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Magnetic field effect on mixed convection in a lid-driven square cavity filled with nanofluids †

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Abstract

A numerical investigation of laminar mixed convection heat transfer in a lid-driven cavity filled with nanofluid under the influence of a magnetic field is executed. The left and right vertical walls of the cavity are insulated while the top and bottom horizontal walls are kept constant but different temperatures. The top wall is moving on its own plane at a constant speed while other walls are fixed. A uniform magnetic field is applied in the vertical direction normal to the moving wall. The governing differential equations are discretised by the control volume approach and the coupling between velocity and pressure is solved using the SIMPLE algorithm. The heat and mass transfer mechanisms and the flow characteristics inside the cavity depended strongly on the strength of the magnetic field. A comparison is also presented between the results obtained from the Maxwell and modified Maxwell models. The results show that the heat transfer is generally higher based on the modified Maxwell model.

Keywords: Lid-driven cavity; Magnetic field; Mixed convection; Nanofluid

1. Introduction

Investigations of mixed convection of a fluid confined in a cavity with moving wall have applications in problems such as manufacturing solar collectors, optimized thermal designing of buildings, and cooling of electronic devices, and have attracted many researchers [1, 2]. Wan and Kuznetsov [3] investigated numerically fluid flow in a rectangular vibrating lid-driven cavity with three different aspect ratios. They found that the sizes of the secondary eddies vary for different aspect ratios of the cavity. A major limitation against enhancing the heat transfer in such engineering systems is the inherently low thermal conductivity of the commonly used fluids, such as, air, water, and oil. In order to enhance the thermal conductivity of conventional heat transfer fluids, it has been tried to develop a new type of modern heat transfer fluid by suspending ultrafine solid particles in base fluids. In 1993, Masuda et al. [4] studied the heat transfer performance of liquids with solid nanoparticles suspension. However, the term of ”nanofluid” was first named by Choi [5] in 1995, and successively gained popularity. Because of the extensively greater thermal conductivity and heat transfer performance of the nanofluids as compared to the base fluids, they are expected to be ideally suited for practical applications.

Due to the lack of a sophisticated theory for estimating the thermal conductivity of nanofluids many models developed that mostly focused on several parameters, such as: temperature or Brownian motion, geometry of nanoparticles and interaction between nanoparticles and base fluid etc. The first model was given by Maxwell [6]; he proposed a model to predict the thermal conductivity of mixtures that contain solid particles. Maxwell's model show that the effective thermal conductivity of suspensions that contain spherical particles increases with the volume fraction of the solid particles.

Heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids is investigated for various pertinent parameters by Khanafer et al. [7] using finite-volume approach along with the alternating direction implicit method. They analyzed the effect of suspended ultrafine metallic nanoparticles on the fluid flow and heat transfer processes within the enclosure. Tiwari and Das [8] used the finite volume method to investigate the flow and heat transfer in a square cavity with insulated top and bottom walls, and differentially-heated, moving sidewalls. The cavity was filled with the Copper water nanofluid. Conducting a parametric study, they investigated the effects of the Richardson number and the volume fraction of the nanoparticles on the heat transfer, and observed that, for the Richardson number equal to unity, the average Nusselt number increased substantially with increasing the volume

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fraction of the nanoparticles. Ghasemi et al. [9] numerically studied the natural convection in an enclosure that is filled with a water-Al_2O_3 nanofluid and is influenced by a magnetic field. They showed that the increase of the solid volume fraction may result in enhancement or deterioration of the heat transfer performance depending on the value of Hartmann and Rayleigh numbers.

