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Transverse vibration of flexible hoisting rope with time-varying length †

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Abstract

The nonlinear vibration of a flexible hoisting rope with time-varying length is investigated. The governing equations of the flexible hoisting rope are developed based on Hamilton's principle. Experiments performed evaluated the theoretical model and found that the experimental data agree well with the theoretical prediction, which validates the mathematical model of the flexible hoisting system. The results of the simulations and experiments show that the flexible hoisting system dissipates energy during downward movement (thus is stabilized) and gains energy during upward movement (thus is unstabilized). In addition, a passage through resonance in the hoisting system with periodic external excitation is analyzed. Due to the time-varying length of the hoisting rope the natural frequencies of the system vary slowly, and transient resonance may occur when one of the frequencies coincides with the frequency of an external excitation.

Keywords: Flexible hoisting rope; Natural frequency; Transient resonance; Transverse vibration

1. Introduction

Ropes, which are extensively used as mine hoists, elevators, cranes etc, are subject to vibration due to their high flexibility and relatively low internal damping characteristics [1, 2]. Usually, these systems are modeled as either an axially moving tensioned beam or a string with time-varying length and a rigid body at its lower end [3, 4]. In general, the vibration energy of the rope changes during elongation and shortening [5, 6]. When the rope length is shortened, the vibration energy increases exponentially with time, causing dynamic instability [7]. Studies on rope vibration problems in flexible hoisting systems have attracted wide attention. For example, Chi and Shu [8] calculated the natural frequencies associated with the vertical vibration of a stationary cable coupled with an elevator car. Terumichi and Ohtsuka et al. [9] studied the transverse vibrations of a string with time-varying length by considering the velocity of the string as a constant and of a mass-spring system at the lower end with theoretical and experimental methods. By taking into account the inertia of the rotor, Fung and Lin [10] analyzed the transverse vibration of an elevator rope with time-varying length, and proposed a variable structure control scheme to suppress the transient amplitudes of vibrations. Kaczmarczyk and Ostachowicz [11] studied the coupled vibration of a deep mine hoisting cable and built a distributed-parameter model. They found that the response of the catenary-vertical rope system might represent a number of resonance phenomena. Zhang and Agrawal [12] derived the governing equations of the coupled vibration of a flexible cable transporter system with arbitrarily varying length. Zhu and Chen [13] investigated the control of an elevator cable with theoretical and experimental methods. A novel experimental method was developed to validate the uncontrolled and controlled lateral responses of a moving cable in a high-rise elevator, which showed good agreement with the theoretical predictions. Chio and Hong et al. [14] investigated the vibration control of a translating tensioned steel strip in the zinc galvanizing line. A right boundary control law based upon the Lyapunov second method was derived. The results of simulation show that a time-varying boundary force and a suitable passive damping at the right boundary can successfully suppress the transverse vibrations. Nguyen and Hong [15] studied transverse vibration control of axially moving membranes by regulation of axial velocity. A novel control algorithm that suppresses the transverse vibrations of an axially moving membrane system was developed. Ngo and Hong et al. [16] investigated the control of an axially moving system. The Lyapunov function taking the form of the total mechanical energy of the system was adopted to ensure the uniform stability of the closed-loop system. The results of experiments show

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the proposed control law was effective. Zhang [17] presented a systematic procedure for deriving the model of a cable transporter system with arbitrarily varying cable length and proposed a Lyapunov controller to dissipate the vibratory energy. Zhang and Zhu et al. [18] derived the governing equation and energy equations of the longitudinal vibration of a flexible hoisting system with arbitrarily varying length.

Although extensive studies focus individually on vibration characteristics of a rope with time-varying length, the dynamic stability of the rope has also been studied by several groups. Kumaniecka and Niziol [19] investigated the longitudinal-transverse vibration of a hoisting cable with slow variability of the parameters. The non-linearity of the cable material was considered and the unstable regions were identified by applying the harmonic balance method. General stability characteristics of the horizontally and vertically translating beams and strings with arbitrarily varying length and boundary conditions were studied by Zhu and Ni [20]. While the amplitude of the displacement can behave in a different manner depending on the boundary conditions, the amplitude of the vibratory energy of a translating medium decreases and increases in general during extension and retraction, respectively. Lee [7] introduced a new technique to analyze the free vibration of a string with time-varying length by dealing with traveling waves. When the string length was shortened, free vibration energy increased exponentially with time, leading to dynamic instability.

Despite numerous research efforts on the flexible hoisting rope with time-varying length in the last few decades, most studies have been restricted to cases with constant transport speed samples. The dynamic characteristics of a flexible hoisting rope with an arbitrarily varying length are the subject of this investigation. The governing equations are developed based on the extended Hamilton's principle. The derived governing equations are shown to be nonlinear partial differential equations (PDEs) with variable coefficients. By choosing proper mode functions that satisfy the boundary conditions, the solutions of the governing equations are obtained using the Galerkin method. To evaluate the mathematical model, an experimental set-up is built and some experiments are conducted. Comparing the experimental data to the simulation, a favorable result is obtained, which indicates that the proposed mathematical model is valid for the flexible hoisting rope. So the modeling methods can well represent the transverse vibration of flexible hoisting rope with time-varying length. The derived mathematical model may illustrate the true dynamic nature of a flexible hoisting rope system, and can be used to predict and analyze resonance phenomena. Based on the proposed fundamental dynamic analyses, further vibration control can be adopted for such flexible hoisting systems in the near future.

