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Chaos synchronization of gyroscopes using an adaptive robust finite-time controller†

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Abstract

The problem of robust finite-time chaos synchronization between two chaotic nonlinear gyroscopes with model uncertainties, external disturbances and unknown parameters is investigated. Appropriate adaptive laws are derived to tackle the unknown parameters. Based on the adaptive laws and the finite-time control technique, suitable adaptive control laws are designed to ensure the stability of the resulting synchronization error system in a given finite time. Numerical simulations and comparative examples are presented to illustrate the applicability and usefulness of the proposed finite-time control strategy.

Keywords: Chaotic gyroscope; Finite-time synchronization; Uncertainty; Unknown parameter

1. Introduction

In recent years, synchronization of chaotic dynamical systems has attracted the attention of many researchers due to its powerful applications in physics and engineering sciences, such as secure communication, chemical reactions, optics, lasers, power convertors, biological systems, mechanical systems, etc (Chen and Dong, 1998). In this line, many effective control methodologies have been successfully applied to realize chaos control and synchronization (Aghababa and Aghababa, 2011; Aghababa and Aghababa, 2012a; Aghababa and Aghababa, 2012b; Aghababa and Heydari, 2012; Aghababa and Akbari, 2012; Aghababa et al., 2011; Poon et al., 2003; Shah et al., 2012; Skowronski et al., 2003).

The gyroscope is one of the most interesting dynamical systems. Gyroscopes have found useful applications in optics, navigation, aeronautics and space engineering fields. Recent research has recognized different kinds of the gyroscope systems with linear or nonlinear damping features. Furthermore, these systems display a diverse range of dynamic behavior including both sub-harmonic and chaotic motions (Chen, 2002; Ge and Chen, 1996; Tong and Mrad, 2001; Van Dooren, 2008). Synchronization of two gyroscopes is usually used in areas of secure communication (Chen and Lin, 2003), attitude control of long-duration spacecrafts (Zhou et al., 2006) and signal processing in optical gyroscopes.

Lei et al. (2005) proposed an active control technique for synchronizing two identical gyroscopes. Hung et al. (2008) investigated the problem of generalized projective synchronization of chaotic gyroscopes with dead-zone nonlinearity in the control input. Yau (2007; 2008) developed a fuzzy rule based controller and a fuzzy sliding mode controller for synchronization of two uncertain chaotic gyroscopes. Salarieh and Alasty (2008) introduced a Markov synchronization control law to synchronize two chaotic gyroscopes with stochastic based excitations and uncertain parameters. Recently, Yan et al. (2006) studied the problem of synchronizing two chaotic gyroscopes with unknown parameters via an adaptive sliding mode controller.

However, all of the aforementioned methods and synchronization strategies guarantee the asymptotic stability of the resulting synchronization error system. In other words, in the previous works, the trajectories of the response gyroscope system can approach to the trajectories of the drive gyroscope system with infinite settling time. Nevertheless, from a practical point of view, it is more valuable to synchronize two chaotic gyroscopes in a given finite time. To obtain faster convergence in a control system, the finite-time control method is an effective technique. The finite-time control techniques have demonstrated better robustness and disturbance rejection properties (Bhat and Bernstein, 2000). On the other hand, in real applications some robust control methods should be taken into account to deal with system uncertainties and external disturbances (Zhang and Shi, 2012; Zhang et al., 2012; Zhang

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et al., 2011a; Zhang et al., 2011b; Zhang et al., 2010a; Zhang et al., 2010b). However, to our best knowledge, there is less work in the literature about the problem of robust finite-time synchronization of two chaotic gyroscopes with model uncertainties, external disturbances and fully unknown parameters.

Therefore, this paper discusses the problem of robust synchronization of two uncertain chaotic gyroscopes in a given finite time. It is assumed that both drive and response chaotic gyroscopes are perturbed by model uncertainties, external disturbances and fully unknown parameters. An adaptive robust finite-time controller is proposed to guarantee that the state trajectories of the response gyroscope system converge to the state trajectories of the drive gyroscope system in a given finite time. Numerical simulations are given to demonstrate the efficiency of the proposed synchronization scheme. The main contributions of this paper are as follows: a) design of a robust adaptive controller for finite-time synchronization of gyroscopes; b) considering the effects of both model uncertainties and external disturbances in both drive and response systems; and c) dealing with the fully unknown parameters of both master and slave chaotic gyroscopes.