Most of the researchers argue that the addition of nanoparticles with relatively higher thermal conductivity to the base fluid results in an increase of the thermal performance of the resultant nanofluid [10-13]. Some researchers, on the other hand, argue that the dispersion of nanoparticles in the base fluid may result in a substantial decrease of the heat transfer [14]. The influence of nanofluids on heat transfer performance is still a controversial issue. Most of the studies on the effective thermal conductivity of the nanofluid, for spherical nanoparticles is evaluated from the Maxwell model. This proposed model is valid only for low dense mixtures and for micro sized particles. Thus several researchers (for example, Yu and Choi [15], Kumar et al. [16], Prasher et al. [17], etc.) tried to improve the Maxwell's model. Some of the models include the contribution of Brownian motion of the nanoparticles to determine the nanofluid properties [18, 19]. Nield and Kuznetsov [20] investigated analytically double-diffusive convection in a porous medium saturated by a nanofluid. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. They considered the base fluid of the nanofluid is itself a binary fluid such as salty water. Kuznetsov and Nield [21] have examined the influence of nanoparticles on the natural convection boundary layer flow past a vertical plate by using a model in which Brownian motion and thermophoresis are accounted for. However, there are still a number of studies which rely on either Maxwell [5] or modified Maxwell model [15].

The aforementioned literature survey reveals that although many numerical efforts have been made to investigate convection heat transfer of nanofluids in a square cavity using Maxwell model, there is a lack of exploration on the modified Maxwell model. Therefore, this paper deals with a numerical study of mixed convection in a lid-driven cavity filled with different nanofluids in the presence of magnetic field using modified Maxwell model. The present investigation can be used to cool automobile engines and welding equipment and to cool high heat-flux devices such as high power microwave tubes and high-power laser diode arrays. A nanofluid coolant could flow through tiny passages in micro-electro-mechanical systems (MEMS) to improve its efficiency. The measurement of nanofluids critical heat flux (CHF) in a forced convection loop is useful for nuclear applications.

2. Mathematical analysis

Consider a steady-state two-dimensional square cavity filled with nanofluid of height H as shown in Fig. 1. It is assumed that the top wall is moving from left to right at a constant

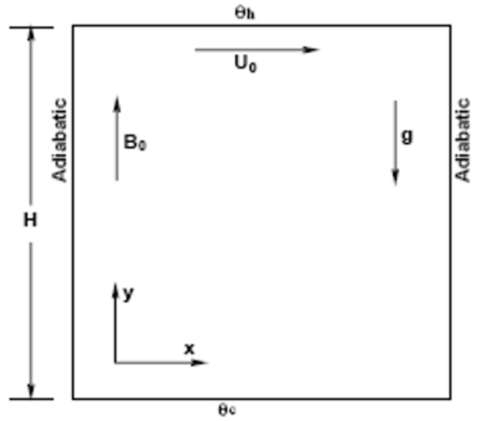


Fig. 1. Flow configuration and coordinate system.

speed U_0 and is maintained at a constant temperature θh. The bottom wall is maintained at a constant temperature θc ($\theta h > \theta c$). The vertical sidewalls are considered to be adiabatic. A uniform magnetic field is applied in the vertical direction normal to the moving wall. The nanofluid in the enclosure is Newtonian, incompressible, and laminar. The nanoparticles are assumed to have uniform shape and size. Also, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state and they flow at the same velocity. The physical properties of the nanofluid, considered in this study, given in Table 1, are assumed constant expect the density variation in the body force term of the momentum equation which is satisfied by the Boussinesq's approximation. Under the above assumptions the system of equations can be written in the following non-dimensional form.

$$\frac{\partial V}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\frac{\mu_{eff}}{\nu_f\rho_{nf}}\nabla^2 U \tag{2}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\frac{\mu_{eff}}{\nu_f\rho_{nf}}\nabla^2 V+\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}RiT-\frac{\alpha_{nf}}{\alpha_f}\frac{Ha^2}{\mathrm{Re}}V \tag{3}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\frac{1}{\mathrm{Pr}.\mathrm{Re}}\nabla^2 T. \tag{4}$$

In the above equations the following non-dimensional parameters are used:

$$X=\frac{x}{H},Y=\frac{y}{H},U=\frac{u}{U_0},V=\frac{v}{U_0},T=\frac{\theta-\theta_c}{\theta_h-\theta_c},Gr=\frac{g\beta\Delta H^3}{{\nu_f}^2}$$

$$Ha^2=\frac{\sigma {B^2}_0 H^2}{\mu},\ P=\frac{p}{\rho {U^2}_0},\mathrm{Re}=\frac{U_0 H}{\nu_f},\mathrm{Pr}=\frac{\nu_f}{\alpha_f}$$

Table 1. Thermophysical properties of water and nanoparticles.