2. Model of the flexible hoisting system

The flexible hoisting system can be simplified as an axially moving string with time-varying length and a rigid body m at its lower end, as shown in Fig. 1. The rail and the suspension of the rail are assumed to be rigid. The string has Young's modulus E, diameter d and the mass per unit length ρ. The origin of the coordinate is set at the top end of the string and the instantaneous length of the string is $l(t)$ at time t. The instantaneous axial velocity, acceleration and jerk of the string are $v(t)=\dot{l}(t)$, $a(t)=\dot{v}(t)$ and $j(t)=\dot{a}(t)$ respectively, where the overdot denotes time differentiation. At any instant t, the transverse displacement of the string is described by $y(x, t)$, at a spatial position x, where $0 \le x \le l(t)$. In an actual flexible hoisting system, the rotational unbalance of the traction motor or the abnormal off-track of the rope possibly causes vibration of the hoisting system. To reproduce this phenomenon, a transverse extrinsic disturbing excitation $s(t)$ is applied at the upper end of the string. In this paper, all the equations and derivations are based on the following assumptions:

(1) The parameters E, d and ρ of the string are always constants;

(2) Only transverse vibration is considered here. The elastic distortion of the string due to the transverse vibration is much less than the length of the string;

(3) The bending stiffness of the string, all the damp and friction, and the influence of air current are ignored.

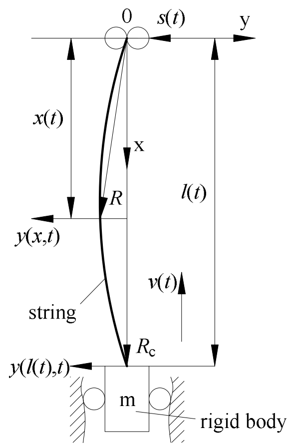

Fig. 1. Schematic of flexible hoisting string with time-varying length.

2.1 Energy of the flexible hoisting system

After the string is deformed, the position vector R of a point at x can be written as:

$$R = x(t)i + y(x,t)j \tag{1}$$

where i and j are the unit vectors along the x-axes and y-axes, respectively. The material derivative of R yields the velocity vector

$$V = v(t)i + (y_t + vy_x)j \tag{2}$$

where the subscript t denotes partial differentiation with respect to time, and subscript x denotes partial differentiation with respect to space. Similarly, the position vector R_c and velocity vector V_c of the rigid body can be written as:

$$R_c = l(t)i + y(l(t),t)j \tag{3}$$

$$V_c = v(t)i + y_t(l(t),t)j . \tag{4}$$

Then, the kinetic energy of the system is computed by

$$E_k(t) = 0.5mV_c \cdot V_c\big|_{x=l(t)} + 0.5\rho\int_0^{l(t)} V \cdot Vdx . \tag{5}$$

The first term on the right of Eq. (5) represents the kinetic energy of the rigid body, the second term represents the kinetic energy of the string. The elastic strain energy of the string is:

$$E_e(t) = \int_0^{l(t)} (T\varepsilon + 0.5ES\varepsilon^2)dx \tag{6}$$

where $T(x, t)$ is the quasi-static tension at spatial position x of the string at time t due to gravity. Since the string is acted upon not only by the weight of the concentrated mass at the lowest end but also its own weight, the tension $T(x, t)$ is expressed as:

$$T = [m + \rho(l(t) - x)]g . \tag{7}$$

And ε represents the strain at spatial position x of the string and can be expressed as:

$$\varepsilon = (ds - dx) / dx . \tag{8}$$

As shown in Fig. 2, ds can be expressed as:

$$ds \approx \sqrt{1 + (dy/dx)^2}dx \approx (1 + 0.5y_x^2)dx . \tag{9}$$

Substituting Eq. (9) into Eq. (8) yields

$$\varepsilon = 0.5y_x^2 . \tag{10}$$

2.2 Free vibration equations

According to the characteristics of top restriction of the string, the boundary conditions at $x(t) = 0$ are

$$y(0,t) = 0 . \tag{11}$$

Substitute Eqs. (5) and (6) into Hamilton's Principle,

$$\int_{t_1}^{t_2} (\delta E_k(t) - \delta E_e(t))dt = 0 \tag{12}$$

and apply the variational operation. Because the length of the

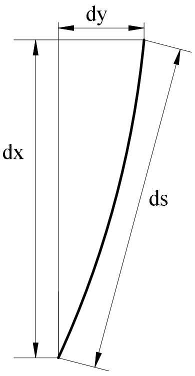

Fig. 2. A small element of the string in a deformed position.

