Springer

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Journal of Mechanical Science and Technology 27 (5) (2013) 1443~1449
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-013-0214-8

Analytical closed form model for static pull-in analysis in electrostatically actuated torsional micromirrors †

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(Manuscript Received May 13, 2012; Revised August 26, 2012; Accepted November 18, 2012)

Abstract

The objective of this work is to create an analytical framework to study the static pull-in and also equilibrium behavior in electrostatically actuated torsional micromirrors. First the equation governing the static behavior of electrostatic torsion micromirrors is derived and normalized. Perturbation method, the method of straight forward expansion is utilized to find the pull-in angle of the mirror. Comparison of the presented results with numerical ones available in the literature shows that the proposed second order perturbation expansion gives very precise approximations for the pull-in angle of the mirror. Then straightforward perturbation expansion method is used again to analytically simulate the voltage dependent behavior in electrostatic torsion micromirrors. The results are compared with numerical and experimental findings and excellent agreement is observed.

Keywords: Electrostatic actuation; Static pull-in; Straightforward perturbation expansion; Torsional micromirror

1. Introduction

Technology of N/MEMS has experienced lots of progress in testing and fabricating new devices recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them even more attractive [1, 2].

Successful MEMS devices rely not only on well-developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [2]. So simulations of micro-machined systems and sensors are becoming increasingly important. Before prototyping a device, one wishes to virtually build the device and predict its behavior. This allows the optimization of various design parameters according to the specifications [3].

The fact that MEMS devices can have emerging roles in optical systems, helped the development of a new class of MEMS devices which is known as miro-opto-electro-mechanical systems (MOEMS). Micromirrors and electrostatic torsional actuators are examples of MOEMS devices. Based on their motion types, micromirrors can be classified into four categories: deformable micromirror [4], movable micromirror [5], piston micromirror [6], and torsional micromirror [7]. These four types of micromirrors have been widely applied in recent years with the torsional micromirror being the most interesting among them [8]. Torsional micromirrors have been widely used for applications because of theirs good dynamic response and small possibility of adhesion, for instance in digital projection displays [7], spatial light modulators [9] and optical crossbar switches [10].

There has been extensive numerical approaches for characterization of electrostatically actuated micromirrors. Primary simulation tools approach the pull-in state by iteratively adjusting the voltage applied across the actuator electrodes. The convergence rate of this scheme gradually deteriorates as the pull-in state is approached. Moreover, the convergence is inconsistent and requires many mesh and accuracy refinements to assure reliable predictions. As a result, the design procedure of electrostatically actuated MEMS devices can be time-consuming [11]. Degani et al. [12] analyzed pull-in for the electrostatic torsion micro-actuator. They derived a polynomial algebraic equation for the pull-in voltage and pull-in angle of a torsion micro-actuator and compared their results with experimental data. Degani et al. [11] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro-actuators. They showed that their presented method converges one hundred times faster than the voltage iteration scheme. Degani and Nemirovsky [13] presented experimental and theoretical study on the effect of various geometrical parameters on the electromechanical response and pull-in parameters of torsion ac-

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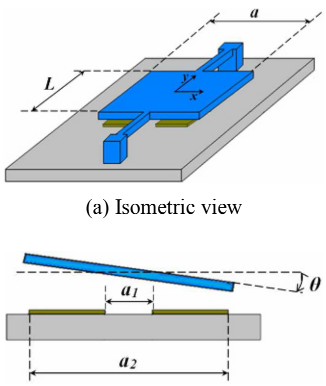
† Recommended by Associate Editor Jong Soo Ko



(a) Isometric view

(b) Side view

Fig. 1. Schematic: (a) isometric view; (b) side view of an electrostatic torsion micromirror.

tuators. They also proposed a novel rapid solver for extracting the pull-in parameters of the torsion actuator. Their proposed solver was based on Newton-Raphson scheme and also their presented DIPIE algorithm. Zhang et al. [14] described the static characteristics of an electrostatic torsion micromirror based on parallel plate capacitor model. They extensively studied the pull-in phenomenon in micromirrors. They used numerical approach for their simulations. Huang et al. [8] presented a general theoretical model considering the coupling effect between the torsion and bending in electrostatic torsional actuators. They also presented experiments to verify their model which was solved numerically.

