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A moment-matching robust collaborative optimization method†

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Abstract

Robust collaborative optimization (RCO) is a widely used approach to design multidisciplinary system under uncertainty. In most of the existing RCO frameworks, the mean of the state variable is considered as auxiliary design variable and the implicit uncertainty propagation method is employed for estimating their uncertainties (interval or standard deviation), which are then used to calculate uncertainties in the ending performances. However, as repeated calculation of the global sensitivity equations (GSE) is demanded during the optimization process of the existing approaches, it is typically very cumbersome or even impossible to obtain GSE for many practical engineering problems due to the non-smoothness and discontinuity of the black-box-type analysis models. To address this issue, a new RCO method is proposed in this paper, in which the standard deviation of the state variable is introduced as auxiliary design variable in addition to the mean. Accordingly, interdisciplinary compatibility constraint on the standard deviation of state variable is added to enhance the design compatibility between various disciplines. The effectiveness of the proposed method is demonstrated through two mathematical examples. The results generated by the conventional robust all-in-one (RAIO) approach are used as benchmarks for comparison. Our study shows that the optimal solutions produced by the proposed RCO method are highly close to those of RAIO while exhibiting good interdisciplinary compatibility.

Keywords: Collaborative optimization; Robust design; Moment matching; Coupled variable

1. Introduction

Complex systems design often involves a large number of design variables and information coupling among subsystems. Multidisciplinary design optimization (MDO) [1-3] methods have been developed to relieve the computational burden by decomposing a system into several manageable subsystems. Collaborative optimization (CO) is one popular approach among them, which decouples various disciplines by introducing the interdisciplinary compatibility constraints on auxiliary design variables and shared design variables. As, each discipline can achieve autonomous design simultaneously, CO is widely utilized to solve coupled MDO problems. On the other hand, it is widely recognized that uncertainty universally exists in engineering systems and often causes unexpected quality loss or catastrophic failure [4]. Therefore, traditional MDO has been extended to robust MDO [5] and reliability-based MDO [6] with the consideration of uncertainties.

Literature has seen many works to solve the robust MDO problem. A worst-case based uncertainty propagation method for evaluating the interval of end performances in robust MDO has been developed by Gu [7]. To accommodate this approach to generic probabilistic representations of uncertainties, Du and Chen proposed three techniques, namely, system uncertainty analysis method (SUA), concurrent subsystem uncertainty analysis method (CSSUA) and modified concurrent subsystem uncertainty analysis method (MCSSUA) [8, 9]. These works greatly facilitate the integration of robust design with MDO in an all-in-one fashion. However, they may not be applicable to robust MDO formulated in a multi-level type for distributed design. To address this problem, lots of robust CO (RCO) approaches with various uncertainty management strategies have been developed by extending the CO framework to robust design. A novel RCO method based on the dual-response surface was proposed, in which the mean and standard deviation estimation of the state variables and end performances are replaced by two response surfaces respectively [10]. However, it is well known that a large amount of sample points are needed to guarantee the accuracy of response surface. In addition, the inaccuracy of metamodels would have large impact on the uncertainty estimation [11]. A RCO framework based on the implicit uncertainty propagation (IUP) method was established, in which the state variable is considered as auxiliary design variable as is typically done in CO, while its interval is estimated by calculating the global

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sensitivity equation (GSE) in the IUP module [12]. Since this approach can only deal with interval uncertainties, based on it another RCO method applicable to problems with probabilistic uncertainties was developed by integrating SUA into the IUP module [13]. In these works, the IUP method is employed to propagate uncertainties, during which repeated GSE calculation is demanded. However, as the IUP method relies upon the first-order Taylor approximation of the state variables, it can cause large error when the state variables are highly nonlinear or have large variations. Most importantly, the sensitivity information for the GSE calculation may not exist at certain design point. For example, for some black-box-type simulated based functions, such as finite element analysis and computational fluent dynamics simulation models, the sensitivity information may not exist due to non-smoothness or discontinuity of performance functions, which however cannot be detected easily before optimization. In this case, the IUP based approaches are lack of robustness and no longer applicable.

