Investigation of balancing problem for a planar mechanism using genetic algorithm[†]

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Abstract

In this study, optimal balancing of a planar articulated mechanism is investigated to minimize the shaking force and moment fluctuations. Balancing of a four-bar mechanism is formulated as an optimization problem. On the other hand, an objective function based on the sub-components of shaking force and moment is constituted, and design variables consisting of kinematic and dynamic parameters are defined. Genetic algorithm is used to solve the optimization problem under the appropriate constraints. By using commercial simulation software, optimized values of design variables are also tested to evaluate the effectiveness of the proposed optimization process. This work provides a practical method for reducing the shaking force and moment fluctuations. The results show that both the structure of objective function and particularly the selection of weighting factors have a crucial role to obtain the optimum values of design parameters. By adjusting the value of weighting factor according to the relative sensitivity of the related term, there is a certain decrease at the shaking force and moment fluctuations. Moreover, these arrangements also decrease the initiative of mechanism designer on choosing the values of weighting factors.

Keywords: Shaking force and moment; Optimal balancing; Four-bar mechanism; Genetic algorithm

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1. Introduction

Balancing of the mechanism is an old problem both reducing the amplitude of vibration on the frame and obtaining nearly constant drive speed. Since any vibration leads to noise, wear, fatigue, etc., in the mechanism, its reduction improves several aspects of mechanical design as well. Over the last two decades, the balancing problem has been faced with new challenges, namely, the balancing of shaking force, shaking moment, and input-torque fluctuations together. Once the balancing problem is formulated as a mathematical optimization problem, it is possible to solve it by using any existing optimization tool (global and local methods such as genetic algorithms, constrained or unconstrained minimizations etc.). Since the dynamic performance characteristics such as shaking force, shaking moment, input-torque, etc., depend on the mass and inertia of each moving link, and its mass center location, it is required to optimally distribute the link masses for dynamic balancing. Minimization both shaking force and shaking moment fluctuations is important for improving the mechanism's fatigue life by reducing vibration, noise and wear.

Many machine designers have paid an attention to solve the balancing problems by using either classical methods or optimal approaches. Assuming that both linear and rotary inertia,

Feng [1] presented a method for the complete shaking force and moment balancing of eight-bar linkages having only revolute joints. In study of Ye and Smith [2], a logical extension to the concept of mass flow was developed in which the effects of inertia moment as well as inertia force of a link were modeled by equivalent simple links. Li [3] presented sensitivity formulation of the shaking force and moment for planar articulating mechanisms. The sensitivity analysis and a robust balancing method, which was sensitive to the processing errors in manufacture, were presented. Objective function was composed of shaking force and shaking moment, and the values of weighting factor were selected as equal to each other. For reducing the shaking force and moment of mechanical presses, Chiou et al. [4] proposed optimum designs by adding disk counterweights. Two-phase optimization technique was presented for the multi-objective optimization. Arakelian and Smith [5] proposed a new solution, considering a pantograph with the crank and coupler, to the problem of complete shaking force and shaking moment balancing of linkages. By using counterweights, complete force balancing of planar linkages was presented by Tepper and Lowen [6]. Esat and Bahai [7] also showed if a linkage can be fully force balanced using the criterion of Tepper and Lowen, then it can be fully force and moment balanced using geared counter-inertias. Feng et al. [8] analyzed the joint forces of planar linkage with joint clearance and presented a new optimization method, which was based on optimizing mass distribution of links to decrease the

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change of joint forces. A critical review of complete shaking moment balancing was implemented in the study of Kochev [9].

Guo et al. [10] proposed a new mixed mass redistribution method to investigate the optimum dynamic design. By using genetic algorithms, optimum dynamic characteristics were obtained more efficiently than the traditional nonlinear optimization techniques. Arakelian et al. [11, 12] presented a solution of the shaking force and shaking moment balancing of planar and spatial linkages. Arakelian [13] also formulated the conditions of shaking moment balancing by using the copying properties of the pantograph linkage and the method of dynamic substitution of distributed masses by concentrated point masses. Alici and Shirinzadeh [14] presented optimum dynamic balancing of planar 2-DOF parallel manipulators. By using an objective function based on the sensitivity analysis of shaking moment with respect to the position, velocity and acceleration of the links, the dynamic balancing was formulated as an optimization problem. Chaudhary and Saha [15] proposed a method based on the maximum recursiveness of the dynamic equations to evaluate the bearing forces. Balancing problem of four-bar linkages was considered as an optimization problem, and mass distribution of linkage was embedded in the constraints to obtain the new linkage. Also, they [16] presented a general mathematical formulation of optimization problem for balancing of planar mechanisms to improve the dynamic performances. Erkaya and Uzmay [17] investigated dynamic behavior of a four-bar mechanism with joint clearances. They used an objective function based on shaking force and shaking moment. Also, they proposed a neural-genetic (NN-GA) approach to minimize the additional effects of joint clearances on shaking force and moment under related constraints. By using a novel and simplified approach, Ilia and Sinatra [18] studied the derivation of design equations and techniques for the dynamic balancing of a five-bar linkage. Balancing of the mechanism was formulated and solved as an optimization problem under equality constraints. Park et al. [19] studied for minimizing the moments excited in a four-stroke seven-cylinder vehicle engine and reducing the forces transmitted to the engine mounts. A computer program was developed to predict the excitation forces and moments.

