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Chaotic behavior in fractional-order horizontal platform systems and its suppression using a fractional finite-time control strategy †

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Abstract

The present paper investigates the dynamical properties of a non-autonomous fractional-order horizontal platform system (FOHPS). According to different parameter settings, we show that the FOHPS can possess stable, chaotic and unstable states. Using the maximal Lyapunov exponent criterion, we show that the FOHPS exhibits chaos. Strange attractors of the system are also plotted to validate chaotic behavior of the system. Since the chaotic behavior of the FOHPS may be undesirable, a fractional finite-time controller is introduced to suppress the chaos of the FOHPS with model uncertainties and external disturbances in a given finite time. We use the fractional Lyapunov theory to prove the finite time stability and robustness of the proposed scheme. Finally, computer simulations are given to illustrate the efficiency and applicability of the proposed fractional control method.

Keywords: Horizontal platform; Chaotic state; Fractional-order equation; Finite-time controller

1. Introduction

Recently, there has been an accelerating level of interest in the use of fractional calculus for modeling and control of dynamical systems. Fractional calculus is a 300-year-old mathematical topic where it generalizes the concept of integer-order differentiation/integration to arbitrary (non-integer) order one. Although it has a long history, for many years it had not been used in physics and engineering. However, during the last 20 years or so, fractional calculus has attracted increasing attention of physicists, chemists and engineers from an application point of view [1]. It has been found that many systems in interdisciplinary fields, such as differential oscillators [2], micro-electro-mechanical systems [3] and gyroscopes [4] can display fractional-order dynamics. The authors of these works have shown that modeling and describing integer-order systems with fractional-order differential equations can be useful in both application and research and can reveal more useful and applied dynamical properties of the system. More recently, there is a new trend to study the chaotic dynamics of fractional-order dynamical systems and to stabilize the unstable fixed points of them [5-11].

In recent years, several mechanical systems with chaotic phenomena have been developed [12-14]. And, control of mechanical systems has attracted the interest of many scholars [15-20]. One of the most interesting and attractive nonlinear dynamical systems is the horizontal platform system (HPS). It is a mechanical device that can freely rotate around the horizontal axis. The horizontal platform devices are widely used in offshore and earthquake engineering. It has been shown that these systems display a diverse range of dynamic behavior including both chaotic and regular motions [21]. Wu et al. [22] have used Lyapunov direct method to achieve a sufficient criterion for global chaos synchronization between two identical HPS coupled by linear state error feedback controller. By means of a linear state error feedback controller, the robust synchronization of the chaotic HPS with phase difference and parameter mismatches has been studied in Ref. [23]. Based on the Lyapunov stability theorem and Sylvester's criterion, some algebraic sufficient criteria for synchronization of two HPS coupled by sinusoidal state error feedback control have been derived in Ref. [24]. Pai and Yau [25] have designed an integral-type sliding mode controller for generalized projective synchronization of two HPS with uncertainties. Pai and Yau [26] have also designed an adaptive sliding mode control scheme for controlling the chaos in the state trajectories of the uncertain HPS. In Ref. [27], a finite-time control scheme has been proposed to synchronize two HPSs with uncertainties. However, to the best knowledge of the authors, there is no work in the literature about the control and dynamical analysis of the fractional-order horizontal platform systems (FOHPS), which remains as an open issue to be investigated.

In recent years, many scholars believe that the fractional

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modeling of the traditional integer-order systems opens a wide door for research in the area of physical and engineering systems, as it truly serves as a generalization of the integer-order case. The traditional models of many systems need to be revisited within the framework of the fractional-order differential equations, where the integer-order transfer functions of systems become simply special cases of the fractional-order ones. Therefore, in this paper, we generalize the integer-order model of the traditional HPS system to the fractional-order one to obtain more characteristics of the dynamics of the system and to discover more dynamical properties of the system. From both modeling and practical application points of view, the findings of this paper are of utility for many engineers and designers dealing with the applications of the gyro system.