With the popularity of simulation tools, practical engineering problems widely involve black-box-type simulated based performance functions. It is necessary to develop a more general and robust RCO approach that can address the above issue of the existing RCO approaches. In this paper, a general RCO framework is developed avoiding the complicated IUP module, thus to make the remedy for IUP-RCO. In addition to the mean, the standard deviation of the state variable is also considered as auxiliary design variable in each discipline rather than being estimated by IUP. Consequently, additional interdisciplinary compatibility constraints on the state variable standard deviation are added at the system-level optimization. During optimization, the targets for the state variables from the system level are not only the mean but also the standard deviation. That is to say, the first two statistic moments of the state variables are matched between various disciplines to ensure the design compatibility. Thus, the proposed RCO method in this paper is named as moment-matching RCO (MM-RCO). The rest of this paper is organized as follows. The proposed MM-RCO method is introduced in Sec. 2 with a brief review of the CO method, followed by case studies on two mathematical examples to demonstrate the effectiveness of MM-RCO in Sec. 3. Conclusions are summarized in Sec. 4.

2. The proposed robust collaborative optimization method

2.1 Review of collaborative optimization

Collaborative optimization (CO) first proposed by Kroo et al. [14, 15] is a bi-level MDO approach specifically created for large-scale distributed-analysis applications, which has been successfully applied to many design problems [16, 17]. The basic architecture of CO [15] for a multidisciplinary system with two coupled disciplines is shown in Fig. 1. CO is designed to allow each discipline to solve its subproblem in parallel with the others by adding auxiliary design variables ($(x_{aux})_{12}$, $(x_{aux})_{21}$) corresponding to the input state variables in each subproblem. The system-level optimization design variables include: its own local design variable (x_{sys}), and x_{sh}^0, y_{12}^0, y_{21}^0 that are then sent to disciplines as target values respectively for $(x_{sh})_1 \cup (x_{sh})_2$, y_{12}, y_{21}. Each discipline determines its local design variables (x_1, $(x_{sh})_1$, x_2, $(x_{sh})_2$) in order to meet the targets (x_{sh}^0, y_{12}^0, y_{21}^0) as closely as possible subject to its local constraints (g_1, g_2). $(x_{sh})_1$ is design variable shared by discipline 1 and system, while $(x_{sh})_2$ shared by discipline 2 and system.

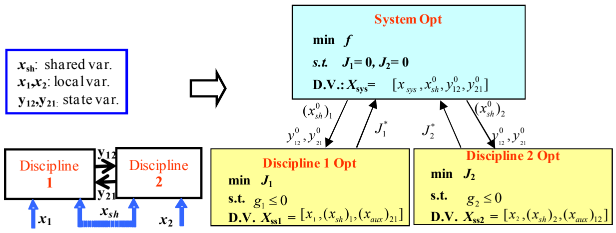

Fig. 1. A system with two coupled disciplines and the corresponding CO architecture.

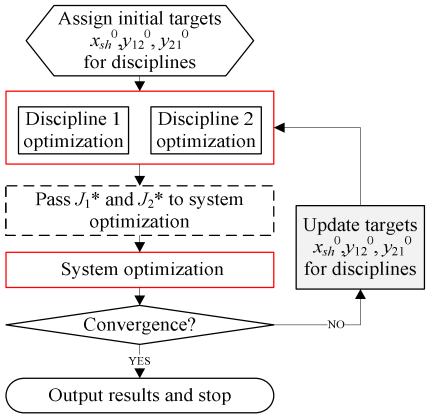

Fig. 2. The general flowchart of the CO algorithm for system with two coupled disciplines.

Through introducing auxiliary design variable $(x_{aux})_{ij}$ as additional design variable in discipline j, concurrent design of each discipline becomes possible in the CO framework. To ensure interdisciplinary consistency, interdisciplinary compatibility constraints $J = 0$ are introduced in the system level. The general flowchart of the CO algorithm is shown in Fig. 2, with the system and discipline optimization formulations displayed in Eqs. (1)-(3), respectively.

The system-level optimization attempts to minimize the system objective f while satisfying all the interdisciplinary compatibility constraints J. After its optimization, the designed variables x_{sh}^0, y_{12}^0 and y_{21}^0 would be assigned to disciplines as targets to be matched (see the shadowed box in Fig. 2). Each discipline operates on its own local design variables x_i with the goal of matching target values posed by the system level as well as satisfying its local constraints g_i. Meanwhile, they would feed back the current optimal objective J^* (i.e.

optimal shared design variables, optimal auxiliary design variables and achievable output state variables) to the system level after optimization (see the dashed box in Fig. 2). The matching can be obtained by gradually minimizing the discrepancy J_i between some of the local design variables and/or local states and their corresponding target values with the increase of optimization iteration. The optimization procedure iterates till it converges.