Former balancing studies, which particularly consider this problem as an optimization problem, have usually chosen the values of weighting factors equal to each other. This arrangement obviously affects the results of optimization. Furthermore, the initiative of the mechanism designer has a crucial role on choosing the values of weighting factors and defining the structure of objective function. The focus of this study is to present a simple approach to constitute the structure of objective function for decreasing the shaking force and shaking moment fluctuations. On the other hand, a simple method is also proposed to reduce the initiative of the mechanism designer on choosing the values of weighting factors. An objective function based on the sub-components of shaking force and shaking moment is constituted. Genetic algorithm is used
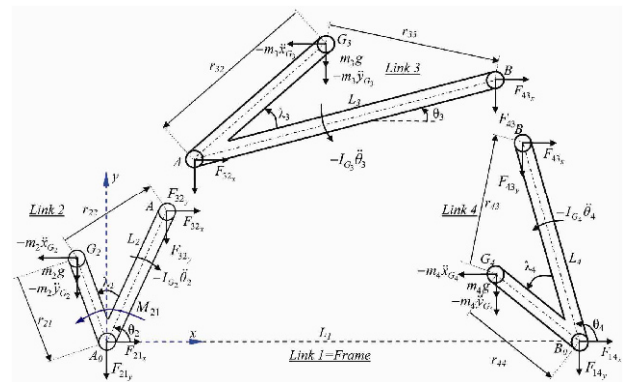

Fig. 1. Force representation of four-bar mechanism.

for solving the optimization problem. Three case studies are implemented to show the effectiveness of the proposed approach. This paper is organized as follows; section 2 outlines the kinematics and dynamics of model mechanism. Optimization process is given in section 3. Results and conclusions are summarized in sections 4 and 5, respectively.

2. Analysis of the model mechanism

A four-bar mechanism, which is frequently used in the former balancing problems, is considered as an example to investigate the effects of shaking force and shaking moment exerted in the frame (Fig. 1).

Kinematic analysis of the model mechanism comprises determining of displacements, velocities and accelerations of moving links. Mass center positions of the moving links relative to the crank pivot (A_o) are given in the following form,

$$\begin{bmatrix} x_{G2} \\ y_{G2} \end{bmatrix} = \overline{A_o G_2} \begin{bmatrix} \cos(\theta_2 + \lambda_2) \\ \sin(\theta_2 + \lambda_2) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} x_{G3} \\ y_{G3} \end{bmatrix} = L_2 \begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix} + \overline{AG_3} \begin{bmatrix} \cos(\theta_3 + \lambda_3) \\ \sin(\theta_3 + \lambda_3) \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} x_{G4} \\ y_{G4} \end{bmatrix} = L_1 \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} + \overline{B_o G_4} \begin{bmatrix} \cos(\theta_4 + \lambda_4) \\ \sin(\theta_4 + \lambda_4) \end{bmatrix} \tag{3}$$

where L_i denote the lengths of corresponding links. x_{Gi} and y_{Gi} are the displacements at the x and y directions for mass center of ith moving link, respectively. θ_3 and θ_4 define the angular positions of coupler and follower links relative to x direction, respectively (see appendix). Mass center velocities and accelerations can also be defined as the time-derivatives of Eqs. (1)-(3).

Dynamic analysis of the model mechanism provides to define the joint forces and torque as a function of input link's position. Dynamic force analysis was carried out considering the inertial effects of the links for determining the joint forces and torque. Force analysis for the model mechanism is given in Eq. (4).