string $l(t)$ changes with time, the standard procedure for integration by parts with respect to the temporal variable cannot apply. Applying Leibniz's rule and partial integration results in the following expressions:

$$\begin{aligned}&\int_0^{l(t)} \rho(y_t + vy_x)\delta y_t dx = \rho\frac{\partial}{\partial t}\int_0^{l(t)} (y_t + vy_x)\delta y dx - \\ &\rho\left[v(y_t + vy_x)\delta y\right]\Big|_{l(t)} - \rho\int_0^{l(t)} \frac{\partial}{\partial t}(y_t + vy_x)\delta y dx .\end{aligned} \tag{13}$$

Following the standard procedure for integration by parts with respect to the spatial variable and invoking Eq. (13), one obtains from Eq. (12),

$$\begin{aligned}&-\int_{t_1}^{t_2}\left[m\frac{\partial}{\partial t}y_t(l,t) + Ty_x + 0.5ESy_x^3\right]\delta y(l,t)dt - \\ &\int_{t_1}^{t_2}\int_0^{l(t)}\left[\rho\frac{\partial}{\partial t}(y_t + vy_x) + \rho v\frac{\partial}{\partial x}(y_t + vy_x)\right]\delta y dx dt + \\ &\int_{t_1}^{t_2}\int_0^{l(t)}\left[\frac{\partial}{\partial x}(Ty_x) + ES\frac{\partial}{\partial x}(0.5y_x^3)\right]\delta y dx dt = 0 .\end{aligned} \tag{14}$$

Setting the coefficients of δy in Eq. (14) to zero yields the governing equations in the forms

$$\begin{aligned}&\rho(y_{tt} + 2vy_{xt} + \dot{v}y_x + v^2y_{xx}) - T_xy_x - \\ &Ty_{xx} - 1.5ESy_x^2y_{xx} = 0\end{aligned}, \ 0 < x < l(t). \tag{15}$$

The first four terms in Eq. (15) correspond to the local, Coriolis, tangential and centripetal acceleration, respectively. The resulting boundary condition from Eq. (14) at $x = l(t)$ is

$$my_{tt} + Ty_x + 0.5ESy_x^3 = 0 , \ x = l(t). \tag{16}$$

The energy associated with the transverse vibration of the system is

$$\begin{aligned}&E_v(t) = 0.5my_t^2(l,t) + 0.5\rho\int_0^{l(t)} (y_t + vy_x)^2 dx + \\ &0.5\int_0^{l(t)} (Ty_x^2 + 0.25ESy_x^4)dx\end{aligned} . \tag{17}$$

2.3 Forced vibration equations

When external excitation occurs at the upper end of the string, the governing Eq. (15) must be adjusted. Compared with Eq. (11), the corresponding boundary conditions are changed into

$$y(0,t) = s(t)\,,\quad y(l,t) = 0\,. \tag{18}$$

Obviously, the boundary conditions are nonhomogeneous and difficult to apply directly. Here, the procedure described in Ref. [5] is used to transfer the governing Eq. (15) with nonhomogeneous boundary conditions into the equation of motion with homogeneous boundary conditions. The transverse displacement is expressed in the form of

$$y(x,t) = w(x,t) + h(x,t) \tag{19}$$

where $w(x, t)$ is the part that satisfies the homogeneous boundary conditions and $h(x, t)$ is the part that satisfies the nonhomogeneous boundary conditions. Substituting Eq. (19) into Eq. (15) yields

$$\begin{aligned}&\rho(w_{tt} + 2vw_{xt} + \dot{v}w_x + v^2 w_{xx}) - T_x w_x - Tw_{xx} - \\ &1.5ESw_x^2 w_{xx} + \rho(h_{tt} + 2vh_{xt} + \dot{v}h_x + v^2 h_{xx}) - \\ &T_x h_x - Th_{xx} - ESw_{xx}h_x(3w_x + 1.5h_x) - \\ &ESh_{xx}(1.5w_x^2 + 3w_x h_x + 1.5h_x^2) = 0 \\ &0 < x < l(t),\end{aligned} \tag{20}$$

where $w(x, t)$ represents displacements. Eq. (20) describes the transverse vibration of the flexible hoisting system under extrinsic disturbing excitation. The corresponding boundary condition is

$$\begin{aligned}&mw_{tt} + Tw_x + 0.5ESw_x^3 + mh_{tt} + \\ &Th_x + 0.5ES(3w_x^2 h_x + 3w_x h_x^2 + h_x^3) = 0\end{aligned}\,,\ x = l(t). \tag{21}$$

Setting the function $h(x, t)$ to the first-order polynomial,

$$h(x,t) = a_0(t) + a_1(t)x/l(t)\,. \tag{22}$$

Then, when $x(t) = 0$ and $x(t) = l(t)$

$$h(0,t) = s(t)\,,\quad h(l(t),t) = 0\,. \tag{23}$$

Substituting Eq. (23) into Eq. (22), the coefficients $a_0(t)$ and $a_1(t)$ can be obtained as:

$$a_0(t) = s(t)\,,\quad a_1(t) = -s(t)\,. \tag{24}$$

Therefore,

$$h(x,t) = s(t) - s(t)x/l(t)\,. \tag{25}$$

Once $h(x, t)$ is known, the solution for $w(x, t)$ is sought from Eq. (20). $y(x, t)$ is obtained subsequently from Eq. (19). Eq. (20) is a partial differential equation which describes the dynamics of the flexible hoisting string. The equation is defined over time-dependent spatial domain rendering the problem non-stationary. Hence, the exact solution to this problem is not available, and recourse must be made to an approximate analysis. In what follows, numerical techniques are employed to obtain approximate solution for the governing equation.

