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Designing a hand rest tremor dynamic vibration absorber using H_2 optimization method †

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Abstract

An optimal single DOF dynamic absorber is presented. A tremor has a random nature and then the system is subjected to a random excitation instead of a sinusoidal one; so the H_2 optimization criterion is probably more desirable than the popular H_∞ optimization method and was implemented in this research. The objective of H_2 optimization criterion is to reduce the total vibration energy of the system for overall frequencies. An objective function, considering the elbow joint angle, θ_2, tremor suppression as the main goal, was selected. The optimization was done by minimization of this objective function. The optimal system, including the absorber, performance was analyzed in both time and frequency domains. Implementing the optimal absorber, the frequency response amplitude of θ_2 was reduced by more than 98% and 80% at the first and second natural frequencies of the primary system, respectively. A reduction of more than 94% and 78%, was observed for the shoulder joint angle, θ_1. The objective function also decreased by more than 46%. Then, two types of random inputs were considered. For the first type, θ_1 and θ_2 revealed 60% and 39% reduction in their rms values, whereas for the second type, 33% and 50% decrease was observed.

Keywords: Frequency response function; Objective function; Vibration absorber; H_2 optimization method

1. Introduction

In 1817, James Parkinson described 'shaking palsy' clinically. This undesirable motion announced Parkinson's disease (PD). The most important tremor in the Parkinson's disease is rest tremor (i.e., when voluntary muscle activity is absent) and movement tremor observed in these patients. The main characteristics of tremor specification are frequency, amplitude and waveform. The frequency of PD tremor varies from 2 to 10 Hz [1]. Parkinson's disease treatment has not been successful because its physiology is not well understood. Usual treatments like drug therapies and surgery have not been successful, also, causing the alternative approaches to arise. To date for the design of tremor suppression devices, active and semi-active strategies are more utilized. In active strategy, tremor or other undesired movements are sensed by a sensor. A control signal is output to the driving motor to oppose the tremor.

Prochazak et al. (1992) proposed closed loop functional electrical stimulation (FES) to activate the tremorogenic muscles out-of-phase [2]. Functional electrical stimulation (FES) is a popular technique in rehabilitation engineering that uses electrical pulses to stimulate the skeletal muscle in order to restore motor function or generate desired movement [3]. HALL (1996) invented a hand-held gyroscopic device to attenuate the effects of tremors [4]. Arnold and Rosen (1993) applied energy-dissipating orthosis for people disabled by pathological intention tremor [5]. Tremor suppression via passive strategies is not common like active strategy. A table-mounted device called the Neater Eater was made by Michaelis Engineering (1988) is passive vibration isolation mechanism. This device is a two DOF damped linkage that supports a utensil to assist eating [6]. Kotovsky et al. (1998) designed a wearable tremor-suppression orthosis which was able to apply viscous resistance to the motion of the wrist in flexion and extension [7]. Damping was provided by a constrained-layer-damping (CLD) system. Hashemi and Golnaraghi (2004) proposed a tuned vibration absorber (TVA), which was effective in the suppression of vibration in an experimental model of the human arm with two degrees of freedom [1]. In their paper, they considered the tremor to be harmonic, while it has a

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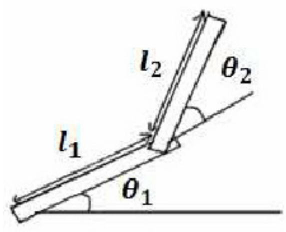


Fig. 1. Schematic diagram of arm model.

completely random nature. In this paper, passive control of the rest tremor of the human arm, caused by PD, is investigated. A three DOF dynamic model, including the absorber, is introduced. Considering the stochastic model for the tremor [3], H_2 optimization method is utilized and the optimal absorber characteristics are found. Then, the optimal system performance is analyzed in both time and frequency domains.