$$
\begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
r_{21_y} & -r_{21_x} & -r_{22_y} & r_{22_x} & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & r_{32_y} & -r_{32_x} & -r_{33_y} & r_{33_x} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & r_{43_y} & -r_{43_x} & -r_{44_y} & r_{44_x} & 0
\end{bmatrix}
\begin{bmatrix}
F_{21_x} \\ F_{21_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_x} \\ F_{14_y} \\ M_{21}
\end{bmatrix}
=
\begin{bmatrix}
m_2 \ddot{x}_{G2} \\
m_2 \ddot{y}_{G2} + m_2 g \\
I_{G2} \ddot{\theta}_2 \\
m_3 \ddot{x}_{G3} \\
m_3 \ddot{y}_{G3} + m_3 g \\
I_{G3} \ddot{\theta}_3 \\
m_4 \ddot{x}_{G4} \\
m_4 \ddot{y}_{G4} + m_4 g \\
I_{G4} \ddot{\theta}_4
\end{bmatrix}.
$$

$$(4)$$

Position vectors from the gravity center of link *i* to joint *j* are read from the Fig. 1 as:

$$
\begin{bmatrix}
r_{21_x} & r_{21_y} \\
r_{22_x} & r_{22_y} \\
r_{32_x} & r_{32_y} \\
r_{33_x} & r_{33_y} \\
r_{43_x} & r_{43_y} \\
r_{44_x} & r_{44_y}
\end{bmatrix}
=
\begin{bmatrix}
\overline{A_o G_2} & 0 & 0 & 0 & 0 & 0 \\
0 & \overline{AG_2} & 0 & 0 & 0 & 0 \\
0 & 0 & \overline{AG_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \overline{BG_3} & 0 & 0 \\
0 & 0 & 0 & 0 & \overline{BG_4} & 0 \\
0 & 0 & 0 & 0 & 0 & \overline{B_o G_4}
\end{bmatrix}
\begin{bmatrix}
-\cos(\theta_2 + \lambda_2) & -\sin(\theta_2 + \lambda_2) \\
\cos(\theta_2 - \beta_2) & \sin(\theta_2 - \beta_2) \\
-\cos(\theta_3 + \lambda_3) & -\sin(\theta_3 + \lambda_3) \\
\cos(\theta_3 - \beta_3) & \sin(\theta_3 - \beta_3) \\
\cos(\theta_4 - \beta_4) & \sin(\theta_4 - \beta_4) \\
-\cos(\theta_4 + \lambda_4) & -\sin(\theta_4 + \lambda_4)
\end{bmatrix}
$$

$$(5)$$

where β_2, β_3 and β_4 define the acute angles for G_2AA_0, G_3BA and G_4BB_0, respectively (see appendix). According to theorem, the shaking force is considered as the reaction of the vector sum of all the inertia forces of moving links associated with the mechanism, and the shaking moment is also the reaction of the resultant of the inertia moment and the moment of the inertia forces. The design algorithm used in this study aims at minimizing the shaking force and shaking moment. Therefore, this force components and the relevant moment relative to the crank pivot can be defined as:

$$\sum \mathbf{F}_{sh_x} = \mathbf{F}_{41_x} + \mathbf{F}_{21_x} \tag{6}$$

$$\sum \mathbf{F}_{sh_y} = \mathbf{F}_{41_y} + \mathbf{F}_{21_y} \tag{7}$$

$$\mathbf{M}_{sh} = L_1 \sin\theta_1 \mathbf{j} \otimes \mathbf{F}_{41_x} \mathbf{i} + L_1 \cos\theta_1 \mathbf{i} \otimes \mathbf{F}_{41_y} \mathbf{j} \tag{8}$$

where $\mathbf{F}_{41}$ and $\mathbf{F}_{21}$ denote the forces at the joints of follower-frame and crank-frame, respectively.

3. Optimization process

In the optimization process, genetic algorithm (GA) approach was used to solve the optimization problem and it was performed on optimization toolbox of MATLAB [20]. Genetic algorithm, or any evolutionary method, differs from classical optimization methods in that there is a non-zero probability of attaining the global optimum [21]. Many gradient-based methods, which are very efficient local optimization methods for parameter optimization, are available. However, these conventional algorithms need the gradient information of the objective function with respect to the design variables and cannot get out of local optimum points when they fall into a false peak (local optimum point). Also, they may miss a
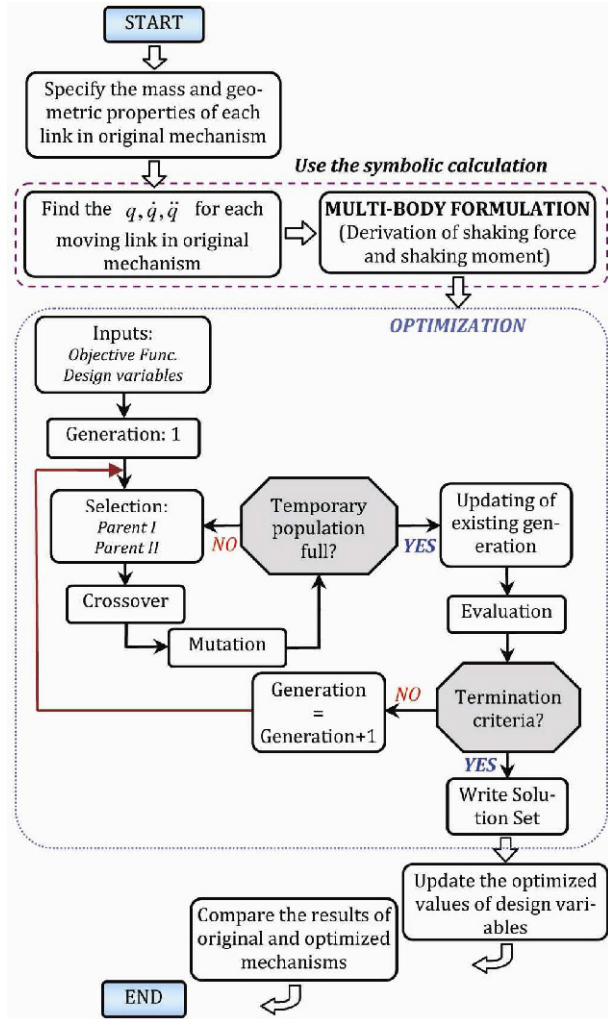