3. Discretization of the governing equation

Eq. (20) is a partial differential equation with infinite dimensions and many parameters are time-variant. It is impossible to obtain an exact analytical solution from Eq. (20). In this section, Galerkin's method is applied to truncate the infinite-dimensional partial differential equation into a nonlinear finite-dimensional ordinary differential equation with time-variant coefficients. Then, Eq. (20) can be solved with numerical methods. To map Eq. (20) onto the fixed domain, a new independent variable $\zeta = x/[l(t)]$ is introduced and the time-variant domain $[0, l(t)]$ for x is converted to a fixed domain $[0, 1]$ for ζ. According to the characteristics of a taut translating string, the solution of the transverse vibration $w(x, t)$ is assumed in the form [12, 13]

$$w(x,t) = \sum_{i=1}^{n} \varphi_i(\zeta)q_i(t) = \sum_{i=1}^{n} \varphi_i(x/l)q_i(t) \tag{26}$$

where $q_i(t)$ ($i = 1, 2, 3,\ldots, n$) are the generalized coordinates respect to $w(x, t)$, n is the number of modes included and $\varphi_i(\zeta)$ is the trial function [12, 13],

$$\varphi_i(\zeta) = \sqrt{2}\sin i\pi\zeta\,. \tag{27}$$

Consequently, expansion of Eq. (26) results in the expressions for partial derivatives of the transverse displacement function:

$$\begin{aligned}&w_x = \frac{1}{l}\sum_{i=1}^{n}\varphi_i' q_i\,,\quad w_{xx} = \frac{1}{l^2}\sum_{i=1}^{n}\varphi_i'' q_i \\ &w_{xt} = \sum_{i=1}^{n}\frac{1}{l}\varphi_i'\dot{q}_i - \sum_{i=1}^{n}\frac{\zeta v}{l^2}\varphi_i'' q_i - \sum_{i=1}^{n}\frac{v}{l^2}\varphi_i' q_i\,, \\ &w_{tt} = \sum_{i=1}^{n}\varphi_i\ddot{q}_i - \frac{2\zeta v}{l}\sum_{i=1}^{n}\varphi_i'\dot{q}_i + \\ &\frac{\zeta}{l}[\frac{2v^2}{l}\sum_{i=1}^{n}\varphi_i' q_i - a\sum_{i=1}^{n}\varphi_i' q_i + \frac{\zeta v^2}{l}\sum_{i=1}^{n}\varphi_i'' q_i]\,.\end{aligned} \tag{28}$$

Substituting Eq. (28) into Eq. (20), multiplying the governing equation by $\varphi_j(\zeta)$ ($j = 1, 2, 3 ,\ldots, n$), integrating it from $\zeta = 0$ to 1, and using the boundary conditions and the orthonormality relation for $\varphi_i(\zeta)$ yields the discretized equation of transverse vibration for the flexible hoisting system

$$M\ddot{Q}+C\dot{Q}+KQ+P(Q)=F \quad (29)$$

where $Q(t) = [q_1(t), q_2(t), \ldots, q_n(t)]^T$ is the vector of generalized coordinates, M, C, K and F are matrixes of mass, damp, stiffness and generalized force respect to Q, respectively. $P(Q)$ is a higher order item of generalized coordinate. The matrices are expressed as follows:

$$\begin{aligned}
&M_{ij}=\rho\delta_{ij}\,,\ \ C_{ij}=\int_0^1\frac{2v}{l}(1-\zeta)\varphi_i'\varphi_j d\zeta \\
&K_{ij}=\frac{\rho a}{l}\int_0^1(1-\zeta)\varphi_i'\varphi_j'd\zeta-\frac{\rho v^2}{l^2}\int_0^1(1-\zeta)^2\varphi_i'\varphi_j'd\zeta+ \\
&\frac{\rho g}{l}\int_0^1(1-\zeta)\varphi_i'\varphi_j'd\zeta-\left(\frac{mg}{l^2}+\frac{3ES}{2l^4}s^2\right)\int_0^1\varphi_i''\varphi_j d\zeta \\
&P_j=-\frac{3ES}{2l^4}\int_0^1(\textstyle\sum_{i=1}^n\varphi_i'q_i)^2\sum_{i=1}^n\varphi_i''q_i\varphi_j d\zeta- \\
&\frac{3ES}{l^4}s\int_0^1\textstyle\sum_{i=1}^n\varphi_i'q_i\sum_{i=1}^n\varphi_i'q_i\varphi_j d\zeta \\
&F_j=-\rho\left(\ddot{s}+\frac{2v}{l}\dot{s}+\frac{a}{l}s-\frac{2v^2}{l^2}s\right)\int_0^1(1-\zeta)\varphi_j d\zeta- \\
&-\frac{\rho g}{l}s\int_0^1\varphi_j d\zeta
\end{aligned} \quad (30)$$