	ρ (kgm^{-3})	Cp (Jkg^{-1}K^{-1})	K (Wm^{-1}K^{-1})	$\beta \times 10^{-5}$ (K^{-1})
H_2O	997.1	4179	0.613	21
Ag	10500	235	429	1.89
Cu	8933	385	401	1.67
CuO	6320	531.8	76.5	1.8
Al_2O_3	3970	765	40	0.85
TiO_2	4250	686.2	8.9538	0.9

Table 2. Applied formulae for the nanofluid properties.

Nanofluid properties	Applied model
Density	$\rho_{nf} = (1-\chi)\rho_{nf} + \chi\rho_s$
Thermal diffusivity	$\alpha_{nf} = K_{eff} / \left(\rho C_p\right)_{nf}$
Heat capacitance	$\left(\rho C_p\right)_{nf} = (1-\chi)(\rho C_p)_f + \chi(\rho C_p)_s$
Dynamic viscosity	$\mu_{nf} = \mu_f / \left(1-\chi\right)^{2.5}$
Thermal expansion coefficient	$(\rho\beta)_{nf} = (1-\chi)\rho_f\beta_f + \chi\rho_s\beta_s$

The non-dimensional boundary conditions, used to solve Eqs. (1)-(4) are as follows.

$$U = 1, V = 0, T = 0\ (Y = 1)$$
$$U = 0, V = 0, T = 0\ (Y = 0)$$
$$U = V = 0, \frac{\partial T}{\partial X} = 0\ (X = 0, 1).$$

The properties of the nanofluids are given in Table 2.

The local and average heat transfer rates of the cavity can be presented by means of the local and average Nusselt numbers. The local Nusselt number is calculated along the top heated wall and the average Nusselt number (Nu_{avg}) is determined by integrating the local Nusselt number.

$$Nu = -\frac{k_{eff}}{k_f}\frac{\partial T}{\partial Y}$$
$$Nu_{avg} = \int_0^1 Nu\, dX. \tag{5}$$

3. Modified Maxwell model

According to the Maxwell [5] model the effective thermal conductivity of the nanofluid, keff , for spherical nanopartilces is

$$\frac{k_{eff}}{k_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)}. \tag{6}$$

This classical model has been cited by many researchers. However, it does not take into account the Brownian motion of the nanoparticles and the effect of solid like nanolayers formed around nanoparticles. The modified Maxwell [15] model takes into account a nano-layer with a solid like structure formed by the liquid molecules close to a solid surface. According to this model the effective thermal conductivity of the nanofluid, k_{eff}, for spherical nanopartilces is

$$\frac{k_{eff}}{k_f} = \frac{(k_{eq} + 2k_f) - 2(k_{eq} - k_f)(1+\sigma)^3\chi}{(k_{eq} + 2k_f) - (k_{eq} - k_f) + (1+\sigma)^3\chi}. \tag{7}$$

In this equation, σ is the ratio of the thickness of nanolayer to the original radius of nanoparticles (h_{nl}/r_s) and k_{eq} is the equivalent thermal conductivity of nanoparticles and their layers:

$$\frac{k_{eq}}{k_s} = \eta\frac{2(1-\eta) + (1+\sigma)^3(1+2\eta)}{-(1-\eta) + (1+\sigma)^3(1+2\eta)}. \tag{8}$$

In this equation, η is the ratio of thermal conductivity of nano-layer upon the thermal conductivity of the nanoparticles $(\eta = k_{nl}/k_s)$. In this study, it is assumed that $h_{nl} = 2$ nm, $r_s = 3$ nm and $k_{nl} = 100\ kf$. Yu and Choi [15] argued that for these conditions, the result of the modified Maxwell model is in good agreement with experimental results.