Fig. 2. Flow chart of design study and genetic algorithm.

global optimum solution because they are dependent on the starting point of searching and converge on the optimum solution that is nearest to the starting point, and cannot find all the global optimum solutions [22]. Genetic algorithms, on the other hand, are simple to implement and involve evaluations of only the objective function and the use of certain genetic operators such as selection, crossover, mutation and reproduction to explore the design space [23]. Moreover, a population of optimum points is obtained that will allow the designer to select a design that satisfies all subjective constraints as well. These characteristics make this approach well suited for finding the optimal solutions. GA operations in a typical optimization procedure are outlined in Fig. 2. In this study, stochastic uniform was applied as selection function for choosing the next generation, and the crossover probability was adjusted as 0,8. The solving of optimization problem using genetic algorithm was performed on a PIV processor with a CPU speed of 3.2 MHz and 1024 Mb Ram.

In order to balance a mechanism completely, it is necessary to eliminate both the shaking force and the shaking moment.

However, complete balancing of any one may result in an increased unbalance in the other one. The shaking force can be eliminated completely by attaching counterweights to the moving links of mechanism. But, this increases overall mass and inertia of the mechanism. Also, this leads to increasing in shaking moment, required driving torque, and reactions at the joints. An alternative way to reduce the shaking force and shaking moment together with other dynamic quantities such as driving torque, bearing reactions, etc., is to optimize all the dynamic quantities. Hence, for improving the overall performance of mechanism, the balancing problem should be treated as an optimization problem [24].

In this study, objective function in the optimization process was constituted as given in Eq. (9) by considering the shaking force and shaking moment [3, 15-17]. This function comprises each sub-component of shaking force and shaking moment:

minimize

$$F(\mathbf{X}) = \sum_{n=1}^{s} \Big[W_1\big(F_{21_{x_n}}\big) + W_2\big(F_{21_{y_n}}\big) \ldots$$
$$\ldots + W_3\big(F_{41_{x_n}}\big) + W_4\big(F_{41_{y_n}}\big) + W_5\big(M_{sh_n}\big) \Big] \qquad (9)$$

subject to
$$g_k(\mathbf{X}) \leq 0$$
$$x_r^{min} \leq x_r \leq x_r^{max}$$
$$x_r \in \mathbf{X}$$

where W_h are weighting factors, s is the number of the considered points during the one cycle of crank link. g_k are the constraints arising from the condition satisfies the crank-rocker motion. The objective function minimizes the related shaking force and shaking moment provided that the generated solution satisfies a set of constraints. These constraints are necessary to have a functional mechanism, although they increase solution complexity. $\mathbf{X}$ is a vector comprising the sixteen independent design variables (x_r). These variables are given as:

$$\mathbf{X} = \begin{bmatrix} L_i & \lambda_i & m_i & I_{G_i} & r_{21} & r_{32} & r_{44} \end{bmatrix}^T \qquad (10)$$

where L_i denote the link lengths as L_1, L_2, L_3 and L_4. λ_i consist of structural angles of moving links as λ_2, λ_3 and λ_4. m_i and I_{Gi} are the masses and inertial moments of moving links, respectively, that is, m_2, m_3, m_4, I_{G2}, I_{G3} and I_{G4}. r_{21}, r_{32} and r_{44} are the position vectors of crank, coupler and follower links, respectively. x^{min} and x^{max} are lower and upper bounds of design variables. These bounds have to be arranged by considering the working space of the mechanism. For the verification of the proposed approach, lower bounds of link lengths were arranged as L_i-0.1*L_i. Similarly, upper bounds of link lengths were arranged as L_i+0.1*L_i. Also, lower and upper bounds for λ_i were considered 0-360 degrees, respectively. Lower and upper bounds for m_i, I_{Gi}, r_{21}, r_{32} and r_{44} were arranged by con-