System

$$\begin{aligned} &\min\ f(X_{sys}) \\ &s.t.\quad J_1=0,\quad J_2=0 \\ &X_{sys}=(x_{sys},x_{sh}^0,y_{12}^0,y_{21}^0). \end{aligned} \tag{1}$$

Discipline 1

$$\begin{aligned} &\min\ J_1=((x_{sh}^0)_1-(x_{sh})_1)^2+(y_{12}^0-y_{12})^2+(y_{21}^0-(x_{aux})_{21})^2 \\ &s.t.\quad g_1\le 0 \\ &X_{ss1}=[x_1,(x_{sh})_1,(x_{aux})_{21}] \\ &y_{12}=y_{12}(X_{ss1}). \end{aligned} \tag{2}$$

Discipline 2

$$\begin{aligned} &\min\ J_2=((x_{sh}^0)_2-(x_{sh})_2)^2+(y_{21}^0-y_{21})^2+(y_{12}^0-(x_{aux})_{12})^2 \\ &s.t.\quad g_2\le 0 \\ &X_{ss2}=[x_2,(x_{sh})_2,(x_{aux})_{12}] \\ &y_{21}=y_{21}(X_{ss2}). \end{aligned} \tag{3}$$

2.2 The proposed moment-matching based robust collaborative optimization method

RCO is actually an extension of CO with the consideration of uncertainties, in which the state variables and ending performances become stochastic. As has been mentioned in the introduction, GSE is demanded in the existing RCO methods employing IUP (shorted to IUP-RCO for simplicity in this paper). In RCO, the completion of one system-level optimization and one discipline-level optimization is defined as one optimization iteration. Specifically, in each optimization iteration of IUP-RCO, each discipline has to conduct local sensitivity analysis at its current obtained optimal design point and then pass the sensitivity information to the IUP module to calculate the GSE. The uncertain information derived from GSE would then be assigned to the system and each discipline to estimate the uncertainty properties in their ending performances. As have been indicated in the introduction, the IUP module in RCO may become paralyzed for certain practical design problems due to the non-smoothness or discontinuity of performance functions thus terminates the optimization process of IUP-RCO. It is necessary to develop a more robust

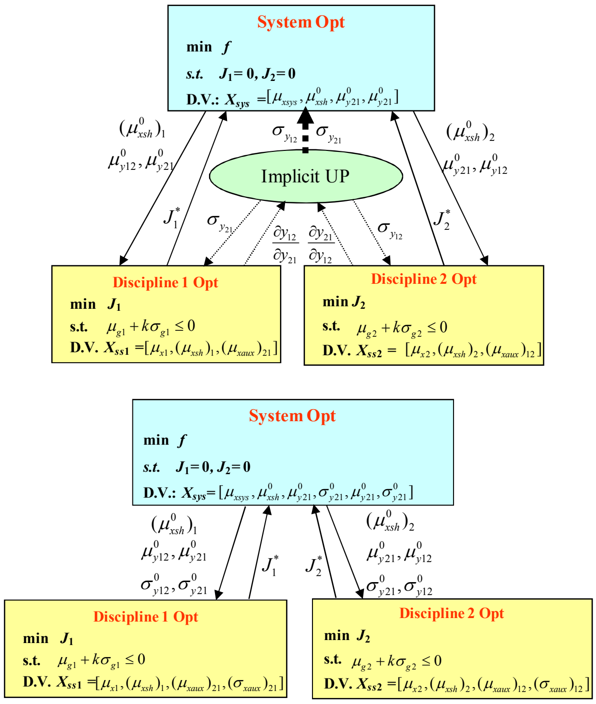

Fig. 3. Design architectures of IUP-RCO (top) and MM-RCO (bottom).

RCO method that is applicable to general design problems. Therefore, in this work, illuminated by the moment matching approach adopted in solving hierarchical MDO problems [18, 19], where the stochastic interrelated responses are quantified by the first two statistic moments and then matched in the hierarchy, a new RCO method (MM-RCO) following the general framework of the existing RCO approach [12] is proposed. With this method, in addition to the mean, the standard deviation of state variable is also introduced as auxiliary design variable during each discipline optimization. Subsequently, additional disciplinary compatibility constraints are correspondingly added in the system-level optimization. During each optimization iteration, the system-level optimization aims to minimize the system objective function while satisfying interdisciplinary compatibility constraints and then assigns targets (the shared variables and the first two statistical moments of state variables) to each discipline. The discipline-level optimization tries to match these targets and then feeds back its achievable values to the system. The iteration process will continue until certain convergence criteria are satisfied.

