w}_i''(1) = \bar{w}_i'''(1) = 0 \quad \text{(nano-cantilever)} \tag{17a}$$

$$\bar{w}_i(0) = \bar{w}_i(1) = \bar{w}_i'(0) = \bar{w}_i'(1) = 0 \quad \text{(nano-bridge)}. \tag{17b}$$

One can use the undeformed beam ($\bar{w}_0 = 0$) as an initial point. This procedure is continued until the convergence is achieved or pull-in has happened. The pull-in parameters can be determined via the slope of the $\bar{w}$ - β graphs.

3.3 Lumped parameter model

In order to develop a lumped parameter model, the elastic response of the structures is modeled by a linear spring. This model assumes uniform attractions along the length of beams. By assuming a trial function for the beam deflection (see Appendix A.2) the relation between β and the maximum eflecttion ($\bar{w}_{\max}$) can be obtained as:

Table 1. Accuracy check of DTM series solution (β = ξ = g/B = 1, α = 0).

	Number of DTM series terms			
	4 Term	8 Term	19 Term	23 Term
$\bar{w}(x=1)$	0.1031	0.1215	0.11757	0.11763
E* (%)	12.3	3.29	0.051	0.002
R**	0.3177	0.1417	0.0073	0.002

*E: Difference with numerical value (0.117631).
**R: Maximum residual error (at x = 1).

$$\beta=\frac{(1-\bar{w}_{\max})^2[7.89(1+\xi)\bar{w}_{\max}-\alpha(1-\bar{w}_{\max})^{-3}]}{1+\gamma(1-\bar{w}_{\max})}$$

$$\beta=\frac{(1-\bar{w}_{\max})^2[379(1+\xi)\bar{w}_{\max}+45\eta\bar{w}_{\max}^3-\alpha(1-\bar{w}_{\max})^{-3}]}{1+\gamma(1-\bar{w}_{\max})} \tag{18}$$

For lumped parameter model, the pull-in parameters can be obtained from Eq. (18) by setting $d\beta / d\bar{w}_{\max}=0$.

4. Result and discussion

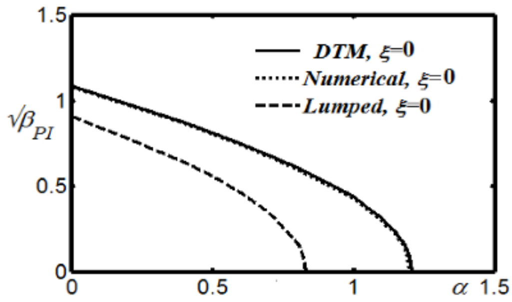
In order to verify the DTM series solution, deflection of typical nanocantilever is determined using different number of series terms. The residual error of the series solution, calculated using Eq. (19) [34], is reported in Table 1. This table also shows the comparison between the numerical solution and the DTM ones for the nanocantilever. As seen, higher accuracy and reduction in residual error can be obtained by considering more terms in series solution $\bar{w}(x)$. Moreover, the difference between numerical and DTM solutions decreases by selecting more series terms.

$$R=(1+\xi)\frac{d^4\bar{w}}{dx^4}-\frac{\alpha}{(1-\bar{w}(x))^3}-\frac{\beta}{(1-\bar{w}(x))^2}-\frac{\gamma\beta}{1-\bar{w}(x)}-\eta\left(\int_0^1\left(\frac{d\bar{w}}{dx}\right)^2dx\right)\frac{d^2\bar{w}}{dx^2}. \tag{19}$$

In the following, typical nanostructures with $\eta=24$, g/H = 2 and g/B = 1 are investigated.

4.1 Effect of vdW force

Fig. 2 shows the influences of the vdW force on dimensionless pull-in voltage ($\sqrt{\beta_{PI}}$) of the nanostructures without considering size effect (ξ = 0). As seen, vdW force decreases the instability voltage of the beam. Note that when α takes the critical value, the beam becomes unstable even with β = 0. This reveals that when the separation is small enough, vdW force can cause the beam to collapse onto the ground plane even without an electrostatic attraction. Fig. 3 depicts the variation of pull-in deflection ($\bar{w}_{PI}$) as a function of α. As seen, vdW force decreases the pull-in deflection of the structures. Comparison between Figs. 2 and 3 reveal that the critical value of vdW force for nanobridge is larger than that of nanocantilever due to the lower elastic rigidity of nanocantilevers. In Fig. 2, the intersection of the curves with horizontal