The rest of this paper is organized as follows. The nonlinear dynamics of a symmetric gyroscope is briefly described in section 2. The synchronization problem is formulated in section 3. In section 4, the design procedure of the proposed adaptive robust finite-time controller is included. Numerical simulations are performed in section 5. Finally, some conclusions are presented in section 6.

2. Dynamics of nonlinear chaotic gyroscopes

The motion of a symmetric gyroscope with linear-plus-cubic damping mounted on a vibrating base (see Fig. 1) in terms of the rotation angle θ, is governed by (Chen, 2002)

$$\ddot{\theta}+\alpha^2\frac{(1-\cos\theta)^2}{\sin^3\theta}-\beta\sin\theta+c_1\dot{\theta}+c_2\dot{\theta}^3= f\sin(\omega t)\sin\theta \tag{1}$$

where the term $f\sin(\omega t)$ is a parametric excitation that models the base excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are linear and nonlinear damping terms, respectively, the term $\alpha^2(1-\cos\theta)^2/\sin^3\theta-\beta\sin\theta$ is a nonlinear resilience, M_g is the gravity force, $\bar{l}$ is the amplitude of the external excitation disturbance, ω is the frequency of the external excitation disturbance and θ, ϕ and ψ are nutation, precession and spin Euler's angles, respectively.

Defining $x_1=\theta$ and $x_2=\dot{\theta}$, the gyroscope system (1) can be transformed into the following normalized form:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\alpha^2\dfrac{(1-\cos x_1)^2}{\sin^3 x_1}-c_1x_2-c_2x_2^3+\\ \qquad\beta\sin x_1+f\sin(\omega t)\sin x_1\,.\end{cases} \tag{2}$$

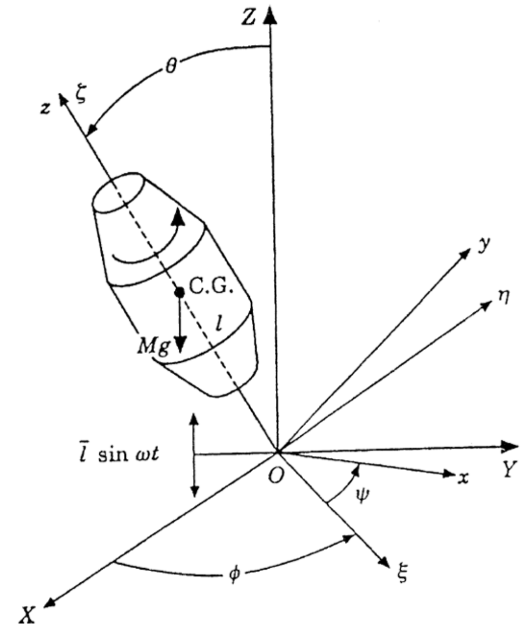

Fig. 1. A schematic picture of the gyroscope system (Chen, 2002).

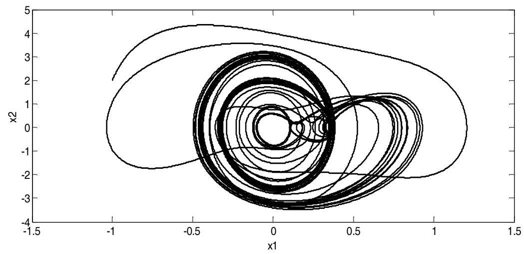

Fig. 2. The phase plane trajectory of the chaotic gyroscope.

The dynamics of this gyroscope system has been extensively studied by Chen (2002) and Van Dooren (2008) for the values of f, in the range $32<f<36$. In particular, for the parameter values of $\alpha^2=100$, $\beta=1$, $c_1=0.5$, $c_2=0.05$, $\omega=2$ and $f=35.5$, this gyroscope exhibits chaotic behavior. The strange attractor of the gyroscope system (1) is illustrated in Fig. 2. More details about the chaotic dynamics of the gyroscope systems can be seen in Chen (2002).