To better show the difference between the proposed MM-RCO method and the existing IUP-RCO method, the design architectures for system with two coupled disciplines as above of both approaches are illustrated and compared side by side in Fig. 3. It is clear that IUP-RCO has to invoke the IUP module within each optimization iteration to calculate the uncertainty properties in the state variables, which will be used for performance uncertainty property quantification. While, in MM-RCO such information can be easily obtained in each discipline optimization via certain uncertainty propagation method since the mean and standard deviation of input state variables are both considered as auxiliary design variables. Moreover, since the state variables are controlled by mean and standard deviation in MM-RCO with the application of the moment-matching strategy, it allocates more design freedom to each discipline. This is largely aligned with CO in that autonomous design can be completely realized in each discipline.

Based on the CO formulation shown in Eqs. (1)-(3), the corresponding MM-RCO formulation for system and discipline optimization is given in Eqs. (4)-(6), respectively. The quantities that discipline i attempts to minimize include: 1) discrepancies between its designed values of the shared variables $(\mu_{sh}^0)_i$ and the system targets $(\mu_{xsh}^0)_i$; 2) discrepancies between its designed values of the auxiliary design variable $(\mu_{xaux}^0)_{ji}$, $(\sigma_{xaux}^0)_{ji}$ and the system targets μ_{yji}^0, σ_{yji}^0; 3) discrepancies between its achievable values of its output state variable μ_{yij}, σ_{yij} and the system targets μ_{yij}^0, σ_{yij}^0. The formulation of the compatibility constraints in the system-level optimization are the same as the optimization objectives in each discipline. As has been noticed in CO, the interdisciplinary compatibility in MM-RCO cannot be guaranteed at the beginning of optimization. However, the discrepancies of mean and standard deviation of the auxiliary variable between different disciplines would become smaller and smaller, which would closely approach to zero at the optimal solution.

Clearly, the key component in robust CO is how to quantify the performance uncertainties in the system (μ_f and σ_f) and discipline optimization (μ_{gi} and σ_{gi}). With the proposed MM-RCO method, at any design point (μ_{xi} , $(\mu_{xsh})_i$, $(\mu_{xaux})_{ji}$, $(\sigma_{xaux})_{ji}$) of discipline i optimization, since σ_{xi} and $(\sigma_{xsh})_i$ are user prespecified and known before optimization, μ_{yij} and σ_{yij} can be easily estimated by many efficient uncertainty propagation methods without resorting to other discipline. Subsequently, performance uncertainties (μ_f, σ_f, μ_{gi}, σ_{gi}) that are functions of μ_{xi}, σ_{xi} , $(\mu_{xsh})_i$, $(\sigma_{xsh})_i$, $(\mu_{xaux})_{ji}$, $(\sigma_{xaux})_{ji}$, μ_{yij}, σ_{yij} can be estimated in the same way. While in IUP-RCO, only the mean of state variable is added as auxiliary design variable. Thus the variance of state variable should be calculated by the IUP module with tedious local sensitivity analysis in each discipline, based on which the performance uncertainties can only be quantified. With the moment-matching strategy, the tedious IUP module is excluded in MM-RCO by introducing additional auxiliary design variables and interdisciplinary compatibility constraints. For some practical engineering problems, the GSE calculation may be paralyzed at certain design point. In this case, MM-RCO is much more robust and preferable.

System optimization

$$\begin{aligned} \min\ & F = \mu_f + k\sigma_f \\ s.t.\ & J_i = 0 \qquad i = 1,2 \\ & X_{sys}^0 = (\mu_{xsys}, \mu_{xsh}^0, \mu_{y_{12}}^0, \sigma_{y_{12}}^0, \mu_{y_{21}}^0, \sigma_{y_{21}}^0)\ . \end{aligned} \tag{4}$$

Discipline 1 optimization

$$\begin{aligned} \min\ & J_1 = ((\mu_{xsh}^0)_1 - (\mu_{xsh})_1)^2 + (\mu_{y_{21}}^0 - (\mu_{xaux})_{21})^2 \\ & \quad + (\sigma_{y_{21}}^0 - (\sigma_{xaux})_{21})^2 + (\mu_{y_{12}}^0 - \mu_{y_{12}})^2 + (\sigma_{y_{12}}^0 - \sigma_{y_{12}})^2 \\ s.t.\ & \mu_{g_1} + k\sigma_{g_1} \le 0 \\ & \mu_{y12} = \mu_{y12}(\mu_{x_1}, \sigma_{x_1}, (\mu_{xsh})_1, (\sigma_{xsh})_1, (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}) \\ & \sigma_{y12} = \sigma_{y12}(\mu_{x_1}, \sigma_{x_1}, (\mu_{xsh})_1, (\sigma_{xsh})_1, (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}) \\ & X_{ss1} = [\mu_{x_1}, (\mu_{x_{sh}})_1, (\mu_{xaux})_{21}, (\sigma_{xaux})_{21}]\ . \end{aligned} \tag{5}$$