This paper investigates the dynamical behavior of a non-autonomous fractional-order horizontal platform system by adopting fractional-order differential equations. We use the maximal Lyapunov exponent (MLE) criterion to show that the nonlinear behaviors of the FOHPS can be chaotic. We show that for some special settings of the system parameters, the state of the system can be stable or unstable. Subsequently, a fractional-order finite-time controller is proposed to suppress the chaotic state of the FOHPS in a given finite time. The effects of model uncertainties and external disturbances are considered and the robustness and finite-time stability of the closed-loop system are proved fractional Lyapunov stability theory [28].

2. Chaos in horizontal platform systems

2.1 Mathematical model of HPS

The HPS is a mechanical device composed of a platform and an accelerometer located on the platform. The platform can freely rotate about the horizontal axis, which penetrates its mass center. The accelerometer produces an output signal to the actuator, subsequently generating a torque to inverse the rotation of the platform to balance the HPS, when the platform deviates from horizon. The motion equations of the HPS are given by [21]

$$A\ddot{x} + D\dot{x} + kg\sin x - \frac{3g}{R}(B - C)\cos x \sin x = F\cos\omega t \tag{1}$$

where A = 0.3, B = 0.5 and C = 0.3 are the inertia moment of the platform, D = 0.4 is the damping coefficient, k = 0.11559633 is the proportional constant of the accelerometer, g = 0.98 is the acceleration constant of gravity, R = 6378000 is the radius of the Earth, x is the rotation of the platform relative to the earth and $F\cos\omega t = 3.4\cos 1.8t$ is the harmonic torque. For the above-mentioned parameters values the non-autonomous HPS Eq. (1) exhibits chaotic behavior [21].

Assuming $x_1 = x$ and $x_2 = \dot{x}$ the non-autonomous system Eq. (1) is transformed into the following normal form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_2 - b\sin x_1 + l\cos x_1 \sin x_1 + h\cos\omega t \end{cases} \tag{2}$$

where a = D/A = 4/3, b = kg/A = 3.776, l = 3g(B-C)/RA = 4.6 × 10^{-6} and h = F/A = 3.4/4 are the HPS parameters.

2.2 Mathematical model of HPS

Definition 1 [1]. The qth-order fractional integration of function $f(t)$ is given by

$${}_{t_0}I_t^q f(t) = {}_{t_0}D_t^{-q} f(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{q-1}} d\tau . \tag{3}$$

Definition 2 [1]. The Caputo fractional derivative of order q of a function $f(t)$ is defined as follows:

$${}_{t_0}D_t^q f(t) = \begin{cases} \dfrac{1}{\Gamma(m-q)}\displaystyle\int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau & m-1 < q < m \\ \dfrac{d^m}{dt^m} f(t) & q = m \end{cases} \tag{4}$$

where m is the smallest integer number larger than q.

Based on the definition of the fractional-order differential equations and using Eq. (2), the following fractional-order model for the HPS is given.

$$\begin{cases} D^q x_1 = x_2 \\ D^q x_2 = -ax_2 - b\sin x_1 + l\cos x_1 \sin x_1 + h\cos\omega t \end{cases} \tag{5}$$

where $q \in (0,1)$ is the fractional order of the system.

In order to investigate the dynamical behavior of the FOGS Eq. (5), we assume that the fractional order q and harmonic torque amplitude F can be changed. The other parameters of the system are set to a = 4/3, b = 3.776, l = 4.6×10^{-6} and $\omega = 1.8$. We use the maximal Lyapunov exponent criterion and apply the numerical method introduced in Ref. [29] to calculate the system's MLE. The initial conditions of the system are chosen as $x_1(0) = -3.4$ and $x_2(0) = 2.4$. In what follows, we explain different parameter settings in two case studies.