3. Synchronization problem formulation

Consider two coupled, chaotic gyroscope systems with model uncertainties, external disturbances and unknown parameters in the following form:

Drive gyroscope:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\alpha^2\dfrac{(1-\cos x_1)^2}{\sin^3 x_1}-c_1x_2-c_2x_2^3+\\ \qquad\beta\sin x_1+f\sin(\omega t)\sin x_1+\\ \qquad\Delta f(x,t)+d^D(t)\,.\end{cases} \tag{3}$$

Response gyroscope:

$$\begin{cases} \dot{y}_1 = y_2 + u_1(t) \\ \dot{y}_2 = -\alpha^2 \dfrac{(1-\cos y_1)^2}{\sin^3 y_1} - c_1 y_2 - c_2 x_2^3 + \\ \quad \beta \sin y_1 + f \sin(\omega t)\sin y_1 + \\ \quad \Delta g(y,t) + d^R(t) + u_2(t) \end{cases} \tag{4}$$

where $x = [x_1, x_2]^T$ is the state vector of the drive system, $\Delta f(x,t)$ and $d^D(t)$ are unknown model uncertainties and external disturbances of the drive system, respectively; $y = [y_1, y_2]^T$ is the state vector of the response system, $\Delta g(y,t)$ and $d^R(t)$ are unknown model uncertainties and external disturbances of the response system, respectively; and $u(t) = [u_1(t), u_2(t)]^T$ is the control input to be designed later.

Assumption 1. Since the trajectories of chaotic gyroscope systems are always bounded (Curran and Chua, 1997), the uncertainties $\Delta f(x,t)$ and $\Delta g(y,t)$ are assumed to be bounded by

$$\begin{aligned} &|\Delta f(x,t)| \le a^D \|x\| + b^D \quad \text{and} \\ &|\Delta g(y,t)| \le a^R \|y\| + b^R \end{aligned} \tag{5}$$

where $\|.\|$ denotes the Euclidean norm in R^n and a^D, b^D, a^R and b^R are given positive constants.

Assumption 2. In general, it is assumed that the external disturbances are norm-bounded in C^1:

$$|d^D(t)| \le D^D \quad \text{and} \quad |d^R(t)| \le D^R \tag{6}$$

where D^D and D^R are known positive constants.

As a result, using Eqs. (5) and (6), one can obtain

$$\begin{aligned} &|\Delta f(x,t) - \Delta g(y,t) + d^D(t) - d^R(t)| \le \\ &a^D \|x\| + a^R \|y\| + \sigma \end{aligned} \tag{7}$$

where $\sigma = b^D + b^R + D^D + D^R$.

Assumption 3. The parameters α^2, c_1, c_2, β, f and ω are fully unknown in advance. Let $\Psi = [\alpha^2, c_1, c_2, |\beta| + |f|]^T = [\psi_1, \psi_2, \psi_3, \psi_4]^T$ be as the unknown vector parameter. Then, the following assumption is made.

Assumption 4. The unknown vector parameter ψ is norm-bounded:

$$\|\psi\| \le \Psi \tag{8}$$

where Ψ is a given positive constant.

To solve the finite-time synchronization problem, the synchronization error between the drive and response systems can be defined as $e(t) = [e_1(t), e_2(t)]^T = x(t) - y(t)$. Therefore, with subtracting Eq. (4) from Eq. (3), the error dynamics is obtained as follows:

$$\begin{cases} \dot{e}_1 = e_2 - u_1(t) \\ \dot{e}_2 = \alpha^2 h(x_1, y_1) - c_1 e_2 - c_2(x_2^3 - y_2^3) \\ +(\beta + f \sin \omega t)(\sin x_1 - \sin y_1) + \Delta f(x,t) \\ -\Delta g(y,t) + d^D(t) - d^R(t) - u_2(t) \end{cases} \tag{9}$$

where $h(x_1, y_1) = \dfrac{(1-\cos y_1)^2}{\sin^3 y_1} - \dfrac{(1-\cos x_1)^2}{\sin^3 x_1}$.