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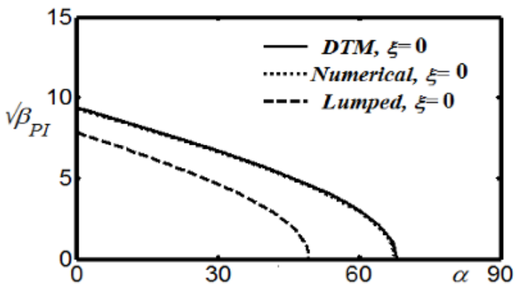
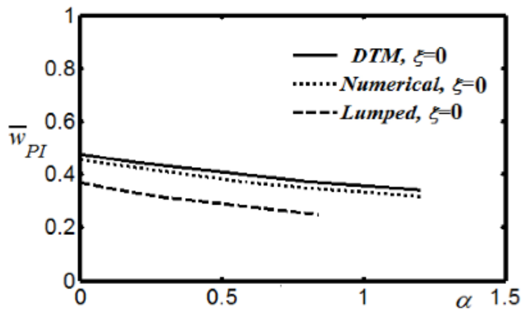


(b)

Fig. 2. Effect of vdW force (α) on the pull-in applied voltage of (a) nanocantilever; (b) nanobridge neglecting size effect.



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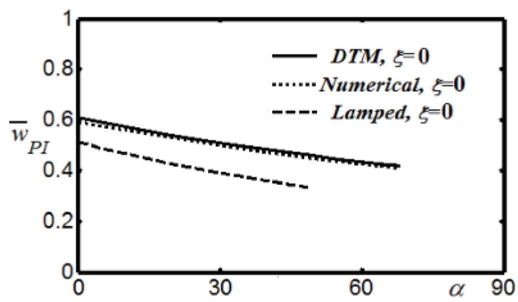
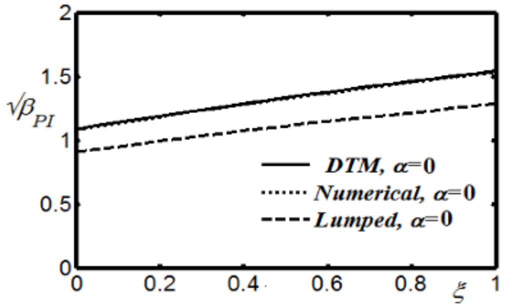


(b)

Fig. 3. Effect of vdW force on the pull-in deflection of (a) nanocantilever; (b) nanobridge neglecting the size effect.



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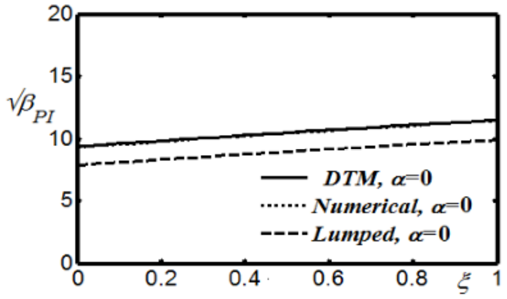
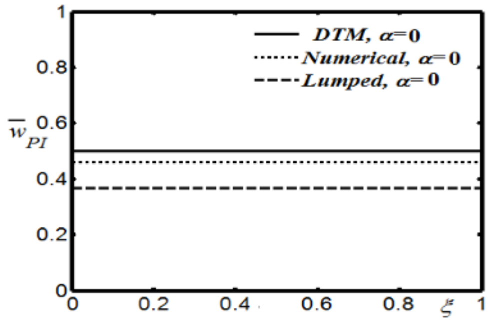


(b)

Fig. 4. Effect of the size dependency (ξ) on the pull-in voltage of (a) nanocantilever; (b) nanobridge neglecting the vdW force.



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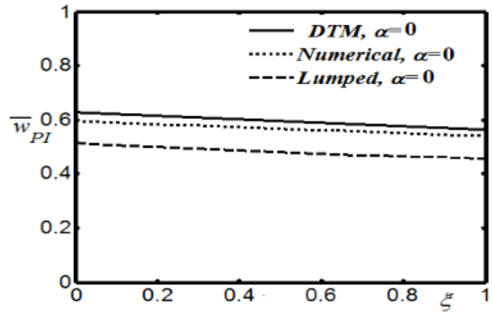


(b)

Fig. 5. Effect of the size dependency on pull-in deflection of (a) nanocantilever; (b) nanobridge neglecting the vdW force.

axis corresponds to the critical value of α (vdW force). Indeed, if α exceed its critical value, the beams adhere the ground even without any applied voltage due to strong vdW attraction. There exist no $\sqrt{\beta_{PI}}$ and $\overline{w}_{PI}$ for α greater than the critical value of intermolecular force (see Figs. 2 and 3). The critical value of α determined by lumped model is lower than those of DTM and numerical methods.