2.2.1 Case1: F = 3.4 and $q \in (0,1)$

In this case, we set $F = 3.4$ and change the fractional order q from 0.01 to 0.99. After running the simulations, we observe that the FOHPS states are unstable for $0.01 \le q < 0.06$ However, when we set $0.06 \le q < 0.75$, we find out that the system behavior becomes chaotic. The MLEs of the HPS Eq. (5) for $0.06 \le q < 0.75$ are shown in Fig. 1. Since the corresponding MLEs are positive, one can conclude that the system state is chaotic. The strange attractor of the HPS Eq. (5) for $q = 0.1$ is revealed in Fig. 2. One can see that the system Eq. (5) exhibits rich and chaotic dynamics. On the other hand, changing q from 0.75 to 0.99, we see that the first state of the HPS con-

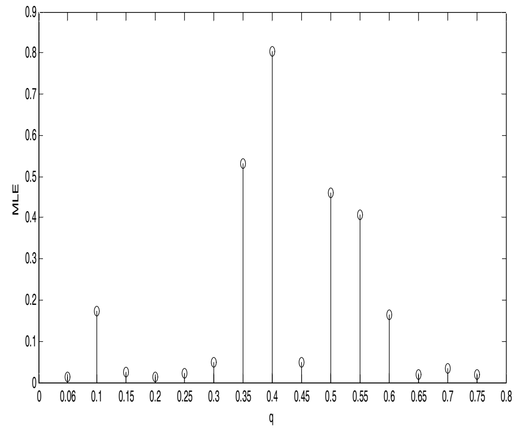

Fig. 1. Maximal Lyapunov exponents of the FOHPS with different values of q.

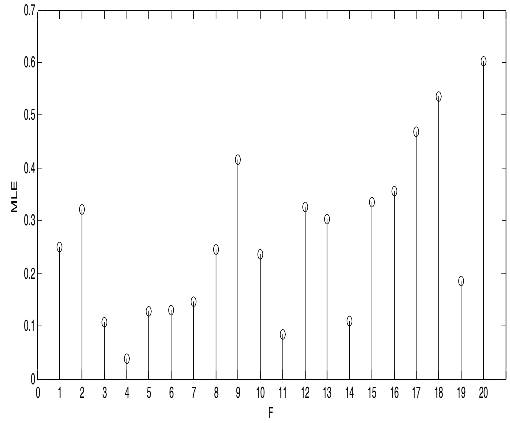

Fig. 3. Maximal Lyapunov exponents of the FOHPS with different values of F.

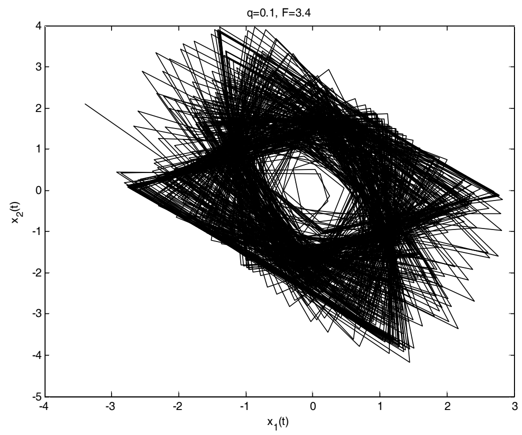

Fig. 2. Strange attractors of the FOHPS with different values of $q = 0.1$.

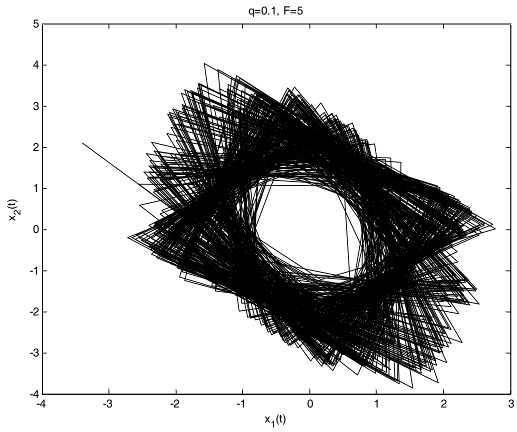

Fig. 4. Strange attractors of the FOHPS with different values of $F = 5$.

verges to zero, while the second state of the HPS shows a periodic behavior.