The main objective of this paper is that for any given drive gyroscope system and response gyroscope system described by Eqs. (3) and (4) respectively, with model uncertainties, external disturbances and unknown parameters, a suitable finite-time control law $u(t)$ is designed such that the finite-time stability of the resulting error system of Eq. (9) is achieved.

4. Design of an adaptive robust finite-time controller

Lemma 1 (Wang et al., 2009). Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -pV^{\varsigma}(t) \quad \forall t \ge t_0, \; V(t_0) \ge 0 \tag{10}$$

where $p > 0$, $0 < \zeta < 1$ are two constants. Then, for any given t_0 , $V(t)$ satisfies the following inequality:

$$V^{1-\zeta}(t) \le V^{1-\zeta}(t_0) - p(1-\zeta)(t - t_0), t_0 \le t \le t_1 \tag{11}$$

and $V(t) \equiv 0 \;\; \forall t \ge t_1$ with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\zeta}(t_0)}{p(1-\zeta)} . \tag{12}$$

To guarantee the finite-time stability of the synchronization error system, the following finite-time control laws are proposed.

$$\begin{aligned} &u_1(t) = e_2 + \lambda\left(\|\hat{\psi}\| + \Psi\right)\left(\frac{e_1}{\|e\|^2}\right) + k_1 \operatorname{sgn}(e_1) \\ &u_2(t) = \lambda\left(\|\hat{\psi}\| + \Psi\right)\left(\frac{e_2}{\|e\|^2}\right) + \\ &(a^D \|x\| + a^R \|y\| + \sigma + k_2)\operatorname{sgn}(e_2) + \\ &\hat{\psi}_1 h(x_1, y_1) - \hat{\psi}_2 e_2 - \hat{\psi}_3 (x_2^3 - y_2^3) + \\ &\hat{\psi}_4 |\sin x_1 - \sin y_1| \operatorname{sgn}(e_2) \end{aligned} \tag{13}$$

where $\hat{\psi}_i, i = 1,2,3,4$ are estimations for ψ_i, $i = 1,2,3,4$, respectively; ($\hat{\psi} = [\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3, \hat{\psi}_4]^T$), sgn(.) is the sign function, k_1 and k_2 are positive gains, $\lambda = \min\{k_1, k_2\} > 0$ is a constant and if $e(t) = 0$, then $\frac{e_i}{\|e\|^2} = 0, i = 1,2$.

Now, let the appropriate adaptive law to be proposed as follows:

$$\dot{\hat{\psi}}(t) = [\dot{\hat{\psi}}_1, \dot{\hat{\psi}}_2, \dot{\hat{\psi}}_3, \dot{\hat{\psi}}_4] = [h(x_1, y_1)e_2, \\ -e_2^2, -(x_2^3 - y_2^3)e_2, |e_2||\sin x_1 - \sin y_1|]. \tag{14}$$

Theorem 1. If the error system (9) is controlled with the control laws in Eq. (13) and adaptive law in Eq. (14), then the trajectories of the error system converge to zero in finite time.

Proof. Choose a positive definite function in the form of

$$V(t) = \tfrac{1}{2}(e_1^2 + e_2^2) + \tfrac{1}{2}\|\hat{\psi} - \psi\|^2. \tag{15}$$

Taking the time derivative of $V(t)$, one has

$$\dot{V}(t) = e_1\dot{e}_1 + e_2\dot{e}_2 + (\hat{\psi} - \psi)^T \dot{\hat{\psi}}. \tag{16}$$

Introducing $\dot{e}_1$ and $\dot{e}_2$ from Eq. (9) into the right hand side of Eq. (16), we have

$$\dot{V}(t) \le e_1(e_2 - u_1(t)) + e_2(\alpha^2 h(x_1, y_1) - \\ c_1 e_2 - c_2(x_2^3 - y_2^3) + (\beta + f \sin \omega t) \\ (\sin x_1 - \sin y_1) + \Delta f(x,t) - \Delta g(y,t) + \\ d^D(t) - d^R(t) - u_2(t)) + (\hat{\psi} - \psi)^T \dot{\hat{\psi}}. \tag{17}$$