where the superscript "′" denotes partial differentiation for normalized variable ζ, δ_{ij} is the Kronecker delta defined by $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$ ($i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, n$). Solving the ordinary differential Eq. (29) with numerical methods may yield the instantaneous values of Q. Substituting these values into Eq. (26) may yield the instantaneous values of the transverse vibration of the string $w(x, t)$. The mathematical model defined by Eq. (29) illustrates the true dynamic nature of the flexible hoisting string, and can be used to predict and analyze the dynamic characteristics of the flexible hoisting string.

4. Natural frequencies of the system

To obtain the natural frequencies of the flexible hoisting rope with time-varying length, the methods suggested by Stylianou [21] are used to reduce the system of governing Eq. (29) to a set of first-order differential equations. The set of reduced equations takes the form

$$A\dot{U}+BU=0 \quad (31)$$

where A and B are matrix differential operators, and

$$A=\begin{Bmatrix}M & 0\\ 0 & K\end{Bmatrix},\ \ B=\begin{Bmatrix}C & K\\ -K & 0\end{Bmatrix}. \quad (32)$$

U is the state vector, and

$$U=\begin{Bmatrix}\dot{Q}\\ Q\end{Bmatrix}. \quad (33)$$

Eq. (31) is the canonical form of the equation of motion and its solution satisfies the appropriate boundary conditions and initial conditions. Substituting Eqs. (32) and (33) into Eq. (31) yields

$$\dot{U}+DU=0 \quad (34)$$

where

$$D=A^{-1}B=\begin{Bmatrix}M^{-1}C & M^{-1}K\\ -I & 0\end{Bmatrix}. \quad (35)$$

Here I is an $n\times n$ identity matrix. To obtain the natural frequencies and mode shapes for the flexible hoisting rope with time-varying length, consider the eigenvalue problem of Eq. (34). Assuming that **U** is periodic, i.e.,

$$U=\Lambda e^{\lambda t} \quad (36)$$

where

$$\lambda=\xi+i\omega \quad (37)$$

is the eigenvalue which is a complex number, $\xi(t) = [\zeta_1(t), \zeta_2(t), \ldots, \zeta_{2n}(t)]^T$ and $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_{2n}(t)]^T$ are the real and imaginary parts of $\lambda(t) = [\lambda_1(t), \lambda_2(t), \ldots, \lambda_{2n}(t)]^T$, and ω is also the natural frequency of the flexible hoisting rope. The real and imaginary parts of the eigenvalue are related to the modal damping coefficients and the natural frequencies of the flexible hoisting rope. Substituting Eq. (36) into Eq. (34) leads to an eigenvalue equation

$$(\lambda I+D)\Lambda=0 \quad (38)$$

where Λ is the corresponding eigenvector. The eigenvalues can be obtained from

$$det(\lambda I+D)=0\,. \quad (39)$$

5. Simulation and experiment

5.1 Experiment setup

To validate the mathematical model, an experimental setup of the flexible hoisting system was designed and built as shown in Fig. 3. The setup, simulating the hoisting system of traction elevator, consists of a traction system, a guide system, an excitation system and data acquisition system (PC-OPTIPLEX755 and Charge Amplifier-B&K2635). A frequency conversion motor DC415 (manufactured by Suntous Shanghai Electromechanical Co., Ltd.) is used in the flexible hoisting system. The rotation speed of the motor may be controlled by adjusting the output of transducer to obtain the anticipant motion curve of the hoisting system. A thin steel rope with a diameter of 3.2 mm was chosen as the hoisting rope. The model car and counterweight are made up of many weights. The mass of the car and the counterweight can be

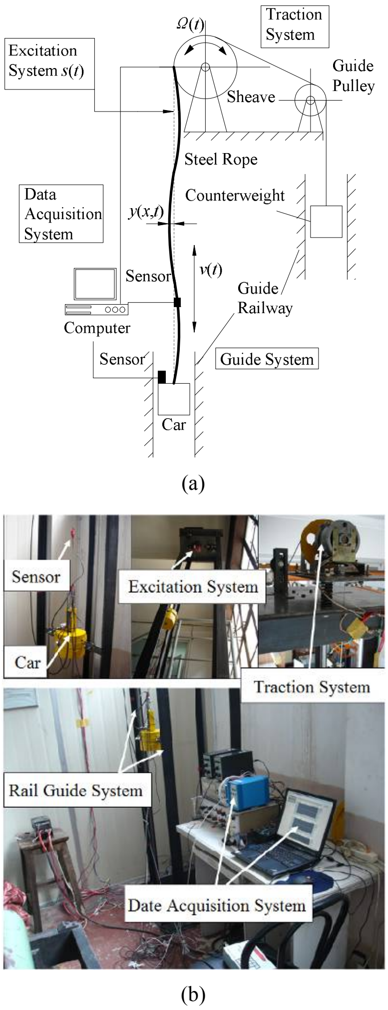

Fig. 3. Experimental setup: (a) schematic diagram of the experimental set-up; (b) actual picture of the experimental setup.