4. Method of solution

The governing equations along with the boundary conditions are solved numerically employing finite volume method using staggered grid arrangement. The semi-implicit method for pressure linked equation (SIMPLE) is used to couple momentum and continuity equations as given by Patankar [22]. The third order accurate deferred QUICK scheme of Hayase et al. [23] is employed to minimize the numerical diffusion for the convective terms for both the momentum equations and energy equation. The solution of the discretized momentum and pressure correction equations is obtained by TDMA line-by-line method. The pseudo-transient approach is followed for the numerical solution as it is useful for situation in which the governing equations give rise to stability problems, e.g., buoyant flows. Euclidean norm of the residual is taken as convergence criteria for each dependent variable in the entire row field. At each step, the solution is iterated until the normalized residuals of the mass, momentum and temperature equation become smaller than 10^{-7}. The grid independence test is performed using successively sized uniform grids, 21 × 21 to 101 × 101. After grid independence check considering the accuracy and the computational time, all the computations are performed with a 61 × 61 grid.

In order to check on the accuracy of the computational code employed for the solution of the problem considered in the present study, it is validated by performing simulation for hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation or absorption

Table 3. Comparison of the average Nusselt number between the present solution and Chamkha [24].

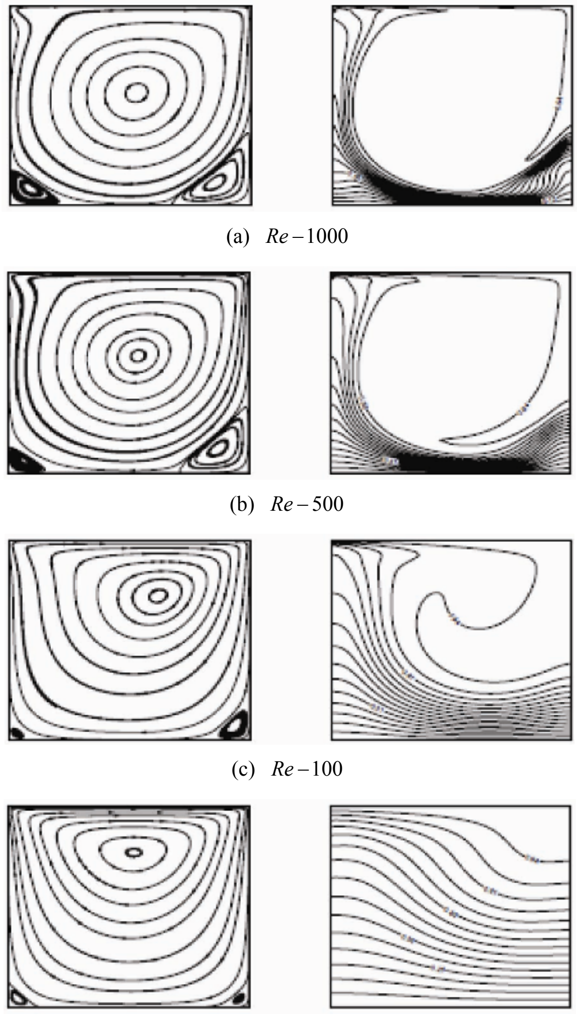
Ha	Present Nuavg	Chamkha [24]
0	2.2511	2.2692
10	2.1149	2.1050
20	1.7523	1.6472
50	1.0692	0.9164

which is reported earlier by Chamkha [24]. Table 3 clearly shows good agreement of the average Nusselt number between the two numerical results.

5. Results and discussion

In this paper the mixed convection flow inside the cavity is numerically investigated using modified Maxwell model. In the present analysis, the Prandtl number and the Grashof number is assumed to be Pr = 6.2 and Gr = 100, respectively. The range of Reynolds number, Hartmann number and solid volume fraction of nanoparticles are $10 \leq Re \leq 1000$, $0 \leq Ha \leq 50$ and $0 \leq \chi \leq 0.06$.