It is obvious that

$$\dot{V}(t) \le e_1(e_2 - u_1(t)) + e_2(\alpha^2 h(x_1, y_1) - \\ c_1 e_2 - c_2(x_2^3 - y_2^3) + (|\beta| + |f|) \\ |\sin x_1 - \sin y_1| \operatorname{sgn}(e_2) + |\Delta f(x,t) - \\ \Delta g(y,t) + d^D(t) - d^R(t)| \operatorname{sgn}(e_2) - \\ u_2(t)) + (\hat{\psi} - \psi)^T \dot{\hat{\psi}}. \tag{18}$$

According to Eq. (7) and using the adaptive laws of Eq. (14), one has

$$\dot{V}(t) \le e_1(e_2 - u_1(t)) + e_2(\alpha^2 h(x_1, y_1) - \\ c_1 e_2 - c_2(x_2^3 - y_2^3) + (|\beta| + |f|) \\ |\sin x_1 - \sin y_1| \operatorname{sgn}(e_2) + (a^D\|x\| + \\ a^R\|y\| + \sigma)\operatorname{sgn}(e_2) - u_2(t)) + \\ (\hat{\psi} - \psi)^T [h(x_1, y_1)e_2, -e_2^2, \\ -(x_2^3 - y_2^3)e_2, |e_2||\sin x_1 - \sin y_1|]. \tag{19}$$

Using $\psi^T \dot{\hat{\psi}} = \alpha^2 h(x_1, y_1)e_2 - c_1 e_2^2 - c_2(x_2^3 - y_2^3)e_2 + (|\beta| + |f|)|e_2||\sin x_1 - \sin y_1|$, one can obtain

$$\dot{V}(t) \le e_1(e_2 - u_1(t)) + e_2((a^D\|x\| + a^R\|y\| + \\ \sigma)\operatorname{sgn}(e_2) - u_2(t)) + \hat{\psi}^T [h(x_1, y_1)e_2, -e_2^2, \\ -(x_2^3 - y_2^3)e_2, |e_2||\sin x_1 - \sin y_1|]. \tag{20}$$

Substituting $u_1(t)$ and $u_2(t)$ from Eq. (13) into the right hand of the above inequality yields

$$\dot{V}(t) \le -k_1|e_1| - k_2|e_2| - \lambda(\|\hat{\psi}\| + \Psi) \\ \left(\frac{e_1}{\|e\|^2}\right) - \lambda(\|\hat{\psi}\| + \Psi)\left(\frac{e_2}{\|e\|^2}\right) - \big(\hat{\psi}_1 h(x_1, y_1) - \\ \hat{\psi}_2 e_2 - \hat{\psi}_3(x_2^3 - y_2^3) + \hat{\psi}_4 |\sin x_1 - \sin y_1| \\ \operatorname{sgn}(e_2)\big) + \hat{\psi}^T [h(x_1, y_1)e_2, -e_2^2, \\ -(x_2^3 - y_2^3)e_2, |e_2||\sin x_1 - \sin y_1|]. \tag{21}$$

Using the fact that $\frac{e_1^2}{\|e\|^2} + \frac{e_2^2}{\|e\|^2} = 1$ and $\hat{\psi}^T [h(x_1, y_1)e_2, -e_2^2, -(x_2^3 - y_2^3)e_2, |e_2||\sin x_1 - \sin y_1|] = \hat{\psi}_1 h(x_1, y_1) - \hat{\psi}_2 e_2 - \hat{\psi}_3(x_2^3 - y_2^3) + \hat{\psi}_4 |\sin x_1 - \sin y_1| \operatorname{sgn}(e_2)$.

We have

$$\dot{V}(t) \le -k_1|e_1| - k_2|e_2| - \lambda(\|\hat{\psi}\| + \Psi). \tag{22}$$

Using assumption 4 and the fact $\|\hat{\psi}\| + \Psi \ge \|\hat{\psi}\| + \|\psi\| \ge \|\hat{\psi} - \psi\|$, one has

$$\dot{V}(t) \le -k_1|e_1| - k_2|e_2| - \lambda\|\hat{\psi} - \psi\| \le -\lambda \\ (|e_1| + |e_2| + \|\hat{\psi} - \psi\|) \le -\sqrt{2}\lambda\Big(\tfrac{1}{2}(e_1^2 + \\ e_2^2) + \tfrac{1}{2}\|\hat{\psi} - \psi\|^2\Big)^{1/2} = -\sqrt{2}\lambda V^{1/2}. \tag{23}$$