2. Biomechanical two DOF arm model

A representative two DOF model is considered as Fig. 1. Assuming small angular displacement and neglecting the higher-order terms, the nonlinear equations of motion can be linearized about the point $\theta_1 = 45^\circ$ and $\theta_2 = 90^\circ$ as

$$[M]\{\ddot{\theta}\} + [C]\{\dot{\theta}\} + [K]\{\theta\} = \{T\} \tag{1}$$

where $[M], [C], [K], \{\theta\}$ and $\{F\}$ represent the symmetric mass, damping and stiffness matrices, the angular displacement and excitation force vectors, respectively. Here,

$$[M] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},\ [C] = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix},\ [K] = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \tag{2}$$

where

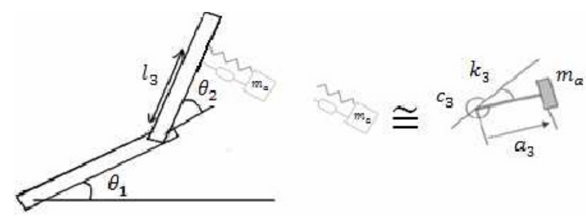
$$M_{11} = \left(I_1 + m_1 a_1^2\right) + \left(I_2 + m_2 a_2^2\right) + m_2 l_1^2,$$
$$M_{12} = \left(I_2 + m_2 a_2^2\right),$$
$$M_{21} = M_{12} \qquad M_{22} = \left(I_2 + m_2 a_2^2\right).$$

I_1, I_2 are the moments of inertia at the center of mass of the upper arm and the forearm; m_1, m_2 are the masses of the upper arm and the forearm; l_1, l_2 represent the lengths of the upper arm and the forearm; a_1, a_2 are the distances from the centers of mass of the upper arm and the forearm to their joints; T_1, T_2 represent the external torques acting at the shoulder and elbow joints, respectively; θ_1, θ_2 represent the angle of flexion at the shoulder and elbow joints, and K_i , C_i are the net stiffness and the damping coefficients of each pair of muscles Based on the system parameters presented in Table 1, the first two natural frequencies of the primary system were found to be

$$\omega_1 = 2.59(Hz), \quad \omega_2 = 7.79(Hz) \tag{3}$$

Table 1. Two DOF Topology [1]

Parameter	Value	Unit	Definition
l_1	0.30	m	Length, upper arm
l_2	0.45	m	Length, forearm
m_1	1.45	kg	Shoulder mass
m_2	2.18	kg	Elbow mass
K_1	180	Nm	Shoulder spring stiffness
K_2	250	Nm	Elbow spring stiffness
C_1	(0.001)* K_1	Ns/m	Shoulder joint damping coefficient
C_2	(0.002)* K_2	Ns/m	elbow joint damping coefficient



(a) (b)

Fig. 2. Two DOF arm model, with vibration absorber as 3rd DOF (a); dynamic absorber and equivalent pendulum model (b).

3. Optimal absorber design

3.1 Three DOF dynamic model

Generally, tremor suppression requires control of motion in all joints simultaneously. However, for simplicity considerations and as the first step, in this research, only one absorber was used. The three DOF dynamic model of the arm with the vibration absorber, modeled as a pendulum one, is illustrated in Fig. 2. Linearization at the point $\theta_1 = 45^\circ$ and $\theta_2 = 90$, the governing equations of motion have the same form as Eq. (1), where $[M]$, $[C]$ and $[K]$ matrices are as

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \qquad K = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix},$$
$$C = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \tag{4}$$

where

$$M_{11} = \left(I_1 + m_1 a_1^2\right) + \left(I_2 + m_2 a_2^2\right) + m_a\left(L_1^2 + a_3^2 + L_3^2 + 2 l_3 a_3\right),$$

$$M_{12} = \left(I_2 + m_2 a_2^2\right) + m_a\left(a_3^2 + L_3^2 + 2l_3 a_3\right)$$
$$M_{13} = m_a\left(a_3^2 + l_3 a_3\right),$$
$$M_{22} = \left(I_2 + m_2 a_2^2\right) + m_a\left(a_3^2 + L_3^2 + 2l_3 a_3\right),$$
$$M_{21} = M_{12}, \quad M_{23} = M_{13}, \quad M_{31} = M_{13}$$
$$M_{32} = M_{23}, \; M_{33} = m_a a_3^2 .$$

l_3 is the distance from the absorber joint to the elbow joint; a_3 is the length of absorber; m_a represents the absorber mass; K_3 and C_3 are the torsional stiffness and damping coefficients, respectively. The rest of the parameters are the same as already described.