4.2 Size dependency in the absence of vdW force

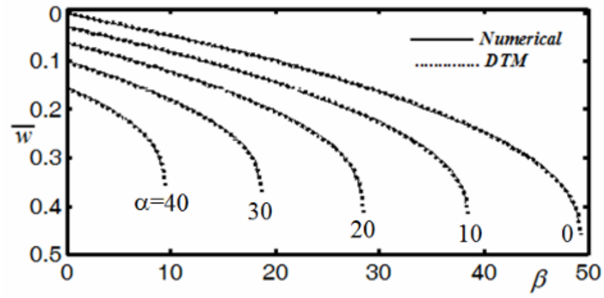
Fig. 4 represents the effect of size dependency on $\sqrt{\beta_{PI}}$ of the nanostructures neglecting vdW force. As seen, increasing ξ results in increasing the pull-in voltage of nano-systems.

This means size effect enhances the elastic resistance and consequent operation voltage of the nano-devices. Figs. 5(a) and (b) demonstrate the influence of size dependency on $\overline{w}_{PI}$ of the nanocantilever and nanobridge, respectively. In the absence of vdW forces, the pull-in deflection of nanocantilever is not sensitive to the size effect. However, $\overline{w}_{PI}$ of a nanobridge slightly reduces by increasing the size effect. This difference is the result of stretching that induces non-linearity in governing equation of nanobridge. Note that $\sqrt{\beta_{PI}}$ is more sensitive to the size effect in comparison with $\overline{w}_{PI}$. It should be noted that the difference between DTM and numerical solution in these figures can be reduced by increasing number of series terms.

In the case of Fig. 5(a), one can achieve less than 1.5% error between $\overline{w}_{PI}$ values determined via DTM (0.4680) and numerical solution (0.4611) by using 23 series terms. The lumped models do not provide precise values due to assuming uniform distribution of electrical and vdW force along the beams. However, lump models are practical in understanding physical aspects of the phenomena without mathematical complexities.

4.3 Coupling between vdW attraction and size effect

At nano-scale distances, both vdW forces and size effect should be accounted. Figs. 6(a) and (b) illustrate the influence of size effect on instability voltage of nanocantilever and nanobridge for various values of α. As seen in the presence of vdW force, $\sqrt{\beta_{PI}}$ increases with increase in size effect. Note that the beam bending rigidity predicted by MCST ($EI + \mu A l^2$) is greater than that of classic theory (EI). Therefore, the size effect provides a hardening behavior (increase in bending rigidity) that enhances the elastic resistance of the nano-beams. The enhanced elastic resistance allowed the nanostructures to tolerate higher applied voltage before the instability occurs. Figs. 7(a) and (b) depict the variation of pull-in deflection as a function of size effect parameter considering vdW attraction. As seen, the $\overline{w}_{PI}$ of nanocantilever slightly increases in the presence of vdW force. This trend is different from what observed in Fig. 5(a) where $\overline{w}_{PI}$ does not change in the absence of vdW force. On the other hand, Fig. 7(b) shows that increasing the size effect decreases $\overline{w}_{PI}$ of the nanobridge in the presence of vdW force (similar to Fig. 5(b)). However, increasing the vdW force decreases the slope of the $\overline{w}_{PI} - \xi$ curves. The obtain results reveal that coupling between vdW force and size effect is a crucial issue to precise



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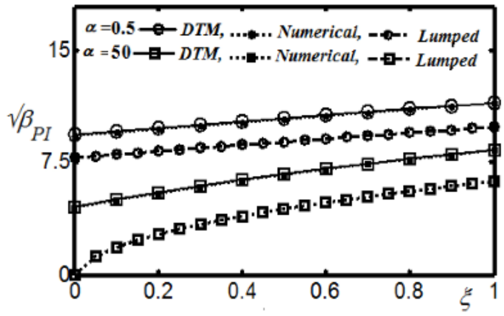