changed by adding or reducing the number of weight. The hoisting rope at the car side in this set-up is the main research object, whose dynamic behavior was studied. To simulate the extrinsic disturbing excitation in an actual hoisting system, a transverse vibration exciter is applied at the top of the objective rope. The output of the exciter is decided by an adjustable signal generator. The exciter is a KDJ-20C. A micro-sensor 8791A250 (KISTLER) with a mass of 4 g is attached at a certain position of the objective rope to acquire the transverse vibration acceleration of the rope. The sensitivity of the sensor is 20 mV/g. The frequency range of the sensor is 0-300 Hz. The signals from the micro-sensor are transmitted to a computer and saved. Fig. 3(a) shows the schematic diagram of the experiment setup and Fig. 3(b) shows the actual experimental setup. The main parameters of the test are shown in Table 1.

5.2 Experiment procedure

The transverse vibration of the flexible hoisting rope can be calculated with theoretical equation and tested with experimental setup, respectively. All the parameters used in the calculation and the test are the same. A downward or upward car movement is prescribed to be the input of the theoretical equations and experimental set-up. At the beginning, the car starts at the top of the flexible hoisting system and goes down. When arriving at the bottom, the car pauses for a moment and turns back to the starting point. Fig. 4 displays the prescribed displacement, velocity, acceleration curves of the flexible hoisting system, where the processes of acceleration, deceleration and uniform speed downwards and upwards are included.

Table 1. Parameters of experimental setup of flexible hoisting system.

Items	Data values
Mass per unit length ρ (kg/m)	0.042
Young's modulus E (N/m^2)	1×10^{12}
Rope diameter d (m)	3.2×10^{3}
Hoisting mass m (kg)	15
Excitation $s(t)$ (m)	$5\times10^{-4}\sin(18\pi t)$
Minimum length of the string $l_{min}(t)$ (m)	0.8
Maximum length of the string $l_{max}(t)$ (m)	4.8
Maximum velocity v_{max} (m/s)	0.55
Maximum acceleration a_{max} (m/s^2)	0.4
Total travel time t (s)	8
Number of transverse modes n	4

In an actual flexible hoisting system such as in elevators, a rotational unbalance of traction motor or the abnormal off-track of traction rope possibly occurs because of the improper installation or the abrasion of correlative components. To reproduce this phenomenon, a transverse extrinsic disturbing excitation $s(t)$ is applied at the top of the rope. The excitation signal is expressed as:

$$s(t) = 5\times10^{-4}\sin(18\pi t) . \quad (40)$$

The extrinsic disturbing excitation disturbs the dynamic behavior of the flexible hoisting rope only when the rope is moving. They are applied to the theoretical equations and experimental set-up. In following calculation, the number of modes included in $w(x, t)$ n is set to 4, which has been proved to be a proper value with many calculation results and comparisons. When $n = 4$, less calculation time and satisfactory veracity of results may be simultaneously obtained. However, the experiment results can validate the mathematical model and the computer simulation if the simulated cases agree with the experimental conditions.

5.3 Free vibration responses

If the initial displacement and velocity of the string are given by $y(x, 0)$ and $y_t(x, 0)$, respectively, where $0 < x < l(t)$, the initial conditions for the generalized coordinate can be obtained from Eqs. (26) and (28),

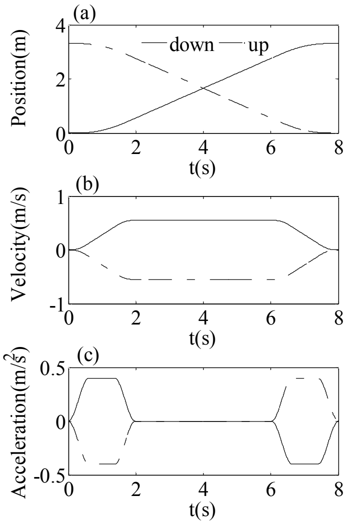

Fig. 4. Movement profile of flexible hoisting system: (a) $l(t)$; (b) $v(t)$; (c) $\dot{v}(t)$.

$$q_i(0) = \int_0^1 y(\zeta l,0)\varphi_i d\zeta\Big|_{t=0} \tag{41}$$

$$\dot{q}_i(0) = \int_0^1 y_t(\zeta l,0)\varphi_i d\zeta + \frac{v}{l}\sum_{i=1}^{n} q_i \int_0^1 \zeta\varphi_i'\varphi_j d\zeta\Big|_{t=0} . \tag{42}$$

Solving the ordinary differential Eq. (29) with numerical methods and setting $s(t) = 0$ may yield the instantaneous values of Q. Substituting these values into Eq. (26) may yield the free vibration responses of the string $y(x, t)$.