2.2.2 Case2: $q = 0.1$ and $F \in (1,20)$

In this case, we select $q = 0.1$ and we change the harmonic torque amplitude F from 1 to 20. After doing the simulations, we understand that the HPS Eq. (5) possesses chaotic state for different values of F. Fig. 3 shows the MLEs of the system for different values of $F \in (1,20)$. It is apparent that MLEs are positive, indicating that the system possesses chaos. The strange attractor of the system Eq. (5) for $F = 5$ is plotted in Fig. 4. One can see that the system behavior is chaotic.

3. Designing a finite-time fractional controller for chaos suppression of FOHPS

Theorem 1 [28]. Let $x = 0$ be an equilibrium point for the non-autonomous fractional-order system

$$D^q x(t) = f(x,t) \tag{6}$$

where $f(x, t)$ satisfies the Lipschitz condition with Lipschitz constant $l>0$ and $q \in (0,1)$ is the fractional order of the system. Assume that there exists a Lyapunov function $V(x(t), t)$ which satisfies the following conditions.

$$\alpha_1 \|x\|^a \le V(x(t),t) \le \alpha_2 \|x\| \tag{7}$$

$$\dot{V}(x(t),t) \le -\alpha_3 \|x\| \tag{8}$$

where α_1, α_2, α_3 and a are positive constants. Then the equilibrium point of the non-autonomous system Eq. (6) is Miattag-Leffler (asymptotically) stable.

Since in real world applications model uncertainties and external disturbances affect the dynamics of the system, the following uncertain FOHPS with a control input is taken into account in this paper.

$$\begin{cases} D^q x_1 = x_2 \\ D^q x_2 = -a x_2 - b \sin x_1 + l \cos x_1 \sin x_1 + h \cos \omega t \\ +\Delta f(t) + d(t) + u(t) \end{cases} \tag{9}$$

where $X(t)=[x_1(t),x_2(t)]^T$ is the state vector of the system and $\Delta f(X)$, $d(t)$ and $u(t)$ represent the system model uncertainty, external disturbance and control input, respectively.

Assumption 1. It is assumed that the uncertainty term and external disturbance are unknown, but bounded as follows:

$$\left\|D^{1-q}\Delta f(t)\right\|_1 \le \alpha,\ \left\|D^{1-q}d(t)\right\|_1 \le \beta \tag{10}$$

where α and β are given positive constants.

Theorem 2. Consider the non-autonomous uncertain chaotic FOHPS Eq. (9). If this system is controlled by the control law Eq. (11), then its chaotic behavior will be suppressed in finite time.

$$\begin{aligned}&u(t)=-f(x,t)-D^{q-1}((\alpha+\beta+l_1|x_1|^c+l_2|x_2|^c+\\&sign(x_1)D^{1-q}(x_2)+k_1|x_1|)sign(x_2)-k_2x_2)\end{aligned} \tag{11}$$

where $f(x,t)=-ax_2-b\sin x_1+l\cos x_1\sin x_1+h\cos\omega t$ and l_1, l_2, k_1, k_2 are positive constants and $0<c<1$ is a real number.

Proof. Choosing a Lyapunov function candidate in the form of $V(t)=\|X(t)\|_1$ and taking its time derivative, we have

$$\begin{aligned}&\dot V(t)=sign^T(X(t))\dot X(t)=sign(x_1)\dot x_1+sign(x_2)\dot x_2=\\&sign(x_1)D^{1-q}(D^q x_1)+sign(x_2)D^{1-q}(D^q x_2)\,.\end{aligned} \tag{12}$$

Substituting $D^q x_1$ and $D^q x_2$ from Eq. (9) into Eq. (12), one has

$$\begin{aligned}&\dot V(t)=sign(x_1)D^{1-q}(x_2)+sign(x_2)D^{1-q}(-ax_2-\\&b\sin x_1+l\cos x_1\sin x_1+h\cos\omega t+\Delta f(t)+d(t)+u(t))\,.\end{aligned} \tag{13}$$