3.2 H_2 Optimization criterion

If the system is subjected to a random excitation instead of a sinusoidal one, the H_2 optimization method is probably more desirable than the popular H_∞ optimization [8]. The objective of H_2 optimization criterion is to reduce the total vibration energy of the system for overall frequencies. As the first step, the complex frequency response function (CFRF) is derived. For the standard system representation of Eq. (1), the CFRF is defined as [9]

$$H(w) = \frac{1}{([K]-[M]w^2) + i[C]w}. \tag{5}$$

Including the system with absorber, $H(w)$ is a 3*3 matrix

$$H(w) = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}. \tag{6}$$

Next, since the main objective of this research is the tremor suppression control of the forearm in the presence of external moment, acting at elbow joint and approximating the tremor source, the objective function is defined as

$$J = \int_{W_1}^{W_2} \left|H_{22}\right|^2 dw \tag{7}$$

where W_1, W_2 define the frequency band in which the optimization should be held and selected to be 0, 2π * 20 rad/sec, respectively. Optimization procedure is followed by minimizing this objective function. Now, this function is dependent on primary system parameters, represented in Table 1, and absorber characteristics, namely m_a, l_3 a_3 k_3 *and* c_3 . It was shown by LI that the absorber performance improves as absorber mass increases, but it should be below 10% of the arm weight (e.g., for a person with a weight of 75 kg, the mass of the arm is approximately 3.75 kg [1]. So the maximum allowable value, 0.375 kg, was considered for the absorber mass. In a similar way, l_3 selected to be 0.45 m. Here, no damping was included, $c_3 = 0$, in the optimization. Considering the patient comfort requirement, the absorber stiffness, k_3, was constrained to be less than 40 N/m, applying the limitation on the transferred moment to the forearm at the absorber joint. Finally the optimized values of a_3, k_3 were found to be 0.2 m, 30 N/m. The appropriate damping coefficient was 0.05* k_3, which will be explained in the following sections. The optimal absorber parameters are listed in Table 2. The objective function for the primary and the optimal systems has the magnitude of 0.0026 and 0.0014. So the optimization reveals 46% reduction in the objective function magnitude.

Table 2. The optimal absorber characteristics.

$m_a\left(kg\right)$	$l_3\left(m\right)$	$a_3\left(m\right)$	$k_3\left(N/m\right)$	$c_3\left(Ns/m\right)$
0.375	0.45	0.2	30	0.05* k_3

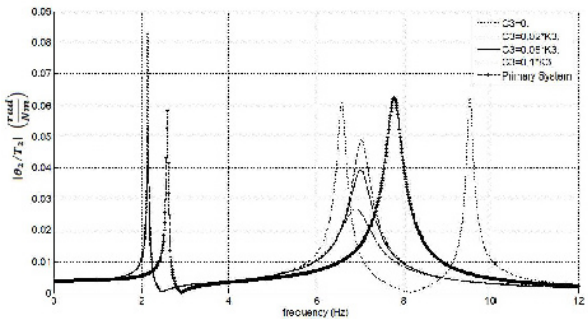


Fig. 3. θ_2 frequency response for different absorber dampings.

4. Optimal system performance analysis

This section considers the performance improvement of the optimal system rather than the primary one in both time and frequency domains. So, first the frequency characteristics and then the time responses to the random input are presented.

4.1 Frequency domain analysis

Fig. 3 represents the frequency response of θ_2 for the primary system and the system including the absorber, for different absorber damping coefficients. Note that only one damping is the optimum. Other absorber parameters, except c_3, are according to the Table 2. It can be observed that the absorber employment, independent of the value of the damping coefficient, can extremely decrease the magnitude of response at the resonance frequencies. It is obvious that the response magnitude decreases with the damping coefficient reduction. However, the best damping coefficient was found to be 0.05*k_3. This matter will be discussed afterwards. Then the frequency responses of the primary, non-optimal (zero damping) and optimal systems are shown in Figs. 4 and 5. It should be pointed out that the most energy content of the hand tremor is concentrated at the natural frequencies of the primary system.

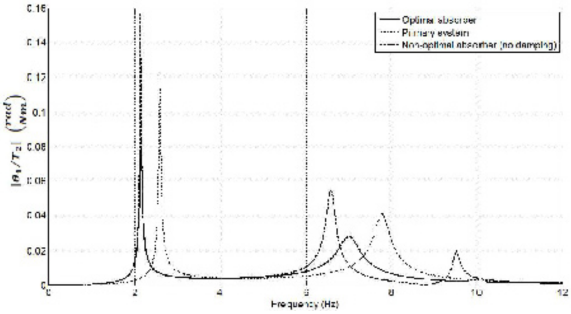


Fig. 4. θ_1 frequency response for optimal and non-optimal designs.