The numerical simulations with the exact experiment parameters are conducted to compare with the experiments, and the experimental results compare favorably with the simulations, as seen in Figs. 5 (downward movement) and 6 (upward movement). Comparing the results of tests with calculations in Figs. 5 and 6, the extent and trend of vibration curves are similar. Therefore, the theoretical equations, proposed in this paper, may be used to evaluate the vibration of the flexible hoisting rope.

Fig. 5 displays reducing vibration amplitudes with increasing length of the rope during downward movement. This is due to the energy of the flexible hoisting system transfers from the transverse vibration to the axial motion by bringing some mass into the domain of effective length, i.e., the axially hoisting rope is dissipative during downward movement, thus leading to a stabilized transverse dynamic response, as shown in Fig. 7(a). A possible physical interpretation of the result is as follows: during downward movement negative external work is required to maintain the prescribed axial motion which, in turn, brings about a convection of mass in the domain of effective length. At the same time, frequencies of the transverse vibration are reduced with increasing length of the rope. This is because the mass of the rope increases and the stiffness of the rope decreases, i.e., the rope becomes somewhat "softer" as shown in Fig. 8(a).

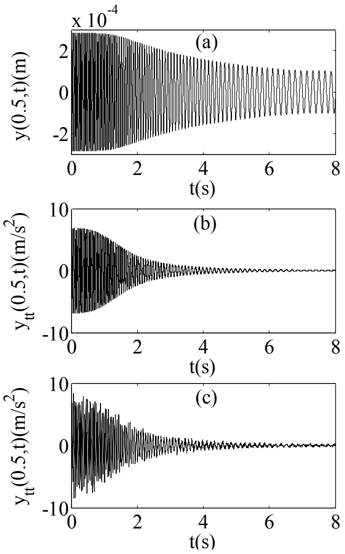

Fig. 5. Free vibration responses of the flexible hoisting rope at 0.5 m above the car during downward movement: (a) displacement curve; (b) acceleration curve (simulation); (c) acceleration curve (experiment).

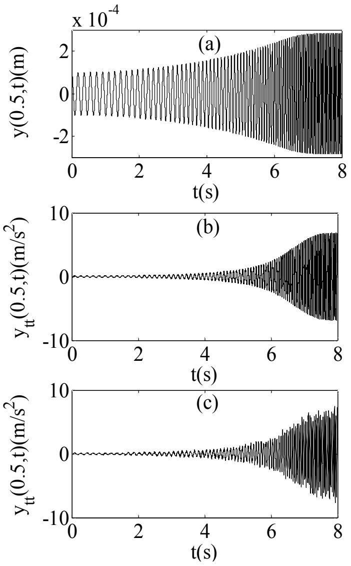

Fig. 6. Free vibration responses of the flexible hoisting rope at 0.5 m above the car during upward movement: (a) displacement curve; (b) acceleration curve (simulation); (c) acceleration curve (experiment).

However, in Fig. 6, vibration amplitudes of the rope increase with decreasing length of the rope during upward movement. This is due to the energy of the system transfers from the axial motion to the transverse vibration by leaving some mass out of the domain of effective length, i.e., the axially hoisting rope gains energy during upward movement, thus leading to an unstabilized transverse dynamic response, as shown in Fig. 7(b). A possible physical interpretation of the result is that during upward movement positive external work is required to maintain the prescribed axial motion which, in

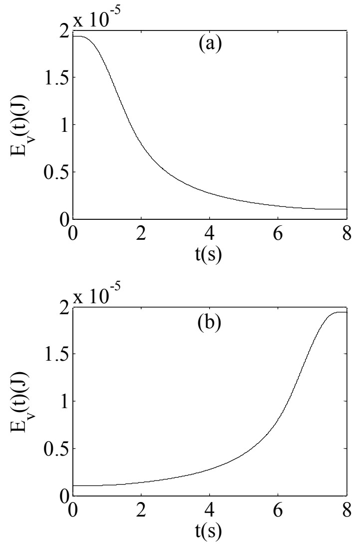

Fig. 7. Total energy associated with the transverse vibration of the system during movement: (a) downward movement; (b) upward movement.

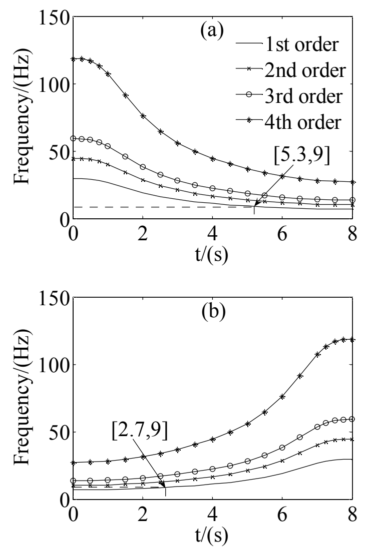

Fig. 8. The lowest four order natural frequencies with time increment: (a) downward movement; (b) upward movement.

turn, brings about a convection of mass out of the domain of effective length. In the meantime, frequencies of the transverse vibration increase with decreasing length of the rope. This is because the mass of the rope decreases and the stiffness of the rope increases, i.e., the rope becomes somewhat "stiffer" as shown in Fig. 8(b).