Discipline 2 optimization

$$\begin{aligned} \min\ & J_2 = ((\mu_{xsh}^0)_2 - (\mu_{xsh})_2)^2 + (\mu_{y_{12}}^0 - (\mu_{xaux})_{12})^2 \\ & \quad + (\sigma_{y_{12}}^0 - (\sigma_{xaux})_{12})^2 + (\mu_{y_{21}}^0 - \mu_{y_{21}})^2 + (\sigma_{y_{21}}^0 - \sigma_{y_{21}})^2 \\ s.t.\ & \mu_{g_2} + k\sigma_{g_2} \le 0 \\ & \mu_{y21} = \mu_{y21}(\mu_{x_2}, \sigma_{x_2}, (\mu_{xsh})_2, (\sigma_{xsh})_2, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{12}) \\ & \sigma_{y21} = \sigma_{y21}(\mu_{x_2}, \sigma_{x_2}, (\mu_{xsh})_2, (\sigma_{xsh})_2, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{12}) \\ & X_{ss2} = [\mu_{x_2}, (\mu_{x_{sh}})_2, (\mu_{xaux})_{12}, (\sigma_{xaux})_{12}]\ . \end{aligned} \tag{6}$$

In the system-level optimization, often it is difficult to strictly satisfy the interdisciplinary compatibility constraints ($J_i = 0$) especially when approaching to the optimal design point. Therefore, a dynamic slack factor s is introduced to loose the interdisciplinary compatibility requirement in MM-RCO [18]. A relatively larger slack factor is employed in the beginning stage of optimization to enhance the global search capability of MM-RCO. While in the end, a relatively smaller one is used to enhance the interdisciplinary compatibility and expedite the convergence process. The dynamic slack factor (s) is defined in Eq. (7), where "a" and "b" are constants and "I" is the number of optimization iteration. Generally, a larger "a" yields an optimal solution with better interdisciplinary compatibility, but more iterations. Therefore, "a" should be carefully selected based on the available computational resources. With the increase of I, s is decreasing, which augments the interdisciplinary compatibility between various disciplines. Actually, the selection of the slack factor s is somewhat subjective and problem independent. However, if no slack factor is employed, the optimal solution may not be able to converge after large numbers of optimization iteration.

$$s = b / (I)^a\ . \tag{7}$$

3. Case studies

3.1 Example 1

The all-in-one optimization formulation of the first mathematical example used to test the effectiveness of MM-RCO is shown in Eq. (8).

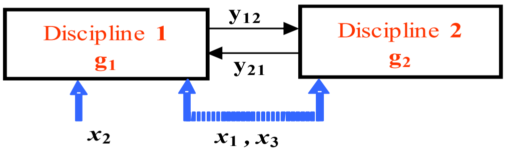

Fig. 4. Structure after decomposition of example 1.

$$\begin{aligned} \min\ & f = x_2^2 + x_3 + y_{12} + e^{-y_{21}} \\ s.t.\ & y_{12} = x_1^2 + x_2 + x_3 - 0.2y_{21} \\ & y_{21} = \sqrt{y_{12}} + x_1 + x_3 \\ & g_1 = 1 - \frac{y_{12}}{8} \le 0, \quad g_2 = \frac{y_{21}}{10} - 1 \le 0 \\ & -10 \le x_1 \le 10 \quad 0 \le x_2 \le 10 \quad 0 \le x_3 \le 10. \end{aligned} \tag{8}$$

This problem can be decomposed into two disciplines, of which x1 and x3 are shared design variables by the two disciplines (see Fig. 4). The MM-RCO formulation is given in Eqs. (9)-(11).