Fig. 2 show typical contour maps for the streamlines and isotherms obtained numerically for different values of Reynolds number Re (Re = 1000, 500, 100, 10) at a fixed χ = 0.06(Cu). As it is clear from the definition of Ri ($Ri = Gr/Re^2$), the Richardson number provides a measure of the importance of buoyancy-driven natural convection relative to the lid-driven forced convection. For high values of Re (Re = 1000, 500, forced convection dominated regime), Figs. 2(a) and (b), indicate that the buoyancy effect is overwhelmed by the mechanical or shear effect due to the movement of the top lid and the flow features are similar to those of a viscous flow of a non-stratified fluid in a lid-driven cavity. The streamline behavior in a two-dimensional lid-driven cavity is characterized by a primary recirculating cell occupying most of the cavity generated by the lid and two secondary eddies near the bottom wall corners with one near the right bottom corner is bigger and stronger than the one in the left bottom corner of the cavity. The isotherms are clustered heavily near the bottom surface of the cavity which indicates steep temperature gradients in the vertical direction in this region. In the remaining area of the cavity the temperature gradients are very weak and this implies that the temperature differences are very small in the interior region of the cavity due to the vigorous effects of the lid-driven circulations. Fig. 2(c) show that the moderate value of Re = 100 the buoyancy effect is of relatively comparable magnitude of the shear effect due to the sliding lid. Fig. 2(d) show that the small value of Re = 10 the isotherm plot indicate that conduction mode heat transfer dominants. The corresponding streamlines show the flow circulation is very weak and small eddies are visible in the bottom corners of the cavity. When the value of Re decreased from 1000 to 10 the bigger and stronger corner eddies becomes small and weaker one due to the effect of Richardson number.



(a) $Re-1000$

(b) $Re-500$

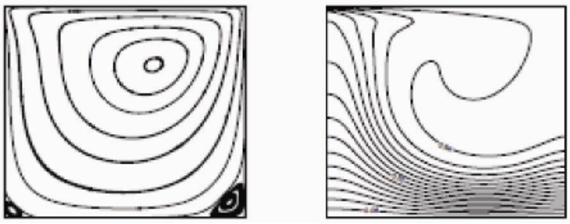
(c) $Re-100$

(d) $Re-10$

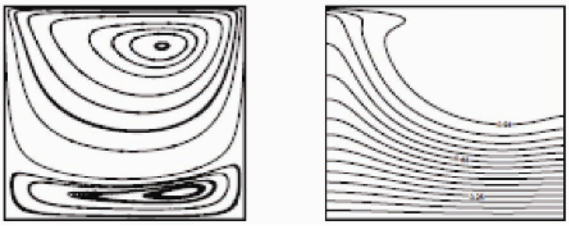
Fig. 2. Streamlines (on the left) and isotherms (on the right) for Ha = 0 and χ = 0.06.

Figs. 3(a)-(d) present the streamlines (on the right) and isotherms (on the left) for Re = 100 and χ = 0.06 (Cu). When Ha = 0 streamlines show the main cell occupied entire cavity and small eddies are visible near the bottom corners. When Ha is increased from 0 to 10 the corner eddies become a small bottom recirculating cell. When Ha is increased to 50 three recirculating cells formed one below one due to the increasing effect of magnetic field. For the highest value of Hartmann number, flow structure changes completely. The corresponding isotherm (Figs. 3(a)-(d)) plots indicate that the thermal boundary layer increases due to the increasing of effectiveness of magnetic field. The temperature gradient is distributed starting from the right corner into the cavity as illustrated in Fig. 3(a). Mostly temperature gradient is parallel to the horizontal wall it indicate that conduction heat transfer dominates entire cavity is shown in Figs. 3(b)-(d).

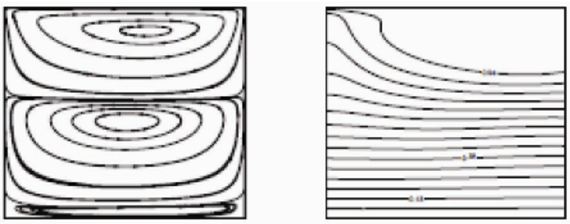
Fig. 4. presents typical x-component of velocity U, y-component of velocity V at the mid-plane of the cavity for Ha