Using Assumption 1, one can obtain

$$\begin{aligned}&\dot V(t)\le sign(x_1)D^{1-q}(x_2)+sign(x_2)D^{1-q}(-ax_2\\&-b\sin x_1+l\cos x_1\sin x_1+h\cos\omega t+u(t))+\|\Delta f(t)\|_1\\&+\|d(t)\|_1\le sign(x_1)D^{1-q}(x_2)+sign(x_2)D^{1-q}(-ax_2\\&-b\sin x_1+l\cos x_1\sin x_1+h\cos\omega t+u(t))+\alpha+\beta\,.\end{aligned} \tag{14}$$

Introducing the control law Eq. (9) into the right hand side of Eq. (14) and after some mathematical manipulations, we have

$$\dot V(t)\le -k_1|x_1|-k_2|x_2|-l_1|x_1|^c-l_2|x_2|^c\,. \tag{15}$$

Using the fact $-(|x_1|^c+|x_2|^c)\le-(|x_1|+|x_2|)^c$ [30], we have

$$\dot V(t)\le -(m(|x_1|+|x_2|)+n(|x_1|+|x_2|)^c)=-m\|X\|_1-n\|X\|_1^c \tag{16}$$

where $m = min\{k_1, k_2\}$ and $n = min\{l_1, l_2\}$ are two positive constants.

As a result, according to Theorem 1, the state trajectories of the uncertain FOHPS Eq. (9) will converge to zero and its chaotic behavior will be suppressed asymptotically. In what follows, we prove that the chaos suppression take places in finite time.

From the inequality Eq. (16), we have

$$\dot V(t)=\frac{d\|X(t)\|_1}{dt}\le -m\|X\|_1-n\|X\|_1^c\,. \tag{17}$$

Using some simple calculations, we get

$$\begin{aligned}&dt\le-\frac{d\|X(t)\|_1}{m\|X(t)\|_1+n\|X(t)\|_1^c}=-\frac{\|X(t)\|_1^{-c}d\|X(t)\|_1}{m\|X(t)\|_1^{1-c}+n}\\&=-\frac{1}{1-c}\frac{d\|X(t)\|_1^{1-c}}{m\|X(t)\|_1^{1-c}+n}\,.\end{aligned} \tag{18}$$

Taking integral of both sides of Eq. (18) from 0 to T and letting $x_1(t_1) = x_2(t_2) = 0$ and $X(T) = 0$ ($T = max\{t_1, t_2\}$), one has

$$\begin{aligned}&T\le-\frac{1}{1-c}\int_{X(0)}^{X(T)}\frac{d\|X(t)\|_1^{1-c}}{m\|X(t)\|_1^{1-c}+n}=-\frac{1}{m(1-c)}\ln(n+\\&m\|X(t)\|_1^{1-c})\Big|_{X(0)}^{X(T)}=\frac{1}{m(1-c)}\ln\left(m\|X(0)\|_1^{1-c}+n\right).\end{aligned} \tag{19}$$

This completes the proof.

4. Simulation of the proposed finite-time fractional-order controller

Here, we assume that the fractional order of the system Eq. (9) is equal to 0.1, where the system has a MLE of 0.1738. Moreover, the following model uncertainty and external disturbance are considered in the simulation.

The parameters of the controller Eq. (11) are selected as $l_1 = l_2 = k_1 = k_2 = 0.1$, $c = 0.2$ and $\alpha=\beta=2$. The initial conditions of the system are set to $x_1(0)=-3.4$ and $x_2(0)=2.4$. The state trajectories of the controlled uncertain FOHPS Eq. (9) are shown in Fig. 5. It is obvious that the chaotic motions of the non-autonomous uncertain chaotic FOHPS system are suppressed quickly in a finite time. The time history of the applied control input is depicted in Fig. 6. It is seen that the control input converges to zero.