sidering the link geometries. Depth, thickness and length of each moving link were used for these definitions. The weighting factor's value has an important effect on the optimum adjusting of design variables. Since the selecting criterion is not obvious, it is always difficult to make the decision on choosing the values of weighting factors [4, 25]. In general, initiative of mechanism designer has a crucial role upon the definition of these values. Each weighting factor must satisfy the condition:

$$0 \leq W_h \leq 1 \quad \text{and} \quad \sum_{h=1}^{5} W_h = 1$$

In this study, by using the total value of the shaking force at the main support, the relative importance of each sub-component inside the shaking force was calculated. These calculated values were considered as weighting factors. In the proposed optimization process, the values of the weighting factors were adjusted as 0.40, 0.24, 0.16, 0.1 and 0.1 for W_1, W_2, W_3, W_4 and W_5, respectively.

4. Results

In the present study, a theoretical model was used to investigate the effects of shaking force and moment. The operation speed of the mechanism was constant and it was adjusted as 300 rpm. Assuming that an objective function based on the sub-components of shaking force and shaking moment, genetic algorithm was used to solve the optimization problem. Design variables which consisted of kinematic and dynamic parameters of the mechanism were also defined. Three case studies were implemented. Proposed structure of objective function and values of weighting factors, which were defined in Section 3, were considered in the first case. By using the different values of weighting factors, second and third cases were also performed. Dimensions and inertial parameters of the original (unbalanced) and optimized (balanced) mechanisms for three case studies are given in Table 1.

By using the optimized values of each case study, dynamic analysis of the mechanism was performed to obtain the force and moment results. The convergence history for Case I is given in Fig. 3. The algorithm shows good convergence. After 111 generations, the best individual fitness stays as 5711.4096 and the average fitness occurs as 5711.6292.

Fig. 4 gives the crank-frame and follower-frame joint forces, which are also the sub-component of shaking force. After the optimization, there is a certain decrease at the force values. X and y components of the crank-frame joint force decrease by 95.52% and 77.18%, respectively. The decreasing ratios for the maximum values are observed as 97.51% and 95.97% for x and y components of the crank-frame joint force, respectively. In the case of minimum values, the decreasing ratios occur as 96.23% and 71.56% for x and y components of that force, respectively.

For the case of follower-frame joint, the force components

Table 1. Original and optimized parameters of four-bar mechanism (**Case I:** W_1=0.40, W_2=0.24, W_3=0.16, W_4=0.1, W_5=0.1, **Case II:** W_1=0.45, W_2=0.27, W_3=0.18, W_4=0.1, W_5=0, **Case III:** W_1=0.2, W_2=0.2, W_3=0.2, W_4=0.2, W_5=0.2).

Parameter	Description	Original value	Optimized values		
			Case I	Case II	Case III
L_1 (mm)	Length of fixed link	600	570	570	570.2
L_2 (mm)	Length of crank link	100	95	95	95
r_{21} (mm)	Position vector of crank link	50	66.7	66.7	72.54
m_2 (kg)	Mass of crank link	0.360	2.027	3.470	1.755
I_{G2} (kgm^2)	Inertial moment of crank link	4.13 10^{-4}	42.30 10^{-4}	98.2810^{-4}	48.93 10^{-4}
λ_2 (Radian)	Structural angle of crank link	0	3.0332	3.065	3.032
L_3 (mm)	Length of coupler link	400	420	420	420
r_{32} (mm)	Position vector of coupler link	200	77.5	88.73	87.98
m_3 (kg)	Mass of coupler link	1.296	1.264	2.06	1.23
I_{G3} (kgm^2)	Inertial moment of coupler link	1.87 10^{-2}	4.87 10^{-2}	9.96 10^{-2}	4.43 10^{-2}
λ_3 (Radian)	Structural angle of coupler link	0	0.1275	0.417	0.1619
L_4 (mm)	Length of follower link	320	329.8	313.9	330
r_{44} (mm)	Position vector of follower link	160	100.4	128	97
m_4 (kg)	Mass of follower link	1.046	0.866	1.425	1.22
I_{G4} (kgm^2)	Inertial moment of follower link	9.85 10^{-3}	14.30 10^{-3}	16 10^{-3}	15 10^{-3}
λ_4 (Radian)	Structural angle of follower link	0	0.0002	0.0013	0.0023
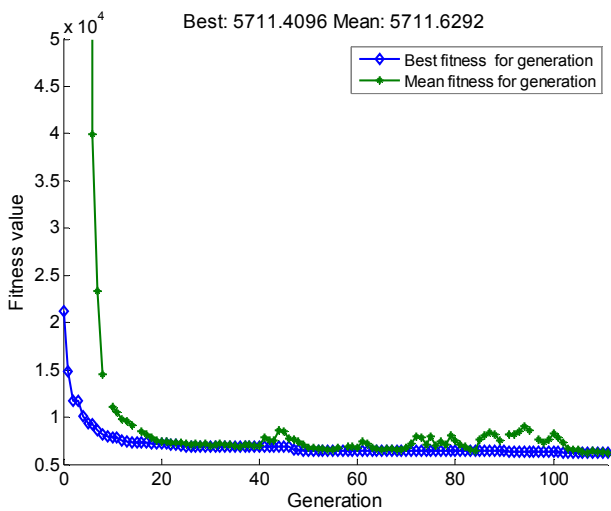
Fig. 3. Convergence history of GA evolution for Case I.