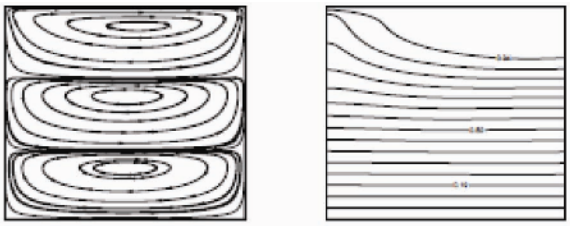
(a) $Ha-0$



(b) $Ha-10$



(c) $Ha-30$

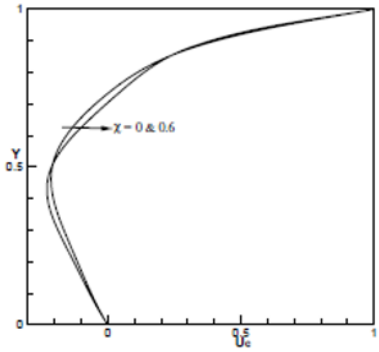


(d) $Ha-50$

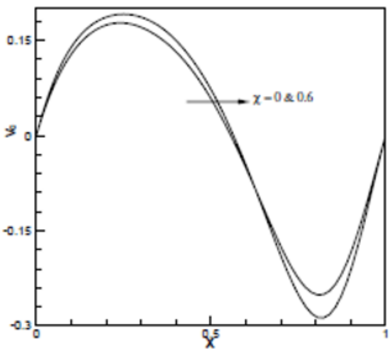
Fig. 3. Streamlines (on the left) and isotherms (on the right) for Re = 100 and = χ 0.06.

= 0 and Re = 100. It is observed that the nanofluid does not affect much in the flow patten inside the cavity. Fig. 5 show the variation of the dimensionless horizontal vertical component of the different nanofluid along the center line of the cavity for fixed χ = 0.06. It is observed that Al_2O_3 nanofluid leads to enhance the flow intensity. So that the maximum value of the velocity is obtained at the Al_2O_3 nanofluid. Five nanoparticles are compared at Ha = 0 and Re = 100 in Fig. 6. The average heat transfer depends strongly on the density of the nanoparticles. So water-Al_2O_3 nanofluid enhances the heat transfer compared with water-TiO_2, water-CuO, water-Cu and water-Ag.

As mentioned in the introduction, the models used to estimate the properties of the nanofluid significantly affect the heat transfer performance of the nanofluid. The analysis thus far is based on the modified Maxwell model. However, in this section, both Maxwell and modified Maxwell models are used.



(a)



(b)

Fig. 4. Velocity profiles at mid-plane of the cavity for Ha = 0 and Re = 100.

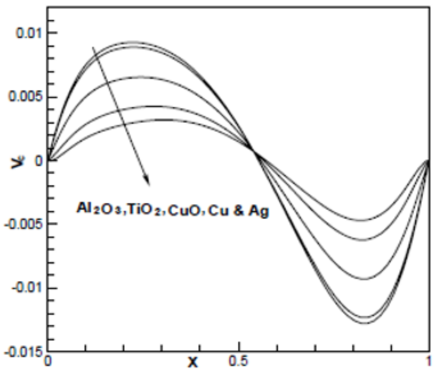


Fig. 5. Velocity profiles at mid-plane of the cavity for χ = 0.06, Ha = 0 and Re = 100.

Fig. 8 presents a comparison between the two models for the variation of the ratio of the nanofluid to the pure fluid average Nusselt number with respect to χ. Both models show an increase in Nu_{avg} at highest Re and χ. However the rate of this increase is more noticeable in the results obtained from modified Maxwell model.

6. Conclusions

This paper presents a numerical study of mixed convective

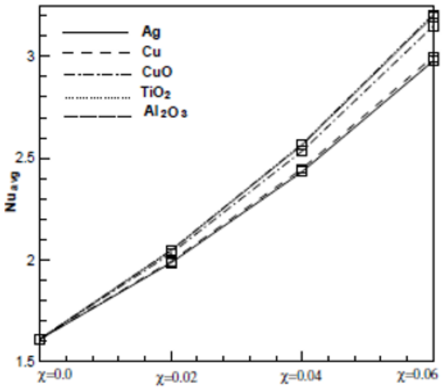


Fig. 6. Average Nusselt number for different nanofluids.