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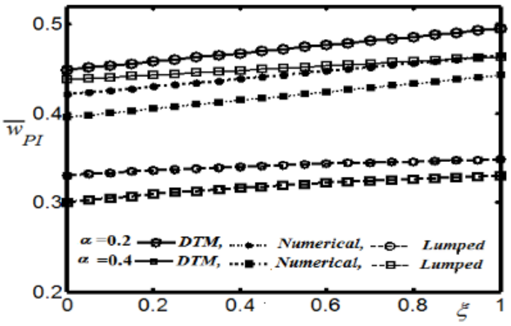
Fig. 6. Effect of the size dependency on the pull-in voltage of (a) nanocantilever; (b) nanobridge considering the vdW force.



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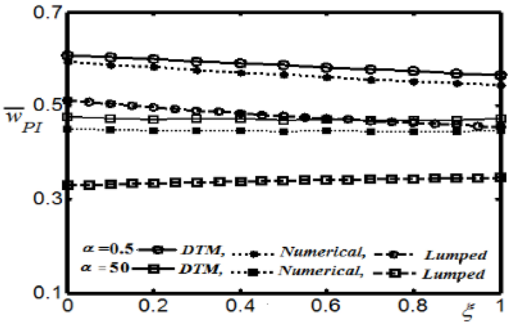


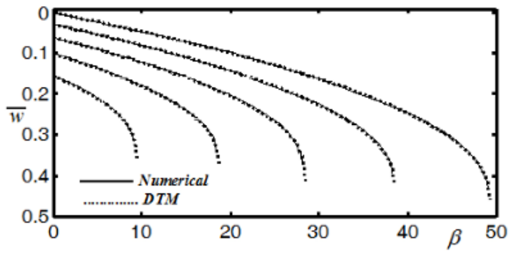
(b)

Fig. 7. Effect of the size dependence on the pull-in deflection of (a) nanocantilever; (b) nanobridge considering the vdW force.

determining the pull-in parameters of the nanostructures and should be included in theoretical models.

Table 2. Geometry and constitutive material properties used in Fig. 8.

	g/B	ξ	η	g/H	l/H	E(GPa)	μ	ν
Ref.[33]	1	0	0	0	0	77	28.95	0.33
Ref.[27]	0.2	0.4695	24	2	1/3	98.49	34.68	0.42



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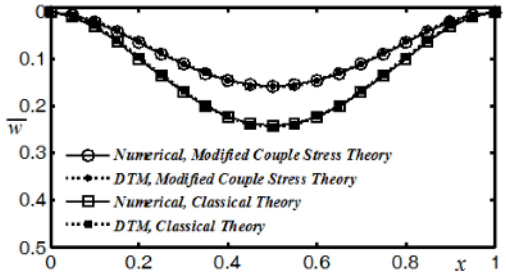

(b)

Fig. 8. (a) Pull-in behavior of nanocantilever neglecting size effect [33]; (b) deflection of nanobridge neglecting vdW force ($\beta = 64$) [27].

4.4 Comparison with literature

The comparison between response of the systems predicted via present work (DTM) and those of the Refs. [27, 33] is shown in Fig. 8. The geometry and the constitutive material of the beams are identified in Table 2. Fig. 8(a) presents the variation of deflection as a function of β for a nanocantilever. Moreover, Fig. 8(b) shows the comparison between deflection of a nanobridge ($\beta = 64$) for two cases e.g. considering the size effect (MCST) and omitting the size effect (classic theory). As seen, the difference between DTM solutions and the literature is within the excellent range.

5. Conclusions

Modified couple stress theory has been used to investigate the effect of vdW force on size dependent pull-in instability of beam-type nanobridges and nanocantilevers. Three different approaches e.g. applying approximated DTM, developing lumped parameter model and numerical solution are applied to solve governing equations. It is found that:

- vdW force reduces the pull-in voltage and deflection of the systems. It induces initial deflection in freestanding structures. Effect of vdW force on instability of nanocantilevers is more pronounce in comparison with nanobridges.

- Neglecting vdW force, size effect enhances the pull-in voltage of nanocantilever without change in the pull-in deflection. On the other hand, size effect increases the pull-in voltage while slightly decrease the pull-in deflection of nanobridge.
- Considering vdW force, both pull-in voltage and deflection of the nanocantilever increase with increasing the size effect. Moreover, increasing the size effect decreases the pull-in deflection while increase the pull-in voltage of the nanobridge.
- DTM is in good agreement with numerical solution. Although the lumped models do not provide precise values, they are practical in understanding physical aspects of the phenomena without mathematical complexities.