System optimization

$$\begin{aligned} \text{given}\ & (\mu_{x1})_1, (\mu_{x2})_1, (\mu_{x3})_1, (\mu_{x1})_2, (\mu_{x3})_2, \mu_{y12}, \sigma_{y12}, \\ & (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}, \mu_{y21}, \sigma_{y21}, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{21} \\ \text{find}\ & D.V. = [\mu_{x1}^0, \mu_{x2}^0, \mu_{x3}^0, \mu_{y12}^0, \mu_{y21}^0, \sigma_{y12}^0, \sigma_{y21}^0] \\ \min\ & F = \mu_f + k\sigma_f \\ s.t.\ & J_1 = (\mu_{x1}^0 - (\mu_{x1})_1)^2 + (\mu_{x2}^0 - (\mu_{x2})_1)^2 + (\mu_{x3}^0 - (\mu_{x3})_1)^2 + \\ & \quad (\mu_{y12}^0 - \mu_{y12})^2 + (\mu_{y21}^0 - (\mu_{x_{aux}})_{21})^2 + (\sigma_{y12}^0 - \sigma_{y12})^2 + \\ & \quad (\sigma_{y21}^0 - (\sigma_{x_{aux}})_{21})^2 = 0 \\ & J_2 = (\mu_{x1}^0 - (\mu_{x1})_2)^2 + (\mu_{x3}^0 - (\mu_{x3})_2)^2 + \\ & \quad (\mu_{y21}^0 - \mu_{y21})^2 + (\mu_{y12}^0 - (\mu_{x_{aux}})_{12})^2 + (\sigma_{y21}^0 - \sigma_{y21})^2 + \\ & \quad (\sigma_{y12}^0 - (\sigma_{x_{aux}})_{12})^2 = 0. \end{aligned} \tag{9}$$

Discipline 1 optimization

$$\begin{aligned} \text{given}\ & \mu_{x1}^0, \mu_{x2}^0, \mu_{x3}^0, \mu_{y1}^0, \mu_{y2}^0, \sigma_{y1}^0, \sigma_{y2}^0 \\ \text{find}\ & D.V. = [(\mu_{x1})_1, (\mu_{x2})_1, (\mu_{x3})_1, (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}] \\ \min\ & J_1 = (\mu_{x1}^0 - (\mu_{x1})_1)^2 + (\mu_{x2}^0 - (\mu_{x2})_1)^2 + (\mu_{x3}^0 - (\mu_{x3})_1)^2 + \\ & \quad (\mu_{y12}^0 - \mu_{y12})^2 + (\mu_{y21}^0 - (\mu_{x_{aux}})_{21})^2 + (\sigma_{y12}^0 - \sigma_{y12})^2 + \\ & \quad (\sigma_{y21}^0 - (\sigma_{x_{aux}})_{21})^2 = 0 \\ s.t.\ & \mu_{g1} + k\sigma_{g1} \le 0 \\ & \mu_{y12} = \mu_{y12}((\mu_{x1})_1, (\mu_{x2})_1, (\mu_{x3})_1, \sigma_{x1}, \sigma_{x2}, \sigma_{x3}, (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}) \\ & \sigma_{y12} = \sigma_{y12}((\mu_{x1})_1, (\mu_{x2})_1, (\mu_{x3})_1, \sigma_{x1}, \sigma_{x2}, \sigma_{x3}, (\mu_{x_{aux}})_{21}, (\sigma_{x_{aux}})_{21}). \end{aligned} \tag{10}$$

Discipline 2 optimization

$$\begin{aligned} \text{given}\ & \mu_{x1}^0, \mu_{x3}^0, \mu_{y1}^0, \mu_{y2}^0, \sigma_{y1}^0, \sigma_{y2}^0 \\ \text{find}\ & D.V. = [(\mu_{x1})_2, (\mu_{x3})_2, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{12}] \\ \min\ & J_2 = (\mu_{x1}^0 - (\mu_{x1})_2)^2 + (\mu_{x3}^0 - (\mu_{x3})_2)^2 + \\ & \quad (\mu_{y21}^0 - \mu_{y21})^2 + (\mu_{y12}^0 - (\mu_{x_{aux}})_{12})^2 + (\sigma_{y21}^0 - \sigma_{y21})^2 \\ & \quad + (\sigma_{y12}^0 - (\sigma_{x_{aux}})_{12})^2 = 0 \\ s.t.\ & \mu_{g2} + k\sigma_{g2} \le 0 \\ & \mu_{y21} = \mu_{y21}((\mu_{x1})_2, (\mu_{x3})_2, \sigma_{x1}, \sigma_{x3}, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{12}) \\ & \sigma_{y21} = \sigma_{y21}((\mu_{x1})_2, (\mu_{x3})_2, \sigma_{x1}, \sigma_{x3}, (\mu_{x_{aux}})_{12}, (\sigma_{x_{aux}})_{12}). \end{aligned} \tag{11}$$