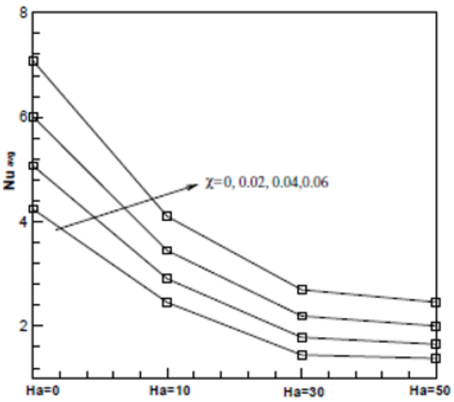


Fig. 7. Average Nusselt number for different *Ha* and χ .

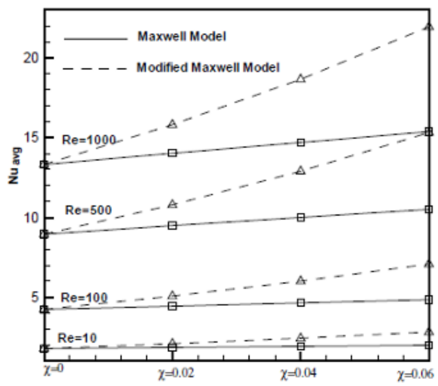


Fig. 8. A comparison between Maxwell and modified Maxwell model with different *Re* and χ.

flow and heat transfer of a five different nanofluid in a lid-driven cavity in the presence of magnetic field. Graphical results for various parametric conditions were presented and discussed. It is found that the flow and heat transfer inside the cavity are strongly dependent on the Reynolds and Hartmann numbers. The suspended nanoparticles remarkably enhance heat transfer process and the nanofluid has larger heat transfer coefficient than that of the original base liquid under the same Reynolds number. This is due to a substantial increase in effective dynamic viscosity compared to that of the base fluid and consequently a stronger convection is induced. Results indicated that the average Nusselt number increases linearly with the increase in the solid volume fraction at given Reynolds number. A comparison study between the results obtained from the Maxwell and modified Maxwell models indicate that the heat transfer rates obtained based on the modified Maxwell model are generally higher than those obtained based on the classical Maxwell model.

Acknowledgment

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Nomenclature

B_0	: Magnetic field strength
g	: Gravitational acceleration, m/s^2
Gr	: Grashof number
H	: Enclosure length, m
Ha	: Hartmann number
h_{nl}	: Tthickness of nano-layer, nm
k_f	: Thermal conductivity of the fluid, W/m K
k_s	: Thermal conductivity of the solid, W/m K
Nu_{avg}	: Average Nusselt number
P	: Pressure
Pr	: Prandtl number
r_s	: Radius of nanoparticles, nm
Re	: Reynolds number
Ri	: Richardson number, Gr/Re^2
T	: Dimensionless temperature
U,V	: Dimensionless velocities in X- and Y-direction
U_0	: Lid velocity, m/s
U_c, V_c	: Dimensionless velocity in X and Y -direction at mid-plane of the cavity
u,v	: Velocities in x- and y-direction respectively
X,Y	: Dimensionless Cartesian coordinates
x,y	: Cartesian coordinates

Greek symbols

α	: Effective thermal diffusivity, m^2/s
β_f	: Coefficient of thermal expansion of fluid, K^{-1}
β_s	: Coefficient of thermal expansion of solid, K^{-1}
$\Delta\theta$	: Temperature difference
θ	: Temperature ^{o}C
μ	: Effective dynamic viscosity kg/ms
υ	: Effective kinematic viscosity, m^2/s
ρ	: Fluid density kg/m^3
χ	: Solid volume fraction

Subscripts

avg	: Average
c	: Cold wall

eff : Effective
eq : Equivalent
f : Fluid
h : Hot wall
nf : Nanofluid
nl : Nano-layer
s : Solid

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