The present model is useful to accurately predict the NEMS behavior in nano-scales where the presence of vdW force highly affects the pull-in parameters of system. The present work can be helpful to precise analysis of nanostructures.

Nomenclature

m	: Deviator part of the stress tensor
χ	: Symmetric curvature tensor
σ	: Stress tensor
ε	: Strain tensor
U_a	: Energy stored due to axial forces
U_b	: Bending strain energy
L	: Length of beam
B	: Wide of beam
H	: Thickness of beam
g	: Initial gap
I	: 2nd moment of cross-section
A	: Beam cross section area
f_{elec}	: Electrostatic force per unit length
f_{vdw}	: vdW force per unit length
α	: Dimensionless van der Waals force
β	: Dimensionless electrostatic force
γ	: Gap to width ratio
ξ	: Size effect parameter
η	: Gap to thickness ratio
λ	: Lame constant
μ	: Shear modulus
ν	: Poisson's ratio
E	: Elastic modulus
l	: Material length scale parameter
$\bar{A}$	: Hamaker constant
ε_0	: Permittivity of vacuum
V	: Applied voltage
$\bar{w}$	: Normalized deflection

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Appendix

A.1 Differential transformation method

The basic idea and the fundamental theorems of the DTM and its applicability are given in Ref. [34]. The differential transform of the kth derivative of arbitrary function y(x) is defined as:

$$Y(k)=\frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{A.1}$$

where y(x) is the original function and Y(k) is the transformed function. The differential inverse transform of Y(k) is defined as

$$\mathrm{y}(x)=\sum_{k=0}^{\infty} Y(k)(x-x_0)^k \ . \tag{A.2}$$

The differential transformation relations for functional operations and boundary conditions are found in Ref. [34]. In this work, deflection of the nanostructures can be considered as:

$$\begin{aligned}\bar{w}(x)&=\sum_{k=0}^{\infty}\bar{W}(k)x^k= \\ &\bar{W}(0)+\bar{W}(1)x+\bar{W}(2)x^2+\bar{W}(3)x^3+\bar{W}(4)x^4+....\end{aligned} \tag{A.3}$$

where $\bar{W}$ is the transformation function.

By applying the transformations on Eq. (11) (see Ref. [34]) and after some elaborations, one can found:

$$\begin{aligned}&(1+\xi)\left\{\sum_{\lambda=0}^{k}\sum_{m=0}^{\lambda}\sum_{p=0}^{m}\sum_{q=0}^{p}\left[\{\delta(p-q)-\bar{W}(p-q)\}\{\delta(q)-\bar{W}(q)\}\right]\times\right. \\ &\left[\delta(m-p)-\bar{W}(m-p)\right]\left[\delta(\lambda-m)-\bar{W}(\lambda-m)\right]\times \\ &\left.\left[(k-\lambda+4)(k-\lambda+3)(k-\lambda+2)(k-\lambda+1)\bar{W}(k-\lambda+4)\right]\right\}= \\ &\left[\eta\sum_{r=1}^{n}\frac{1}{r}\sum_{L=0}^{r-1}(L+1)\bar{W}(L+1)(r-L)\bar{W}(r-L)\right]\times \\ &\left\{\sum_{\lambda=0}^{k}\sum_{m=0}^{\lambda}\sum_{p=0}^{m}\sum_{q=0}^{p}\left[\{\delta(p-q)-\bar{W}(p-q)\}\{\delta(q)-\bar{W}(q)\}\right]\times\right. \\ &\left[\delta(m-p)-\bar{W}(m-p)\right]\left[\delta(\lambda-m)-\bar{W}(\lambda-m)\right]\times \\ &\left.\left[(k-\lambda+2)(k-\lambda+1)\bar{W}(k-\lambda+2)\right]\right\} \\ &+\alpha\left(\delta(k)-\bar{W}(k)\right)+\beta\left(\sum_{\lambda=0}^{k}[\delta(\lambda)-\bar{W}(\lambda)][\delta(k-\lambda)-\bar{W}(k-\lambda)]\right)+\gamma\beta \\ &\left(\sum_{\lambda=0}^{k}\sum_{m=0}^{\lambda}\left[\delta(m)-\bar{W}(m)\right]\left[\delta(\lambda-m)-\bar{W}(\lambda-m)\right]\left[\delta(k-\lambda)-\bar{W}(k-\lambda)\right]\right)\end{aligned} \tag{A.4}$$