Perturbation based methods have been widely used to analytically solve the nonlinear problems in MEMS. For example Younis and Nayfeh [15] investigated the response of a resonant micro-beam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [16] used the same method to model secondary resonances in electrically actuated micro-beams. Moeenfard et al. [17] used homotopy perturbation method (HPM) to model the nonlinear vibrational behavior of Timoshenko micro-beams. They [18] also applied the HPM to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces. Moeenfard et al. [19] used HPM to investigate the static behavior and pull-in of nano/micromirrors under the effect of Casimir force.

The current paper makes use of the perturbation technique to analytically find the pull-in angle of electrostatic torsion micromirrors. Then, the same method is utilized to analytically investigate the voltage-angle relationship in electrostatically actuated torsional micromirrors. For verification purpose, the analytical results are compared with numerical and experimental ones and an excellent agreement is observed.

2. Problem formulation

Fig. 1 shows an electrostatically actuated torsion micromirror. There are two electrodes underneath the mirror. By adding electrostatic potential between one of the electrodes and the mirror, the mirror is made to rotate. The mirror can rotate in the reverse direction by imposing the voltage difference between the mirror and the other electrode.

The electrostatic force per unit area is computed using Eq. (1).

$$\hat{F} = \frac{\varepsilon_0 V^2}{2D^2} \tag{1}$$

where $\hat{F}$ is the electrostatic force per unit area, ε_0 is the permittivity of the air, V is the applied voltage and D is the distance between the areas with potential difference.

So the electrostatic torque applied to the micromirror shown in Fig. 1 can be calculated by integrating the differential electrostatic torque as:

$$M_e = \int_{a_1/2}^{a_2/2} x.dF = \int_{a_1/2}^{a_2/2} x.\frac{\varepsilon_0 V^2}{2\left(h - x\tan\theta\right)^2} L.dx \tag{2}$$

where M_e is the total electrostatic torque applied to the mirror, θ is the tilting angle of the mirror, h is the initial distance between the mirror and the electrodes, a_1 and a_2 are some geometrical parameters defining the position of the electrodes as depicted in Fig. 1 and L is the length of the mirror. Since θ is usually small, $\tan\theta$ can be approximated by θ in the Eq. (2). So Eq. (2) can be rewritten as:

$$M_e = \int_{a_1/2}^{a_2/2} x\frac{\varepsilon_0 V^2}{2\left(h - x\theta\right)^2} Ldx = \frac{\varepsilon_0 V^2 L}{2\theta^2}\left(\frac{1}{1-\left(a_2/(2h)\right)\theta} - \frac{1}{1-\left(a_1/(2h)\right)\theta} + \ln\left(\frac{1-\left(a_2/(2h)\right)\theta}{1-\left(a_1/(2h)\right)\theta}\right)\right). \tag{3}$$

For convenience, the following normalized parameters are introduced.

$$\alpha = \frac{a_1}{a} \tag{4}$$

$$\beta = \frac{a_2}{a} \tag{5}$$

$$\Theta = \frac{\theta}{\theta_{\max}} \tag{6}$$

where a is the width of the mirror and $\theta_{\max} = \tan^{-1}(2h/a) \approx 2h/a$ is physically the maximum possible rotation angle of the mirror. Using the above mentioned normalized parameters, Eq. (3) is simplified to

$$M_e = \frac{\varepsilon_0 V^2 L}{2\theta_{\max}^2 \Theta^2}\left(\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right). \tag{7}$$

When the mirror rotates, the torsion beams connected to the mirror apply an opposing mechanical torque to the mirror. Assuming small rotation angles, one can assume that the mentioned mechanical torque is linearly proportional with the angle of rotation

$$M_{mech} = K_\theta \theta \tag{8}$$

where K_θ is the overall torsional stiffness of the torsion beams. In terms of normalized parameters, Eq. (8) can be restated as:

$$M_{mech} = K_\theta \theta_{\max} \Theta . \tag{9}$$

In the static equilibrium state, the mechanical and electrical torques applied to the mirror are equal with each other. In other words

$$M_e = M_{mech} \tag{10}$$

or

$$\begin{aligned}\Xi(\Theta) &= \Theta - \frac{\bar{V}^2}{\Theta^2}\left(\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right) \\ &= 0\end{aligned} \tag{11}$$

where $\Xi(\Theta)$ is the equilibrium equation and $\bar{V}$ is the normalized applied voltage and is defined as

$$\bar{V} = \frac{V}{\theta_{\max}}\sqrt{\frac{\varepsilon_0 L}{2K_\theta \theta_{\max}}} . \tag{12}$$