5.4 Forced vibration responses

During movement, the flexible hoisting system is subjected to vibration caused by various sources of excitation, which

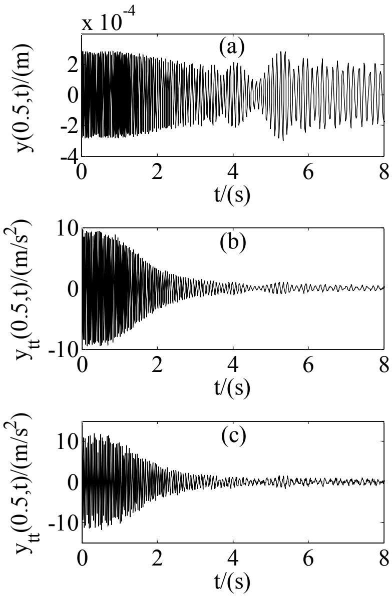

Fig. 9. Forced vibration responses of the flexible hoisting rope at 0.5 m above the car during downward movement: (a) displacement curve; (b) acceleration curve (simulation); (c) acceleration curve (experiment).

include excitations due to the irregularities of the guiding system and rotational unbalance of the traction motor as well as environmental phenomena such as air current. The system parameters are changing due to the time-varying length of the rope. The rate of variation of the length is, however, slow, and the vibrations represent waves in a slowly varying domain. Hence, the hoisting rope is essentially a nonstationary vibration system with slowly varying frequencies. Therefore, a passage through resonance may occur when one of the slowly varying frequencies coincides with the frequency of the extrinsic disturbing excitation at some critical time instant.

Forced vibration responses for the hoisting rope with extrinsic disturbing excitation are illustrated in Figs. 9 (downward movement) and 10 (upward movement). From Figs. 9 and 10, it can be seen that transient resonance occurs during movement of the hoisting system. The amplitudes exhibit oscillatory behavior before the resonance, and near the resonance the amplitudes increase rapidly and decline afterwards due to damping, developing damped beat phenomena. This is because one of the time-varying frequencies of the hoisting rope coincides with the frequency (9Hz) of the extrinsic disturbing excitation during the movement of the hoisting system. Coordinate values [5.3, 9] and [2.7, 9] represent the point of transient resonance in Fig. 8. Note that the adverse dynamic response in the hoisting system promotes large oscillations in rope tension. The phenomenon cannot be ignored, as the high amplitude in the tension contributes directly to rope fatigue. Fatigue often results in the hoisting ropes being discarded after lower working cycles. Therefore, suitable strategy can be sought to minimize the effects of adverse dynamic response of the system.

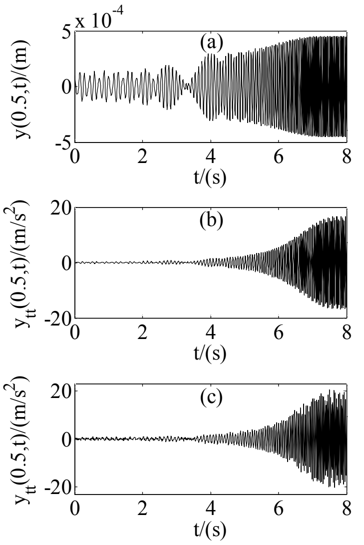

Fig. 10. Forced vibration responses of the flexible hoisting rope at 0.5 m above the car during upward movement: (a) displacement curve; (b) acceleration curve (simulation); (c) acceleration curve (experiment).

6. Conclusions

The nonlinear dynamic characteristics for a flexible hoist rope with time-varying length considering coupling of axial movement and flexural deformation are analyzed in this paper. The flexible hoisting system is modeled as an axially moving string with time-varying length and a rigid body at its lower end. The governing equations are derived by using Leibniz's rule and Hamilton's principle. The Galerkin method is used to truncate the infinite-dimensional partial differential equations into a set of nonlinear finite-dimensional ordinary differential equations with time-variant coefficients. To validate the theoretical model, an experimental setup of a flexible hoisting system was built and experiments were performed. By comparing the experimental results with the numerical simulation, a good agreement between the simulation and experiment is obtained, thus validating the mathematical model of the flexible hoisting system. Based on the simulations and experiments, the following conclusions can be obtained:

(1) A flexible hoisting rope with time-varying length experiences instability during upward movement, the natural frequencies increase because of the reduction of mass and the increase of stiffness of the rope, and the energy is transformed from the axial movement into the flexible deformation. By contrast, it is stable during downward movement, the natural frequencies decrease because of the increase of mass and the reduction of stiffness of the rope, and the energy is converted from flexible deformation into axial movement.

(2) The flexible hoisting rope is a nonstationary oscillatory system with slowly varying frequencies. Transient resonance may occur when one of the time-varying frequencies of the hoisting rope coincides with the frequency of the extrinsic disturbing excitation.

(3) The proposed theoretical model and analyses on the dynamic characteristics of the flexible hoisting system in this paper will be helpful for researchers to comprehend the dynamic behavior and to develop the proper method to suppress vibration in practice.

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