Fig. 4. Original and optimized values of joint forces for Case I; (a) and (b) Crank-frame joint, (c) and (d) Follower-frame joint.

for x and y directions decrease by 84.69% and 74.95%, respectively. Maximum values for x and y components of the follower-frame joint force are reduced as 93.45% and 89.81%, respectively. The decreasing ratios for the minimum values are also obtained as 79.07% and 80.96% for x and y components of that force, respectively.

As a natural result of the optimization, shaking force and shaking moment at the optimized mechanism are more close to zero than that of the original mechanism. As shown in Figs. 5(a) and (b), shaking force decreases by 90.96% and 77.54% for x and y directions, respectively. During the one period of the crank link, the maximum values of this force components
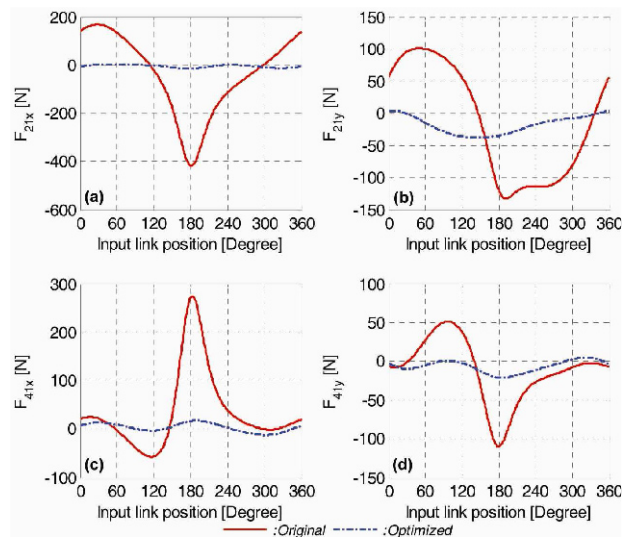
decrease 91.11% and 97.78% for x and y directions, respectively. In the case of minimum values, the decreasing ratios occur as 85.36% and 76.25% for x and y directions, respectively. As seen from Fig. 5(c), shaking moment decreases by 76.21% as well. This ratio is better than that of Ref. [3]. The decreasing ratios for the maximum and minimum values occur as 90.32% and 81.91%, respectively.

After the optimum adjusting of design variables, driving torque decreases by 73.46%. Decreasing ratios for the maximum and minimum values are read from Fig. 5(d) as 76.76%
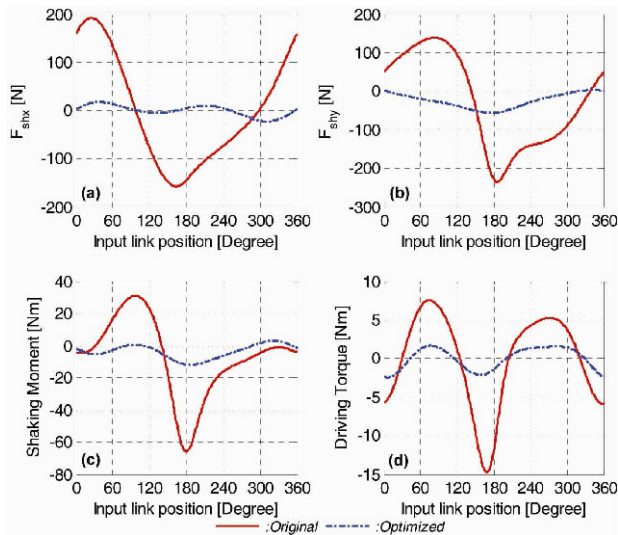
Fig. 5. Original and optimized values for Case I; (a) and (b) Shaking force components, (c) Shaking moment, (d) Driving torque.
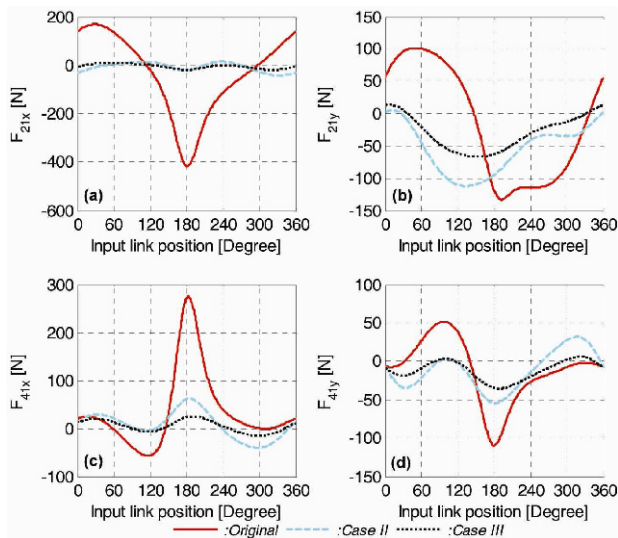