Eq. (11) can be solved for $\bar{V}$ and the result would be as

$$\bar{V} = \sqrt{\frac{\Theta^3}{1/(1-\beta\Theta) - 1/(1-\alpha\Theta) + \ln\left((1-\beta\Theta)/(1-\alpha\Theta)\right)}} . \tag{13}$$

3. Pull-in analysis

Theoretically, the pull-in point can be obtained by solving the following Eq. (14).

$$\frac{d\bar{V}}{d\Theta} = 0 . \tag{14}$$

By substituting Eq. (13) into Eq. (14), the equation governing the pull-in angle of the mirror would be simplified as

$$\begin{aligned}\Lambda(\alpha,\beta,\Theta_p) &= \frac{3}{1-\beta\Theta_p} - \frac{3}{1-\alpha\Theta_p} - \frac{\beta^2\Theta_p^2}{(1-\beta\Theta_p)^2} \\ &+ \frac{\alpha^2\Theta_p^2}{(1-\alpha\Theta_p)^2} + 3\ln\left(\frac{1-\beta\Theta_p}{1-\alpha\Theta_p}\right) = 0\end{aligned} \tag{15}$$

where $\Lambda(\alpha, \beta, \Theta_p)$ is the equation governing the pull-in angle of the mirror and Θ_p is the normalized tilting angle at the pull-in state. It is observed that the pull-in angle is only an implicit function of the normalized electrode parameters (i.e. α and β). Generally Eq. (15) does not have exact closed form solution. However, Zhang et al. [14] reported the following solution for the equation $\Lambda(0,\beta,\Theta_p) = 0$ (i.e. exceptional case of $\alpha = 0$).

$$\beta\Theta_p = 0.4404 . \tag{16}$$

Here in this paper, the above solution for the exceptional case of $\alpha = 0$ is used to find highly precise analytical solution for the pull-in angle for the general case of $0 \le \alpha < 1$. In order to do so, α is used as a perturbation parameter and the straight forward perturbation expansion method [20] is employed to solve the Eq. (15).

Since α is small, Θ_p can be perturbed using α as a perturbation parameter.

$$\Theta_p = \Theta_p^{(0)} + \alpha\Theta_p^{(1)} + \alpha^2\Theta_p^{(2)} + O(\alpha^3) . \tag{17}$$

In Eq. (17), $\Theta_p^{(i)}$, $i \ge 0$ is the i'th component of the perturbation expansion of Θ_p.

In the next step, Eq. (17) is substituted into Eq. (15).

$$\Lambda\left(\alpha,\beta,\Theta_p^{(0)} + \alpha\Theta_p^{(1)} + \alpha^2\Theta_p^{(2)}\right) = 0 . \tag{18}$$

The left hand side of the Eq. (18) can be substituted by its Taylor series expansion with respect to α. The result is

$$\begin{aligned}&\Lambda(0,\beta,\Theta_0) + \left(\left.\frac{\partial\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'}\right|_{\alpha'=0} + \Theta_1\frac{\partial\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0}\right) \\ &\times\alpha + \left(\frac{1}{2}\left.\frac{\partial^2\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'^2}\right|_{\alpha'=0} + \right. \\ &\Theta_1\frac{\partial}{\partial\Theta_0}\left(\left.\frac{\partial\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'}\right|_{\alpha'=0}\right) + \frac{1}{2}\Theta_1^2\frac{\partial^2\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0^2} \\ &\left. + \Theta_2\frac{\partial\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0}\right)\alpha^2 + O(\alpha^3) = 0 .\end{aligned} \tag{19}$$

The left hand side of Eq. (19) must be unified with zero for all values of α. As a result the above equation can be satisfied if and only if the coefficients of α^i $(0 \le i)$ in the Eq. (19) are zero. This, would lead to the following equations.