Therefore, from Lemma 1, the error trajectories e_1 and e_2 will converge to zero in the finite time

$$T = \tfrac{\sqrt{2}}{\lambda}\Big(\tfrac{1}{2}\big(e_1^2(0) + e_2^2(0)\big) + \tfrac{1}{2}\|\hat{\psi}(0) - \psi\|^2\Big)^{\frac{1}{2}}.$$

Hence, the trajectories of the response gyroscope system will approach to the trajectories of the drive gyroscope system in the finite time T and the proof is achieved completely.

5. Numerical simulations

5.1 Example 1

The simulations are carried out using the Matlab software. We set $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$ and $f = 35.5$. As a result, Ψ is chosen equal to 160. Subsequently, the following model uncertainties and disturbances

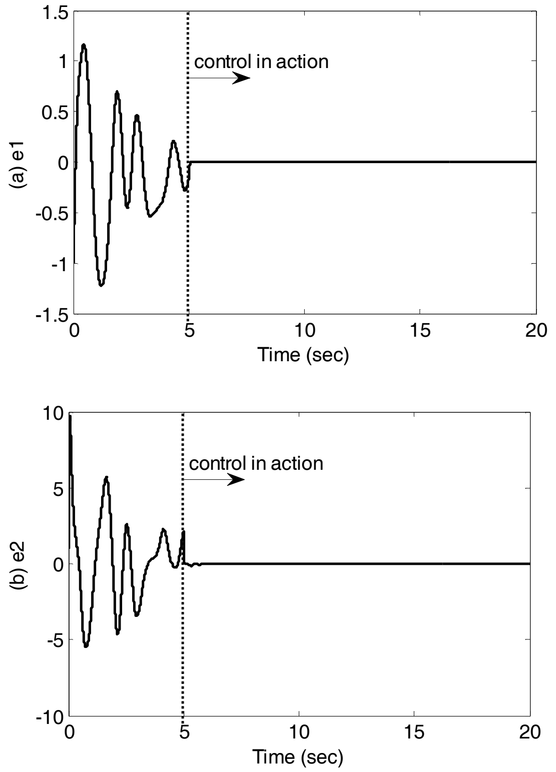

Fig. 3. Synchronization errors of the drive and response chaotic gyroscope systems.

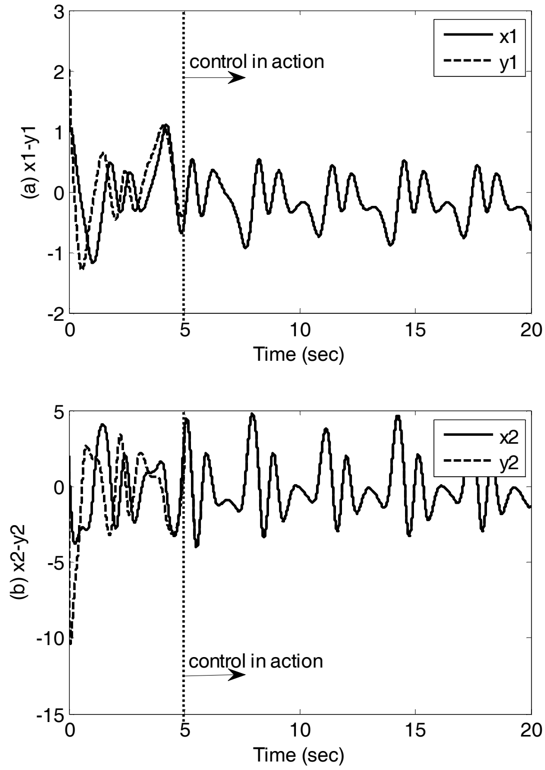

Fig. 4. Trajectories of the drive and response chaotic gyroscope systems.