Fig. 6. Original and optimized values of joint forces for Case II and III; (a) and (b) Crank-frame joint, (c) and (d) Follower-frame joint.

Table 2. Decreasing ratios for three case studies.

	Decreasing ratio (%)		
	Case I	Case II	Case III
F_{21x}	95.52	88.04	93.50
F_{21y}	77.18	31.66	59.10
F_{41x}	84.69	51.48	78.28
F_{41y}	74.95	21.59	56.58
F_{shx}	90.96	69.35	86.30
F_{shy}	77.54	37.61	61.54
M_{sh}	76.21	25.51	58.73
M_{21}	73.46	57.65	70.49

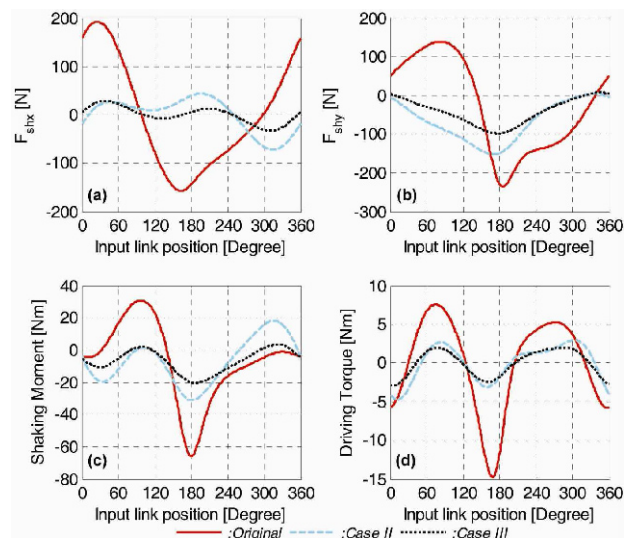
Fig. 7. Original and optimized values for Case II and III: (a) and (b) Shaking force components; (c) Shaking moment; (d) Driving torque.

and 82.48%, respectively. The commercial simulation software is also used to model the mechanism and to test the optimized values of design variables [26]. Simulation results for original and optimized values of forces and moments are given in appendix. Force and moment results of Case II and III are also given in Figs. 6 and 7.

By using the different values of weighting factors, decreasing ratios at the total values of forces and moments are outlined in Table 2 to evaluate the results of three case studies.

Case I shows the decreasing ratios for the proposed structure of the objective function and the values of weighting factors in section 3. Contrary to Case I, objective function of Case II is constituted by using only sub-components of shaking force, that is, shaking moment is eliminated due to the

values of the fifth weighting factor ($W_5=0$). The evaluations of Case II with respect to Case I show that the objective function should comprise both shaking force and shaking moment while their dimensions do not match. When the values of weighting factors are selected equals to each other as in Case III [3, 14], the decreasing ratios are smaller than that of Case I. So, the weighting factor's value has to be defined by considering the relative sensitivity of the related term.

In addition to three case studies, if the objective function only consists of F_{shx}, F_{shy} and M_{sh}, that is, not comprising their sub-components, the obtained decreasing ratios are worse than that of the proposed Case I. So, this proves that the proposed structure of the objective function is very effective for the optimum balancing.