In discipline 1 optimization, $(\mu_{xaux})_{21}$ and $(\sigma_{xaux})_{21}$ corresponding to the input state y_{21} are auxiliary variables. At any design point ($(\mu_{x1})_1$, $(\mu_{x2})_1$, $(\mu_{x3})_1$, $(\mu_{xaux})_{21}$, $(\sigma_{xaux})_{21}$), since $(\sigma_{xaux})_{21}$ is design variable which is evidently known for discipline 1, and σ_{x1}, σ_{x2}, σ_{x3} are constant given before optimization, the uncertainties in its state (μ_{y12}, σ_{y12}) and output performance (μ_{g1}, σ_{g1}) can be directly calculated by uncertainty propagation approaches, such as Monte Carlo simulation (MCS) [20], numerical integration [21] or polynomial chaos expansion [22]. Since this paper aims to verify the effectiveness of the proposed RCO formulation, MCS is utilized to implement this task to ensure high accuracy although many efficient uncertainty propagation methods can be employed. In discipline 2 optimization, μ_{y21}, σ_{y21} , μ_{g2} and σ_{g2} can be estimated in the same way. In this example, x_1 and x_3 are considered as random variables following normal distribution with distribution parameter $\sigma = 0.1$, i.e. $\sigma_{x1} = 0.1$, $\sigma_{x2} = 0$, $\sigma_{x3} = 0.1$.

In all the tested examples, the required reliability level is 99.865% ($k = 3$) for all the probabilistic constraints, and "a" and "b" in Eq. (7) are set as $a = 1.2$, $b = 1$. The convergence criterion of MM-RCO is that the discrepancy of the objective value between the current iteration and the previous iteration is less than a pre-specified value. The results from RAIO are used as benchmarks for validation. The optimal design variables X obtained by MM-RCO are plugged into the RAIO formulation to obtain the confirmed values of objective (μ_f and σ_f) and constraints g. All the results for example 1 are shown in Table 1. To clearly show how the disciplinary consistency is satisfied at the optimal solution, the mean and standard deviation (μ_y and σ_y) of the two state variables are calculated by conducting system analysis (SA) at the optimal design point of MM-RCO, which are validated by those from RAIO, and the results are listed in Table 2.

From Table 1, it is observed that the results produced by MM-RCO are very close to those of RAIO, which demonstrates the effectiveness of MM-RCO. From Table 2, evidently the optimal solutions for the auxiliary design variables from MM-RCO show great agreements to those generated by the SA at the optimal design point, which indicates that the discipline compatibility is well ensured at the optimal design point by MM-RCO. Meanwhile, both the optimal designed

Table 1. Optimal solutions from MM-RCO and PAIO (example 1).

	$X(x_1,x_2,x_3)$	μ_f	σ_f	$g(g_1,g_2)$
MMRCO	3.2643 0.0960 0	9.9863	0.6288	-0.0108 -0.2904
RAIO	3.3259 0.0997 0	9.8902	0.6259	0 -0.2933

Table 2. Comparison of auxiliary design variables (example 1).

	μ_{y12}	σ_{y12}	μ_{y21}	σ_{y21}
MMRCO	9.9957	0.6287	6.4498	0.1207
MMRCO (SA)	9.9737	0.6291	6.4969	0.1997
RAIO	9.8786	0.6262	6.4675	0.1998

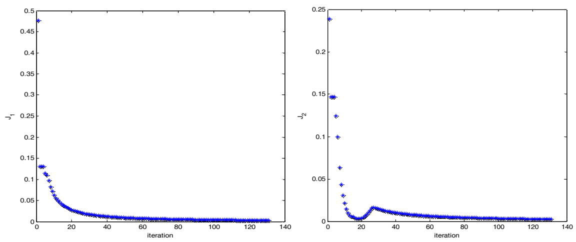

Fig. 5. The disciplinary discrepancy in each discipline of example 1.

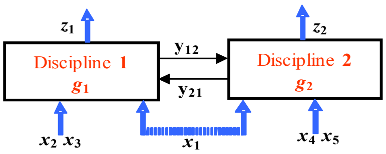

Fig. 6. Structure after decomposition of example 2.

auxiliary design variables and those obtained by SA are very close to those of RAIO, which further demonstrates the effectiveness of MM-RCO. To clearly show how the interdisciplinary discrepancies evolve with the optimization iteration of MM-RCO, values for the two compatibility constraints in the system-level optimization are plotted in Fig. 5. It is clear that both discrepancies become smaller and smaller as the iteration proceeds, meaning that the disciplinary compatibility is getting better and better.