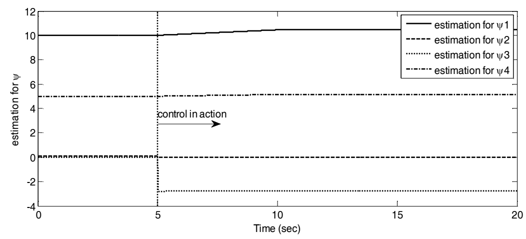

Fig. 5. Time responses of the adaptive parameters ψ_i, $i = 1,2,3,4$.

are considered in the simulations.

$$\Delta f(x,t) = -0.3\cos(2t)x_2 + 0.25\sin(6t)$$
$$\Delta g(y,t) = 0.35\cos(6t)y_2 + 0.25\sin(3t)$$
$$d^D(t) = 0.5\cos(2t)\,,\ \ d^R(t) = 0.5\sin(2t)\,. \tag{24}$$

Consequently, $a^D = 0.3$, $a^R = 0.35$, $\sigma = 1.5$. The initial values of the drive gyroscope and response gyroscope systems are chosen as $x_1(0) = 1$, $x_2(0) = 2$ and $y_1(0) = 2$, $y_2(0) = 1$. Both gains k_1 and k_2 are chosen equal to 1.

Synchronization errors between the drive gyroscope and the response gyroscope systems are depicted in Fig. 3, where the control inputs are applied at t = 5 s. It can be seen that the synchronization errors converge to zero quickly. This means that the trajectories of the response gyroscope reach to the trajectories of the drive gyroscope in a finite time, as illustrated in Fig. 4. Fig. 5 shows the time histories of the adaptive parameters $\psi_i, i = 1,2,3,4$. Obviously, all adaptive parameters converge to some bounded values. The simulation results indicate that the proposed finite-time controller is robust against model uncertainties, external disturbances and unknown parameters and can synchronize two uncertain gyroscopes as quickly as possible.

5.2 Example 2

In this example, the efficiency of the proposed method is compared to the proposed adaptive sliding mode controller in Yan et al. (2006). All the system parameters, external disturbances and initial conditions are chosen same as those in Yan et al. (2006). In other words, the gyroscope system parameters are chosen as follows: $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$ and $f = 35.5$. Moreover, the external disturbance in response system (4) is defined as $0.2\cos 2t$. The initial states of the master system are selected as $x_1(0) = 0.5$, $x_2(0) = 1$ and initial states of the slave system are $y_1(0) = 1$, $y_2(0) = 2$.

Fig. 6 shows the synchronization errors obtained by our method. The trajectories of the response gyroscope reach the trajectories of the drive gyroscope in a finite time, as illustrated in Fig. 7. The state trajectories of the master and slave systems obtained by the proposed method in Yan et al. (2006) are plotted in Fig. 8. One can see that the adaptive sliding mode controller proposed in Yan et al. (2006) can synchronize the gyroscope systems. However, its convergence time is longer than that of our method. This means that our proposed

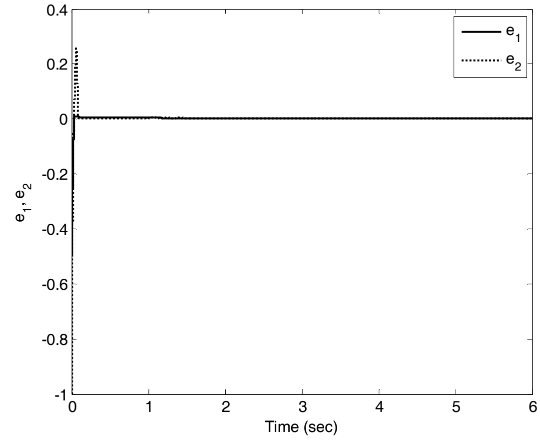

Fig. 6. Synchronization errors of the drive and response chaotic gyroscope systems.