5. Conclusions

The focus of this study is to minimize the shaking force and moment fluctuations at the planar mechanism. This phenomenon is considered as an optimization problem. In addition to

the similar studies in literature, the sub-components of shaking force and shaking moment are considered together to constitute the objective function. Also, relative importance of the force component inside the total shaking force is evaluated to define the value of related weighting factor. Therefore, it is possible to reduce the negative reflection on the optimization process arising from mechanism designer's initiative.

Three case studies indicate that both the structure of the objective function and the value of the weighting factor have a crucial role to minimize the shaking force and moment fluctuations. Objective function should comprise both shaking force and shaking moment while their dimensions do not match. Although the objective functions have the same structure, definition of the weighting factors' values is very important for the optimization process. By using the shaking force and moment in objective function, and evaluating the values of weighting factors according to their relative importance of the related forces, Case I gives the better results for solving the present optimization problem than that of the other cases. The obtained results show that the proposed structure of the objective function and the values of weighting factors are very effective to decrease the force and moment fluctuations, and power consumption for driving torque. Due to the flexibility of the proposed approach, mechanism designer can individually decrease each sub-component of force, and this approach can also be applied to other planar and spatial mechanisms.

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Appendix

Angular positions of coupler and follower links in Eqs. (2-3) are defined as:

$$\theta_3 = 2\tan^{-1}\left[-\frac{B}{2A} \pm \frac{1}{2A}\left(B^2 - 4AC\right)^{1/2}\right] \tag{A1}$$

$$\theta_4 = \cos^{-1}\left[\frac{1}{L_4}\left(L_2\cos\theta_2 + L_3\cos\theta_3 - L_1\cos\theta_1\right)\right] \tag{A2}$$

where A, B and C are given as:

$$A = 2L_3L_1\cos\theta_1 - 2L_2L_3\cos\theta_2...$$
$$...+ L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_2L_1\cos\left(\theta_2 - \theta_1\right)$$
$$B = 4L_3\left(L_2\sin\theta_2 - L_1\sin\theta_1\right)$$
$$C = 2L_2L_3\cos\theta_2 - 2L_3L_1\cos\theta_1...$$
$$...+ L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_2L_1\cos\left(\theta_2 - \theta_1\right)$$

β_2, β_3 and β_4 angles in Eq. (5) are given as:

$$\beta_2 = a\cos\left(\frac{\overline{AG_2}^2 + L_2^2 - \overline{A_oG_2}^2}{2\overline{AG_2}L_2}\right)$$

$$\beta_3 = a\cos\left(\frac{\overline{BG_3}^2 + L_3^2 - \overline{AG_3}^2}{2\overline{BG_3}L_3}\right)$$

$$\beta_4 = a\cos\left(\frac{\overline{BG_4}^2 + L_4^2 - \overline{B_oG_4}^2}{2\overline{BG_4}L_4}\right).$$

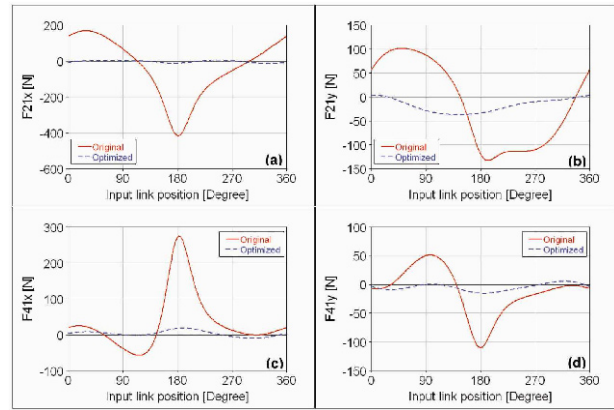Simulation results of force and moment characteristics for Case I are given in Figs. A1 and A2.

Fig. A1. Simulation results for Case I; (a) and (b) Crank-frame joint force, (c) and (d) Follower-frame joint force.
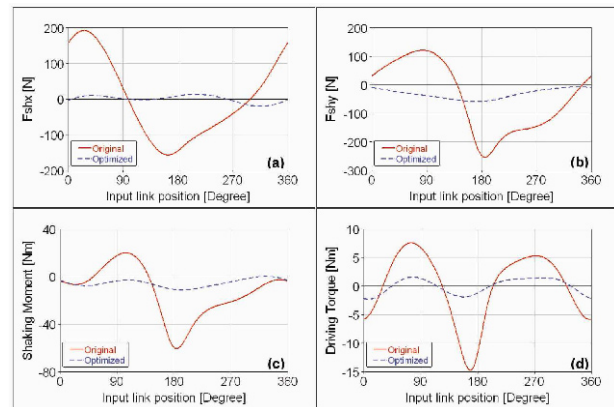

Fig. A2. Simulation results for Case I: (a) and (b) Shaking force components; (c) Shaking moment; (d) Driving torque.

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