To compare the performance of the proposed fractional finite-time technique, the fractional sliding mode control strategy introduced in Ref. [31] is applied to stabilize the uncertain chaotic FOHPS Eq. (9). Fig. 7 reveals the state trajectories of the controlled uncertain chaotic FOHPS Eq. (9) via the proposed method in Ref. [31]. It is seen that the second state suffers from undesirable oscillations. Furthermore, the time history of the applied control input via the proposed sliding mode approach in Ref. [31] is appeared in Fig. 8. One can see that the control input has permanent chattering which restricts the

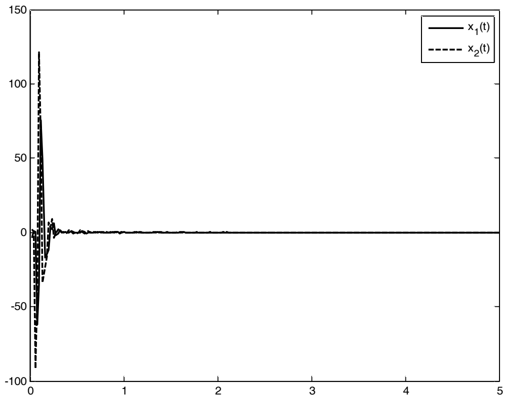

Fig. 5. State trajectories of the controlled FOHPS Eq. (9) via the proposed method.

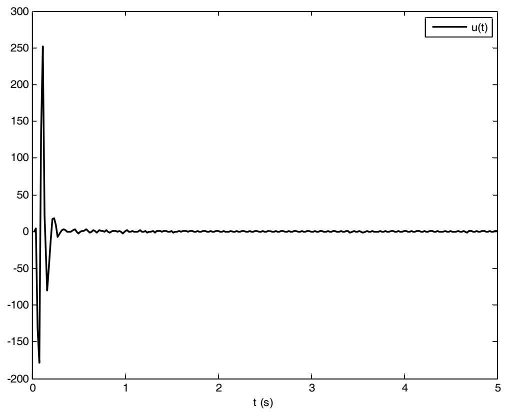

Fig. 6. Time histories of the applied control input Eq. (11) via the proposed method.

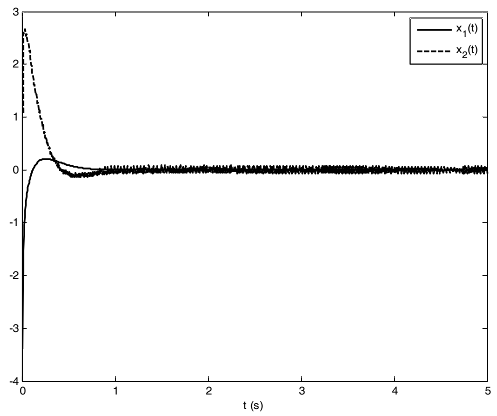

Fig. 7. State trajectories of the controlled FOHPS Eq. (9) via the method in Ref. [31].

practical implementation of the controller proposed in Ref. [31]. However, the method proposed in our paper can quickly stabilize the uncertain chaotic FOHPS in a robust manner.

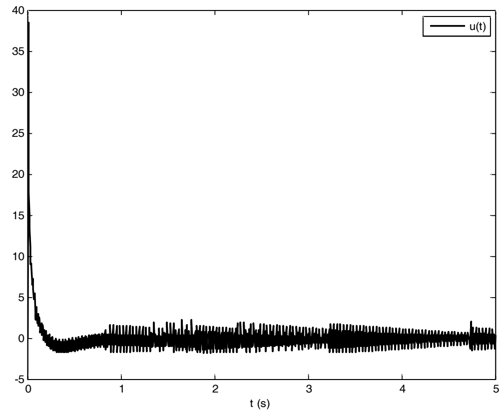

Fig. 8. Time histories of the applied control input via the method in Ref. [31].

5. Conclusions

This paper studies the possible existence of the chaos for a non-autonomous fractional-order horizontal platform system (FOHPS). Using the maximal Lyapunov exponent criterion, it is shown that the FOHPS exhibits chaotic behavior. A robust finite-time controller is then designed to attenuate the chaotic behavior of the system in the presence of both model uncertainty and external disturbance. Using the fractional Lyapunov stability theory, the finite-time stability and robustness of the proposed scheme are mathematically proved. A numerical simulation illustrates the superiority of the proposed technique compared to the sliding mode approach existing in the literature.

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