$$\alpha^0: \quad \Lambda(0,\beta,\Theta_0) = 0 \tag{20}$$

$$\alpha^1: \quad \Theta_1\frac{\partial\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0} + \left.\frac{\partial\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'}\right|_{\alpha'=0} = 0 \tag{21}$$

$$\begin{aligned}\alpha^2: \quad &\Theta_2\frac{\partial\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0} + \frac{1}{2}\left.\frac{\partial^2\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'^2}\right|_{\alpha'=0} \\ &+ \Theta_1\frac{\partial}{\partial\Theta_0}\left(\left.\frac{\partial\Lambda(\alpha',\beta,\Theta_0)}{\partial\alpha'}\right|_{\alpha'=0}\right) \\ &+ \frac{1}{2}\Theta_1^2\frac{\partial^2\Lambda(0,\beta,\Theta_0)}{\partial\Theta_0^2} = 0 .\end{aligned} \tag{22}$$

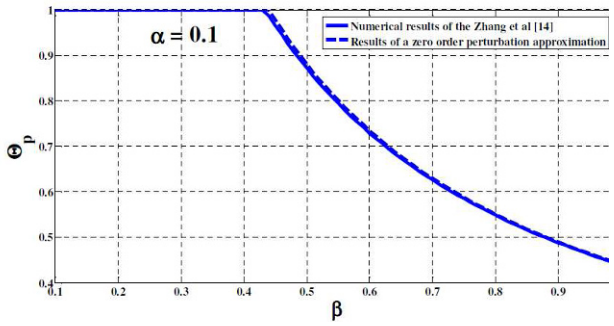


Fig. 2. Pull-in angle versus normalized electrode parameter β at α = 0.1.

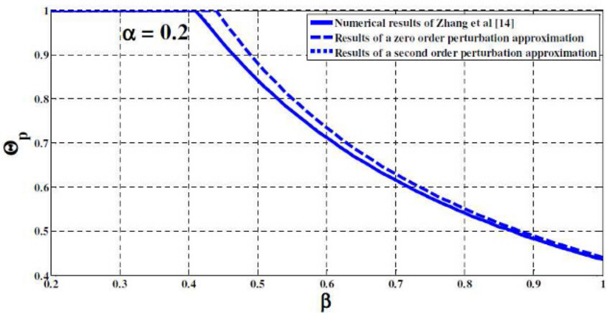


Fig. 3. Pull-in angle versus normalized electrode parameter β at α = 0.2.

Eq. (20) is the same as Eq. (15) within which α has been set equal with zero. So as it was stated earlier, its solution would be as follows.

$$\Theta_0 = \frac{0.4404}{\beta}. \tag{23}$$

Eq. (21) is a linear function of Θ_1. Since $\partial\Lambda(\alpha',\beta,\Theta_0)/\partial\alpha'|_{\alpha'=0} = 0$, the solution of this equation would be

$$\Theta_1 = -\frac{\partial\Lambda(\alpha',\beta,\Theta_0)/\partial\alpha'\big|_{\alpha'=0}}{\partial\Lambda(0,\beta,\Theta_0)/\partial\Theta_0} = 0. \tag{24}$$

Now, using Eqs. (22)-(24), Θ_2 is easily obtained as

$$\Theta_2 = -\frac{1}{2}\frac{\partial^2\Lambda(\alpha',\beta,\Theta_0)/\partial\alpha'^2\big|_{\alpha'=0}}{\partial\Lambda(0,\beta,\Theta_0)/\partial\Theta_0} = -\frac{0.1201}{\beta^3}. \tag{25}$$

By substituting Eqs. (23)-(25) into Eq. (17), the following simple expression is obtained for the pull-in angle of an electrostatically actuated micromirror.

$$\Theta_p = \frac{0.4404}{\beta} - 0.1201\frac{\alpha^2}{\beta^3}. \tag{26}$$

Physically, in a torsion micromirror, Θ_p cannot exceed unity.