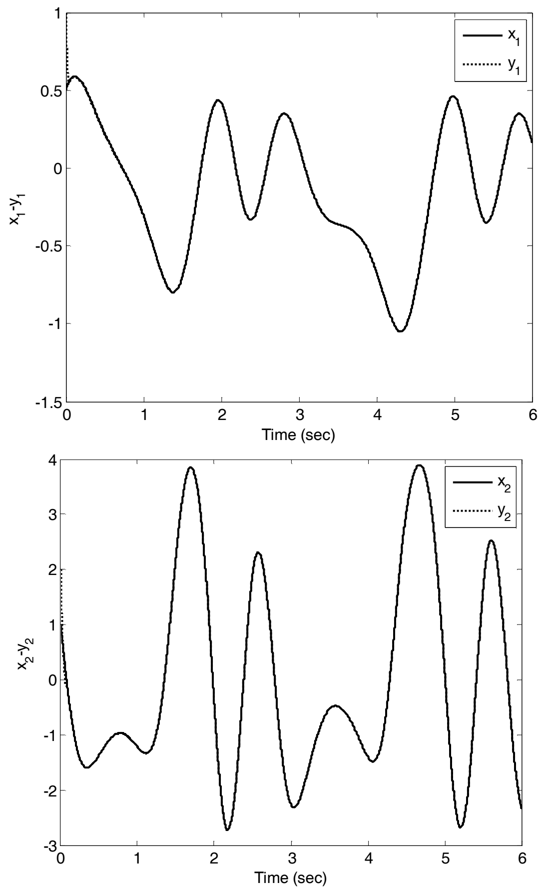

Fig. 7. Trajectories of the drive and response chaotic gyroscope systems.

finite-time controller outperforms the proposed sliding mode controller by Yan et al. (2006). The time histories of the selected adaptive parameters in Yan et al. (2006) are illustrated in Fig. 9. One can see that similar to our results the adaptive parameters converge to some constants.

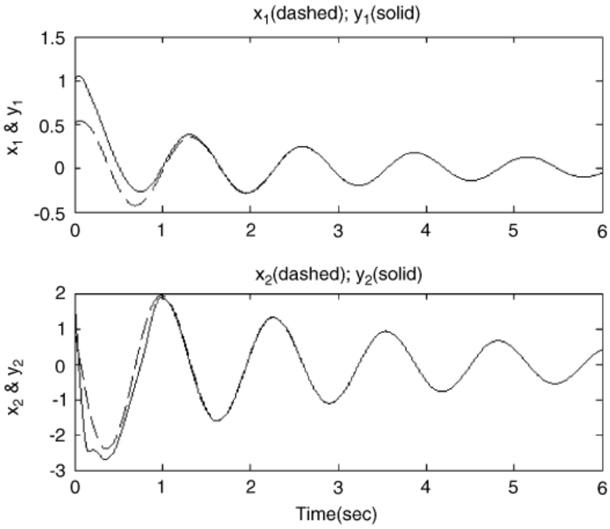

Fig. 8. Trajectories of the drive and response chaotic gyroscope systems (Yan et al., 2006).

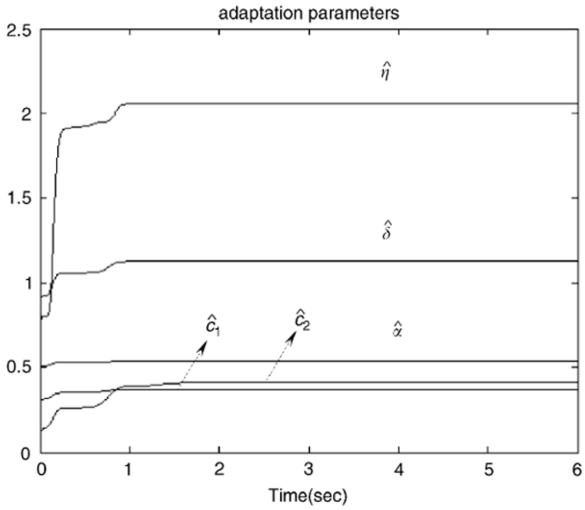

Fig. 9. Time responses of the adaptive parameters (Yan et al., 2006).

6. Conclusions

An adaptive robust finite-time controller is designed to synchronize two chaotic gyroscopes with model uncertainties, external disturbances and fully unknown parameters. Suitable adaptive laws are proposed to undertake the unknown parameters. For ensuring the convergence of the response gyroscope's state trajectories to those of the drive gyroscope in a given finite time, appropriate finite-time control laws are derived. The finite-time stability of the proposed method is mathematically proved. Simulation results and comparative studies reveal that the proposed controller works well for finite-time synchronization of two uncertain chaotic gyroscopes.

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