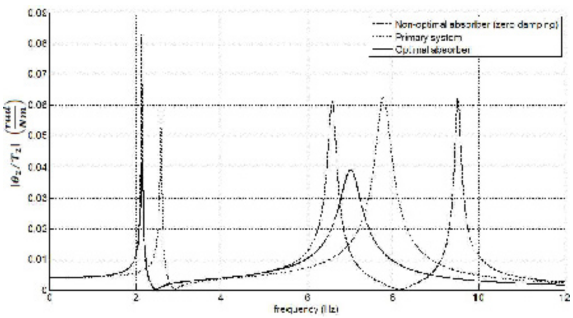


Fig. 5. θ_2 frequency response for optimal and non-optimal designs.

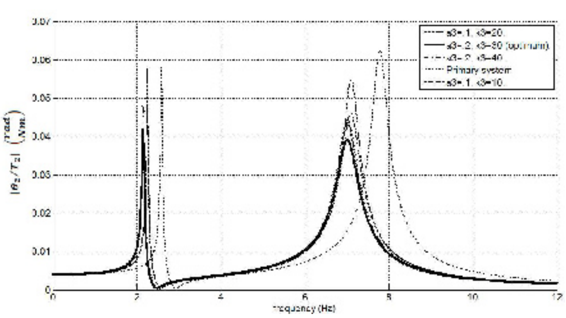


Fig. 6. θ_2 frequency response for optimal and different non-optimal designs.

So the main objective of implementing the absorber is the response magnitude reduction at the neighborhood of the natural frequencies, as much as possible. The absorber achieves this goal by displacing the system's natural frequencies. Furthermore, the response magnitude at new natural frequencies is anticipated to be less than those of the primary system natural frequencies. It can be seen that using the proposed optimal absorber, the elbow angle frequency response is reduced by more than 98% and 80% at the first and second natural frequencies of the primary system, respectively (see Fig. 5). Also, the shoulder angle frequency response decreases by more than 94% and 78% at the first and second resonance frequencies (see Fig. 4). Fig. 6 compares the elbow angle, for the instance, frequency response of the optimal design to the different non-optimal designs.

Table 3. Rms reduction percent for various absorber damping in comparison with primary system (first input).

Response	$c_3 = 0$	$c_3 = 0.02 * k_3$	$c_3 = 0.05 * k_3$	$c_2 = 0.1 * k_3$
θ_1	–30%	–46%	–60%	–49%
θ_2	+14	–48%	–39%	–35%

Table 4. Response rms reduction percent for various absorber damping in comparison with primary system (second input).

Response	$c_3 = 0$	$c_3 = 0.02 * k_3$	$c_3 = 0.05 * k_3$	$c_2 = 0.1 * k_3$
θ_1	+20%	–38%	–33%	–38%
θ_2	–8%	–32%	–50%	–47%

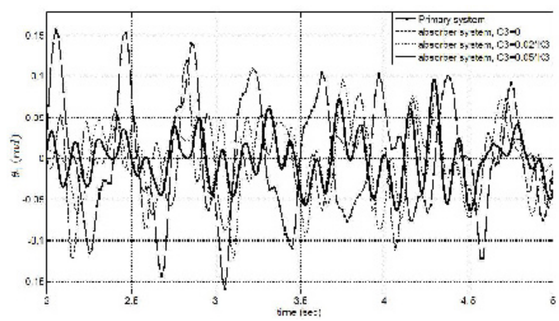


Fig. 7. θ_1 Random response for primary and absorber systems (first input).

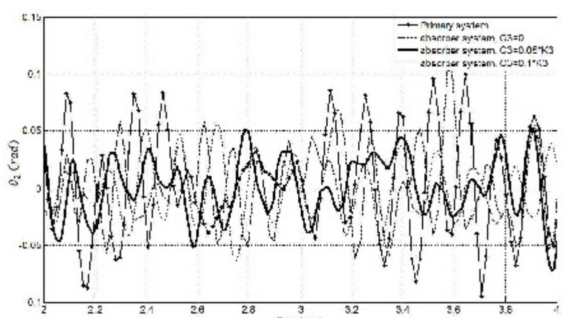


Fig. 8. θ_2 Random response for primary and absorber systems (second input).