3.2 Example 2

The second example can be also decomposed into two disciplines with two coupled state variables (see Fig. 6) and the all-in-one formulation is shown in Eq. (12).

$$
\begin{aligned}
\min\ & f=\sqrt{x_1}+x_4+x_5(0.4y_{12}) \\
s.t.\ & y_{12}=x_1^2+x_2-x_3+2\sqrt{y_{21}} \\
& y_{21}=x_1x_4+x_4^2+x_5+y_{12} \\
& z_1=x_1^2+2x_2+x_3+x_2e^{-y_{21}} \\
& z_2=\sqrt{x_1}+x_4+x_5(0.4y_{12}) \\
& g_1=1-\frac{z_1}{11}\le 0, g_2=1-\frac{z_2}{12}\le 0 \\
& 0\le x_i\le 10 \quad i=1,2,\ldots,5.
\end{aligned}
\tag{12}
$$

Table 3. Optimal solutions from MM-RCO and PAIO (example 2).

	$X(x_1,x_2,x_3,x_4,x_5)$	μ_f	σ_f	$g(g_1,g_2)$
MM RCO	1.6 1.6 9.4 8.0413 1.6	19.6008	2.5601	-0.0429 -0.0066
RAIO	1.6 1.6 9.4 8.0847 1.6	19.7004	2.5668	-0.0429 0

Table 4. Comparison of auxiliary design variables (example 2).

	μ_{y12}	σ_{y12}	μ_{y21}	σ_{y21}
MMRCO	15.3246	1.5459	95.0928	6.5000
MMRCO(SA)	15.8927	1.5503	95.1074	6.9027
RAIO	15.9806	1.5497	95.9651	6.9282

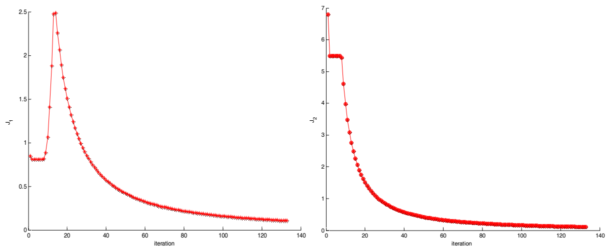

Fig. 7. The disciplinary discrepancy in each discipline of example 2.

The MM-RCO formulation for this example is not shown here since it is very easy to derive it by following the general formulation of MM-RCO in Eqs. (4)-(6). All the design variables x_i (i = 1,…,5) are considered as random variables following normal distribution and the standard deviation is $\sigma_{xi}=0.2$(i = 1,…,5). During the optimization process of MM-RCO, the first two statistic moments of y_{21} are considered as auxiliary design variables in discipline 1. Similarly, the first two statistic moments of y_{12} are considered as the auxiliary design variables in discipline 2. The optimal design variables X (x_1, x_2, x_3, x_4, x_5) produced by MM-RCO are plugged into the RAIO formulation to obtain the confirmed values of objective f and constraints g. All the results are shown in Table 3. The mean and standard deviation of auxiliary design variables from MM-RCO, SA and RAIO are listed in Table 4. It is observed that X, μ_f and σ_f from MM-RCO are very close to those of RAIO. Meanwhile, the first two statistic moments of state variables produced by MM-RCO show great agreements to those from SA and RAIO.

Values for the two compatibility constraints in the system optimization are illustrated in Fig. 7, which exhibits similar trend of example 1 that the discrepancies become smaller and smaller with the increase of optimization iteration, thus the disciplinary compatibility becomes better and better. It is also noticed that J_1 for discipline 1 jumps to a relatively large value in the first few iterations. However, it quickly becomes smaller and smaller and approaches to zero at the last few iterations.

It means that the disciplinary compatibility cannot be ensured in the beginning. However, it will be satisfied gradually, which further demonstrate the effectiveness of MM-RCO.

4. Conclusions

The existing RCO approaches employ the IUP method to estimate the uncertainties of the state variables, which require repeated tedious GSE calculation. However, it is cumbersome to estimate the GSE and the most important is that GSE may do not exist at certain design point especially for black-box-type simulation based model due to its non-smoothness and discontinuity. It significantly prohibits the applicability of the IUP-RCO approaches to solving general robust multidisciplinary systems. To address this issue, a new RCO method based on the moment-matching strategy (MM-RCO) is developed in this paper, in which the standard deviation of the state variable is also considered as auxiliary design variable in addition to the mean. Therefore, uncertainties of the ending performance in the system and discipline-level optimization can be conveniently obtained rather than by employing the IUP module. The proposed MM-RCO approach is tested via two mathematical examples to demonstrate its applicability and effectiveness. Based on our empirical study, it is observed that the optimal solutions produced by MM-RCO show great agreements to those obtained from RAIO. Meanwhile, the optimal designed mean and standard deviation of the state variables are very close to those obtained from the system analysis, indicating good disciplinary compatibility. Future work will focus on the application of the proposed MM-RCO method to some practical engineering problems.

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