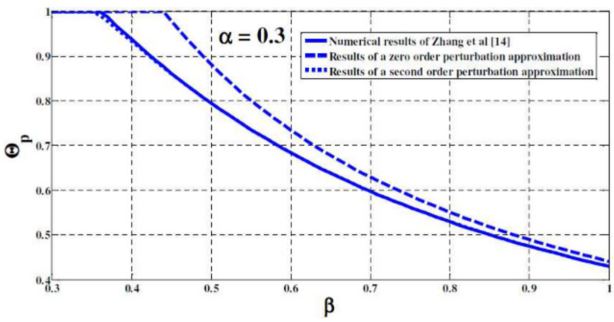


Fig. 4. Pull-in angle versus normalized electrode parameter β at α = 0.3.

So, the value of the pull-in angle in a torsion mirror would be

$$\Theta_p = \min\left(\frac{0.4404}{\beta} - 0.1201\frac{\alpha^2}{\beta^3}, 1\right). \tag{27}$$

In order to verify the accuracy of the presented closed-form solution for the pull-in angle of the mirror, the proposed analytical results have been compared with their corresponding numerical ones in Figs. 2-4. In these figures, the normalized pull-in angle of the mirror, Θ_p has been plotted versus β. It is observed that for small values of α, the second order perturbation expansion results perfectly matches the numerical results of Zhang et al. [14].

4. Voltage-angle relationship

In order to explicitly find the rotation angle of the micromirror in terms of the applied voltage, Eq. (11) has to be solved for Θ. Since this equation has no exact solution even for the specific case of $\alpha = 0$, straightforward perturbation expansion method is utilized for finding analytical closed form expressions for the solution of this equation. To do so, the linear part of $\Xi(\Theta)$ is calculated using the Taylor series expansion of the Eq. (11) as

$$L(\Theta) = L_1 + L_2\Theta \tag{28}$$

where $L(\Theta)$ is the linear part of $\Xi(\Theta)$ and L_1 and L_2 are defined as follows

$$L_1 = \frac{\bar{V}^2}{2}\left(\alpha^2 - \beta^2\right) \tag{29}$$

$$L_2 = 1 + \frac{2}{3}\bar{V}^2\left(\alpha^3 - \beta^3\right). \tag{30}$$

The nonlinear part of $\Xi(\Theta)$ is simply obtained by subtracting $L(\Theta)$ from $\Xi(\Theta)$. The result would be

$$N(\Theta) = \bar{V}^2\bar{N}(\Theta) \tag{31}$$

where

$$\bar{N}(\Theta)=\frac{1}{1-\alpha\Theta}-\frac{1}{1-\beta\Theta}-\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)+\frac{2}{3}\left(\beta^3-\alpha^3\right)\Theta+\frac{1}{2}\left(\beta^2-\alpha^2\right). \quad (32)$$

The normalized voltage $\bar{V}$ can be scaled as

$$\bar{V}=\sqrt{\vartheta}\hat{V} \quad (33)$$

where ϑ is a small book-keeping parameter and will be used as perturbation parameter.

$\Xi(\Theta)$ can be reconstructed as follows by adding its linear and nonlinear counterparts:

$$\Xi(\Theta)=L(\Theta)+N(\Theta). \quad (34)$$

Using Eqs. (31) and (33), Eq. (34) can be restated as

$$\Xi(\Theta)=L(\Theta)+\vartheta\hat{V}^2\bar{N}(\Theta). \quad (35)$$

In the next step, Θ is expanded in terms of ϑ as follows.

$$\Theta=\Theta_0+\vartheta\Theta_1+\vartheta^2\Theta_2+O\left(\vartheta^3\right). \quad (36)$$

By substituting Θ from Eq. (36) into Eq. (35) and computing the Taylor series expansion of the resulting equation with respect to ϑ, one would get

$$\begin{aligned}\Xi(\Theta)&=L\left(\Theta_0+\vartheta\Theta_1+\vartheta^2\Theta_2\right)+\vartheta\hat{V}^2\bar{N}\left(\Theta_0+\vartheta\Theta_1+\vartheta^2\Theta_2\right)=L(\Theta_0)+\left(\Theta_1\frac{dL(\Theta_0)}{d\Theta_0}+\hat{V}^2\bar{N}(\Theta_0)\right)\vartheta\\&+\left(\Theta_2\frac{dL(\Theta_0)}{d\Theta_0}+\Theta_1\hat{V}^2\frac{d\bar{N}(\Theta_0)}{d\Theta_0}\right)\vartheta^2+\ldots.\end{aligned} \quad (37)$$