4.2 Time domain analysis

Here, the time responses of θ_1 and θ_2 to the stochastic input are presented. So, the effectiveness of implementing the proposed absorber will be proved. The input torque, acting at the elbow joint, is modelled as a sinusoid one with stochastic amplitude and frequency. The amplitude is considered to have a mean of 10 Nm and variance of 5 Nm. So, the amplitude rms is equal to 10.25 Nm. Two types are selected for the frequency mean. In the first type, the mean is 2.8 Hz and for the second one, it equals 8 Hz. The variance for the two types is 20% of the mean. So, for the first and second types, the frequency rms equals to 2.9 and 8.1 Hz, respectively. Here the rms of θ_1 and θ_2 for two kinds of the input are presented.

Table 5. Rms reduction percent for different absorber characteristics in comparison with primary system.

$a_3(m)$	$k_3(N/m)$	θ_1	θ_2
0.1	10	−36%	−31%
0.1	20	−3%	−10%
0.2	30	−60%	−39%
0.2	40	−17%	−15%

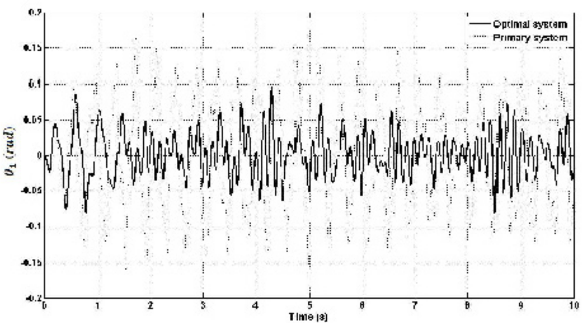


Fig. 9. θ_1 random response for primary and optimal absorber system (first input).

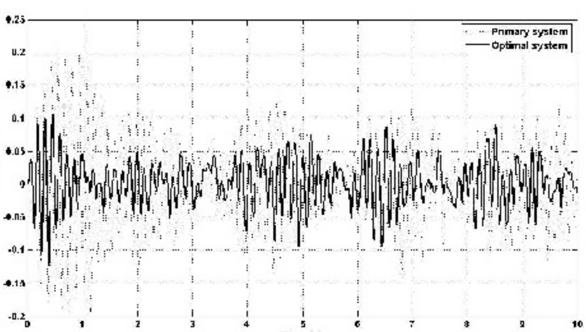


Fig. 10. θ_2 random response for primary and optimal absorber system (second input).

Now, we are going to explain the reason that the damping coefficient of the optimal absorber was selected to be 0.05* k_3. Tables 3 and 4 list the rms reduction of θ_1 and θ_2 of the optimal absorber, in comparison with the primary system, for different damping coefficients, where the negative sign reveals the reduction and effectiveness of implementing the proposed absorber. In these two tables, the input stochastic frequency means are equal to 2.8 and 8 Hz, respectively. Figs 7 and 8 also show the time response of θ_1 and θ_2 for the primary and absorber system with various damping.

Although the optimal absorber with the damping of 0.05* k_3 does not have the best frequency response in Fig. 3, it can be seen from the Tables 3 and 4 that this damping coefficient has appropriated the best performance to itself. So, the mentioned damping coefficient was considered to be the optimum one. Table 5 specifies the performance improvement of the system, with the first type input for example, for the optimal and various non-optimal kinds of absorbers. It is obvious that the optimal proposed absorber has the best performance, having the maximum rms reduction of the responses. Lastly, primary and optimal absorber response to the both random inputs are presented in Figs. 9 and 10. The decrease of 60% in θ_1 rms and 50% in θ_2 rms, see Tables 3 and 4, can be sensed in these figures.

5. Conclusions

Our goal was to design a single DOF dynamic absorber that could effectively suppress the tremor of the human arm undergoing involuntary rhythmic oscillations. Because of the random nature of the arm tremor, H_2 optimization criterion was selected. Then, the optimum absorber characteristics were found. Implementing the optimal absorber, the optimal system revealed great attenuation in the frequency response curves in the neighbourhood of the resonance frequencies of the primary system. This is the main objective of designing the absorber. In the time domain analysis also, considering the random inputs, the optimal system caused a very considerable reduction in the response root mean square, rms, and the arm tremor was highly suppressed. So, the effectiveness of the proposed dynamic absorber was proved in both time and frequency domains.

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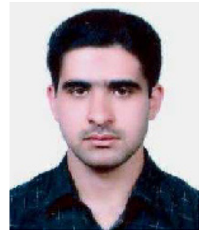

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