and the transformations of boundary conditions is:

$$\bar{W}(0)=\bar{W}(1)=0 \qquad \text{nanocantilever}$$
$$\sum_{k=0}^{n} k(k-1)\bar{W}(k)=\sum_{k=0}^{n} k(k-1)(k-2)\bar{W}(k)=0 \tag{A.5}$$
$$\bar{W}(0)=\bar{W}(1)=0 \qquad \text{nanobridge}$$
$$\sum_{k=0}^{n} k\bar{W}(k)=\sum_{k=0}^{n} \bar{W}(k)=0 \quad . \tag{A.6}$$

By using two boundary condition of Eq. (A.5) i.e. $\bar{W}(0)=\bar{W}(1)=0$, assuming $\bar{W}(2)=a$ and $\bar{W}(3)=b$, we obtain higher terms from Eq. (A.4) as:

$$\bar{W}(4)=\frac{-8a^3\eta-18a^2b\eta+3\beta\gamma+3\alpha+3\beta}{72(1+\xi)}$$
$$\bar{W}(5)=\frac{b\eta}{900(1+\xi)^2}(-16a^4\eta-36a^3b\eta-60a^2\xi-135ab\xi$$
$$+6a\beta\gamma-81b^2\xi-60a^2-135ab+6a\alpha+6a\beta-81b^2)$$
$$.... \tag{A.7}$$

Appendix

A.2 Lumped parameter model

To develop a lumped model, the trial solutions can be chosen as the first mode shape of beam free vibration e.g. relations Eq. (A.8) for nanocantilever and Eq. (A.9) for nanobridge:

$$w(X)=\frac{w_{max}}{2}\left[\cosh\left(\lambda\frac{X}{L}\right)-\cos\left(\lambda\frac{X}{L}\right)-\left(\frac{\cosh(\lambda)+\cos(\lambda)}{\sinh(\lambda)+\sin(\lambda)}\right)\left(\sinh\left(\lambda\frac{X}{L}\right)-\sin\left(\lambda\frac{X}{L}\right)\right)\right] \tag{A.8}$$

$$w(X)=\frac{w_{max}}{1.59}\left[\cosh\left(\lambda\frac{X}{L}\right)-\cos\left(\lambda\frac{X}{L}\right)-\left(\frac{\cosh(\lambda)-\cos(\lambda)}{\sinh(\lambda)-\sin(\lambda)}\right)\left(\sinh\left(\lambda\frac{X}{L}\right)-\sin\left(\lambda\frac{X}{L}\right)\right)\right] \tag{A.9}$$

where $\lambda=1.875$ and $\lambda=4.73$ for nanocantilever and nanobridge, respectively.

Now, for nanocantilever the total energy of system can be written as:

$$\Pi=U_b-W_e=(0.13BEH^3+1.55\mu Al^2)\frac{w_{max}^2}{L^3} -0.39[f_{elec}+f_{vdW}]Lw_{max} \tag{A.10}$$

and for nanobridge we have:

$$\Pi=U_b+U_a-W_e=(8.3BEH^3+99.2\mu Al^2+3BHEw_{max}^2)\frac{w_{max}^2}{L^3} -0.52[f_{elec}+f_{vdW}]Lw_{max} \tag{A.11}$$

Taking the derivative with respect to w_{max} (e.g. $d\Pi/dw_{max}=0$), using $\bar{w}_{max}=w_{max}/g$ and substituting f_{elec} and f_{vdW}, one can obtain Eq. (A.12) for nanocantilever and Eq. (A.13) for nanobridge:

$$7.89(1+\xi)\bar{w}_{max}-\beta\frac{1+\gamma(1-\bar{w}_{max})}{(1-\bar{w}_{max})^2}-\frac{\alpha}{(1-\bar{w}_{max})^3}=0 \tag{A.12}$$

$$379(1+\xi)\bar{w}_{max}+45\eta\bar{w}_{max}^3-\beta\frac{1+\gamma(1-\bar{w}_{max})}{(1-\bar{w}_{max})^2}-\frac{\alpha}{(1-\bar{w}_{max})^3}=0 \ . \tag{A.13}$$

By rearranging relations Eqs. (A.12) and (A.13), the relations between applied voltage and the maximum deflection (Eq. (18)) can be obtained.

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