By equating the coefficients of all powers of ϑ with zero, the following equations are obtained.

$$L(\Theta_0)=L_1+L_2\Theta_0=0 \quad (38)$$

$$\Theta_1\frac{dL(\Theta_0)}{d\Theta_0}+\hat{V}^2\bar{N}(\Theta_0)=0 \quad (39)$$

$$\Theta_2\frac{dL(\Theta_0)}{d\Theta_0}+\Theta_1\hat{V}^2\frac{d\bar{N}(\Theta_0)}{d\Theta_0}=0. \quad (40)$$

These equations can be solved consecutively and the result would be as

$$\Theta_0=-\frac{L_1}{L_2} \quad (41)$$

$$\Theta_1=-\frac{\hat{V}^2\bar{N}(\Theta_0)}{dL(\Theta_0)/d\Theta_0}=-\frac{\hat{V}^2}{L_2}\bar{N}\left(-L_1/L_2\right) \quad (42)$$

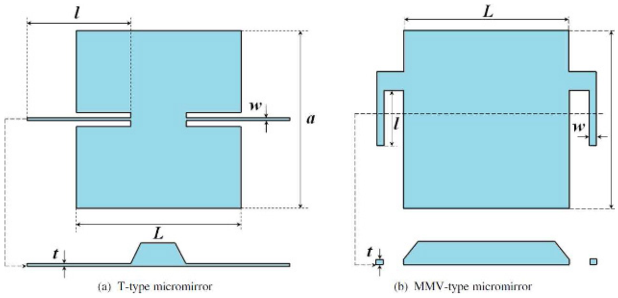


Fig. 5. Schematic view of a (a) T-type and a; (b) MMV-type micromirror fabricated by Degani et al [12].

$$\begin{aligned}\Theta_2&=-\Theta_1\hat{V}^2\frac{d\bar{N}(\Theta_0)/d\Theta_0}{dL(\Theta_0)/d\Theta_0}\\&=\hat{V}^4\left.\frac{\bar{N}(\Theta')}{L_2^2}\frac{d\bar{N}(\Theta')}{d\Theta'}\right|_{\Theta'=-L_1/L_2}.\end{aligned} \quad (43)$$

Now, a second order perturbation approximation for the normalized rotation angle Θ, is obtained by substituting Eqs. (41)-(43) into Eq. (36).

$$\begin{aligned}\Theta&=-\frac{L_1}{L_2}-\frac{\vartheta\hat{V}^2}{L_2}\bar{N}\left(-L_1/L_2\right)\\&+\vartheta^2\hat{V}^4\left.\frac{\bar{N}(\Theta')}{L_2^2}\frac{d\bar{N}(\Theta')}{d\Theta'}\right|_{\Theta'=-L_1/L_2}.\end{aligned} \quad (44)$$

By using Eq. (33), Eq. (44) can be simplified as follows.

$$\begin{aligned}\Theta&=-\frac{L_1}{L_2}-\frac{\bar{V}^2}{L_2}\bar{N}\left(-L_1/L_2\right)\\&+\bar{V}^4\left.\frac{\bar{N}(\Theta')}{L_2^2}\frac{d\bar{N}(\Theta')}{d\Theta'}\right|_{\Theta'=-L_1/L_2}.\end{aligned} \quad (45)$$

It is observed that the value of Θ is independent of the book-keeping parameter ϑ and in fact ϑ acts like a dummy parameter. The accuracy of the closed form solution presented for Θ can be improved by using higher order perturbation expansions in Eq. (36).

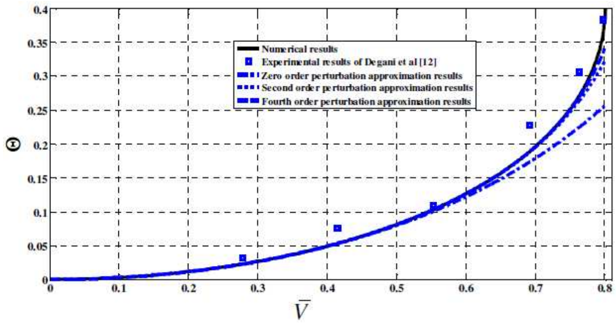
In order to verify the accuracy of the presented method, two torsion T-type and MMV type micromirrors shown in Fig. 5 with characteristics given in Table 1 are considered. Torsional stiffnesses of the supporting structure of the mirror would be as Eqs. (46) and (47) for a T-type [8] and a MMV-type [21] mirror respectively.

$$K_\theta=\frac{2G}{l}\left(\frac{1}{3}t.w^3-\frac{64w^4}{\pi^5}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi t}{2w}\right) \quad (46)$$

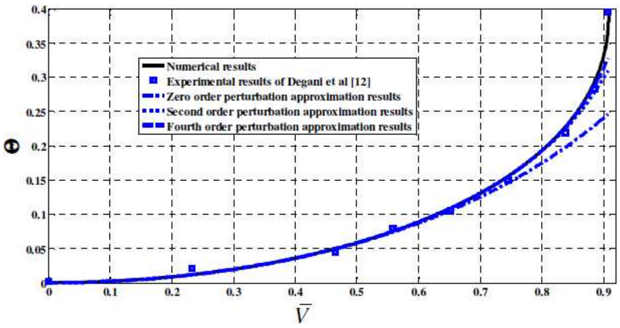
$$K_\theta=\frac{Ew.t^3}{6l}. \quad (47)$$

Table 1. Parameters of the T-type and MMV-Type torsion mirror fabricated by Degani et al. [12].

Parameter	Value	
	T-type	MMV-type
a_1	860 μm	860 μm
a_2	1360 μm	1160 μm
a	1400 μm	1200 μm
L	1300 μm	1300 μm
t	14 μm	16 μm
w	31 μm	32 μm
l	400 μm	200 μm
h	4.55 μm	3.42 μm
E	-----	131 *Gpa*
G	73 *Gpa*	-----



(a) T-type



(b) MMV-type

Fig. 6. Voltage angle relationship in electrostatically actuated: (a) T-type; (b) MMV-type micromirrors with properties given in table (1), comparison of numerical, experimental and analytical results.

In Eqs. (46) and (47), G and E are the shear modulus and Youngs modulus of elasticity of the supporting structure's material and t and w are the thickness and width of the torsion beams respectively as illustrated in Fig. 5.

In Fig. 6, the presented analytical results have been compared with numerical ones and also with experimental findings of Degani et al. [12]. It is observed that for both T-type and MMV-type mirrors, even a second order perturbation approximation closely follows the numerical results as well as the experimental findings, but in order to make the analytical results more accurate, a higher order approximation has to be used. It is also observed that increasing the degree of perturbation approximation more than four would not appreciably improve the accuracy of the analytical results.

5. Conclusion

The importance of analytically studying the static pull-in and also voltage-angle behavior of torsional micromirrors to better inform their design and optimization has been well recognized. However such an investigation is complicated by the fact the governing equations of the mentioned phenomenon are strongly nonlinear. The current paper however, presents closed form solutions for the problem of electrostatically actuated micromirrors. First a straight forward perturbation expansion method is utilized to find closed form expressions for the pull-in angle of the micromirrors. It was observed that the presented second order perturbation solution matched very well the numerical results available in the literature. Again, straight forward perturbation expansion method was used to analytically investigate the voltage dependent behavior in electrostatically actuated micromirrors. In order to verify the accuracy of the proposed analytical solutions, they were compared with the numerical solutions as well as experimental findings and a close match was observed. The analytical models presented in this paper can be used in design optimization and determination of the stable operation range of torsional micro-actuators.

Nomenclature

a : Length of micromirror in x direction
a_1 : Distance between the starting points of the electrodes
a_2 : Distance between the ending points of the electrodes
E : Young's modulus of elasticity
G : Shear modulus of elasticity
h : Initial distance between the mirror and the electrodes
l : Length of the supporting torsion beams
L : Width of micromirror in y direction
t : Thickness of the supporting torsion beams
w : Width of the supporting torsion beams

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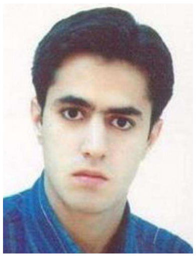

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